A Supply Chain Model with Direct and Retail Channels

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Abstract

We study a dual channel supply chain in which a manufacturer sells to a retailer as well as to consumers directly. Consumers choose the purchase channel based on price and service qualities. The manufacturer decides the price of the direct channel and the retailer decides both price and order quantity. We develop conditions under which the manufacturer and the retailer share the market in equilibrium. We show that the difference in marginal costs of the two channels plays an important role in determining the existence of dual channels in equilibrium. We also show that demand variability has a major influence on the equilibrium prices and on the manufacturer's motivation for opening a direct channel. Our numerical results show that an increase in retailer's service quality may increase the manufacturer's profit in dual channel and a larger range of consumer service sensitivity may benefit both parties in the dual channel. Our results suggest that the manufacturer is likely to be better off in the dual channel than in the single channel when the retailer's marginal cost is high and the wholesale price, consumer valuation and the demand variability are low.

Keywords: e-commerce, supply chain, dual-channel, game theory

1. Introduction

Internet has become an important retail channel. In 2004, online retail sales comprised of about 5.5% of all retail sales excluding travel (Mangalindan 2005). Recognizing the great potential of the Internet to reach customers, many brand name manufacturers, including Hewlett-Packard, IBM, Eastman Kodak, Nike, and Apple, have added direct channel operations (Wilder 1999, Tsay and Agrawal 2004). More companies are weighing the option to sell directly to consumers. The largest English-language publisher Random House has publicly said that it may sell books directly to readers, putting them in direct competition with Barnes and Noble and Amazon.com (Trachtenberg 2004). Meanwhile, traditional online-only companies are expanding their presence at retail stores. Dell has installed kiosks in shopping malls and now sells its computers through Costco (McWilliams and Zimmerman 2003). Gateway also sells its products at the electronic retailer Best Buy and plans to sign up other retailers, including Wal-Mart and Circuit City to carry its computers (Palmer 2004).

Early reports suggested some retailer resistance against their suppliers' direct channel initiatives (Hanover 1999). It is doubtful, however, that such resistance is effective and helpful over time. When Levi Strauss decided to sell its jeans to J.C. Penney and Sears, it promoted a boycott from Macy. It took 10 years for Macy to realize the folly of denying its customers a product they wanted and driving its customers elsewhere to buy (Hanover 1999). Similarly today, as consumers grow accustomed to multiple channels, they expect to have the choice of buying from a store or buying direct. Studies suggest that more consumers are embracing multiple channels to satisfy their shopping needs (Stringer 2004). Supply chains must reorganize to meet this consumer expectation rater than resist it. Examples of dual channels, cited above, in various industries suggest that many retailers and manufacturers have already learnt this lesson. The evidence suggests that dual channel supply chains already exist. Given such a supply chain, our focus in this paper is on analyzing its performance in equilibrium.

Dual channels could mean more shopping choices and price savings to customers. To traditional retailers and manufacturers, however, the implications for their strategic and operational decisions are not all that clear. How should they make the pricing and quantity decisions and what will be the outcome in equilibrium? As a manufacturer is both a supplier of and competitor with a retailer, traditional supply chain models are not sufficient for developing insights into the equilibrium performance of such supply chains. In this paper, our first objective is to develop a model and analysis that offers an answer to the above question.

Mangalindan (2005) observes that consumers are more likely to purchase certain product categories than others via direct channel. We also see that some industries have seen a faster growth in dual channel supply chains than others. Such observations suggest that difference in product/cost characteristics of the two channels as well as consumer preference for different channels deeply influence the performance of such supply chains. Our next objective is to incorporate such factors into our model and develop managerial insights into their influence. We analyze how such factors that are important in shaping consumer behavior and determining channel efficiency affect the model. Specifically, we examine the effects of service quality, consumer sensitivity to service, cost, and wholesale price on pricing and equilibrium outcomes. In addition, we investigate how demand uncertainty affects the equilibrium.

Several studies have examined dual channel supply chains. Rhee and Park (2000) study a hybrid channel design problem, assuming that there are two consumer segments: a price sensitive segment and a service sensitive segment. Chiang *et al.* (2003) examine a price-competition game in a dual channel supply chain. Their results show that a direct channel strategy makes the manufacturer more profitable by posing a viable threat to draw customers away from the retailer, even though the equilibrium sales volume in the direct channel is zero. Their results depend on the assumption that customer's acceptance of online channel is homogeneous. Boyaci (2004) studies stocking decisions for both the manufacturer and retailer and assumes that all the prices are exogenous and demand is stochastic. Tsay and Agrawal (2004) provide an excellent review of recent work in the area and examine different ways to adjust the manufacturer-reseller relationship. In a similar setting, Cattani *et al.* (2005) study pricing strategies of both the manufacturer and the retailer.

Our model differs from prior studies in the following areas: (i) The demand functions in this study are derived by modeling consumers' choice between direct and retail channels based on both price and service quality, and consumer's sensitivity to service quality is heterogeneous. (ii) The two parties make simultaneous decisions; the manufacturer decides the direct price and the retailer makes both price and stocking decisions. (iii) We assume demand is stochastic and analyze the effects of demand uncertainty on the equilibrium results. Incorporation of these new features in our model allows us to focus on the questions we posed earlier about the effect of product characteristics and consumer preference in our model. Other details of our single-period model include a produce-to-order manufacturer, standard inventory costs at the retailer and different selling costs in the two channels.

Unlike other studies, our model leads to outcomes where both channels are active in the market. Analysis of each party's problem shows that the retailer's optimal price and stocking level increase in the manufacturer's price and the manufacturer's optimal price increases in the retailer's price. We then establish conditions under which both the manufacturer direct channel and the retail channel co-exist in equilibrium. We show that a product characteristic like demand variability strongly influences the outcome; an increase in variability results in a decrease in equilibrium prices. We show that the difference in marginal costs of the two channels is a major factor determining the existence of dual channel supply chains. In addition, industries with lower demand variability are more likely to see a dual channel supply chain structure.

Our numerical results show that an increase in retailer's service quality may increase the manufacturer's profit in dual channel. A larger range of consumer service sensitivity may benefit both parties in the dual channel. We show that dual channel equilibrium may exist in both cases: fixed exogenous wholesale price and manufacturer-set wholesale price. In addition, the manufacturer is likely to be better off in the dual channel than in the single channel when the retailer's marginal cost is high and the wholesale price, consumer valuation and the demand variability are low. We believe that these new insights will be useful for retailers and manufacturers in such supply chains.

The rest of the paper is organized as follows: Section 2 sets up the decentralized dual channel supply chain model. We examine the equilibrium results of our model in Section 3. Section 4 presents the numerical results. We conclude in Section 5.

2. The Model

We consider a single period, single product model with a manufacturer and a retailer. The manufacturer sells to the retailer as well as to the consumers directly. Consumers may choose the retailer (retail channel) or the manufacturer (direct channel) to obtain the good. We begin with describing the consumer choice process.

Empirical studies have shown that transaction costs (Liang and Huang 2001) and service qualities (Devaraj *et al.* 2002, Rohm and Swaminathan 2004) are the major determinants of consumers' channel choice decisions. The demand model in this study captures these two major factors in consumer's channel choice decision. The first factor is simply represented by different prices in two channels. Let p_r and p_d denote the unit price at the retailer and the direct channel, respectively.

The second factor is also important; different service characteristics of online channel and conventional retail stores affect consumer behavior. Studies have found that availabilities of product varieties and product information (Hoffman and Novak 1996, Rohm and Swaminathan 2004), the desirability of immediate possession (Balasubramanian 1998), social interactions gained from shopping (Alba *et al.* 1997), and shopping as a recreational experience (Rohm and Swaminathan 2004) are important factors that influence a consumer's channel choice decision. In our model, we represent service quality as an integrated representation of these different characteristics of the two channels. The service quality at the retailer is s_r, and the service quality at the direct channel is s_d. Let $\Delta s = s_r - s_d$.

Different consumers have different sensitivity to the service quality offered by the two channels. For example, some consumers may put a higher value on the ability to physically experience the good than the others. We represent this sensitivity by θ. For different consumers, θ is randomly drawn from a uniform distribution with support on $[\underline{\theta}, \overline{\theta}]$. Let $\Delta\theta = \overline{\theta} - \underline{\theta}$. The consumer's valuation of the product is v.

We model an individual customer's utility u_i as a function of both price and service quality at channel i from which the product is purchased: $u_i = v + \theta s_i - p_i$, $i \in \{r, d\}$. The two channels are the retailer (r) or the direct channel (d). The consumer chooses the channel that maximizes its utility.

Furthermore, we assume that the valuation of the good is homogeneous among all consumers. A more complex model may allow v to vary across the consumers. Then both the consumers' valuation of the product and their sensitivity of service quality will be random variables in the utility function. However, if the resulting distribution of consumer utility follows uniform distribution, the new model is equivalent to the current model with v being homogeneous (Cattani et al. 2005). We also assume that $s_r > s_d$. In most service characteristics, such as desire for immediate gratification, retailer provides better experience. Therefore it seems reasonable to argue that the overall service quality at retailer is higher. It may be that the assumption does not hold for some types of goods. Limiting ourselves to the goods that satisfy this condition, however, allows us to be brief and clear in much of our presentation. The opposite case is equally tractable to our methods of analysis but adds considerable duplication in our presentation. We close with noting that our choice model follows the general structure of the models for products with *vertical differentiation* (Shaked and Sutton 1983, 1987); vertical differentiation suggests that products or services have different levels of quality. It is different from the *horizontal differentiation* models such as Hotelling's local framework (Hotelling 1929).

We next present the development of demand functions based on the consumer choice model. A consumer will be indifferent between the two channels if and only if $u_r = u_d$, or $v + \theta s_r - p_r = v + \theta s_d - p_d$. Thus, a consumer with $\theta^* = (p_r - p_d)/(s_r - s_d)$ is indifferent between the two channels. Consumers with $\theta < \theta^*$ choose the direct channel, and consumers with $\theta > \theta^*$ use the retail channel. (See Figure 1.) Therefore, the normalized deterministic demand functions for the retail channel is

$$\bar{D}_r = \frac{1}{\Delta\theta}(\bar{\theta} - \theta^*) = \frac{\bar{\theta}}{\Delta\theta} - \frac{p_r}{\Delta\theta\Delta s} + \frac{p_d}{\Delta\theta\Delta s},$$

and the demand for the direct channel is

$$\bar{D}_d = \frac{1}{\Delta\theta}(\theta^* - \underline{\theta}) = -\frac{\underline{\theta}}{\Delta\theta} + \frac{p_r}{\Delta\theta\Delta s} - \frac{p_d}{\Delta\theta\Delta s}.$$

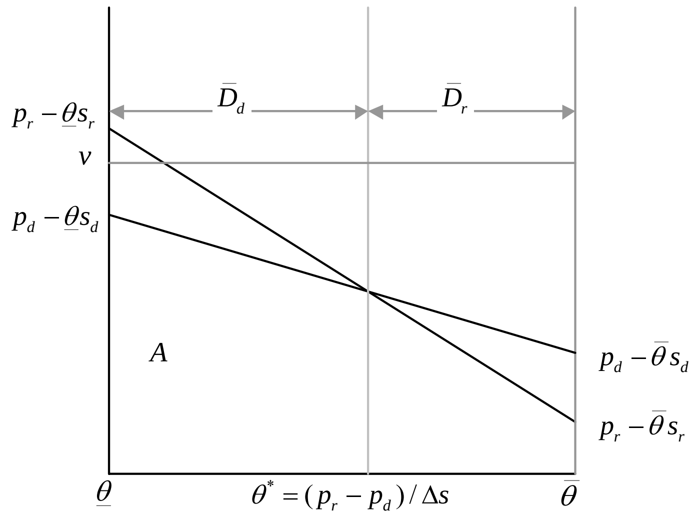

Figure 1: Demand Model

In addition to the deterministic demand functions above, we also include randomness in our demand models. The inventory literature (Mills 1959, Petruzzi and Dada 1999) offers two ways to accomplish this, multiplicative and additive cases. We consider the additive case here in which an exogenous random variable is added to the deterministic demand. A reason for choosing the additive case is that it offers tractability. It is also a reasonable model of reality.   We have the following stochastic demand functions:

$$D_r = \bar{D}_r + \varepsilon = \frac{\bar{\theta}}{\Delta\theta} - \frac{p_r}{\Delta\theta\Delta s} + \frac{p_d}{\Delta\theta\Delta s} + \varepsilon$$

$$D_d = \bar{D}_d + \varepsilon_d = -\frac{\underline{\theta}}{\Delta\theta} + \frac{p_r}{\Delta\theta\Delta s} - \frac{p_d}{\Delta\theta\Delta s} + \varepsilon_d$$

Where $\varepsilon \in [A, B]$ is a random variable with the mean μ and cumulative distribution function $F(\cdot)$. Later, we will discuss that, given our assumptions, the random variable ε_d does not play a significant role in our analysis.

