Public pension, joy-of-giving and housing in China

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Abstract: By employing an overlapping generations model with altruism and lifetime uncertainty, this paper investigates Chinese basic public pension system. We examine the effects of the contribution rates, population growth rate and life expectancy on the capital-labor ratio, intentional bequests, house investment and consumption. The results are as follows: Raising the firm contribution rate decreases the capital-labor ratio, intentional bequests, house investment and per capita consumption. A rise in the population growth rate decreases the four endogenous variables, too. An increase in the life expectancy increases the four endogenous variables. In the post financial crisis era, it is better to reduce the firm contribution rate, continue the special population (family planning) policy and improve the living and medical conditions.

Keywords: Public pension; Joy-of-giving; Housing

1. Introduction

China published its first Insurance Law in 2010, which regulates the basic public pension, medical, unemployment, industrial injury and maternity insurances. The basic public pension has been actualizing in urban areas: The government establishes an individual account for each employee and a social pool for all employees and retirees. Each firm contributes 20% of its payroll to the social pool, while each employee contributes 8% of her wage to her individual account. The social pool fund is used to pay the current retirees as pay-as-you-go (PAYG) pension benefits, while the accumulation in the individual account is used to pay the individual herself when she retires as fully funded pension benefits. Each retiree receives funded pension benefits from her individual account and PAYG pension benefits from the social pool.

How will the public pension system affect Chinese economy in the post financial crisis era? It is worth studying the effects in Chinese real environment: In general, the people are joy-of-giving because they leave bequests when they pass away to their children. The people work hard to buy their own house for not only marking their achievement when young but also investing for old-age material. The population growth rate has been falling mainly because of the special population (family planning) policy. The Chinese life expectancy has been rising because of improved living and medical conditions.

Some literature use overlapping generations (OLG) model with altruism and uncertain lifetime to study public pensions. Sheshinski and Weiss (1981) examine the annuity aspect of social security within the framework of an OLG model, in which the duration of life is assumed to be uncertain. Abel (1987) solves the consumption and portfolio decision problem of a consumer who lives for either one period or two periods and who can hold his wealth in the form of riskless bonds and actuarially fair annuities. Abel and Warshawsky (1988) analyze a joy-of-giving bequest motive in which the utility obtained from leaving bequests depends only on the size of the bequests. Abel (1989) demonstrates the effect of a lump-sum tax on debt neutrality in an OLG model with lifetime uncertainty. There is not any production sector in these models. Fuster (2000) studies how the lack of an annuities market affects savings behavior and intergenerational transfers in a dynastic OLG economy. Zhang *et al*. (2001) examine how mortality decline affects long-run steady state growth by assuming actuarially fair annuity markets in an OLG model with uncertain lifetime and social security.

Utilizing an OLG model with altruism and uncertain lifetime, this paper investigates Chinese basic public pension system. We examine the effects of pension contribution rates, population growth rate and life expectancy on the capital-labor ratio, intentional bequests, house investment and consumption. Instead of only individuals make pension contributions in the literature; both individuals and firms make contributions in this model. Individuals buy house in working period instead of save disposable income excess of consumption.

2. The model

We introduce a production sector into Abel's (1987) two-period OLG model with lifetime uncertainty and replace the fully funded public pension system with China's partially funded one. A closed economy is composed of numerous individuals and firms and a government. Each individual survives to the end of her working period certainly, but survives in retirement period with probability $p \in (0,1]$. The generation born at the beginning of period t is called generation t. The population grows at rate $n=N_t/N_{t-1}-1$, where N_t is the population size of generation t.

2.1 Individuals

In the working period, each individual earns wage by supplying inelastically one unit of labor and makes pension contributions. She gets intentional or unintentional bequests from her parent, consumes part of the incomes and invests the rest into real estate such as house. If she survives in the retirement period, she distributes her house with return, individual account benefits and social pool benefits between her consumption and bequests to her children. If she dies at the beginning of her retirement period, her house with return and individual account benefits are inherited equally by her children as unintentional bequests.

