A Multi-Echelon Inventory System with Information Exchange

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1. Introduction

In recent years, the field of supply chain management has gone through drastic changes. Numerous companies in various industries have undertaken major initiatives such as re-engineering efforts and investment in Information Technology (IT) to better manage their channels and reduce inefficiencies in their supply system. Re-engineering efforts have resulted in more coordination and cooperation between parties in the supply chain in the form of alliances and partnerships, emphasis on logistical issues when designing a product (*e.g.,* delayed differentiation and postponement), modular product and process design, agile supply networks (See Feitzinger and Lee 1997), among others. Investment in IT, however, has resulted in the availability of more information on channel activities to decision maker(s), who, in turn, must find ways of incorporating them in their day-to-day decisions in order to achieve better material flow and on-time deliveries.

Examples of industries that have adopted and incorporated these concepts as vehicles for improvement are numerous. Kurt Salmon Associates (1993) report the adoption of *Efficient Consumer Response* (ECR) by the grocery industry, which requires information sharing between suppliers and the stores. Stalk *et al.* (1992) discuss how the distribution centers at Wal-Mart, a discount retailer, obtain information on the inventory status of various products stocked at their stores and use such information in making replenishment/delivery decisions. A recent article in Forbes (1997) reports that the distributors for Heineken, a Dutch beer manufacturer, can increase or decrease their orders via Heineken's Web page, and Heineken can quickly reroute shipments or adjust production. Another example that is somewhat related to the model studied here is Campbell Soup and its *Continuous Replenishment* Program. As Fisher (1997) puts it:

> *"Campbell establishes electronic data interchange (EDI) links with retailers. Every morning, retailers electronically inform the company of their demand for all Campbell products and of the level of inventories in their distribution centers. Campbell uses that information to forecast demand and to determine which*

products require replenishment based on upper and lower inventory limits previously established with each retailer."

In lieu of the availability of information about demand activities and inventory status of the products to parties in a supply chain, inventory and replenishment policies that incorporate and utilize such information need to be considered and analyzed. In this paper, we consider a standard supply chain consisting of a single product, one supplier and *M* retailers. Demand at each retailer is random, but stationary. Each retailer places her orders to the supplier according to the well known *(Q,R)* policy. We assume that the supplier has on-line information about the demand as well as inventory activities of the product at each retailer, which he uses in making his order/replenishment decisions. We first propose a supplier's replenishment policy, which incorporates the information about the inventory position of the retailers. Then, we provide an exact analysis of the operating measures of such systems. Assuming the inventory/replenishment decisions are made centrally for the system, we compare the performance of our model to those that do not utilize information in their decision making, namely, systems, which employ installation stock policies. (See Deuermeyer and Schwarz 1981, Lee and Moinzadeh 1987, Moinzadeh and Lee 1986, Svoronos and Zipkin 1988 and Axsater 1993 for examples of such systems.)

The literature on incorporating information in supply chains is rather limited and relatively recent. Milgrom and Roberts (1990) identified information as a substitute for inventory in economic terms. Lee and Whang (1998) discuss the use of information sharing in supply chains in practice, relate it to academic research and outline the challenges facing the area. Cheung and Lee (1998) examine the impact of information availability in order coordination and allocation in a Vendor Managed Inventory (VMI) environment. They consider a continuous review inventory/distribution system consisting of one supplier and multiple retailers. Assuming that the supplier has real-time information about the retailers' inventory positions and retailers are identical, they consider coordinated shipment and stock re-balancing initiatives. In the coordinated shipment initiative, when the cumulative demand over all retailers reaches the truckload size, Q, then an order is issued to the supplier to replenish the retailers and restore their inventory position to the target level. In the re-balancing initiative, which is combined with shipment coordination, prior to its arrival, the supplier's order will be allocated to the retailers in a manner to balance retailers' inventory levels. Cheung and Lee analyzed the model with the coordinated shipment and derive bounds for the expected total cost of the model with re-balancing when the probability of the supplier being out of stock is negligible. In a numerical experiment they show how shipment coordination and stock re-balancing can result in significant benefits. It should be mentioned that the policies considered in Cheung and Lee will be effective only when retailers are in close geographical proximity to each other.

Cachon and Fisher (1998) consider a setting similar to that of ours with one supplier and N identical retailers. Inventories are monitored periodically and the supplier has information about the inventory position of all retailers. All locations follow an (R,nQ) ordering policy with the supplier's batch size being an integer multiple of that of the retailers. Cachon and Fisher show how the supplier can use such information to better allocate stock to retailers. Specifically, they devise and analyze an optimal allocation policy in which, when deciding on the allocation of the stock to the retailers, the supplier assigns a higher priority to those retailers who are most *needy (i.e.*, retailers with lower inventory positions). They also consider a policy, which utilizes information to improve the replenishment/ordering decisions at the supplier. However, unlike the one to be proposed in this paper, their policy is a complicated one as it depends on the vector of inventory positions of all retailers in the system in a complex way. In summary, the main contribution of Cachon and Fisher is the use of information in a more efficient allocation of stock to downstream facilities. In this paper, we focus on the impact of utilizing information by the supplier in order to improve his replenishment/ordering decisions. We believe that our study complements Cachon and Fisher (1997). As discussed earlier, in our model the retailers follow a standard (Q,R) policy; however, the supplier uses a replenishment policy that is based on retailers' inventory positions. As will be discussed later, under our proposed supplier's policy, as a result of information exchange, the supplier attempts to predict the time the future order to be placed by retailer(s). This, we will show later, will result in more efficient supply chains.

Chen (1998) examines the value of information in a continuous review serial system. He shows that the optimal inventory policies in such systems have a simple structure similar to that of Clark and Scarf (1960). Other relevant studies that discuss the impact of the use of information in inventory/distribution systems are Graves (1996), Moinzadeh and Aggarwal (1997), Moinzadeh and Bassok (1998), Lee *et al.* (1996), Hariharan and Zipkin (1995), Gavirneni *et al*. *(*1998) and Cachon and Lariviere (1999 and 2001).