In order to have non-zero demands in both channels, we need the following conditions. First, for consumers with the highest valuation of service, $\bar{\theta}$, we need to have $v + \bar{\theta} s_r - p_r > v + \bar{\theta} s_d - p_d$; otherwise, no consumers will buy from the retail channel. Similarly, at $\underline{\theta}$, we need to have $v + \underline{\theta} s_r - p_r < v + \underline{\theta} s_d - p_d$; otherwise, no consumers will purchase from the direct channel. Combining the two, we have the condition:

$$p_d + \underline{\theta}(s_r - s_d) < p_r < p_d + \bar{\theta}(s_r - s_d). \tag{1}$$

Second, consumer valuation has to be higher than a certain level in order to sustain two channels. The following is the condition that both channels have full coverage on the market:

$$v \geq p_d - \underline{\theta} s_d, \tag{2}$$

The rest of the model describes the policies and costs at the manufacturer and the retailer. At the beginning of the period, the retailer decides to buy q_r units from the manufacturer at the given wholesale price w. For each purchased unit, the retailer incurs a marginal cost of selling c_r per unit. Any leftovers at the end of the period incur a holding cost h per unit. Any shortages at the end of the period incur a shortage cost π per unit. The manufacturer delivers q_r units to the retailer at the beginning of the period. For direct channel demand, it produces against orders but incurs a marginal cost of selling c_{ds} per unit. The production cost at the manufacturer is c_d per unit. Both retailer and the manufacturer decide their own prices. In addition, the retailer also decides the order quantity. They make these decisions simultaneously.

Most of the above costs are standard in inventory literature. The marginal costs of selling at the two channels represent different activities each channel undertakes and therefore, are intended to differentiate between the two channels. The retailer cost c_r includes back office costs, merchandizing costs and shelving costs. The manufacturer cost c_{ds} includes the cost of maintaining a website and a distribution system. We also note that in this paper, we focus on the situations where the whole market is covered. In such situations, the production cost c_d does not have any effect on the results. However, including c_d in the model facilitates comparison with the single channel case in which the total demand may be less than the whole market. Finally, there may be a fixed cost for the manufacturer to start a direct channel. This cost is not included in our model but we do not expect a fixed cost to have an impact on the direction of our results.

We briefly discuss the simplifying assumptions inherent in the model above. We assume that the shortages at the retailer are lost, not directed towards the manufacturer. We believe that this is a reasonable assumption, especially in competitive settings, where a consumer may simply switch to another brand that is available. If a consumer discovers the shortage only after a visit to the store, he is likely to substitute another brand for it rather than go back and order it from the manufacturer. Another assumption concerns produce-to-order system at the manufacturer. This is mainly for tractability but this, too, finds parallels in practice. Computer manufacturers like Dell and Gateway produce against direct orders they receive on their Web sites. Next, we assume that the wholesale price is fixed and is exogenous to our model. The assumption reflects the practice of setting contractual prices that remain fixed for medium term. If the manufacturer is operating in a highly competitive market, we can think of wholesale price as being determined by this competition that is exogenous to our model. Later, in Section 4, we discuss how we can endogenize the wholesale price decision. Finally, our model of supply chain structure is similar to the setups in Chiang et al. (2003), Tsay and Agrawal (2004), and Cattani et al. (2005). Our demand model is different in that consumers choose either the retail or the direct channel based on price and service qualities. Unlike other models, we assume that the demand is stochastic.

3. Analysis of the Dual Channel Model

In this section, we analyze the case where the manufacturer and the retailer simultaneously make their decisions. We begin with determining each party's optimal decisions.

3.1 Retailer's Problem

The retailer decides the price p_r and the order quantity q_r. We work with the transformation $z = q_r - \bar{D}_r$ where z represents the quantity ordered to satisfy the stochastic portion of the demand. The retailer pays w and c_r for each unit purchased, and earns p_r for each unit sold. $\Lambda(z)$ represents expected overages and $\Theta(z)$ represents expected shortages at the end of the period. In a manner similar to newsboy model, the retailer's expected profit is:

$$E[\Pi_r(z, p_r)] = (p_r - c_r - w)(\bar{D}_r + \mu) - (c_r + w + h)\Lambda(z) - (p_r + \pi - c_r - w)\Theta(z) . \qquad (3)$$

where $\Lambda(z) = \int_A^z (z-u) f(u) du$ and $\Theta[z] = \int_z^B (u-z) f(u) du$. The retailer's objective is to maximize expected profit for a given manufacturer's price p_d. First order conditions give us the following result.

PROPOSITION 1. *Given manufacturer's price* p_d *, retailer's optimal decisions* p_r^*, z^* *satisfy the following two simultaneous equations:*

$$F(z^*) = \frac{p_r^* + \pi - (c_r + w)}{p_r^* + h + \pi}, \tag{4}$$

$$p_r^* = p^0 - \frac{\Theta(z^*)}{2b}, \tag{5}$$

where $p^0 = \frac{1}{2}(w + c_r + \bar{\theta}\Delta s + \Delta\theta\Delta s\mu) + \frac{p_d}{2}$ *and* $b = \frac{1}{\Delta\theta\Delta s}$.

All proofs are available in the Appendix. Petruzzi and Dada (1999) have considered the newsboy model with pricing and their results, applied to our model, show that p_r^* and z^* are uniquely determined if $F(\cdot)$ is a cumulative distribution satisfying the following condition,

$$2r(z)^2 + dr(z)/dz > 0, \tag{6}$$

where $r(z) = f(z)/(1 - F(z))$ is the hazard rate. It is a mild condition and many distributions used in inventory modeling satisfy it. Any increasing failure rate distribution automatically satisfies the condition. In rest of the paper, we assume that this condition holds in our model. Our next two results aim at better understanding the retailer's optimal decision.

PROPOSITION 2. *The retailer's optimal stocking decision* z *increases in the retailer's price* p_r.

For a given p_d, as retailer increases p_r, both its share of the demand, $\bar{D}_r$, and the size of its order meant to satisfy the deterministic demand decrease. At the same time, the retailer increases the size of its order meant to satisfy the stochastic demand as a higher price increases the average shortage cost, thereby increasing the optimal service level. We next consider the impact of p_d on retailer.

PROPOSITION 3. *(i) The retailer's optimal price* p_r^* *increases in the manufacturer price* p_d. *(ii) The retailer's profit increases in the manufacturer price* p_d.

As the manufacturer's price increases, the retailer sees an increase in its demand without any reduction in its price. This allows the retailer to increase its price while serving the same deterministic demand resulting in a higher profit for the retailer.

We close this section with a brief look at a single channel version of our model, in which the manufacturer only sells to the retailer and does not have a separate direct channel. We will use this analysis later in Sections 3.4 and 4 to draw insights into the manufacturer's motivation in opening a direct channel. The retailer's expected profit is:

$E[\Pi_r^s(z^s, p_r^s)] = (p_r^s - c_r - w)(\bar{D}_r + \mu) - (c_r + w + h)\Lambda(z^s) - (p_r^s + \pi - c_r - w)\Theta(z^s)$.

Given retailers' decisions, the manufacturer's profit is a constant:

$\Pi_d^s = (w - c_d)(\bar{D}_r + z^s)$.

Clearly, we only need to determine the retailers' optimal decisions in this case: the retailer price p_r^s and the stocking level z^s. In the original dual channel model, however, the manufacturer must decide its price. We consider this decision in the next section.

3.2 The Manufacturer's Problem

The manufacturer decides its price p_d. For a given retailer decision, the manufacturer's profit function is:

$$E[\Pi_d(p_d)] = (p_d - c_d - c_{ds})\bar{D}_d + (w - c_d)(\bar{D}_r + z). \quad (7)$$

Note that (7) does not depend on ε_d. This is because, given our assumption of a produce-to-order manufacturer, only the expected value of ε_d influences the profit function. Effectively, this can be treated as a constant demand term and it does not influence manufacturer's decisions. To keep the presentation simple, we assume that expectation of ε_d is zero. To maximize its profit, the manufacturer sets the following price.

PROPOSITION 4. *For a given retailer's price p_r, the manufacturer's optimal price p_d^* is:*

$$p_d^* = \frac{1}{2}(w + c_{ds} - \underline{\theta}\Delta s) + \frac{1}{2} p_r. \quad (8)$$

Examining the manufacturer's response to the retailer, we have the following result.

PROPOSITION 5. *(i) The manufacturer's optimal price p_d^* increases in the retailer's price p_r. (ii) When $c_r > c_{ds} + \underline{\theta}\Delta s$, the manufacturer's profit increases in the retailer's price p_r.*

As retailer price increases, it allows the manufacturer to set a higher price. The impact of retailer's price on manufacturer's profit, however, is not similarly direct; it has two components. First, as the retailer's demand decreases, the manufacturer's profit due to retailer orders will decrease. Second, the manufacturer's direct channel profit will increase due to higher demand. Whether the manufacturer is better off depends on the tradeoff of the two components. When $c_r > c_{ds} + \underline{\theta}\Delta s$, the manufacturer's profit margin at the direct channel is high and the second component dominates.

3.3 Equilibrium

We now analyze the outcome of a simultaneous move game between the retailer and the manufacturer. In the previous two sections, we have outlined the response of each party given the other party's pricing strategy. The intersections of the response functions will be the equilibrium point of this game. As we discussed earlier, we are interested in the case where both parties see positive demand and fully cover the market. We call this the dual channel equilibrium and now focus on finding conditions under which such equilibrium exists.

At an equilibrium point, each party must respond optimally given the other party's price. Therefore, such a point must satisfy both Propositions 1 and 4. We first show that such a point exists.

LEMMA 6. *There exists a unique solution to (4), (5) and (8).*

According to Lemma 6, we can find the unique intersection point, by jointly solving the three equations. Substituting Equation (8) into (5), we obtain

$$p_r^* = \frac{1}{3}(3w + c_{ds} + 2c_r + \bar{\theta}\Delta s + \Delta\theta\Delta s + 2\Delta\theta\Delta s\mu) - \frac{\Theta(z^*)}{2\left(\frac{3}{4\Delta\theta\Delta s}\right)}.$$

We can rewrite the above equation as:

$$p_r^* = \tilde{p}^0 - \frac{\Theta[z^*]}{2\tilde{b}}, \tag{9}$$

where $\tilde{p}^0 = \frac{1}{3}(3w + c_{ds} + 2c_r + \bar{\theta}\Delta s + \Delta\theta\Delta s + 2\Delta\theta\Delta s\mu)$, and $\tilde{b} = \left(\frac{3}{4\Delta\theta\Delta s}\right) > 0$.

Therefore, we can arrive at the solution by first jointly solving Equations (4) and (9), and then find p_d^* using Equation (8).