Each individual derives utility from her working-period consumption c_{1t}, possible retirement-period consumption c_{2t+1}, and intentional bequests B_{t+1}. The utility is described by an additively separable function. Thus, the utility maximization problem is:

$$\max_{\{S_t, c_{1t}, c_{2t+1}, B_{t+1}\}} U_t = u(c_{1t}) + \theta p u(c_{2t+1}) + \gamma p v\big((1+n)B_{t+1}\big) \tag{1}$$

$$s.t. \quad c_{1t} = (1-\tau)w_t - S_t + pB_t + (1-p)b_t \tag{2}$$

$$c_{2t+1} = (1-r_{t+1})S_t + I_{t+1} + P_{t+1} - (1+n)B_{t+1} \tag{3}$$

$$(1+n)b_{t+1} = (1+r_{t+1})S_t + I_{t+1} \tag{4}$$

where $\theta \in (0,1)$ denotes the individual discount rate, $\gamma \in (0,1)$ the altruism intensity, w_t the wage, τ the individual contribution rate, S_t the house, r_{t+1} the interest rate, I_{t+1} the individual account benefits, P_{t+1} the social pool benefits, and b_{t+1} the unintentional bequests. Suppose $\theta > \gamma$ since the limitation of altruism. $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave functions: $u'(\cdot)>0$, $u''(\cdot)<0$; $v'(\cdot)>0$, $v''(\cdot)<0$.

Substituting equations (2)-(4) into (1) and differentiating it with respect to S_t and B_{t+1} gives the first-order conditions for the utility maximization problem:

$$-u'(c_{1t})+\theta p(1+r_{t+1})u'(c_{2t+1})=0, \tag{5}$$

$$-\theta u'(c_{2t+1})+\gamma v'((1+n)B_{t+1})=0. \tag{6}$$

Equation (5) implies that the utility loss from reducing one unit of working-period consumption is equal to the utility gain from increasing $(1+r_{t+1})$ units of retirement-period consumption. Equation (6) describes the tradeoff between the marginal utility of intentional bequests and that of retirement-period consumption.

2.2. Firms

Firms produce a homogenous commodity in competitive markets. The production function $Y_t = F(K_t, N_t)$ or $y_t = f(k_t)$ is homogeneous of degree one, where Y_t is the output, K_t the capital stock, $k_t = K_t/N_t$ the capital-labor ratio and y_t the output-labor ratio. The production function $f(k_t)$ exhibits not only constant returns to scale, but also positive and diminishing marginal product, i.e., $f'(k_t)>0$, $f''(k_t)<0$. The marginal product of capital approaches infinity as capital goes to 0 and approaches 0 as capital goes to infinity: $\lim_{k_t\to 0} f'(k_t)=\infty$ and $\lim_{k_t\to\infty} f'(k_t)=0$.

Firms make pension contributions at the rate of $\eta\in(0,1)$ on their payroll. According to the product distribution, one can get $Y_t = r_t K_t + (1+\eta)w_t N_t$. Firms act competitively, renting capital to the point where the marginal product of capital is equal to its rental rate, and hiring labor to the point where the marginal product of labor is equal to $(1+\eta)w_t$:

$$r_t = f'(k_t), \tag{7}$$

$$w_t = \frac{f(k_t)-k_t f'(k_t)}{1+\eta}. \tag{8}$$

2.3. The government

The social pool fund is paid to the retirees in the current period as PAYG pension benefits: $pN_{t-1}P_t = \eta w_t N_t$, or

$$P_t = \frac{1+n}{p}\eta w_t. \tag{9}$$

The accumulation in the individual account is used to pay the individual when she retires in the next period as funded pension benefits:

$$I_{t+1} = (1+r_{t+1})\tau w_t. \tag{10}$$

2.4 The goods market

The house and the individual pension contributions in period *t* generate the capital stock in period *t*+1 (See Blanchard and Fischer, 1989, or Barro and Sala-i-Martin, 2004, for details):

$$s_t + \tau w_t = (1+n)k_{t+1}. \tag{11}$$

2.5 Dynamic Equilibrium

Given the initial condition (k_0, B_0) and the parameters τ and η, a competitive equilibrium for the economy is a sequence as $\{c_{1t}, c_{2t+1}, w_t, r_{t+1}, S_t, b_t, B_{t+1}, P_t, k_{t+1}\}_{t=0}^{\infty}$ that satisfies equations (1)-(11) for all *t*.

Substituting equations (2)-(4) and (7)-(11) into equations (5)-(6) and rearranging gives a dynamic equilibrium system described by the following difference equation:

$$\begin{aligned}&\left(\frac{f(k_t)-k_t f'(k_t)}{1+\eta} - (1+n)k_t + pB_t + (1-p)(1+f'(k_t))k_t\right) + \\ &\theta p\left(1+f'(k_t)\right)u'\left((1+n)(1+f'(k_t))k_{t+1} + \frac{p}{1+\eta}\eta\frac{f(k_{t+1})-k_{t+1}f'(k_{t+1})}{1+\eta} - (1+n)B_{t+1}\right) - \gamma v'\left((1+n)B_{t+1}\right) = 0\end{aligned} \tag{12}$$

$$\theta\left(1+f'(k_t)\right)u'\left((1+n)(1+f'(k_t))k_{t+1} + \frac{p}{1+\eta}\eta\frac{f(k_{t+1})-k_{t+1}f'(k_{t+1})}{1+\eta} - (1+n)B_{t+1}\right) - \gamma v'\left((1+n)B_{t+1}\right) = 0 \tag{13}$$

Assume that there exists a unique, stable and nonoscillatory steady-state equilibrium. In order to find the stability condition, we linearize the dynamic system around the steady state (*k*, *B*). Some manipulation gives

$$\mathrm{a}\hat{\mathrm{k}}_{t+1} + o\hat{\mathrm{k}}_t + g\hat{\mathrm{B}}_{t+1} + h\hat{\mathrm{B}}_t = 0 \tag{14}$$

$$\mathrm{x}\hat{\mathrm{k}}_{t+1} + z\hat{\mathrm{B}}_{t+1} = 0 \tag{15}$$

where

$$a = (1+n)\, u'_1 + \theta p f''\, u'_2 + \theta p(1+f')\, u''_2\left[(1+n)(1+f+kf'') - \frac{\eta}{p(1+\eta)}kf''\right]$$

$$o = u''_2\left[\frac{kf''}{1+\eta} - (1+p)(1+f+kf'')\right]$$

$$g = -\theta p(1+f')(1+n)\, u''_2$$

$$h = -u''_1 p$$

$$x = \theta\, u''_2\left[(1+n)(1+f+kf'' + \frac{\eta}{p(1+\eta)}kf''\right]$$

$$z = -(1+n)(\theta u'' + \gamma v'')$$

Rearranging equations (14) and (15) gives

$$i\hat{k}_{t+1} + j\hat{k}_t = 0$$

where

$$i = a - \frac{gx}{z}$$

$$= \frac{[(1+n)u''_1 + \theta p f'' u''_2](\theta u''_1 + \gamma v'') + \theta p(1+f)\gamma v'' u''_2\left[(1+n)\left(1+f+kf'' + \frac{\eta}{p(1+\eta)}kf''\right)\right]}{\theta u''_2 + \gamma v''}$$

$$j = o - \frac{hx}{z} = \frac{-\theta u''_1 u''_2(1+f') + \gamma u''_1 v''\left[\left(\frac{1}{1+\eta} - (1-p)\right)kf'' - (1-p)(1+f')\right]}{\theta u''_2 + \gamma v''}$$

$j < 0$ if $p > \frac{\eta}{1+\eta}$. The assumption that the equilibrium is unique, stable and nonoscillatory is equivalent to

$$0 < \frac{\hat{k}_{t+1}}{\hat{k}_t} = -\frac{j}{i} < 1$$

Therefore the stability condition is $i + j < 0$ if $p > \frac{\eta}{1+\eta}$.