As discussed in Cachon and Fisher (1998), the reported benefits of information sharing vary considerably. Cahon and Fisher reported an average savings resulting from information sharing of 2.2% and a maximum of 12.1%. Chen's study indicates an average savings of 1.8% and a maximum of 9%. Lee *et al.* found that the savings in their model can be as high as 23%. In this study, we found that the average savings due to the use of information is around 3.2% with a maximum savings as high as 34.9%.

The rest of the paper is organized as follows: In the next section, we describe the model and its assumptions. Then we propose a supplier replenishment/ordering policy that incorporates information about the status of retailers' inventory positions. In section 3 we derive expressions which enable us to evaluate the operating characteristics of such a system. In section 4, we compare the performance of the policies which use information sharing to those without information sharing (*i.e.*, Svoronos and Zipkin 1988 and Axsater 1993) and study the magnitude of the savings as well as behavior of their operating measures as system parameters are varied. Finally, we close by summarizing the results and suggestions for future research.

2. The Model

Consider an inventory/distribution system consisting of a single item, a supplier and M identical retailers. Demand at each retailer follows a Poisson process with a mean rate λ. Each retailer carries inventory and replenishes her stock from the supplier according to a (Q,R) policy; that is, when inventory position at the retailer reaches R, an order for Q units is placed. We assume that excess demand is backordered at the retailers. When an order is placed by a retailer, the supplier satisfies the full order immediately upon availability (*i.e.*, no partial shipment of the order is allowed); otherwise the order will experience a random delay. Delayed retailer orders are satisfied on a first-come, first-served basis. In addition to the possible random delay at the supplier, the transit time from the supplier to each of the retailers is constant and is assumed to be equal to L.

We assume that the supplier has on-line information on the inventory status and demand activities of all retailers and replenishes his stock from an outside source in batches of Q. Furthermore, we assume that the replenishment leadtime from the outside source to the supplier is constant and is equal to L_0 (*i.e.,* the outside source has ample capacity). In order to take advantage of the available information, we propose the following replenishment policy for the supplier:

Supplier's Inventory/Order Policy: Starting with m initial batches (of size Q), an order (of size Q) is placed to the outside source immediately after a retailer's inventory position reaches $R+s$ ($0 \leq s \leq Q-1$).

Note that m and s define the supplier's inventory/replenishment policy. Since the supplier has on-line access to the inventory position of each retailer, as the inventory position of a retailer gets closer to R, the supplier may choose to act by ordering from the outside source in anticipation of the placement of a retailer's order in the *near future*. This is accomplished by setting s to a positive value. By doing so, the supplier can provide more reliable replenishments to the retailers which results in overall reduction of cost in the system. Thus, s, in a sense, gauges the pro-activity of the supplier due to information availability. By definition, when s=0, the supplier orders immediately after he receives an order from a retailer. In other words, the supplier uses a classical installation policy and the system behaves in a fashion similar to that considered by Axsater (1993) and Svoronos and Zipkin (1988). In the context of this study, the value of information is zero in such cases as monitoring inventory position of the retailers at all times does not give the supplier an edge. Finally, we observe that when both m and s are set to zero, the supplier does not hold any inventory and places his order immediately after he receives a retailer order and his policy is in effect *make to order.* In such situations, retailer orders face a constant delay of L_0. Observe that for negative values of m, the possible delay experienced by a retailer's order will be larger than L_0, which result in inferior policies. Therefore, we only consider non-negative values of m in our analysis.

Before proceeding any further, we would like to point out that the form of the optimal supplier policy in the context of our model is an open question and is possibly a complex function of the different combinations of inventory positions at all the retailers in the system. Although the policy adopted in this paper only considers the inventory position of retailers independently when making ordering decisions and basically ignores the inventory positions of all other retailers at such times, we conjecture that our proposed policy will capture most of the benefits due to information sharing. Clearly, the classical installation policy employed in many earlier models is subsumed in our policy. We cannot make the same statement about our policy in comparison to systems that employ echelon Policies where the supplier's order is triggered when the system's inventory position reaches a certain level. Previous research that compares the pure installation and echelon policies provides some hints. Axsater and Rosling (1993) show that echelon policies dominate installation policies in serial systems. However, the dominance does not extend to distribution systems. For instance, echelon policies should perform poorly in systems where the supplier's batch size is small compared to that of the retailers (*i.e.,* it is equal to those of retailers). Axsater (1996) compares the performance of echelon and installation policies in distribution systems via a simulation study. He concludes that echelon policies dominate installation policies in settings where the transportation time from the outside source to the supplier is large compared to that of supplier to retailers. Cheung and Lee (1998) conclude that echelon policies will become more attractive in systems with large batch sizes and/or many retailers.

Discussion: In developing our model, we have made a few critical assumptions that merit some further discussion. First, we assumed that all retailers are identical. This assumption is used in many of the previous works in the literature (see Svoronos and Zipkin 1988, Axsater 1993, Deuermeyer and Schwarz 1981, Lee and Moinzadeh 1987 and Moinzadeh and Lee 1986, Cachon and Fisher 1998 and Chueng and Lee 1998) and significantly simplifies the analysis and exposition. In fact, one can apply a similar approach to that of ours to evaluate systems with non-identical retailers. As can be seen later, both the analysis and the computational effort will be tedious in such systems, as each retailer (or group of retailers) must be considered separately in the analysis. Second, we have assumed that the order lot size, Q, is given and fixed. The assumption is reasonable in many practical situations due to packaging or shipping requirements and captures some of the economies of scale in shipping and handling. Furthermore, it is commonly used in the literature (see Svoronos and Zipkin 1988, Axsater 1993, Deuermeyer and Schwarz 1981, Cachon and Fisher 1998, Cheung and Hausman 1998 and Cheung and Lee 1998). Third, we have assumed that the supplier's order quantity is equal to that of the retailers. This is probably the most restrictive assumption of our model as it only represents cases where there are essentially no economies of scale in ordering at the supplier (*i.e.*, the ratio of order cost to unit holding cost is negligible at the supplier). This assumption is made for the sake of analytical tractability, as there are many situations where the assumption is violated. Those include situations where the economies of scale in shipping and handling are significant. In cases where shipping costs are small and electronic commerce and on-line ordering is used in ordering (Forbes 1997, Fisher