However, this solution is the dual channel equilibrium only if the prices at this point satisfy the conditions for the validity of the demand equations. That is, if the demand is strictly positive in both channels at this point. In order to derive the conditions that the dual channel equilibrium exists, we first examine the case where the demand is deterministic.

In the certainty-equivalent model, we let the standard deviation σ be 0 and normalize μ, the mean of ε, to be 0. Thus, the profit functions for the manufacturer and the retailer are:

$$\Pi_r(z, p_r) = (p_r - c_r - w)\bar{D}_r ,$$

$$\Pi_d(p_d) = (p_d - c_d - c_{ds})\bar{D}_d + (w - c_d)\bar{D}_r .$$

From the first order conditions of the above problems, we have the following response functions. We use two asterisks in the superscript for deterministic model.

$$p_r^{**}(p_d) = \frac{1}{2}(w + c_r + \bar{\theta}\Delta s) + \frac{1}{2} p_d \tag{10}$$

$$p_d^{**}(p_r) = \frac{1}{2}(w + c_{ds} - \underline{\theta}\Delta s) + \frac{1}{2} p_r \tag{11}$$

The solution to the above two equation gives:

$$p_r^{**} = \frac{1}{3}(3w + 2c_r + c_{ds} + \Delta\theta\Delta s + \bar{\theta}\Delta s) ,$$

$$p_d^{**} = \frac{1}{3}(3w + c_r + 2c_{ds} + \Delta\theta\Delta s - \underline{\theta}\Delta s) .$$

If the above solution satisfies conditions (1) and (2), then the demand in either channel is non-zero and we will have dual channel equilibrium.

LEMMA 7. *When demand is deterministic, the equilibrium demand for either channel is non-zero under the following conditions:*

$$v \geq \frac{1}{3}(3w + c_r + 2c_{ds} + \bar{\theta}\Delta s - \underline{\theta}s_d - 2\underline{\theta}s_r) \text{ and } \Delta s(\underline{\theta} - \Delta\theta) < c_r - c_{ds} < \Delta s(\Delta\theta + \bar{\theta}) .$$

According to Lemma 7, consumer valuation has to be above a critical level in order to have demands in both channels covering the full market. This critical level is increasing in the wholesale price and the two marginal costs. This makes sense because high costs in the system would require a high consumer valuation for ensuring a level of demand high enough for two channels. This critical level increases with the difference in two service qualities. In addition, the marginal cost difference between the two channels cannot be too big or too small. If the marginal cost of the retailer is too high, the retailer has to charge a high price and the manufacturer can easily compete and capture whole market profitably. The marginal cost difference between the two channels cannot be too narrow because this means the manufacturer has relatively high cost and cannot set competitive price. Thus, the retailer can easily set its price to capture the whole market. In Figure 2, the shaded region is the feasible region for dual channel equilibrium.

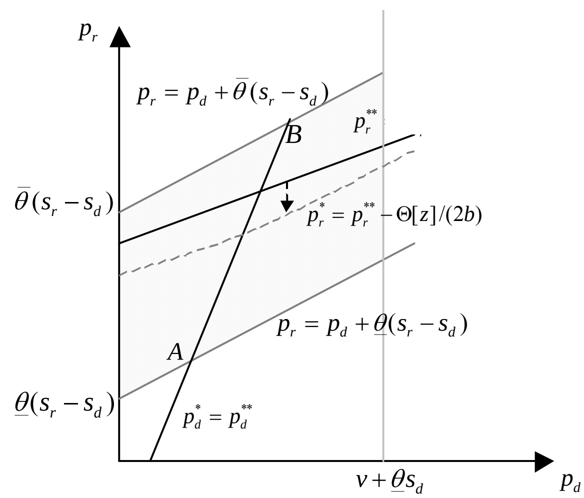

Figure 2: Feasible Region for Dual Channel Equilibrium

Now we analyze the equilibrium when the demand is uncertain. To ensure the existence of dual channel equilibrium in the stochastic demand case, we need the following conditions.

THEOREM 8. *The dual channel equilibrium exists if the following conditions are satisfied:* $v > w + c_{ds} + \Delta s \Delta\theta - \underline{\theta} s_d$ *and* $\Delta s \underline{\theta} + \Delta s \Delta\theta\mu < c_r - c_{ds} < \Delta s(\Delta\theta + \bar{\theta} - \Delta\theta\mu)$.

We can compare the stochastic case with the deterministic case by setting $\mu = 0$. The second condition in Theorem 8 is derived from the application of conditions (1), yielding the following intermediate condition (see the proof for details): $\Delta s \underline{\theta} - \Delta s \Delta\theta(1 - \Theta(z^*)) < c_r - c_{ds} < \Delta s(\Delta\theta + \bar{\theta} + \Delta\theta\Theta(z^*) - \Delta\theta\mu)$. Incorporating minimum and maximum bounds on optimal shortage $\Theta(z^*)$ converts this intermediate step into the final form given in the theorem. When $\mu = 0$, the intermediate condition in the stochastic case differs from the deterministic condition in Lemma 7 by the term $\Delta s \Delta\theta\Theta(z^*)$ on both sides. This implies that the dual channel equilibrium in the stochastic case requires $\Delta s \Delta \theta \Theta(z^*)$ more in the difference in marginal costs than the deterministic case. The higher the expected shortage (*e.g.* from higher variability), the larger the gap between c_r and c_{ds} that is required for the dual channel existence. When the expected shortage is higher, the retailer sets price more aggressively. This puts pressure on the manufacturer to set its price even lower and thus there is less room for it to capture low valuation consumers profitably. As a result, the manufacturer requires a bigger cost advantage to compete and exist in dual channel.

The discussion of the intermediate condition helps us bring shortages in the picture. But the same point can be made by starting from the final condition in the Theorem and substituting $\mu = 0$. The resulting condition has a tighter lower bound than the deterministic condition while the upper bound is the same. Clearly, in the space of the marginal costs, the stochastic condition is harder to satisfy than the deterministic condition. Furthermore, when the condition on cost difference in Theorem 8 holds, the condition in Proposition 5 is not required and Proposition 5 is always true in dual channel. That is, in dual channel equilibrium, the manufacturer's profit increases in the retailer's price p_r.

Based on Figure 2, we can make another interesting comparison between the stochastic and deterministic cases. When $\mu = 0$, comparing the retailer's response function in (5) when demand is stochastic with the retailer's certainty-equivalent response function in (10) yields $p_r^{**} = p^0$. Thus, we can express the retailer's price response when demand is stochastic as follows:

$$p_r^* = p_r^{**} - \frac{\Theta(z^*)}{2b}, \text{ if } \mu = 0. \tag{12}$$

This means that in stochastic case the best response p_r^* shifts downward from the response in deterministic case by $\frac{\Theta(z^*)}{2b}$. Since optimal stocking decision z^* is a function of p_r^* and p_d, the exact amount by which the retailer's price moves downward from p_r^{**} depends on the distribution of demand. Comparing manufacturer's response functions in stochastic and deterministic cases, *i.e.* Equations (8) and (11), we note that $p_d^* = p_d^{**}$. This means that the manufacturer's best response is the same for the two cases. The above argument leads to the following result.

PROPOSITION 9. *In the stochastic demand case with μ=0, both the retailer's and the manufacturer's equilibrium prices are lower compared to the equilibrium prices when demand is deterministic.*

This result puts us on the path to think about the effect of demand variability in our model. We further explore this concern in the next section.

3.4 Effect of Demand Variability

The variability of demand is a major driver of inventory costs and therefore, we are particularly interested in understanding how it affects the dual channel equilibrium. In Proposition 9, we presented a general result that summarized the effect on equilibrium prices, irrespective of the choice of demand distribution. As demand becomes uncertain, the retailer incurs additional overstocking and under-stocking costs. Intuitively, the retailer seeks to diminish the impact of variability. One way to do it is to make the uncertain demand a smaller part of the total demand. The retailer accomplishes this by reducing price and thus, increasing the deterministic part of the demand. From Proposition 5, we know that the manufacturer's price increases in the retailer's price. Therefore, both equilibrium prices decrease.

The above discussion suggests a business strategy to deal with variability by securing more deterministic portion of demand. For example, Blockbuster initiated unlimited rental with a fixed monthly fee. Using this promotion, the price is, in effect, reduced and at the same time more deterministic portion of demand is secured. With this kind of promotion the retailer can mitigate the variability of demand and it is more effective than reducing the price alone.

It is interesting to see whether the same dynamic holds if, starting from the uncertain demand setting, we further increase the demand variability. It turns out that analyzing this case requires us to assume a specific demand distribution. We assume ε follows normal distribution with mean μ and standard deviation σ. The results in the rest of this section follow this assumption.

PROPOSITION 10. *The equilibrium prices of both the retail and direct channels decrease in* σ. *The manufacturer's equilibrium price decreases at half the rate of retailer's. The retailer's demand increases in* σ *and manufacturer's demand through direct channel decreases in* σ

As uncertainty in retailer demand increases, the retailer tries to control the overall variability by increasing the deterministic portion of the demand. To achieve this, it aggressively competes for the customers by lowering the price. In response, the manufacturer also reduces its price but it does so at a lower rate and this results in a higher market share for retailer. For the other retailer decision z, it is uncertain if z increases or decreases with higher variability. This is because there are two opposite effects. First, with higher uncertainty, given a z, expected shortage increases and therefore, we expect the stocking level should increase. However, with lower retailer price, the penalty for not having enough stock is lower and the stocking level should decrease. The combined effect could go in either direction.

We now take the manufacturer's perspective and address the effect of demand variability on the manufacturer's motivation. In many cases, the manufacturer faces the decision whether it should open a direct channel or not. Such a decision requires comparing manufacturer's profits under a traditional single channel (see Section 3.1) with its profits in a dual channel supply chain. We are interested in the way inherent product characteristics such as demand variability may influence this decision. Like Section 3.3, we find it easier to begin with the deterministic case.

PROPOSITION 11: *When the demand is deterministic, manufacturer prefers dual channel to single channel when:*

$$v < \tilde{v} \text{ where } \tilde{v} = w + c_r - \underline{\theta} s_r + s_r \Delta\theta + \frac{2 s_r \Delta\theta \left(c_r - c_{ds} + \Delta s(\Delta\theta - \underline{\theta})\right)^2}{9 \Delta s \Delta\theta (w - c_d)}$$

In the single channel case, the lowest valuation of product among all customers is $v + \underline{\theta} s_r$ and therefore, it is the maximum price that the retailer can charge and still keep 100% of market in single channel ($\bar{D}_r$ = 1). In this setting the retailer's profit is $(v + \underline{\theta} s_r - w - c_r)1$. The condition in Proposition 11 can be rearranged to say that this profit must be lower than a critical value. When this condition is met, the retailer may benefit by setting its price higher than $v+\underline{\theta}s_r$, leaving some unsatisfied demand in the market. In such a situation, the manufacturer in a single channel suffers from the retailer behavior of ordering less than the market demand. Opening a direct channel provides the manufacturer with a way to tap into the rest of the market. Thus, when the above condition holds, the manufacturer is better off in dual channel. We are now ready to consider the stochastic demand case.

PROPOSITION 12: *When $v<\tilde{v}$, there exists a range $\sigma\in[0,\tilde{\sigma}]$ in which the manufacturer prefers dual channel to single channel.*

Based on our earlier discussions, the result can be intuitively explained. As variability grows, the retailer's response is to counter it by cutting prices and increasing the deterministic portion of the demand. As a result, the manufacturer sees larger retailer orders in both channel structures. In the dual channel structure, however, the price reduction by the retailer forces the manufacturer to cut prices and leaves it with access to smaller direct demand (as also suggested in Proposition 10). The result is that, as variability increases, the manufacturer sees smaller profits in dual channel structure and at some point $\tilde{\sigma}$, single channel becomes better for the manufacturer. Indeed, it is possible to extend the above result to show that for all $\sigma>\tilde{\sigma}$, the manufacturer will prefer single channel. The proof of this extension, however, requires additional conditions on parameters. We provide these conditions and the proof in Appendix (Note 13). These results suggest that the demand variability may be a major factor driving the evolution of dual channel supply chains.