3. Comparative Statics

The individual contribution rate does not appear in the dynamic system thereby has no effect on the capital-labor ratio because the mandatory savings (the individual contributions) crowd out the voluntary savings by one-for-one.

Totally differentiating the dynamic system around the steady state gives

$$(a+o)\,dk + (g+h)dB + mdp + ldn + \beta d\eta = 0 \tag{16}$$

$$xdk + zdB + \varepsilon dp + \mu dn + \delta d\eta = 0 \tag{17}$$

where

$$m = -u_1''(B-(1+f')k) + \theta(1+f')(u_2' - u_2''\frac{1+n}{p}\eta\frac{f-kf'}{1+\eta})$$

$$l = u_1''k + \theta p(1+f')u_2''\left((1+f')k + \frac{\eta}{p}\frac{f-kf'}{1+\eta} - B\right)$$

$$\beta = \frac{(f-kf')\left[u_1' + \theta(1+f')u_2''(1+n)\right]}{(1+\eta)^2}$$

$$\varepsilon = -\theta u_2''\frac{(1+n)\eta(f-kf')}{p^2(1+\eta)}$$

$$\mu = \theta u_2''\left((1+f')k + \frac{\eta}{p}\frac{f-kf'}{1+\eta} - B\right) - \gamma v''B$$

$$\delta = \theta u_2''\frac{1+n}{p}\frac{f-kf'}{(1+\eta)^2}$$

Equations (16) and (17) can be written as

$$\begin{pmatrix} a+o & g+h \\ x & z \end{pmatrix}\begin{pmatrix} dk \\ dB \end{pmatrix} = -\begin{pmatrix} m \\ \varepsilon \end{pmatrix}dp - \begin{pmatrix} l \\ \mu \end{pmatrix}dn - \begin{pmatrix} \beta \\ \delta \end{pmatrix}d\eta \tag{18}$$

$$\Delta = \begin{vmatrix} a+o & g+h \\ x & z \end{vmatrix} = (a+o)z - (g+h)x = az - gx - oz - he$$

$\Delta < 0$ because $i + j = \frac{az-gx-oz-he}{z} < 0$ and z>0.

Partially differentiating the capital-labor ratio and intentional bequests with respect to the firm contribution rate, survival probability in retirement and population growth rate gives the effects of the three exogenous variables on the two endogenous variables. It can be shown that the effects depend on the parameter values. Hence, we check the effects of the exogenous variables through simulations.

4. Simulations

Firstly, specify the utility and production functions. To satisfy the property and simplify the calculation, we choose constant relative risk aversion utility function， $u(c)=\frac{c^{1-\sigma}-1}{1-\sigma}$, $v((1+n)B)=\frac{((1+n)B)^{1-\rho}-1}{1-\rho}$, where $-\sigma$ and $-\rho$ are the elasticities of the marginal utilities with respect to consumption and intentional bequests, respectively. In the simulation process below, to simplify the calculation, let $\sigma=\rho$. To satisfy the character of production function and simplify calculation, we take the Cobb-Douglas production function, $Y_t=AK_t^{\xi}N_t^{1-\xi}$ or $y_t=AK_t^{\xi}$。