1997), our assumption becomes more reasonable. Needless to say, a more general model where the supplier's batch size is larger than that of the retailers can constitute an important avenue for future research. In the last section, we provide some closing thoughts on how our model can be extended to such cases.

Next, we define the following notation. Let:

$N_j(t)$	: number of orders placed by retailer j $(j= 1,..,M)$ in $(0,t]$ at steady state,
$I_j(t)$	: inventory position (net inventory + on order) of retailer j $(j= 1,..,M)$ at time t in steady state,
$N_0(t)$	: total number of orders placed in the system in $(0,t]$ at steady state,
X_k	: time to have k units demanded at a retailer,
τ	: elapsed time between the placement of a supplier's order until it is requested to fill a retailer's order,
L	: transit time from the supplier to each of the retailers,
L_0	: replenishment leadtime from the outside source to the supplier,
ω	: random delay experienced by a retailer's order,
υ	: random time a batch (of size Q) spends in supplier's inventory until it is used to fill a retailer's order,
$f(.)$	: probability density of τ,
$F(.)$	: complementary cumulative distribution function of τ,
$G(.)$	: complementary cumulative distribution function of demand at the retailer during her order leadtime,
$p(x, \mu)$	: probability mass function of Poisson with mean μ.,
h	: unit holding cost/time at a retailer,
h_0	: unit holding cost/time at the supplier,
π	: unit backorder cost/time at a retailer,

m	: number of batches (of size Q) initially allocated to the supplier,
$TC(R,m,s)$	: expected total cost of the system/time.

Furthermore, let $g(.)$ denote the distribution of a random variable. Then, we define $g^{(k)}(.)$ as the k-fold convolution of $g(.)$. Finally, denote:

$$P(x,\mu) = \sum_{j=x}^{\infty} p(j,\mu),$$

and,

$$(x)^{+} = Max(0,x).$$

In the next section, we analyze the model and provide expressions for computing the steady state operating characteristics of the system.

3. Analysis

We now proceed with the exact analysis of steady state behavior of the operating characteristics of the system. Our approach is to focus on a supplier's order and evaluate its early and tardy times. By early time, we mean the time an order spends in supplier's inventory until it is used to satisfy a retailer's order. In contrast, tardy time is the amount of time a delayed retailer's order must wait until it is filled by an incoming supplier's order.

Recall that once a retailer's inventory position reaches $R+s$ (suppose this occurs at time t_0), the supplier immediately places an order of size Q which he receives at t_0+L_0. This order will be eventually used to satisfy an order, which is to be placed by one of the retailers at some random time in the future, say, $t_0+\tau$. Note that τ is a random variable and has been defined as the time from the supplier's placement of an order until it is used to fill a retailer's order. As it will be seen later, once the probability distribution function of τ is obtained, the operating characteristics of the system can be computed.

We first derive expressions for the key operating measures of the system. It becomes clear that in order to be able to compute these measures, one needs to compute the distribution of τ. We then proceed by outlining the steps to calculate the distribution of τ.

Based on our supplier's inventory/replenishment policy and considering only non-negative values for *m*, at steady state, we observe that:

a) If $\tau \geq L_0$, then the supplier must carry *Q* units for $\tau - L_0$ time units.

b) If $\tau \leq L_0$, then the retailer's order experiences a delay equal to $L_0 - \tau$ time units.

Thus, the delay experienced by a retailer's order, ω, can be expressed as:

$$\omega = (L_0 - \tau)^+ . \tag{1}$$

Furthermore $E(\omega)$ is given by:

$$E(\omega) = L_0 - \int_0^{L_0} F(\zeta)d\zeta . \tag{2}$$

Similarly, the total time a batch (of size *Q*) is held in inventory at the supplier before it is used to fill a retailer's order, υ, can be written as:

$$\upsilon = (\tau - L_0)^+ , \tag{3}$$

with its expected value, $E(\upsilon)$, given by:

$$E(\upsilon) = E(\tau) - \int_0^{L_0} F(\zeta)d\zeta . \tag{4}$$

Since orders are placed at a rate of $M\lambda/Q$ at the supplier, the expected total inventory cost at the supplier will be $h_0\ (M\lambda)\ E(\upsilon)$.

Because the supplier fills orders on a first-come, first-served basis and the transportation leadtime, *L*, is constant, orders do not cross. Furthermore, the total leadtimes faced by the retailers are iid as demand at the retailers is stationary and a a lot-for-lot ordering policy is employed by the supplier.