4. Numerical Results

Our objective in this section is to draw managerial insights based on a numerical analysis of our model. We consider several scenarios related to different parties in our model. First, we consider the effect of changes in service qualities offered by the parties. Second, we focus on the difference in the two parties' costs of selling. Third, we focus on the consumer and consider the effect of the service sensitivity. We also revisit the influence of demand variability on the equilibrium. Finally, we consider relaxing the assumption that wholesale price is exogenously fixed.

In the rest of this section, we illustrate our results with the help of a selected numerical example. The parameters for this example are: v=0.7, $\bar{\theta}$=1, $\underline{\theta}$=0.2, s_r=0.75, s_d=0.25, c_d=0.25, w=0.35, c_r=0.025, c_{ds}=0.0125, μ=0, σ=0.1, h=0.05 and π= 0.1. We also assume that the additive stochastic component follows a Normal distribution. In each section, we draw and interpret figures by varying a parameter while keeping others constant. The trends we observe, however, are supported by a large numerical study. Specifically, our numerical study consists of the following parameter combinations. Together the combinations yield 729 instances. The rest of parameters are fixed as in the example given above.

$\underline{\theta}$	c_r	s_r	w	σ	c_d
0.1	0.0125	0.375	0.175	0.05	0.125
0.2	0.025	0.75	0.35	0.1	0.25
0.3	0.0375	1.125	0.525	0.15	0.375

Table of parameter values

4.1 Effect of Service Quality

Figures 3 and 4 represent the effect of retailer service quality on the equilibrium prices and profits for the two retailers. As retailer's service quality increases, both the retailer and manufacturer's equilibrium prices go up, and their profits increase as well. Recall that the consumer sensitivity to service quality is uniformally distributed. Therefore, for a given service quality in the direct channel, an increase in the retailer's service quality allows the retailer to concentrate at those consumers that have a higher sensitivity. This leaves the manufacturer free to focus on the other extreme, low sensitivity consumers. The overall effect is that increasing retailer service quality further differentiates the two channels and reduces the direct competition between them. Thus, it allows both the retailer and the manufacturer to charge higher prices and make higher profits. The same dynamic is at work if we reduce the manufacturer service quality while keeping the retailer service quality constant and a similar result is observed in that case. In the interest of brevity, we only focus on the retailer service quality in this section.

The above observation suggests a not-so-obvious implication of the retailer's service quality choice. As a response to the manufacturer's presence in the market, the retailer's first impulse may be to increase its service quality and give the consumer something that the manufacturer cannot offer. For example, unlimited browsing and in-shop café at some book retailers allow them to increase the service quality over an on-line competitor. Our results, however, suggest that any such move, though profitable to the retailer, is unlikely to deter the manufacturer as it may also increase the manufacturer's profit.

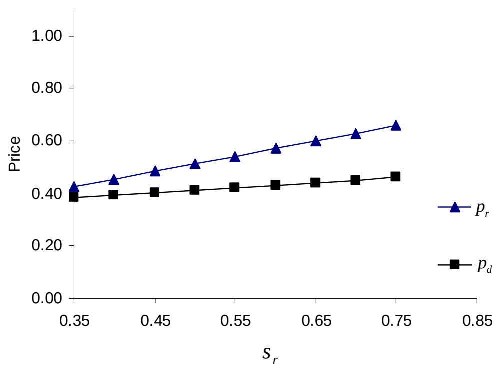

Figure 3: Effect of Retailer's Service Quality on prices

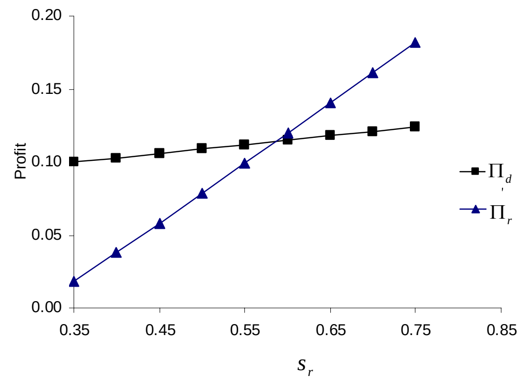

Figure 4: Effect of Retailer's Service Quality on profits

Another interesting observation from Figure 4 is that as s_r increases, the retailer profit at equilibrium increases at a faster rate than the manufacturer profit. Thus, it is indeed possible for the retailer to focus on increasing its service quality in order to be more profitable than the manufacturer.

4.2 Consumer's Service Sensitivity

The parameter θ represents service sensitivity of consumers. We suggest that average value $\left(\frac{\underline{\theta}+\overline{\theta}}{2}\right)$ and range $\left(\overline{\theta}-\underline{\theta}\right)$ characterize the nature of the product under consideration. While the average value measures the importance a consumer gives to service quality with respect to its valuation, the range represents the level of heterogeneity in consumers. For numerical experimentation, average θ and $\Delta\theta$ take values over the range assigned to $\underline{\theta}$ in the table of parameters presented above.

For a $\Delta\theta$ (equal to 0.5 in this example), higher average θ means that the consumers have higher value for service. That should benefit the retailer as it provides better service. Figures 5 and 6 show that the retailer can charge higher price and reap higher profit for products with higher average θ. Correspondingly, the manufacturer is worse off. As the higher average θ leaves smaller lower portion for manufacturer's demand and thus manufacturer has to set lower price to compete for the smaller demand, thereby reducing its overall profit.

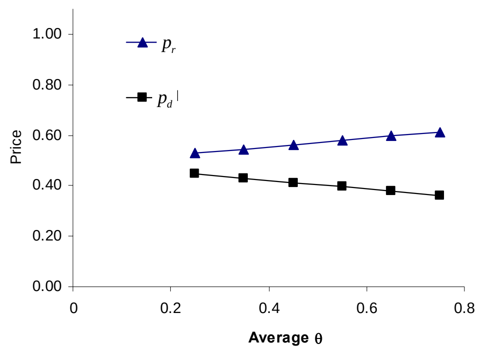

Figure 5: Effect of Average of Consumer's Service Sensitivity on prices

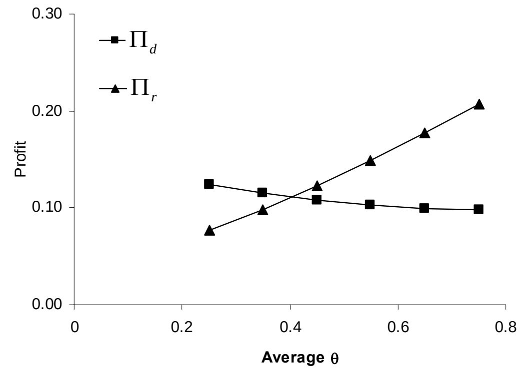

Figure 6: Effect of Average of Consumer's Service Sensitivity on profits

For a fixed average θ, an increase in the range of θ spreads out the consumer sensitivity. This means consumers are more heterogeneous, which allows both the retailer and the online channel to target different segments of consumers. Retailer focuses on higher θ consumers and the manufacturer focuses on lower θ consumers. This reduces the competition on each extreme of the consumer choice. Both channels charge higher prices and gain higher profits as shown in Figures 7 and 8.

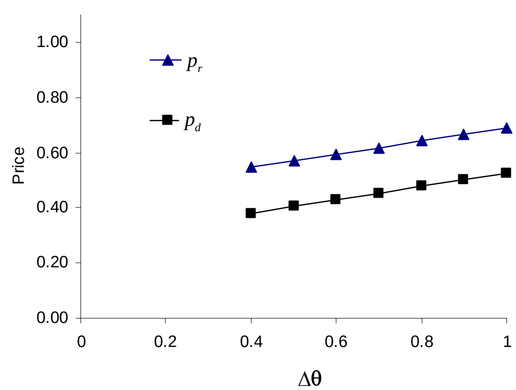

Figure 7: Effect of the difference of Consumer's Service Sensitivity on prices

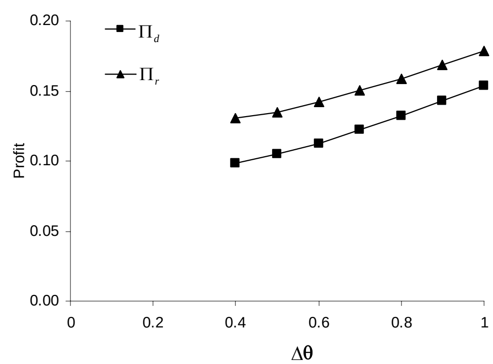

Figure 8: Effect of the difference of Consumer's Service Sensitivity on profits

4.3 Marginal Cost of Selling

As we discussed earlier, one of the fundamental differences in two channels is the marginal cost of selling; the retailer is likely to incur a higher unit cost than the manufacturer. We suggest that this cost, within limits, is controllable by the retailer. Figures 9 and 10 display the effect on equilibrium prices and profits as c_r changes.

Clearly, an increase in retailer cost will negatively affect the retailer's profit. The retailer will increase its price and will see smaller deterministic demand. This will have two contradictory effects on the manufacturer. The manufacturer will suffer a loss of sales to the retailer but will benefit by decreased price competition from the retailer. The result of these two effects, in Figure 10, is increasing profit for the manufacturer. Figure 10 also plots the manufacturer profit in a single channel model discussed in Section 3.4. In the single channel model, the manufacturer has no access to the market and, therefore, suffers from the decrease in retailer orders. Thus, the manufacturer profits in these two settings, dual channel and single channel, follow different trends. This observation suggests that given c_d, there exists a critical c_r, beyond which the manufacturer will always benefit from opening a direct channel. We hypothesize that this difference in marginal costs may be a factor determining which industries are more likely to have dual channel supply chains.

Figure 10 also plots the total manufacturer and retailer profit in the dual channel and single channel models. Note that for high values of c_r, the total profit is higher in the dual channel model. This argues that even though the retailer will always see its profit drop by the manufacturer's opening a direct channel, there may be some cases where a profit sharing arrangement can increase both parties' profits.

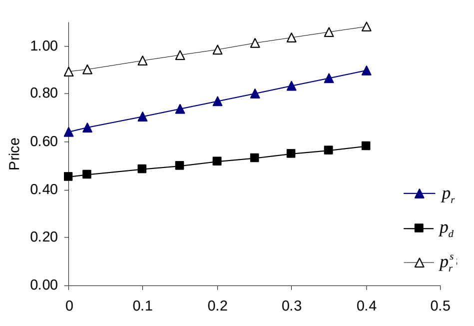

Figure 9: Effect of the Marginal Cost of Selling on prices

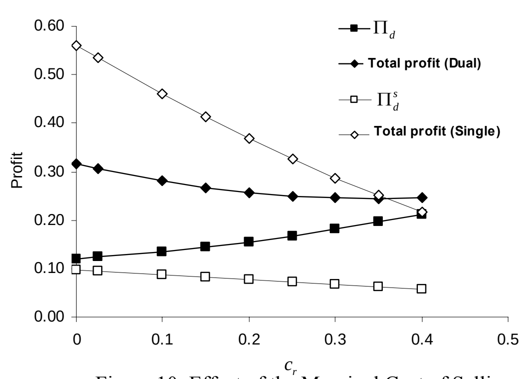

Figure 10: Effect of the Marginal Cost of Selling on profits

4.4 Demand Variability

We have studied the impact of demand variability on the equilibrium in Section 3.4. This section briefly highlights those results numerically. The figures in this section are drawn for a slightly different example; s_r =0.65, s_d =0.35, c_d =0.1 and the rest of the parameters are as before.