We can get the per capita consumption and house investment by using the specified functions:

$$c=f-nk=Ak^{\xi}-nk$$

$$S=(1+n)k-\tau Ak^{\xi}\left(\frac{1-\xi}{1+\eta}\right)$$

Using the specified utility and production function, equations (12) and (13) can be written as:

$$\left[\frac{Ak^{\xi}(1-\xi)}{1+\eta}-(1+n)k+pB+(1-p)(k+A\xi k^{\xi})\right]^{-\sigma}=\theta p(1+A\xi k^{\xi-1})\left[(1+n)(k+A\xi k^{\xi})+\frac{1+n}{p}\eta\frac{Ak^{\xi}(1-\xi)}{1+\eta}-(1+n)B\right]^{-\sigma} \quad (19)$$

$$\theta\left[(1+n)(k+A\xi k^{\xi})+\frac{1+n}{p}\eta\frac{Ak^{\xi}(1-\xi)}{1+\eta}-(1+n)B\right]^{-\sigma}=\gamma((1+n)B)^{-\sigma} \quad (20)$$

Rearranging equations (19) and (20) gives

$$\left(p(1+A\xi k^{\xi-1})\right)^{\frac{1}{\sigma}}=\frac{(1+n)\left(\left(k(1+A\xi k^{\xi-1})\right)+\frac{\eta Ak^{\xi}(1-\xi)}{p(1+\eta)}\right)}{r^{\frac{1}{\sigma}}\left(\frac{Ak^{\xi}(1-\xi)}{(1+\eta)}\left(1+\left(\frac{\theta}{r}\right)^{\frac{1}{\sigma}}+\eta\right)+k(1+A\xi k^{\xi-1})\left(p+(1-p)\left(1+\left(\frac{\theta}{r}\right)^{\frac{1}{\sigma}}\right)\right)-k(1+n)\left(1+\left(\frac{\theta}{r}\right)^{\frac{1}{\sigma}}\right)\right)} \quad (21)$$

$$B=\frac{k(1+A\xi k^{\xi-1})+\frac{\eta Ak^{\xi}(1-\xi)}{p(1+\eta)}}{1+\left(\frac{\theta}{r}\right)^{\frac{1}{\sigma}}} \quad (22)$$

4.1 Estimation of parameter values

In this model, we assume one period length is 30 years because that the length is usually in the interval of 25-30 years in the literature on OLG model. Analogous to

Pecchenino and Pollard, we assume that the individual discount rate per year is 0.985, the altruism intensity per year 0.965. Hence, in a period, θ=0.985^{30}, and γ=0.975^{30}.

The capital share of income is usually to be estimated as 0.3 in developed countries (e.g. Barro and Sala-i-Martin). The labor in China is comparatively cheaper, thus the labor share of income is lower, while the capital share of income is higher than that in developed countries. Hence, we assume that α in China could be 0.35. According to the economic circumstance of China and the research of Li and Liu (2011), the constant *A* should be 2. And the constant relative risk aversion value σ=2 (Blanchard and Fischer, 1989).

According to the urban public pension system in China, the firm contribution rate is η=20%, and the individual contribution rate is τ=8%.

The survival probability in retirement period is estimated by the life expectancy. According to the World Population Prospect from UN Secretariat: The 2006 Revision,① the life expectancy of Chinese people in 2005-2010 is 73 years old. Since one period length is 30 years, the life-span from birth to the end of working-period is 60 years (Even if childhood period is omitted in the model, it must be taken into account when we practically compute the life-span). The life-span from birth to the end of retirement period is 90 years. The concept of life expectancy, $(1-p)\times60+p\times90=73$, gives $p\approx43\%$. Although the choice of period length is arbitrary, it has to obey the following rule: Three times of the period length should be longer than or equal to the life expectancy to ensure $p \leq 1$.

There are several calibers for population statistics in China. Since the public pension system in urban area is different from that in rural area, and only the former is studied in this paper, so the caliber of “Urban Population” is selected. The population growth rate during 1979-2009 is computed as $n\approx224.92\%$ according to the “Population and Its Composition” in China Statistical Yearbook, which has in fact reflected immigration from countryside.