Since demand at the retailers is Poisson, using (1), with some effort (using integration by parts), one can show that the complementary cumulative distribution of demand during a retailer's order leadtime, *G(.)*, comprised of the random order delay, ω, and the fixed transit time, *L*, can be written as:

$$G(x) = P(x,\lambda L)\Pr(\tau \ge L_0) + \int_0^{L_0} P(x,\lambda(L + L_0 - \zeta)) f(\zeta) d\zeta$$

$$= P(x,\lambda(L + L_0)) - \lambda \int_0^{L_0} p(x-1,\lambda(L + L_0 - \zeta)) F(\zeta) d\zeta \,. \qquad (5)$$

Finally, using the results in Hadley and Whitin (1963), we can write the average total cost rate in the system (Hadley and Whitin 1963, pp. 181- 187, 200-204) comprised of the costs at each retailer and the holding cost at the supplier as:

$$TC(R,m,s) = M\left\{ h[\frac{Q+1}{2} + R - \lambda(E(\omega) + L)] + (h+\pi)B(R,m,s)\right\}$$

$$+Mh_0\lambda E(\upsilon), \qquad (6)$$

where,

$$B(R,m,s) = \frac{1}{Q}\left\{ \sum_{u=R+1}^{\infty}(u-R-1)G(u) - \sum_{u=R+Q+1}^{\infty}(u-R-Q-1)G(u)\right\}. \qquad (7)$$

Replacing (4) in (6) and using (2), we get:

$$TC(R,m,s) = M\{h[\frac{Q+1}{2} + R - \lambda(L + L_0)] + (h+\pi)B(R,m,s) + h_0\lambda E(\tau)$$

$$+(h-h_0)\lambda\int_0^{L_o} F(\zeta)d\zeta\}. \qquad (8)$$

As mentioned before, we assume that Q is given and fixed; therefore, we have ignored the retailers' order cost term in (8). We have also excluded terms like the variable retailers' replenishment costs and the supplier's replenishment costs, as they are all constants and play no role in finding the parameters of the optimal policy. However, in order to avoid non-negative cost figures, the term $hM(Q+1)/2$ is included in the average total cost expression in (8).

In order to be able to evaluate (7) and then (8), we need to compute the complementary cumulative distribution function of τ, $F(.)$. Recall that a supplier's order is placed immediately after the inventory position at one of the retailers reaches $R+s$. At that instant, the inventory positions at all $(M-1)$ other retailers are either *(i)* less than or equal to or *(ii)* greater than $R+s$. Let,

$$\begin{aligned} A_0(t) &= \{ j \quad (1,2,..,M): I_j(t) \leq R+s\}, \\ A_1(t) &= \{ j \quad (1,2,..,M): I_j(t) > R+s\}. \end{aligned} \tag{9}$$

Therefore, when a retailer's inventory position reaches $R+s$, all other retailers are either in A_0 or A_1. Note that, prior to this time, a supplier's order was placed for each of those retailers in A_0. However, these retailers are yet to place their orders to the supplier. This means that the inventory position at the supplier will be equal to m plus the number of retailers in A_0. Since the number of retailers is M, then the inventory position at the supplier can take on the values $\{m, m+1,.., m+M\}$.

Consider an instance when retailer's inventory position reaches $R+s$ (suppose this occurs at time zero). This will be immediately followed by the supplier placing an order of size Q which he receives at L_0. The order will be then used to satisfy an order to be placed by one of the retailers at some random time in the future, say, τ. Suppose there are k $(k=0,1,..,M-1)$ retailers in A_0 other than the one whose inventory position just reached $R+s$ (thus, $M-1-k$ retailers in A_1). Then, the order just placed by the supplier will be used to satisfy the $(m+k+1)$-*St* future order to be placed by the retailers (Recall that the inventory position at the supplier takes on the values *{m, m+1,.., m+M})*. Therefore, τ, defined as the time from the supplier's placement of an order until it is used to fill a retailer's order, will be the time until *(m+k+1)* orders are placed at the supplier. As can be seen, in order to derive the distribution of τ, namely *F(.)*, we need to characterize the order process of the retailers in A_0 and A_1 from the time an order is placed by the supplier (instances when inventory position at a retailer reaches *R+s*) until the *(m+k+1)-St* subsequent retailers' order is placed at the supplier.

Suppose at time zero the inventory position at retailer *j* reached *R+s*; this immediately triggers a supplier's order. Denote $\alpha(i,t)$ as the probability of having *i* orders placed by a retailer, say retailer *n*, in *t* time units and that $n \quad A_0(0)$, given that the inventory position at retailer *j* ($j \neq n$) hit *R+s* at time *0*. Similarly, let $\gamma(i,t)$, be the probability of having *i* orders placed by a retailer, say retailer *n*, in *t* time units and that $n \quad A_1(0)$, given that the inventory position at retailer *j* ($j \neq n$) hit *R+s* at time *0*. This means that:

$$\alpha(i,t) = \Pr\{N_n(t) = i, n \quad A_0(0) \mid I_j(0) = R+s\},$$
$$\gamma(i,t) = \Pr\{N_n(t) = i, n \quad A_1(0) \mid I_j(0) = R+s\}.$$

Furthermore, let $\beta(i,t)$ be the probability of having *i* orders placed by retailer *j* in *t* time units and that her inventory position at time zero hit *R+s* . Since all retailers are identical, $\alpha(.,.)$,$\gamma(.,.)$ and $\beta(.,.)$ are independent of the choice of the retailer, *j*.

It can be easily verified that:

$$\alpha(i,t) = \frac{1}{Q}\sum_{l=1}^{s} \Pr\{X_{l+(i-1)Q} \le t < X_{l+iQ}\}$$
$$= \frac{1}{Q}\sum_{l=1}^{s}\{P(l+(i-1)Q,\lambda t) - P(l+iQ,\lambda t)\}, \qquad (10)$$

$$\gamma(i,t) = \frac{1}{Q}\sum_{l=s+1}^{Q} \Pr\{X_{l+(i-1)Q} \le t < X_{l+iQ}\}$$

$$= \frac{1}{Q}\sum_{l=s+1}^{Q} \{P(l+(i-1)Q,\lambda t) - P(l+iQ,\lambda t)\}. \tag{11}$$