As demand gets more uncertain, Figure 11 shows that the prices in general decrease in both single channel and dual channel supply chains. Figure 12 reinforces the main result in Section 3.4. At low values of σ, the manufacturer is better off in dual channel. As σ increases, the manufacturer's profit in dual channel decreases and its profit in the single channel increases. As a result, beyond a threshold σ, the manufacturer is better off in single channel. Section 3.4 discussed the intuition behind these observations.

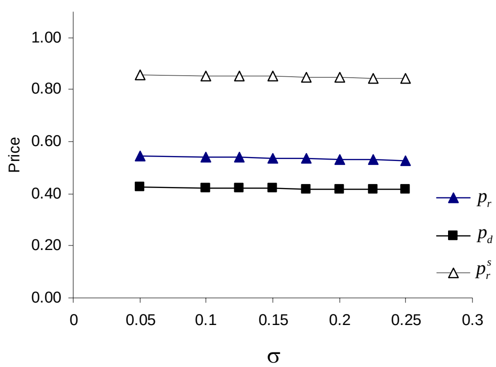

Figure 11: Effect of the Demand Variability on prices

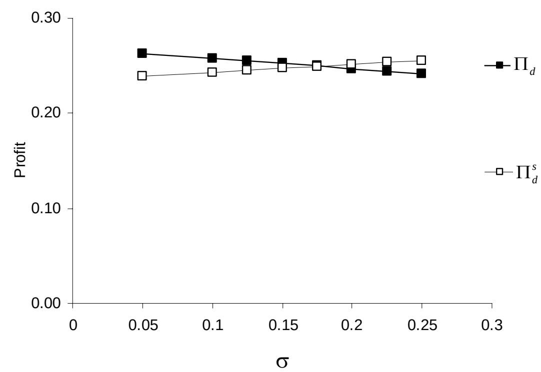

Figure 12: Effect of the Demand Variability on profits

4.5 Wholesale Price as Decision Variable

Thus far, we have assumed that the wholesale price w is given as fixed in a contract. We now consider the case where the manufacturer sets the wholesale price. Our approach is to consider a two-stage process. In the first stage, the manufacturer sets w and in the second stage, the manufacturer and the retailer simultaneously set their pricing decision. While setting w in the first stage, the manufacturer anticipates the outcome of the second-stage game as analyzed in Section 3. Manufacturer then decides w to maximize its second-stage profit.

Figures 13 and 14 present the effect on prices and profits as w increases. Focusing on manufacturer profit in dual channel case, we see that, in the beginning, the profit increases with w. In this portion of the graph, an increase in w forces higher prices in both channels and results in an increase in retailer demand and a decrease in manufacturer demand. This is because even as p_r increases, p_d increases at a higher rate and therefore there is a net increases in retailer demand and a corresponding decrease in manufacturer demand. As the trend continues, beyond a critical value of w, the manufacturer has no demand and the system starts behaving like a single channel system with no direct channel. In the case of this example, this occurs at $w = 0.66$. This is what is responsible for the kinks in the graphs. The kinks occur because the demand function coefficients change from dual channel to single channel case. For $w > 0.66$, the manufacturer profit in dual channel follows the concave curve obtained in single channel case. The nature of the profit function, as described above, suggests that there may be a unique solution to the problem where w is endogenously decided by the manufacturer.

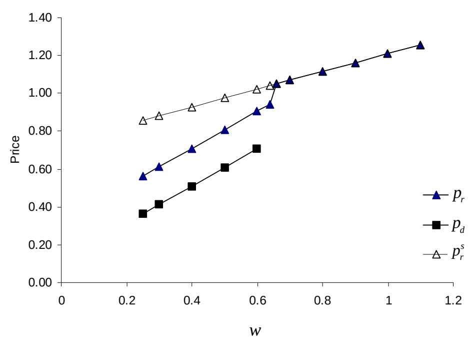

Figure 13: Effect of the wholesale price on prices

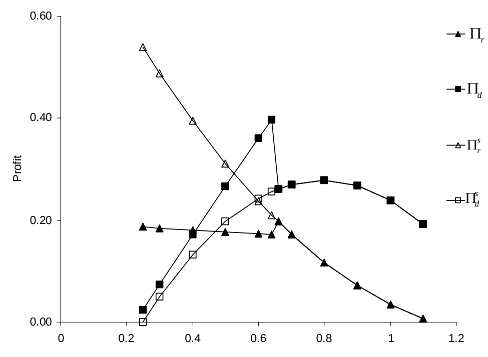

Figure 14 Effect of the wholesale price on profits

In this case, the optimum value of w that maximizes the manufacturer's profit in dual channel is 0.64. At this point, the demands in both channels are non-zero and together they cover the whole market. Thus, we observe that dual channel equilibrium exists even when w is endogenous to the model. From the single-channel manufacturer profit graph, the manufacturer's optimal w is 0.8. Comparing the optimal manufacturer profits in dual and single channel settings, the manufacturer is better off in dual channel. We observe that this is an example where the manufacturer, if it had the choice of channel structure and w, will choose to open a direct channel.

5. Conclusions

We study a dual channel supply chain where a manufacturer sells the same good to a retailer as well as directly to consumers and consumers choose a channel to buy the good accordingly. Based on examples in business press, we suggest that such supply chains already exist in many industries. We build a model to capture the major features of such supply chains. Our objective is to use the model to understand how different product, cost or service characteristics influence the equilibrium behavior of such supply chains.

New features in our model include different costs and service qualities at the two channels, heterogeneous service sensitivity in consumer population, and stochastic additive demand. An exact analysis leads us to conditions for dual channel equilibrium. Further results show the effect of demand variability on the supply chain structure. We show that below a threshold value of demand variability manufacturer will have reason to start a direct channel.

Our numerical results lead to several insights. We find that an increase in retailer's service quality may actually increase the manufacturer's profit in dual channel. A larger range of consumer service sensitivity may benefit both parties in the dual channel. We show that the difference in marginal costs of the two channels is a major factor determining the existence of dual channel supply chains. We also show that even if the manufacturer sets the wholesale prices, the outcome may still be dual channel equilibrium. In addition, the manufacturer is likely to be better off in the dual channel than in the single channel when the retailer's marginal cost of selling is high and the wholesales price, the consumer valuation and the demand variability are low. We believe that these insights are new to the literature and that they will be useful for managers in such supply chains.

References

Alba, J., J. Lynch, B. Weitz, C. Janiszewski, R. Lutz, S. Wood. 1997. Interactive home shopping: consumer, retailer, and manufacturer incentives to participate in electronic marketplaces. *Journal of Marketing* **61** 38-53.

Balasubramanian, S. 1998. Mail versus mall: a strategic analysis of competition between direct marketers and conventional retailers. *Marketing Science* **17** 181-195.

Boyaci, T. 2004. competitive stocking and coordination in a multiple-channel distribution system. Working paper, McGill University.

Cachon, G. and S. Netessine. 2004. Game theory in Supply Chain Analysis. in *Handbook of Quantitative Supply Chain Analysis: Modeling in the eBusiness Era*. edited by David Simchi-Levi, S. David Wu and Zuo-Jun (Max) Shen. Kluwer.

Cattani, K., W. Gilland, H.S. Heese, J. Swaminathan. 2005. Boiling frogs: pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel. Working Paper, The Kenan –Flagler Business School, The University of North Carolina at Chapel Hill.

Chiang, W.K., D. Chhajed, J.D. Hess. 2003. Direct marketing, indirect profits: a strategic analysis of dual-channel supply-chain design. *Management Science* **49** 1-20.

Devaraj S., Fan M., Kohli R. 2002. Antecedents of B2C Channel Satisfaction and Preference: Validating E-Commerce Metrics. *Information Systems Research* **13** (3) 316-333.

Hanover, D. 1999. It's not a threat just a promise. *Chain Store Age* 75 (9) 176.

Hoffman, D.L., T.P. Novak. 1996. Marketing in hypermedia computer-mediated environments: conceptual foundations. *Journal of Marketing* **60** 50-68.

Hotelling, H. 1929. Stability in competition. Economic Journal **39** 41-57.

Liang, T., J Huang. 1998. An empirical study on consumer acceptance of products in electronic markets: A transaction cost model. *Decision Support Systems* **24** 29-43

Mangalindan, M. 2005. Online retail sales are expected to rise to $172 billion this year. *The Wall Street Journal* May 24.

Mas-Colell, A., Whinston M.D., Green,J.R. 1995. *Microeconomic Theory*. Oxford University Press.

McWilliams, G., A. Zimmerman. 2003. Dell plans to peddle PCs insider Sears, other large chains. The Wall Street Journal January 30.

Mills, E. S. 1959. Uncertainty and price theory. *Quart. J. Econom.* **73** 116-130

Palmer, J. 2004. Gateway's Gains. *Barron's. Chicopee*. **84**(48) 12-14

Petruzzi, N. C., M. Dada. 1999. Pricing and the Newsvendor Problem: A Review with Extensions. *Operations Research* **47**(2) 183-194

Rhee, B., S.Y. Park. 2000. Online stores as a new direct channel and emerging hybrid channel system. Working paper, HKUST.

Rohm, A.J., V. Swaminathan. 2004. A typology of online shoppers based on shopping motivations. *Journal of Business Research* **57** 748-757.

Shaked, A., J. Sutton. 1983. Natural oligopolies. *Econometrica* **51** (5) 1469-1483.

Shaked, A., J. Sutton. 1987. Product differentiation and industrial structure. *The Journal of Industrial Economics* **36** (2) 131-146.

Stringer, K. 2004. Style & substance: Shopper who blend store, catalog, and web spend more. *The Wall Street Journal* September 3.

Trenchtenberg, J. 2004. Random House considers online sales of its books. *The Wall Street Journal* December 15.

Tsay, A., N. Agrawal. 2004 Channel conflict and coordination in the E-commerce age. *Production and Operations Management* **13** (1) 93-10.

Wilder, C. 1999. HP's online push. *Information Week* May 31.

Appendix

PROOF OF PROPOSITION 1: Application of first order conditions to the retailer's problem (1), that is, $\frac{\partial E[\Pi_r(z, p_r)]}{\partial z} = 0$, $\frac{\partial E[\Pi_r(z, p_r)]}{\partial p_r} = 0$, and simple algebra gives the result. □

PROOF OF PROPOSITION 2: Retailer's p_r^* and z^* is the intersection point of Equations (4) and (5). We show that $\frac{dz^*}{dp_r} > 0$ for both (4) and (5).

Taking derivative with respect to p_r on both sides of Equation (4), we have:

$$f(z^*)\frac{dz^*}{dp_r} = \frac{1}{(p_r + h + \pi)} - \frac{p_r + \pi - (c_r + w)}{(p_r + h + \pi)^2} = \frac{1}{(p_r + h + \pi)}\left(1 - \frac{p_r + \pi - (c_r + w)}{(p_r + h + \pi)}\right)$$
$$= \frac{1}{(p_r + h + \pi)}\left(1 - F(z^*)\right)$$

$$\frac{dz^*}{dp_r} = \frac{1}{r(z^*)(p_r + h + \pi)} > 0, \text{ where } r(z) = \frac{f(z)}{1 - F(z)}.$$

From Equation (5), we have $\Theta(z) = 2b(p^0(p_d) - p_r^*)$. Taking derivative with respect to p_r, we have $\frac{dz^*}{dp_r} = -\frac{2b}{\Theta'(z)} > 0$ as $\Theta'(z) = -(1 - F(z)) < 0$. □

PROOF OF PROPOSITION 3: First, we prove that p_r^* increases in p_d. From Equation (4), we can express p_r as function of z : $p_r^*(z, p_d) = \frac{1}{1-F(z)}\left(F(z)(h+\pi)+c_r+w-\pi\right)$ (A1)

From Equation (5): $p_r^*(z,\ p_d) = \frac{1}{2}(w+c_r+\bar{\theta}\Delta s+\Delta\theta\Delta s\mu)+\frac{p_d}{2}-\frac{\Theta(z)\Delta\theta\Delta s}{2}$. (A2)

These identities are always true for any given p_d and we can express $z^*(p_d)$ and $p_r^*(p_d)$ as functions of p_d. Thus, we can differentiate them with respect to p_d and get:

$$\frac{dp_r^*(p_d)}{dp_d} = \frac{f(z^*(p_d))\frac{dz^*(p_d)}{dp_d}(p_r+h+\pi)}{(1-F(z^*(p_d))} \quad \text{and} \quad \frac{dp_r^*(p_d)}{dp_d} = \frac{1}{2} - \frac{\Theta'(z)\frac{dz^*(p_d)}{dp_d}}{2b}.$$

Solving them simultaneously gives: $\frac{dp_r^*(p_d)}{dp_d} = \frac{bf(z^*)(p_r^*+h+\pi)}{2bf(z^*)(p_r^*+h+\pi)+(1-F(z^*))\Theta'(z^*)}$

and $\frac{dz^*(p_d)}{dp_d} = \frac{b(1-F(z^*))}{2bf(z^*)(p_r^*+h+\pi)+(1-F(z^*))\Theta'(z^*)}$.