4.2 Effects of firm contribution rate

Let the firm contribution be 20%, substituting the baseline parameter values into equation (21) and calculating repeatedly until the equation holds, we get the capital-labor ratio, $k\approx0.02676$. Substituting k into equation (22), we can get $B\approx0.168912$. Then, raise the

① http://earthtrend.wri.org

firm contribution rate from 10% to 25% continuously. Repeating the above procedure yields the result shown in Figure 1.

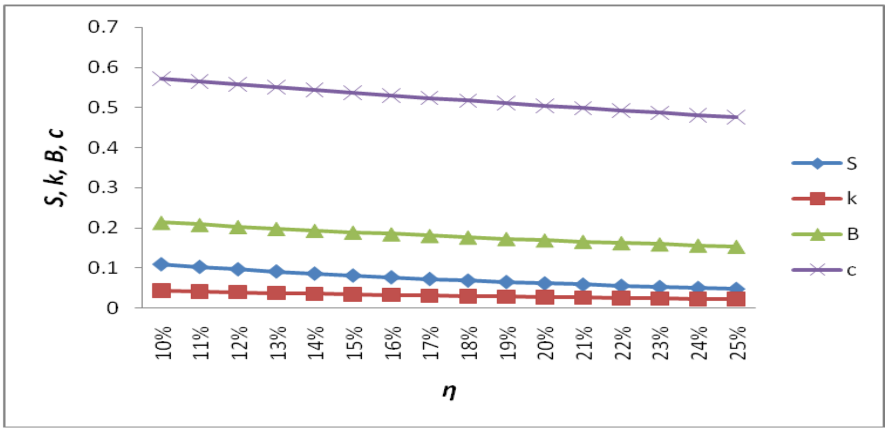

Figure 1 Effects of η on k, B, S and c

As shown in Figure 1, raising the firm contribution rate decreases the capital-labor ratio, intentional bequests, house and per capita consumption. We can explain it through equation (8): Raising the firm contribution rate decreases the wage and income, which reduces the per capita consumption, house investment and intentional bequests.

4.3 Effects of population growth rate

China Population Society and the China Family Planning Association announced in March 10, 2005: China's population growth rate will decrease gradually until 0 in 2040. Consider that China will continue imply the Family Plan policy and relax it in the future to keep the population in a proper scale, we take the value of population growth rate n from 0 to 5 consistently in order to observe its effects (Figure 2). The other parameters take the baseline values.

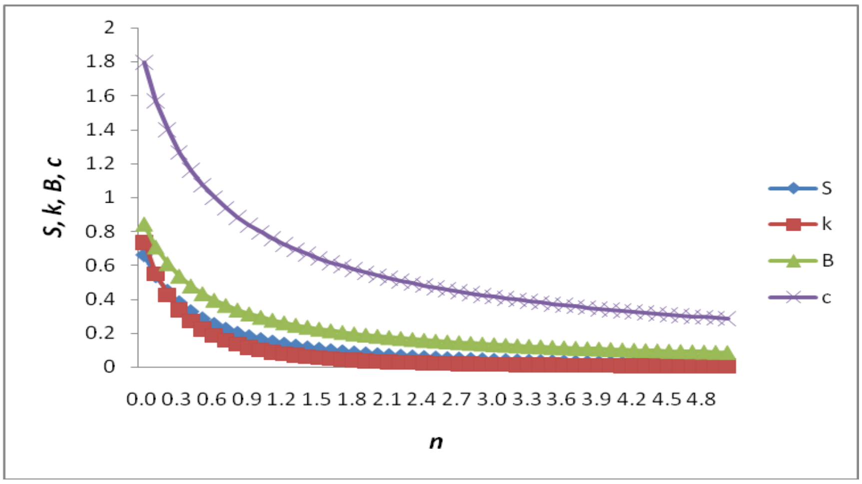

Figure 2 Effects of n on k, B, S and c

Figure 2 shows that the increase of population growth rate will decrease the capital-labor rate, intentional bequests, house investment and per capita consumption.