and,

$$\beta(i,t) = P\{X_{s+(i-1)Q} \le t < X_{s+iQ}\}$$

$$= P(s+(i-1)Q,\lambda t) - P(s+iQ,\lambda t). \tag{12}$$

Consider one of such instances described above where at time *0*, the inventory position at retailer *M* reaches $R+s$. Suppose at this time, inventory positions of retailers *1* through *k* are smaller or equal to *R+s* and inventory positions at retailers *(k+1)* through *(M-1)* are greater than *R+s* $(A_0(0) = \{j = 1,2,..,k\}; A_1(0) = \{j = k+1, k+2,.., M-1\})$. Then, as discussed before, τ will be the time from *0* until the *(m+k+1)-St* subsequent system order is placed. Since all retailers are identical, the probability distribution of τ for the above case is the same for all cases with *k* retailers in $A_0(0)$ and *(M-k-1)* retailers in $A_1(0)$, irrespective of which retailers belong to either $A_0(0)$ or $A_1(0)$. Thus, we can write the complementary cumulative distribution of τ, *F(t)*, by considering all the possible combinations of *k* ranging from *0* to *M-1* as:

$$F(t) = \sum_{k=0}^{M-1} Pr\{k \text{ retailers belong to } A_0(0), (M-1-k) \text{ retailers belong to } A_1(0), N_0(t) \le k+m \mid I_M(0)=R+s\}$$

$$= \sum_{k=0}^{M-1}\left\{\binom{M-1}{k} Q_k(k+m,t)\right\}. \tag{13}$$

In (13), $Q_k(k+m,t)$ is the probability of having less than or equal to $k+m$ system orders in t time units when there are k retailers in $A_0(0)$, given that retailer M's inventory position reached $R+s$ at time zero, and can be expressed by summing the probabilities of all the possible combinations as:

$$Q_k(k+m,t) = \sum_{l=0}^{k+m} q_k(l,t), \tag{14}$$

where,

$$\begin{aligned} q_k(l,t) &= \Pr\{N_0(t) = l, (j \quad A_0(0), j = 1,.., k; k+i \quad A_1(0), i = 1,.., M-k-1 \,|\, I_M(0) = R+s)\} \\ &= \sum_{i=0}^{l} \sum_{j=0}^{l-i} \alpha^{(k)}(i,t)\gamma^{(M-k-1)}(j,t)\beta(l-i-j,t), \end{aligned} \tag{15}$$

Replacing (12) in (15), (14) reduces to:

$$Q_k(k+m,t) = \sum_{i=0}^{k+m} \sum_{j=0}^{k+m-i} \alpha^{(k)}(i,t)\gamma^{(M-1-k)}(j,t)\{1 - P[s+(k+m-i-j)Q, \lambda t]\}. \tag{16}$$

Using (16), we are now able to compute $F(.)$ from (13) and eventually evaluate (6), (7) and (8). The analysis of the model is now complete.

4. Computational Results

In order to examine the impact of information sharing achieved by our model when compared to systems which are based on pure installation stock ordering, we resorted to a numerical experiment, the results of which will be discussed later in this section. We like to clarify that, although we provide an exact framework for the operating characteristics of our model, to evaluate them, we resorted to numerical approximation. This mainly involved evaluating those terms, which required the evaluation of integral values. While, it is clear that our system will out-perform the traditional systems that are based on installation stock policies (*i.e.,* Axsater 1993, Svoronos and Zipkin 1988), we are most interested in identifying the settings where the savings resulting from our model would be most significant. In doing so, we used the best average system cost rate found in our model as the benchmark and compared it to that where no information sharing is employed. Unfortunately, due to the complex nature of the problem, it is not clear that the average total cost rate is convex in the policy parameters. Therefore, to find the best policy parameters and its associated average total cost rate in our model, we resorted to a standard search procedure, which we will next describe. Note that it can be shown that for fixed values of m and s, the average total cost rate in the system is convex in R and the optimal reorder point at the retailer can be found easily. The optimal values of m and s were found by examining values of m starting with $m = 0$, and $s=0,1,\ldots,Q-1$ and obtaining the optimal values of R for each combination of m and s. The search continued by incrementing m in units of one until a local minimum was achieved. In order to ensure that the solution found is the best solution to the problem under consideration we continued examining our search up to twice the value of m, which resulted in the best solution. The best solution to the systems with no information sharing was found in the same fashion by limiting the search to values of $s = 0$. In all the problems examined, the local optimum found was the best solution.

Before summarizing our findings, we would like to point out two important factors that impact the choice of supplier's stocking policy: a) *Risk Pooling* and b) *Information Sharing*. In our model, both information sharing and risk pooling impact the choice of the supplier's policy; in contrast, in systems with no information exchange, only risk pooling is present. In all, we considered 600 problems comprised of all the combination of the following parameters:

λ:	1,5;
L_0:	1;
L:	0, 0.5,1;
h:	1;
h_0/h:	0.1,0.2,0.4,0.6,0.8;
π/h:	10;
Q:	2,5,10,20;

M: 1,2,4,8,16.

For each problem studied, we expressed our findings as *% deviation* between average total cost rates between the two systems where:

$$\%deviation = 100 * \frac{E(TC^*_{w/o\ information}) - E(TC^*_{w/information})}{E(TC^*_{w/information})}.$$