Petruzzi and Dada (1999) showed that if $F(\cdot)$ is a cumulative distribution satisfying $2r(z)^2+dr(z)/dz>0$ for $z\in[A,B]$, there will be at most two values of z that simultaneously satisfy (A1) and (A2). The larger of these two, say z^*, corresponds to the maximum of retailer's profit function. Therefore, the following must be true:

$$\frac{d}{dz}\left[\frac{dE[\Pi_r(z, p_r^*(z)]}{dz}\right] = -\frac{f(z)}{2b}[2b(p^0+\pi+h)-\Theta[z]-\frac{1-F(z)}{r(z)}]<0 \text{ at } \qquad z=z^*.$$

$$\Rightarrow 2b(p^0+\pi+h)-\Theta[z^*]-\frac{1-F(z^*)}{r(z^*)}>0 \Rightarrow 2bf(z^*)(p_r^*+h+\pi)+(1-F(z^*))\Theta'(z^*)>0.$$

This shows that the denominators of $\frac{dp_r^*(p_d)}{dp_d}$ and $\frac{dz^*(p_d)}{dp_d}$ are both positive. The numerators are clearly positive. Therefore, we have $\frac{dp_r^*(p_d)}{dp_d}>0$ and $\frac{dz^*(p_d)}{dp_d}>0$.

Next, we prove that the retailer's profit increases in p_d. From retailer's profit function (1), using Envelope Theorem, $\frac{dE[\Pi_r(z^*(p_r^*(p_d)), p_r^*(p_d), p_d)]}{dp_d}$

$$= \frac{\partial E[\Pi_r(z^*(p_r^*(p_d)), p_r^*(p_d), p_d)]}{\partial p_d} = (p_r^* - c_r - w)(\frac{1}{\Delta\theta\Delta s}) > 0. \qquad \square$$

PROOF OF PROPOSITION 4: The first order condition of manufacturer's problem (7) is

$$\frac{dE[\Pi_d(p_d)]}{dp_d} = \left(-\frac{\underline{\theta}}{\Delta\theta} + \frac{p_r}{\Delta\theta\Delta s} - \frac{p_d}{\Delta\theta\Delta s}\right) - (p_d - c_d - c_{ds})\frac{1}{\Delta\theta\Delta s} + (w - c_d)\frac{1}{\Delta\theta\Delta s} = 0,$$

which gives $p_d^*(p_r) = \frac{1}{2}(w + c_{ds} - \underline{\theta}\Delta s) + \frac{1}{2}p_r$. As the second derivative is negative, $\frac{\partial^2 \Pi_d}{\partial p_d^2} = \frac{-2}{\Delta\theta\Delta s} < 0$, the manufacturer profit function is strictly concave. $\square$

PROOF OF PROPOSITION 5: (i) From (8), we have $\frac{dp_d^*(p_r)}{dp_r} = \frac{1}{2} > 0$.

(ii) From the manufacturer's problem (7), we have $\frac{dE[\Pi_d(z^*(p_r), p_r, p_d^*(p_r))]}{dp_r}$

$$= \frac{\partial E[\Pi_d(z^*(p_r), p_r, p_d^*(p_r))]}{\partial p_d}\frac{dp_d^*}{dp_r} + \frac{\partial E[\Pi_d(z^*(p_r), p_r, p_d^*(p_r))]}{\partial z}\frac{dz^*}{dp_r} + \frac{\partial E[\Pi_d(z^*(p_r), p_r, p_d^*(p_r))]}{\partial p_r}$$

$$= \frac{\partial E[\Pi_d(z^*(p_r), p_r, p_d^*(p_r))]}{\partial z}\frac{dz^*}{dp_r} + \frac{\partial E[\Pi_d(z^*(p_r), p_r, p_d^*(p_r))]}{\partial p_r}$$

$$= (w - c_d)(\frac{dz^*}{dp_r}) + (p_d^* - c_{ds} - w)\frac{1}{\Delta\theta\Delta s} = (w - c_d)(\frac{dz^*}{dp_r}) + \frac{1}{2}\left(p_r - (w + c_{ds} + \underline{\theta}\Delta s)\right)\frac{1}{\Delta\theta\Delta s}$$

We require $p_r^* \geq c_r + w$ for retailer's profit to be non-negative. We have $\frac{dz^*}{dp_r} > 0$ from Proposition 2. Thus, when $c_r \geq c_{ds} + \underline{\theta}\Delta s$, we obtain:

$$\begin{aligned} \frac{dE[\Pi_d(z^*(p_r), p_r, p_d^*(p_r))]}{dp_r} &\geq (w - c_d)(\frac{dz^*}{dp_r}) + \frac{1}{2}(w + c_r - (w + c_{ds} + \underline{\theta}\Delta s))\frac{1}{\Delta\theta\Delta s} \\ &= (w - c_d)(\frac{dz^*}{dp_r}) + \frac{1}{2}\left((c_r - (c_{ds} + \underline{\theta}\Delta s)\right)\frac{1}{\Delta\theta\Delta s} > 0 \end{aligned} . \square$$

PROOF OF LEMMA 6: We first show that there exist at most two intersections of (4), (5) and (8). Substituting Equation (8) into (5), we obtain (9). Plugging in (9) into (4), we get: $F(z^*) = \dfrac{\tilde{p}^0 - \dfrac{\Theta[z^*]}{2\tilde{b}} + \pi - (c_r + w)}{\tilde{p}^0 - \dfrac{\Theta[z^*]}{2\tilde{b}} + h + \pi} \Rightarrow -(c_r + w + h) + (\tilde{p}^0 - \dfrac{\Theta[z^*]}{2\tilde{b}} + h + \pi)(1 - F(z^*)) = 0$

Let R(z) = $-(c_r + w + h) + (\tilde{p}^0 - \dfrac{\Theta[z\ ]}{2\tilde{b}} + h + \pi)(1 - F(z\ ))$. Note that we can also show that

R(z) $= \dfrac{dE[\Pi_r(z, p_r^*(z)]}{dz}$ after plugging in (9) into the partial derivative of retailer profit with respect to z. Zeroes of R(z) correspond to intersections of (4), (5) and (8).

$$\frac{dR(z)}{dz} = \frac{d}{dz}\left[\frac{dE[\Pi_r(z, p_r^*(z)]}{dz}\right] = -\frac{f(z)}{2\tilde{b}}[2\tilde{b}(\tilde{p}^0 + \pi + h) - \Theta[z] - \frac{1 - F(z)}{r(z)}],$$

$$\frac{d^2R(z)}{dz^2} = \left[\frac{dR(z)/dz}{f(z)}\right]\frac{df(z)}{dz} - \frac{f(z)}{2b}\left\{[1 - F(z)] + \frac{f(z)}{r(z)} + \frac{[1 - F(z)][dr(z)/dz]}{r(z)^2}\right\},$$

and $\left.\dfrac{d^2R(z)}{dz^2}\right|_{dR(z)/dz=0} = \dfrac{-f(z)[1 - F(z)]}{2\tilde{b}r(z)^2}(2r(z)^2 + dr(z)/dz)$.

If $2r(z)^2 + dr(z)/dz > 0$, $R(z)$ is monotone or unimodal, implying that $R(z)$ or $\dfrac{dE[\Pi_r(z, p_r^*(z)]}{dz}$ has at most two roots. At $z = B$, $R(B) = -(c_r + w + h) < 0$. If $R(z)$ has one root, there is a change of sign of $R(z)$ from positive to negative at the root or $\left.\dfrac{dR(z)}{dz}\right|_{R(z)=0} < 0$ indicating the local maximum of $E[\Pi_r(z, p_r^*(z)]$ at the root. If $R(z)$ has two roots, this means that the sign of $R(z)$ changes twice from negative to positive ($\left.\dfrac{dR(z)}{dz}\right|_{R(z)=0} > 0$) at the smaller root and positive to negative ($\left.\dfrac{dR(z)}{dz}\right|_{R(z)=0} < 0$) at the larger root. Thus, the smaller root corresponds to a local minimum and the larger root corresponds to a local maximum.

Now that we know that there are at most 2 roots of R(z). We can further show that having the minimum point as one of the roots is not possible because having a minimum point contradicts the second order necessary conditions. Thus, the only possibility left is that R(z) has one root at which retailer maximizes its profit. Consider the Hessian matrix of retailer profit function given manufacturer's price at (z^*, p_r^*)

$H = \begin{pmatrix} \frac{\partial^2 E[\Pi_r]}{\partial p_r{}^2} & \frac{\partial^2 E[\Pi_r]}{\partial z \partial p_r} \\ \frac{\partial^2 E[\Pi_r]}{\partial p_r \partial z} & \frac{\partial^2 E[\Pi_r]}{\partial z^2} \end{pmatrix} = \begin{pmatrix} -2b & 1-F(z^*) \\ 1-F(z^*) & -f(z^*)(p_r^* + \pi + h) \end{pmatrix}$. When the first derivatives of retailer profit function are restricted to manufacturer's response, the Hessian becomes $H_1 = \begin{pmatrix} -2\tilde{b} & 1-F(z^*) \\ 1-F(z^*) & -f(z^*)(p_r^* + \pi + h) \end{pmatrix}$. However, we know from the second order necessary condition of 2-variable case that, at stationary point ($z^*, p_r(z^*)$), Hessian is negative semi-definite for maximum and positive semidefinite for minimum (Mas- Colell, Whiston and Green (1995) Theorem M.J.2). This requires $\frac{\partial^2 E[\Pi_r]}{\partial p_r{}^2}$ and $\frac{\partial^2 E[\Pi_r]}{\partial z^{\,2}}$ to be non-negative at (z^*, p_r^*) for a minimum point. But these two terms are negative for both H and H_1 and thus having minimum point at (z^*, p_r^*) is ruled out. Hence, it is further shown that we can find at most one root which will be a maximum point.

