Likelihood Ratio Tests for Monotone Functions: a New Limit Distribution

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OUTLINE:

1. **Introduction: the problem(s)**
2. **Current status data**
3. **The asymptotic Gaussian problem**
4. **Further problems**

1. Introduction: the problem(s)

- **Example 1.** Monotone density function on $[0, \infty)$.
  Test $H_0 : f(t_0) = \theta_0$ versus $H_1 : f(t_0) \neq \theta_0$.
- **Example 2.** Interval censoring, current status data.
  Test $H_0 : F(t_0) = \theta_0$ versus $H_1 : F(t_0) \neq \theta_0$.
- **Example 3.** Panel count data.
  Test $H_0 : \Lambda(t_0) = \theta_0$ versus $H_1 : \Lambda(t_0) \neq \theta_0$.
- **Example 4.** Monotone hazard function with right-censored data.
  Test $H_0 : \lambda(t_0) = \theta_0$ versus $H_1 : \lambda(t_0) \neq \theta_0$.
- **Example 5.** Monotone regression function.
  Test $H_0 : r(t_0) = \theta_0$ versus $H_1 : r(t_0) \neq \theta_0$.

2. Current status data

2.1. The Model for Current Status Data.

$$X \sim F, \qquad T \sim G\,.$$

We observe $(T, 1\{X \le T\}) \equiv (T, \Delta) \equiv Y$.

$$p_F(t, \delta) = F(t)^{\delta}(1 - F(t))^{\delta}\,.$$

Suppose that $Y_i \equiv (T_i, \Delta_i)$ are i.i.d. as (T, Δ).
Test

$$H_0 : F(t_0) = \theta_0 \qquad \text{versus} \qquad H_1 : F(t_0) \ne \theta_0\,.$$

Unconstrained MLE: with $\mathbb{G}_n(t) = n^{-1}\sum_{i=1}^{n} 1\{T_i \le t\}$,

$$\widehat{\mathbb{F}}_n(t) = \text{left derivative of cumsum diagram at } \mathbb{G}_n^{-1}(t)$$

If $f(t_0) > 0$ and $g(t_0) > 0$, then

$$n^{1/3}(\widehat{\mathbb{F}}_n(t_0) - F(t_0)) \to_d \left\{ \frac{F(t_0)(1 - F(t_0))f(t_0)}{2g(t_0)} \right\}^{1/3} 2\mathbb{Z}$$

where $\mathbb{Z} \equiv \text{armin}(W(t) + t^2)$ and W is two-sided Brownian motion starting from 0.

Confidence Interval for $F(t_0)$?

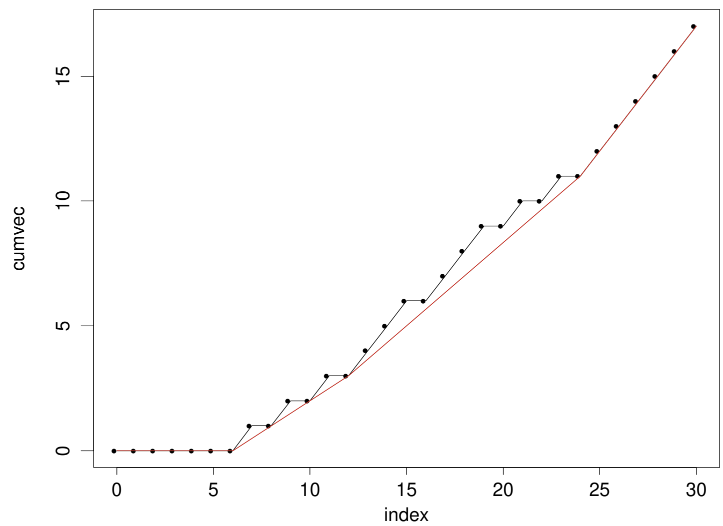

Figure 1: Cumulative sum diagram and Greatest Convex Minorant.

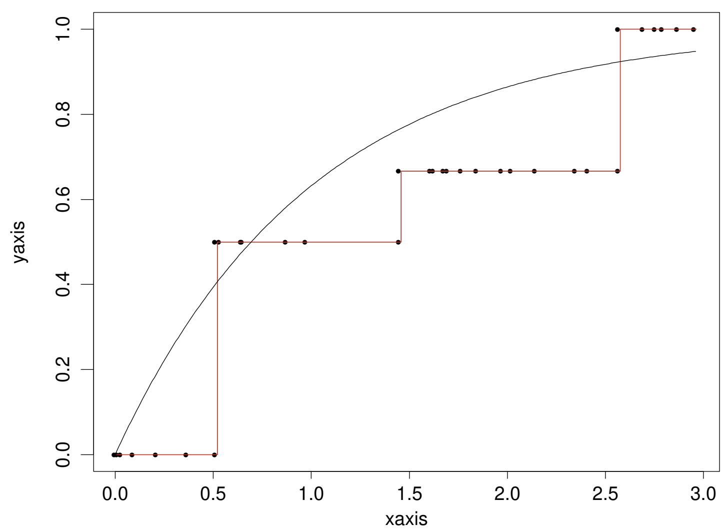

Figure 2: The unconstrained estimator (red).