First, we examined the benefits resulting from our model for different values of the supplier's leadtime as the number of the retailers in the system was varied. Figure 1, depicts the typical behavior for different demand levels (λ =1 and 5) at the retailers. As can be seen, in cases with a small number of retailers in which the effect of risk pooling is relatively small, information sharing has more impact when transit time from the supplier to the retailers is small compared to the maximum possible leadtime for an order, $L+L_0$ (Note that the supplier's leadtime was set to unity in all the problems considered). In other words, the savings is decreasing in $L/(L+L_0)$. The behavior is intuitive as in systems with few retailers and small transit times (relative to the maximum possible leadtime), most of the safety stock in the system will be allocated to the supplier who has the smaller unit holding cost/time and only a small portion of safety stock will be kept at the retailers to hedge against the demand uncertainty during the relatively small transit times. Therefore, the savings resulting from reduction of supplier's safety stock (which is accomplished through information sharing) will have a larger impact. Similarly, the savings resulting from our policy will decrease as the maximum possible leadtime for an order, $L+L_0$, increases. We also note from Figure 1 that, the savings resulting from information sharing diminishes for systems with many retailers (In Figure 1, the *% deviation* is zero for all cases when $\lambda=1$ and $M=16$). This observation has interesting managerial implications with respect to supply chain design as it provides one with the settings (in terms of the number of retailers in a system) where the savings due to utilizing information while employing risk pooling is maximized. For instance, in systems with only one retailer where there is no risk pooling, all the resulting savings from our model are solely due to information exchange. Conversely, in systems with many retailers, risk pooling at the supplier will dictate the form of the supplier's policy and information exchange will have minimal (or no) impact at all (In fact, as the number of retailers approaches infinity, the savings due to our model goes to zero).

Figure 1 About Here

Next, we examined the benefits of information sharing with respect to the order quantity (Q). From Figure 2, we observe that the value of information sharing will be minimal when Q is either small or very large compared to the mean demand rate. In fact, it can be shown that the *% deviation* between costs goes to zero either when Q=1 (when the order-for-order policy is in effect) or as Q approaches infinity. This observation can be explained as follows: Recall that through information sharing, the supplier obtains real time information on the inventory position of all retailers. Now, in systems with no information exchange, the supplier's information about inventory position at a retailer becomes available only at times when a retailer places an order (inventory position at the retailer is $Q+R$). Thus, in such systems, the supplier lacks information about the status of a retailer's inventory position during the time between placement of orders. Thus, in systems with small order quantity, the time between retailer's orders is relatively small and the supplier, in effect, gets frequent updates on the status of the retailers' inventory position and the role of having information is reduced. In systems with very a large order quantity, the times between retailer's orders are long and thus, there is opportunity to take advantage of information exchange. However, inventory levels in such systems tend to be very large (as Q is large) and the cost associated with safety stocks in the system play a very small role in the total cost figure. Clearly, depending on system parameters, there are intermediate values of Q which information sharing is most advantageous.

Figure 2 About Here

Another important issue that needs some investigation is the benefits of information sharing in relationship to the unit holding cost at the supplier to that of the retailer (h_0/h). Figure 3 depicts a typical behavior of the *% deviation* between costs. We observe in Figure 3 that as the unit holding cost at the supplier is approaching the unit cost at the retailers (h_0/h approaches one), the value of information sharing becomes marginal as the *% deviation* between costs goes to zero. This can be explained as follows; when the unit holding cost at the supplier is large compared to that at the retailers (*i.e.*, the ratio h_0/h approaches one), then inventory will be mostly held at the retailers. Thus, the use of information, which tends to shift inventory from the retailers to the supplier, will not result in significant cost savings.

Figure 3 About Here

We also observe a similar behavior as h_0/h approaches zero which, can be explained in the following way: Consider the system with no information sharing when the holding cost at the supplier is low. Then, it is optimal to keep most of the inventory at the supplier and very little inventory at the retailers. Although the use of information will result in less inventory costs in the system, however, this reduction mainly occurs at the supplier, which has low holding cost. Therefore, the reduction in the expected total cost will be small. In fact, when h_0/h is zero, then one can eliminate the possible order delay at the warehouse by allocating an infinite amount of inventory at no cost. In that case, the system will be equivalent to M independent (Q,R) systems with constant leadtime, L, and clearly, information will have no value. The savings due to information sharing is most significant for intermediate values of h_0/h as proper allocation of inventory between supplier and the retailers becomes critical. The reader may note that, as discussed earlier, the *% deviation* values in Figures 1, 2 and 3 will be far greater if the constant term $hMQ/2$ is excluded in the computations. We would like to point out that while Figures 2 and 3 summarize the results for λ=5, we observed similar behavior for λ =1 as well.

5. Conclusions

In this paper, we studied the benefits of information sharing in a supply chain characterized as the availability of on-line information of retailers' inventory positions to the supplier. Specifically, we considered a standard supply chain model consisting of a single product, one supplier and *M* identical retailers. Demand at each retailer follows a Poisson process. Each retailer places her orders to the supplier according to the *(Q,R)* policy. We assumed that the supplier has on-line information about the demand as well as inventory activities of the product at each retailer, which he uses in making his order/replenishment decisions. We first proposed a supplier's replenishment policy, which incorporates the information about the inventory position of the retailers. Then, we provided an exact analysis for the operating measures of such systems. Assuming that inventory/replenishment decisions are made centrally for the system, we compared the performance of our model to those that do not utilize information in their decision making, namely, systems, which employ installation stock policies via a numerical experiment. Based on our numerical results, we found that information sharing is most beneficial in systems, which exhibit the following characteristics:

a) Systems where the supplier's leadtime is long compared to other leadtimes (*i.e.*, transit times from the supplier to the retailers) in the system.
b) Systems where the number of retailers is not large (this is clearly a function of system parameters such as costs, demand rates and leadtimes).
c) Systems where the order quantities are either not too small or too large.
d) Systems where the ratio of the unit holding cost of the retailers to that of the supplier is either not too small or too large.

There are a number of important future extensions to this work. One includes the analysis of the case when the order quantity at the supplier is a multiple integer to that of the retailers. In such cases, our policy can be extended in the following fashion. Define the batch size as Q. Let $Q_0 = qQ$ be the order quantity at the supplier. Then, the supplier who starts with m initial batches of size Q will place orders of size Q_0 to the outside supplier immediately after the inventory position of the *q-th* retailer since supplier's last order reaches $R+s$. The analysis of this policy is much more complicated as, in order to compute the distribution of τ, one needs to derive the distribution of the times between the *i-th* retailer's inventory position ($i= 1,2,..,q$) in the supplier's order reaching $R+s$ until the supplier's order is placed.