The above argument leaves open the possibility that $R(z)$ is always negative and therefore does not have a zero which would mean (4), (5) and (8) do not intersect. The following argument shows why that is not possible. Because the retailer profit function has only one peak, it is quasi-concave. In Proposition 4, we show that the manufacturer profit function is strictly concave and thus it is quasi-concave as well. For p_r and p_d, we will show later in Theorem 8 that there is a closed and bounded set that will make both demand non-negative and the bounds of p_r will determine the bounds of z. Then we can conclude that each player strategy space is compact and convex. Also, both profit functions are continuous. Because the profit functions are continuous and quasi-concave with respect to each player's own strategy and the strategy space is convex and compact set, using Theorem 1 of Cachon, G. and S. Netessine (2004), we can conclude that there exists at least one pure strategy equilibrium in the game. □

PROOF OF LEMMA 7: The conditions for non-zero demand in both channels are $v \geq p_d - \underline{\theta} s_d$ and $p_d - \underline{\theta}(s_r - s_d) < p_r < p_d - \overline{\theta}(s_r - s_d)$. Substituting the expressions for equilibrium prices p_r^{**} and p_d^{**} in the above conditions, we can obtain the results. □

PROOF OF THEOREM 8: If the intersection specified in Lemma 6 satisfies conditions (1) and (2), the demand is non-zero in both channels and the market is covered; we can call it dual channel equilibrium. The feasible area in which (1) and (2) are satisfied is shown in Figure 2. We use Figure 2 to develop bounds under which the intersection in Lemma 6 lies in feasible area. To be feasible, the intersection between two responses must occur on the section of manufacturer's response (8) in the dual channel feasible area. This means the retailer price in equilibrium must be below p_r at point B in Figure 2 that is the retailer price at the intersection of manufacturer's response (8) and the upper bound $p_r = p_d + \overline{\theta}\Delta s$, that is, $p_r^{\max} = w + c_{ds} + \Delta s \Delta\theta + \overline{\theta}\Delta s$. In addition, the retailer price must be higher than p_r at point A in Figure 2 that is the retailer price at the intersection of manufacturer's response (8) and the lower bound $p_r = p_d + \underline{\theta}\Delta s$, that is, $p_r^{\min} = w + c_{ds} + \underline{\theta}\Delta s$. Combining lower and upper bounds, $w + c_{ds} + \underline{\theta}\Delta s < p_r^* < w + c_{ds} + \Delta s\Delta\theta + \overline{\theta}\Delta s$. With this range, we can find the lower bound and upper bound of p_d^* using (8): $w + c_{ds} < p_d^* < w + c_{ds} + \Delta s \Delta\theta$.

To make sure that the upper bound determined by point B is not outside dual channel feasible region or more specifically p_d at B $< v + \underline{\theta} s_d$, we need an additional condition: $v > w + c_{ds} + \Delta s\Delta\theta - \underline{\theta} s_d$.

Plugging (9) into the bounds, we obtain: $w + c_{ds} + \underline{\theta}\Delta s < p_{r,(9)}^* < w + c_{ds} + \Delta s\Delta\theta + \overline{\theta}\Delta s$ and

$$\Delta s\underline{\theta} - \Delta s\Delta\theta(1 - \Theta(z^*)) < c_r - c_{ds} < \Delta s(\Delta\theta + \overline{\theta} + \Delta\theta\Theta(z^*) - \Delta\theta\mu) \tag{A3}$$

To eliminate z^* from (A3), we make the condition more restrictive by replacing $\Theta(z^*)$ with the maximum value, $\Theta^{\max}(z)$, on the left hand side inequality and the minimum value, $\Theta^{\min}(z) = 0$ the right hand side inequality. Because $D_r = \overline{D_r} + \varepsilon$ and $\varepsilon \in [A, B]$ and the retailer demand is non-negative so $D_r = \overline{D_r} + \varepsilon \geq 0$ and then $\overline{D_r} + A \geq 0$. The maximum of retailer demand is 1 so minimum value of A = -1 to ensure $\overline{Dr} + A \geq 0$. The maximum expected shortage will occur when, we keep lowest stock z=A and then $\Theta^{\max} = \Theta(A) = \int_A^B (u-A) f(u) du = \mu - A$. The worst case of $\Theta^{\max}(z)$ happens when A=-1 so $\Theta^{\max} = \mu - A = 1 + \mu$. Plugging $\Theta^{\min}(z) = 0$ and $\Theta^{\max}(z) = 1 + \mu$ into the condition, we get: $\Delta s \underline{\theta} + \Delta s \Delta \theta \mu < c_r - c_{ds} < \Delta s(\Delta \theta + \overline{\theta} - \Delta \theta \mu)$. □

PROOF OF PROPOSITION 9: As indicated in Equation (12), the stochastic response function of the retailer shifts down from deterministic case, which is the less steep curve (slope = ½) , with the introduction of uncertainty (Figure 2). Because the best response of the manufacturer, Equation 8 (slope = 2), is upward sloping, and steeper than the best response of retailer (slope=1/2), the new intersection of both curves occurs at lower retailer's and manufacturer's prices regardless of the probability distribution. □

PROOF OF PROPOSITION 10: Normal distribution has an increasing failure rate and therefore, it satisfies $2r(z)^2 + dr(z)/dz > 0$, the condition required for unique retailer solution. Let p_d^*, p_r^*, z^* denote the equilibrium solution. The equilibrium is determined by these 3 identities and they are true for all value of σ:

$$\Phi(z_1^*(\sigma)) = \frac{p_r^*(\sigma) + \pi - (c_r + w)}{p_r^*(\sigma) + h + \pi}, \quad p_r^*(\sigma) = p^0 - \frac{\Theta(z_1^*(\sigma))}{2b},$$

and $p_d^*(\sigma) = \frac{1}{2}(w + c_{ds} - \underline{\theta}\Delta s) + \frac{1}{2} p_r^*(\sigma)$ where $p^0 = \frac{1}{2}(w + c_r + \overline{\theta}\Delta s + \Delta\theta\Delta s\mu) + \frac{p_d^*(\sigma)}{2}$.

Differentiate these identities with respect to σ and solving them simultaneously:

$$\frac{dp_r^*(\sigma)}{d\sigma} = \frac{-2\phi(z_1^*)(c_r + h + w)\Delta s \Delta\theta(\Theta(z_1^*)/\sigma)}{K},$$

$$\frac{dp_d^*(\sigma)}{d\sigma} = \frac{-\phi(z_1^*)(c_r + h + w)\Delta s \Delta\theta(\Theta(z_1^*)/\sigma)}{K}, \quad \frac{dz_1^*(\sigma)}{d\sigma} = \frac{-2(1 - \Phi(z_1^*))^2 \Delta s \Delta\theta(\Theta(z_1^*)/\sigma)}{K}$$

Where $K = 3\phi(z_1^*)\left(c_r + h + w\right) + 2(1 - \Phi(z_1^*))^2 \Theta'(z_1^*)\Delta\theta\Delta s$

$$= 3\phi(z_1^*)\left(p_r^* + h + \pi\right)(1 - \Phi(z_1^*)) + 2(1 - \Phi(z_1^*))^2 \Theta'(z_1^*)\Delta\theta\Delta s$$

$$= (1 - \Phi(z_1^*))\left(3\phi(z_1^*)\left(p_r^* + h + \pi\right) + 2(1 - \Phi(z_1^*))\ \Theta'(z_1^*)\Delta\theta\Delta s\right)$$

To prove $\frac{dp_r^*(\sigma)}{d\sigma} < 0$, $\frac{dp_d^*(\sigma)}{d\sigma} < 0$ and $\frac{dz_1^*(\sigma)}{d\sigma} < 0$, it will be sufficient to prove that $K > 0$ because all three numerators are negative.

In the proof of Lemma 6, it is shown that $R(z)$ or $\frac{dE[\Pi_r(z, p_r^*(z)]}{dz}$ is monotone or unimodal and it has only one root which is maximum point. Because R(z) is monotone or unimodal, at the maximum point, the slope $\frac{dE[\Pi_r(z, p_r^*(z)]}{dz}$ changes sign from positive to negative or $\left.\frac{dR(z)}{dz}\right|_{z=z^*} < 0$. Rearrange the condition to get: $|H_1| = 2\tilde{b}(\tilde{p}^0 + \pi + h) - \Theta(z^*) - \frac{(1-F(z^*))}{r(z^*)} = \frac{3}{2}bf(z^*)(p_r^* + \pi + h) + (1-F(z^*))\Theta'(z^*) > 0$. For the Normal distribution, this is equivalent to $3\phi(z_1^*)(p_r^* + \pi + h) + 2(1-\Phi(z_1^*))\Theta'(z_1^*)\Delta\theta\Delta s > 0$. Using this, we can show that K is positive: $K = (1-\Phi(z_1^*))\left(3\phi(z_1^*)\left(p_r^* + h + \pi\right) + 2(1-\Phi(z_1^*))\ \Theta'(z_1^*)\Delta\theta\Delta s\right) > 0.$

At equilibrium, the identity $p_d^*(\sigma) = \frac{1}{2}(w + c_{ds} - \underline{\theta}\Delta s) + \frac{1}{2}p_r^*(\sigma)$ always holds. Thus, $\frac{dp_d^*(\sigma)}{d\sigma} = \frac{1}{2}\frac{dp_r^*(\sigma)}{d\sigma}$ or the rate of change of p_d is half of p_r's. Finally,

$$\frac{d\bar{D}_d}{d\sigma} = \frac{1}{\Delta\theta\Delta s}\left(\frac{dp_r^*(\sigma)}{d\sigma} - \frac{dp_d^*(\sigma)}{d\sigma}\right) = \frac{1}{\Delta\theta\Delta s}\left(\frac{dp_r^*(\sigma)}{d\sigma} - \frac{1}{2}\frac{dp_r^*(\sigma)}{d\sigma}\right) = \frac{1}{2\Delta\theta\Delta s}\left(\frac{dp_r^*(\sigma)}{d\sigma}\right) < 0$$

and $\frac{d\bar{D}_r}{d\sigma} = \frac{1}{\Delta\theta\Delta s}\left(-\frac{dp_r^*(\sigma)}{d\sigma} + \frac{dp_d^*(\sigma)}{d\sigma}\right) = -\frac{1}{2\Delta\theta\Delta s}\left(\frac{dp_r^*(\sigma)}{d\sigma}\right) > 0$. □

PROOF OF PROPOSITION 11: Using the optimal prices for the deterministic case (see text above Lemma 7), the optimal manufacturer's profit is:

$$\Pi_d^{**}(p_r^{**}, p_d^{**}) = w - c_d + \frac{\left(c_r - c_{ds} + \Delta s(\Delta\theta - \underline{\theta})\right)^2}{9\Delta s\Delta\theta}.$$

Next, we analyze decentralized single channel. The manufacturer sells only to retailer and retailer acts as monopoly and sells to customers. A customer buys the good from retailer when the utility from buying is non-negative or $v + \theta s_r - p_r^s \geq 0$. Let

$v+\theta^1 s_r - p_r^s = 0$ or $\theta^1 = \dfrac{p_r^s - v}{s_r}$ where θ^1 determines the last customer on distribution of $\theta \in [\underline{\theta}, \overline{\theta}]$ that will buy from retailer and the demand for retailer is $\overline{D}_r^s = \dfrac{\overline{\theta}-\theta^1}{\Delta\theta} = (\dfrac{\overline{\theta}+v/s_r}{\Delta\theta} - \dfrac{p_r^s}{s_r\Delta\theta})$. The corresponding retailer profit is:

$E[\Pi_r^s(z^s, p_r^s)] = (p_r^s - c_r - w)(\overline{D}_r^s + \mu) - (c_r + w + h)\Lambda(z^s) - (p_r^s + \pi - c_r - w)\Theta(z^s)$.

Because the profit function is the same as the one in dual channel except that the demand coefficients change, we can get unique optimal decision similar to the result of dual channel as shown below: $p_r^s = p_0^s - \dfrac{\Theta(z^s)}{2b^s}$ (A6)

$$F(z^s) = \frac{p_r^s + \pi - (c_r + w)}{p_r^s + h + \pi} \tag{A7}$$

where $p_0^s = \dfrac{(\overline{\theta} + \mu\Delta\theta)s_r + (v + c_r + w)}{2}$ and $b^s = \dfrac{1}{s_r\Delta\theta}$. Note that p_0^s is the optimal retailer price when demand is deterministic and μ = 0 and p_0^s in single channel is $\dfrac{v+(\overline{\theta}+\mu\Delta\theta)s_d}{2}$ greater than p_0 in dual channel. For manufacturer, the profit is from selling to retailer only which is: $\Pi_d^s = (\overline{D}_r^s + z^s)(w - c_d)$. For deterministic case, $z^{s*} = 0$ and optimal manufacturer's profit is $\Pi_d^{s*} = \dfrac{(w-c_d)\left(v+\overline{\theta}s_r - w - c_r\right)}{2s_r\Delta\theta}$.