The increase of population growth rate will result in an increase in labor supply and dilute the capital-labor ratio. On the other hand, the increase also means an increase in the number of children that will decrease the intentional bequests per child. The increase in population growth rate will consume part of house investment and decrease the consumption per person at the same time.

4.4 Effects of life expectancy

According to UN Secretariat, the life expectancy of Chinese people in 2005-2010 is 73.0 years old, and that in 2010-2015 is 74.0 years old. With the development of economic and the improvement of health conditions, the Chinese life expectancy will further increase. Taking account of the factors above, we simulate the effects of life expectancy with the value from 72 to 80. Hence, the survival probabilities in retirement period are from 40% to 66.67% (Figure 3).

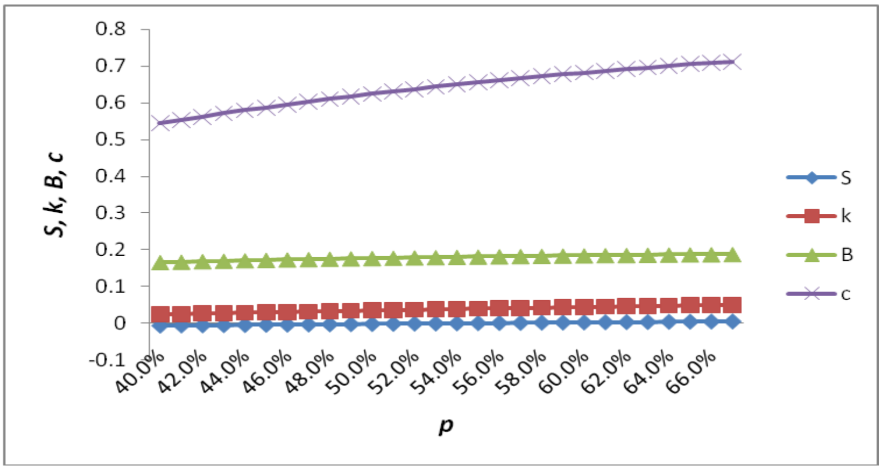

Figure 3 Effects of p on k, B, S and c

As showed in Figure 3, a rise in the life expectancy induces the increase in the capital-labor ratio, intentional bequests, house and per capita consumption.

In the working period, the rise in the life expectancy induces the increase in wage and the individual account benefits. In order to satisfy the consumption demand after retirement, people will consume less during working period. Both the effects will increase the house investment. In the retirement period, house investment can be used to the per capita consumption and intentional bequests for children.

5. Conclusions

Providing that individuals have altruism and uncertain lifetime, this paper employs the OLG model to study the urban public pension in China. We examine the effects of the individual contribution rate, firm contribution rate, life expectancy and population growth rate on the capital-labor ratio, intentional bequests, house investment and per capita consumption. Raising the individual contribution rate has no effect on the capital-labor ratio. Raising the firm contribution rate decreases the capital-labor ratio, intentional bequests, house investment and per capita consumption. A rise in the population growth rate decreases the four endogenous variables, too. An increase in the life expectancy increases the four endogenous variables.

These results have some policy implications. In the post financial crisis era, investment and consumption are very helpful for each country to recover from the crisis. In order to promote the investment and consumption in China, it is necessary to reduce the firm contribution rate and population growth rate and raise the life expectancy. The fall in the population growth rate is mainly because of the special population policy. The rise in the life expectancy is mainly the result of the improved the living and medical conditions. Therefore, China should reduce the firm contribution rate, continue the special population policy and improve the living and medical conditions.

Reducing the firm contribution indicates the increase in resident disposable income. It helps in increasing house investment and per capita consumption, and plays a very important role in exploding domestic demand and will help China’s economic transformation. The government should increase the investment on medical and health industry to enhance people’s health and improve life expectancy, which will not only improves the life quality of resident but also increases consumption and promotes economic development. The fall in the population growth rate will also increase house investment and per capita consumption.

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