Another interesting extension is the study of a decentralized system and examining whether channels can be coordinated followed by devising incentive compatible mechanisms under which channel coordination can be achieved. Finally, considering the benefits of information sharing in systems when demand among retailers are correlated is an important extension to this work.

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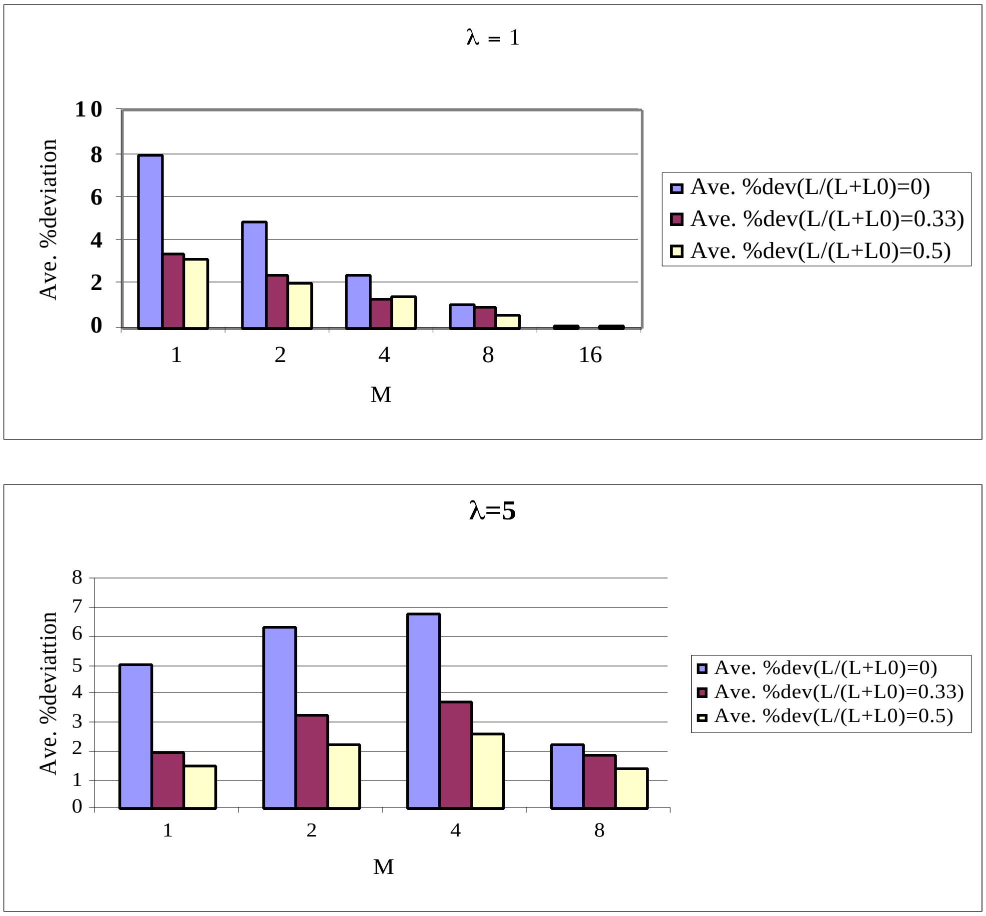

Figure 1*: Performance of systems without information exchange compared to that of ours for different average demand rates as the supplier's leadtimes and the number of retailers in the system are varied.*