Hence, for deterministic case, the optimal manufacturer's profit for single channel is less than optimal manufacturer's profit for dual channel case if:

$$\Pi_d^*(p_r^*, p_d^*) = w - c_d + \frac{\left(c_r - c_{ds} + \Delta s(\Delta\theta - \underline{\theta})\right)^2}{9\Delta s\Delta\theta} > \Pi_d^{s*} = \frac{(w-c_d)\left(v+\overline{\theta}s_r - w - c_r\right)}{2s_r\Delta\theta}$$

After simplifications, $v < w + c_r - \underline{\theta}s_r + s_r\Delta\theta + \dfrac{2s_r\Delta\theta\left(c_r - c_{ds} + \Delta s(\Delta\theta - \underline{\theta})\right)^2}{9\Delta s\Delta\theta(w-c_d)}$. □

PROOF OF PROPOSITION 12: In the deterministic case, when σ=0, the condition in Proposition 11 requires that manufacturer is better off in dual channel. As σ increases, if manufacturer's profit in dual channel decreases and converges to that of single channel, there will be a critical positive $\tilde{\sigma}$ where both channels yield the same profit for manufacturer. However, even though the manufacturer profit in single and dual channel diverges, there will still be a range of positive σ where manufacturer is better off in dual channel anyway. Thus, it does not matter how the manufacturer profit reacts to the increased variability; as long as the rate of change of manufacturer's profit with respect to σ is finite for both settings (and thus the difference of the rates of change is finite), it requires at least some positive change of σ (in case that they converges and $\tilde{\sigma}$ approaches infinity if they diverges) before the manufacturer profits are the same in both cases or there will be $\sigma \in [0, \tilde{\sigma}]$ such that manufacturer is better off in dual channel.

We show that the $\dfrac{d\Pi_d^*(z^*, p_r^*, p_d^*)}{d\sigma}$ is finite in both cases. For dual channel case, let $E[\Pi_d^*(z^*, p_r^*, p_d^*)] = \Pi_{dd}(p_r^*, p_d^*) + \Pi_{dr}(z^*, p_r^*, p_d^*)$ where $\Pi_{dd}(p_r, p_d) = (p_d - c_d - c_{ds})\bar{D}_d$ and $\Pi_{dr}(z, p_r, p_d) = (w - c_d)(\bar{D}_r + z) = (w - c_d)(\bar{D}_r + z_1\sigma + \mu)$.

$\dfrac{dE[\Pi_d^*(z^*, p_r^*, p_d^*)]}{d\sigma} = \dfrac{d\Pi_{dd}(p_r^*, p_d^*)}{d\sigma} + \dfrac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma}$. $\Pi_{dd} = (p_d - c_d - c_{ds})\bar{D}_d$ is manufacturer's profit through direct channel. $\dfrac{d\Pi_{dd}(p_r^*, p_d^*)}{d\sigma} = \dfrac{dp_d^*}{d\sigma}\bar{D}_d + (p_d^* - c_d - c_{ds})\dfrac{d\bar{D}_d}{d\sigma}$. Because $\dfrac{dp_d^*}{d\sigma}$ and $\dfrac{dp_r^*}{d\sigma}$ are finite because K is not zero and thus $\dfrac{d\bar{D}_d(p_r^*, p_d^*)}{d\sigma}$ is also finite. Consider $\dfrac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma}$ next.

$$\frac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma} = (w - c_d)\left(\frac{d\bar{D}_r(p_r^*, p_d^*)}{d\sigma} + \frac{d(z_1\sigma + \mu)}{d\sigma}\right) = (w - c_d)\left(\frac{d\bar{D}_r}{d\sigma} + \sigma\frac{dz_1}{d\sigma} + z_1\right)$$

After some simplification and using $\dfrac{dp_d}{d\sigma} = \dfrac{1}{2}\dfrac{dp_r}{d\sigma}$ from Proposition 10, we get

$$\frac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma} = (w - c_d)\left(\left(\frac{\Theta(z_1^*)}{\sigma K}\right)\left(\phi(z_1^*)(c_r + h + w) - 2\sigma(1 - \Phi(z_1^*))^2 \Delta s \Delta\theta\right) + z_1\right) \qquad \text{(A8)}$$

Similarly, because K is not zero and all other terms are finite, $\dfrac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma}$ is finite.

Because $\dfrac{d\Pi_{dd}(p_r^*, p_d^*)}{d\sigma}$ and $\dfrac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma}$ are finite, $\dfrac{dE[\Pi_d(z^*, p_r^*, p_d^*)]}{d\sigma}$ is finite.

For decentralized single channel, we change the optimal decisions identities in Equations (A6) and (A7) for Normal distribution: $p_r^{s*}(\sigma) = p_0^s - \frac{\Theta(z_1^{s*}(\sigma))}{2b^s}$, $\Phi(z_1^{s*}(\sigma)) = \frac{p_r^{s*}(\sigma) + \pi - (c_r + w)}{p_r^{s*}(\sigma) + h + \pi}$. Let $p_r^{'}(\sigma) = \frac{dp_r^{s*}(\sigma)}{d\sigma}$ and $z_1^{'}(\sigma) = \frac{dz_1^{s*}(\sigma)}{d\sigma}$.Differentiate these two identities both sides with respect to σ:

$$\phi(z_1^{s*}(\sigma))z_1^{'}(\sigma) = \frac{p_r^{'}(\sigma)}{h+\pi+p_r(\sigma)}\left(1-\Phi(z_1^{s*}(\sigma)\right) \quad \text{(A9)}$$

$$p_r^{'}(\sigma) = -\frac{1}{2b}\frac{d\Theta^{'}\left(z_1(\sigma)\right)}{d\sigma}$$
$$= -\frac{1}{2b}\left[\left(\phi(z_1(\sigma)) - z_1(\sigma) + z_1(\sigma)\Phi(z_1(\sigma)\right) + \sigma\left(-z_1^{'}(\sigma) + z_1^{'}(\sigma)\Phi(z_1(\sigma))\right)\right] \quad \text{(A10)}$$

Solving Equations (A9) and (A10) simultaneously for $p_r^{'}(\sigma)$, $z_1^{'}(\sigma)$ and simplifying:

$$p_r^{'}(\sigma) = -\frac{\phi(z_1^{s*})(h+p_r^{s*}+s)\Theta(z_1^{s*})}{\sigma K_1} < 0,\ z_1^{'}(\sigma) = -\frac{(1-\Phi(z_1^{s*}))\Theta(z_1^{s*})}{\sigma K_1} < 0,$$

where $K_1 = 2b\phi(z_1^{s*})(h+p_r^{s*}+\pi) + (1-\Phi(z_1^{s*}))\Theta^{'}(z_1^{s*})$. Because the retailer profit function of single channel and its optimal decisions are similar to those of Proposition 3, we can similarly show that $\frac{dR^s(z^s)}{dz^s} = \frac{d}{dz}\left[\frac{dE[\Pi_r^s(z^s, p_r^{s*}(z^s)]}{dz^s}\right]$ $= -\frac{f(z^s)}{2b}[2b^s(p_0^s+\pi+h) - \Theta[z^s] - \frac{1-F(z^s)}{r(z^s)}] < 0$ at maximum point and this leads to $K_1 = 2b\phi(z_1^{s*})(h+p_r^{s*}+\pi) + (1-\Phi(z_1^{s*}))\Theta^{'}(z_1^{s*}) > 0$ for Normal distribution.

Next, consider $\frac{d\Pi_d^s(z_1^{s*}, p_r^{s*})}{d\sigma}$. $\frac{d\Pi_d^s(z_1^{s*}, p_r^{s*})}{d\sigma} = (w-c_d)(\frac{d\overline{D}_r(z_1^{s*}, p_r^{s*})}{d\sigma} + \frac{d(z_1^{s*}(\sigma)\sigma+\mu)}{d\sigma})$

$$= (w-c_d)\left[\sigma\Theta(z_1^{s*})\left(\frac{\frac{1}{\sigma\Delta\theta s_r}\phi(z_1^{s*})(h+p_r^{s*}+\pi) - (1-\Phi(z_1^{s*}))}{K_1}\right) + z_1^{s*}\right] \quad \text{(A12)}$$

Because all the terms are finite and $K_1 \neq 0$, $\frac{d\Pi_d^s(z_1^{s*}, p_r^{s*})}{d\sigma}$ is finite. □

APPENDIX NOTE 13: We show that, under certain conditions, the manufacturer profit in dual channel is better than that of single channel where $\sigma \in [0, \tilde{\sigma}]$ and lower than that of single channel otherwise. To show this, we needs three sufficient conditions: $v < \tilde{v}$ from Proposition 11, $\frac{d\Pi_d(z^*, p_r^*, p_d^*)}{d\sigma} > 0$ and $\frac{d\Pi_d^s(z^*, p_r^*)}{d\sigma} < 0$. With the conditions, the dual channel manufacturer profit will be higher than that of single channel at $\sigma = 0$ and both profits will converge and intersect at $\tilde{\sigma}$ and then diverge after that.

From Proposition 12, for the dual channel case, $\frac{dE[\Pi_d^*(z^*, p_r^*, p_d^*)]}{d\sigma} = \frac{d\Pi_{dd}(p_r^*, p_d^*)}{d\sigma} + \frac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma}$ and

$\frac{d\Pi_{dd}(p_r^*, p_d^*)}{d\sigma} = \frac{dp_d^*}{d\sigma}\bar{D}_d + (p_d^* - c_d - c_{ds})\frac{d\bar{D}_d}{d\sigma}$. Because $\frac{d\bar{D}_d(p_r^*, p_d^*)}{d\sigma} < 0$, $p_d^* - c_d - c_{ds} > 0$ and $\frac{dp_d^*(\sigma)}{d\sigma} < 0$ from Proposition 10, the manufacturer's profit through direct channel decreases or $\frac{d\Pi_{dd}(p_r^*, p_d^*)}{d\sigma} < 0$.

From Equation (A8), if $z_1^* < 0$ and $\phi(z_1^*)(c_r + h + w) - 2\sigma(1 - \Phi(z_1^*))^2 \Delta s \Delta\theta < 0$, $\frac{d\Pi_{dr}(z^*, p_r^*, p_d^*)}{d\sigma} < 0$ and using , $\frac{r(0)}{1-\Phi(0)}$=1.595769, both of them can be simplified to:

$$0.5(w + c_{ds} + \Delta\theta\Delta s + \bar{\theta}\Delta s + h + \pi) < (c_r + w + h) < 1.2533\sigma\Delta s\Delta\theta \quad \text{(A13)}$$

For single channel, from Equation (A12), if $\frac{1}{\sigma\Delta\theta s_r}\phi(z_1^{s*})(h + p_r^{s*} + \pi) - (1 - \Phi(z_1^{s*}))$>0 and z_1^{s*}>0, $\frac{d\Pi_d^s(z^*, p_r^*)}{d\sigma}$>0. Similarly using $\frac{r(0)}{1-\Phi(0)}$=1.595769, both of them can be simplified:

$$0.6267\sigma\Delta\theta s_r < c_r + w + h < 0.5(v + s_r\bar{\theta} - s_r\Delta\theta + h + \pi) \quad \text{(A14)}$$

Under the conditions (A13) in dual channel, Equation (A14) in single channel and $v < \tilde{v}$, the manufacturer's profit in dual channel will be higher than that of single channel during $\sigma \in [0, \tilde{\sigma}]$ and lower than that of single channel otherwise.