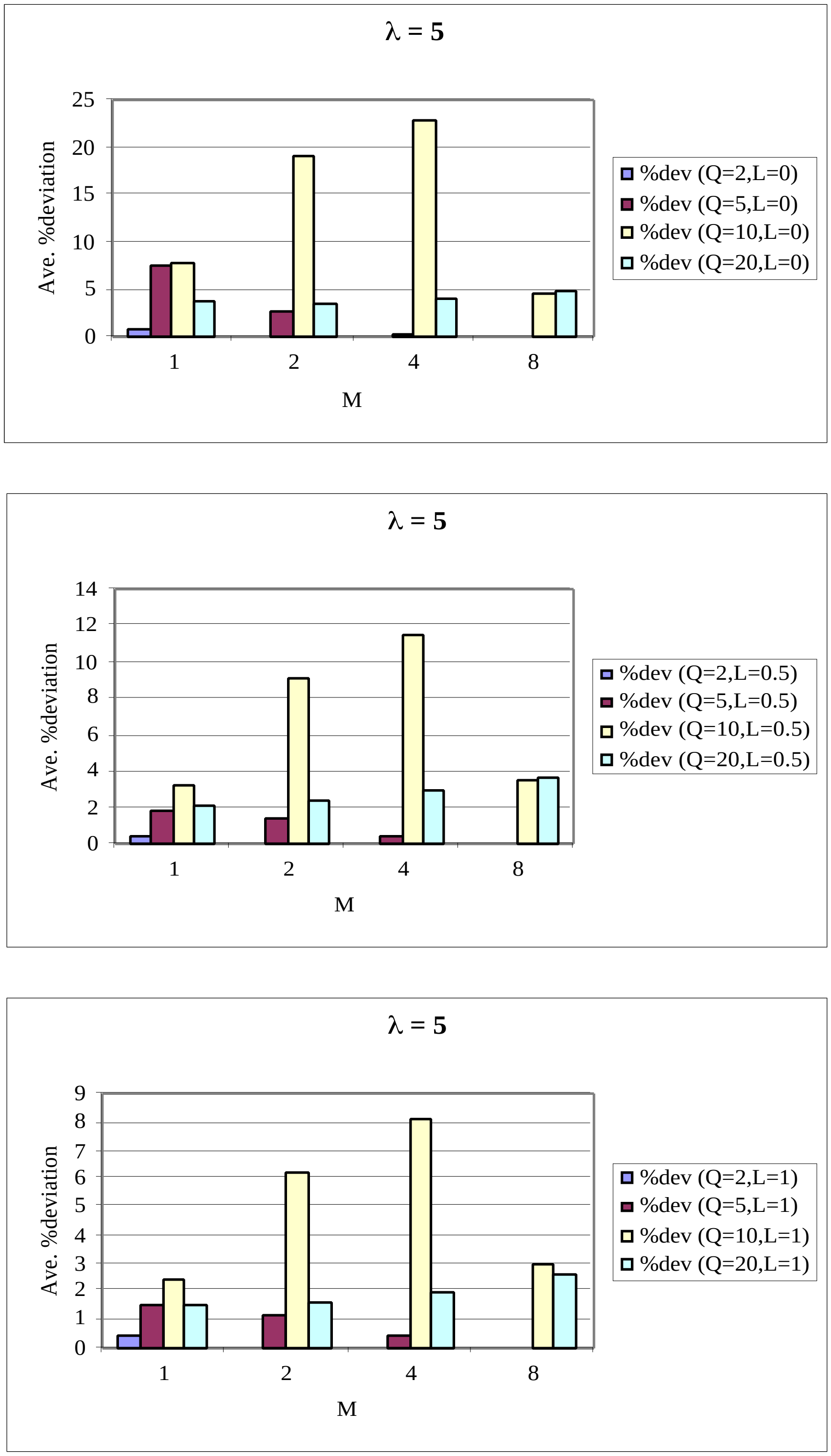

Figure 2*: Performance of systems without information exchange compared to that of ours for order quantities (Q) as the supplier's leadtimes and the number of retailers in the system are varied.*

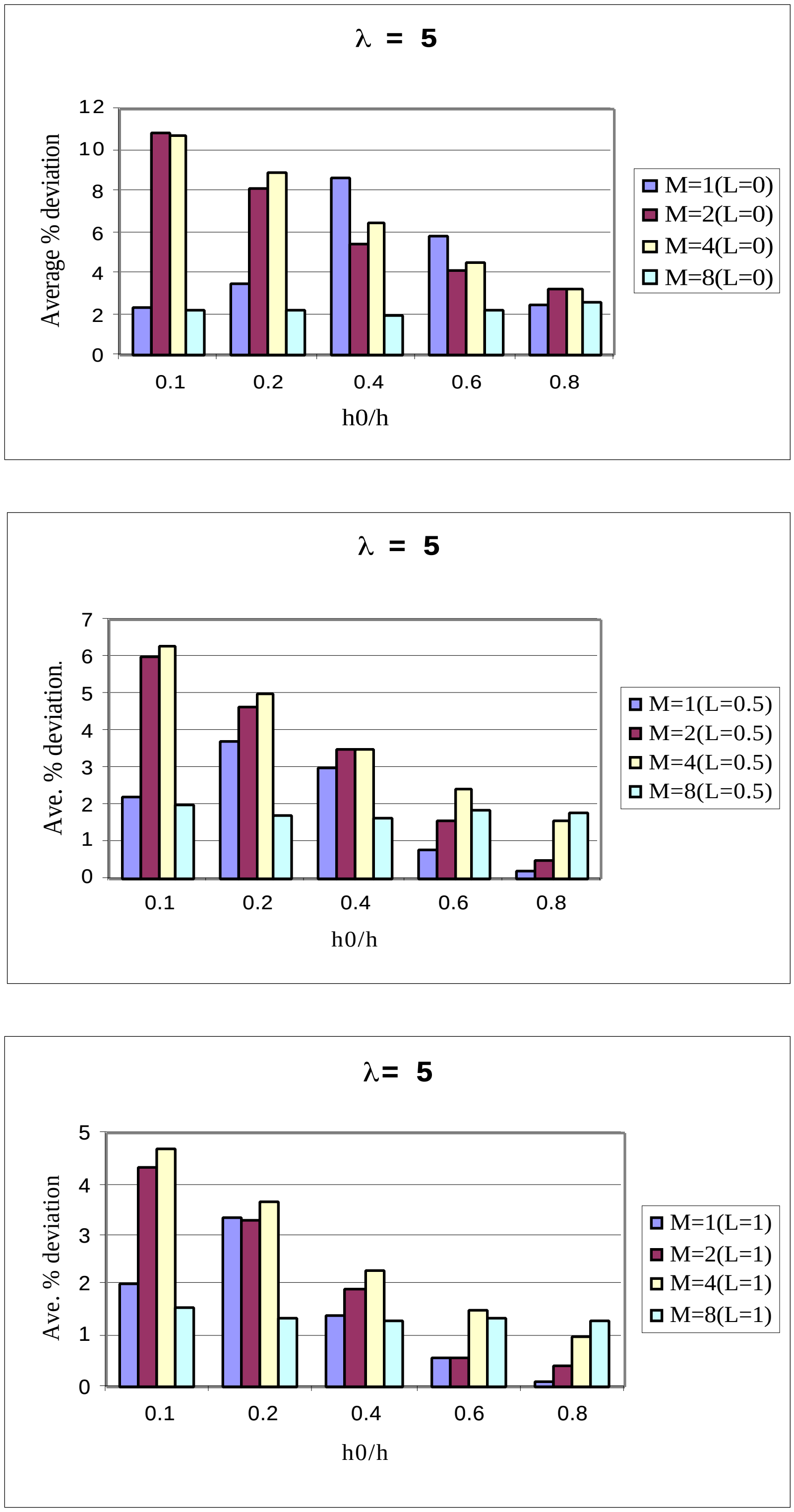

***Figure 3**: Performance of systems without information exchange compared to that of ours for systems with different number of retailers as the supplier's leadtimes and the ratio of holding cost of the supplier to that of the retailers are varied.*