Sandwich-Structured Woodpile Metamaterials for Impact Mitigation

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The objective of this study was to investigate 3D woodpile metamaterials for mitigating impact-induced vibrations by leveraging their local resonant and nonlinear contact characteristics. For experimental demonstrations, we designed, fabricated and tested prototypes of sandwich-structured woodpile metamaterials consisting of two plates, slender cylindrical rods and fasteners. We experimentally and numerically obtained impact responses of sandwich-structured woodpile metamaterials under various geometries and boundary conditions. We found that sandwich-structured woodpile metamaterials could efficiently manipulate and attenuate the impact vibrations due to their local bending motions and nonlinear contact between members. In addition, sandwich-structured woodpile metamaterials could have high damping as well as high stiffness by controlling the rod spacing. The findings from this study suggest sandwich-structured woodpile metamaterials can be used as structural components for impact-induced vibration mitigation.

Keywords: Impact mitigation; sandwich; woodpile; metamaterial.
1. Introduction

Over the last several decades, considerable research efforts have been directed towards techniques for efficient suppression of vibrations and impact applied to engineering systems such as aircraft, machinery, automotive and civil infrastructures [Alkhatib and Golnaraghi, 2003; Kandasamy et al., 2016; Preumont, 2011; Muhammad et al., 2006]. To control and attenuate vibrations, developed techniques have two main approaches by focusing on materials and structures. To achieve active vibration control, various systems such as dynamic absorbers [Hunt and Nissen, 1982], tuned dampers [Sun et al., 1995], and Helmholtz resonators [Kela, 2009] have been used to attenuate vibration and impact. However, these systems are generally effective only at a narrow target frequency. In addition, they are very complex. Passive damping materials such as elastomers, polymers and foam are very efficient in reducing vibrations and impact. However, their high damping characteristics result in low structural stiffness [Chung, 2001].

Recent trend on impact or vibration mitigation has focused on high-damping and high-stiffness system. From this standpoint, composites and sandwich structures are very attractive materials. However, these materials could not suppress low-frequency vibrations or impact [Chandra et al., 1999; Hu et al., 2008; Zhu and Lu, 2007]. New types of damping material systems that can reduce low-frequency impact or vibrations without relying on material damping or complex devices are needed. To achieve high-damping and high-stiffness with low-frequency handling capabilities, researchers are fabricating totally new hierarchical and structural materials instead of working on existing material systems. By exploiting scalability of nano/micro unit-cell structures, the strength of structures can be significantly enhanced for constructing highly efficient impact-mitigating systems [Babaee et al., 2013; Cuan-Urquizo et al., 2015]. Likewise, we can leverage structural buckling behavior of cellular structures without relying on plasticity to maximize energy absorption efficiency [Kolken and Zadpoor, 2017; Montemayor, 2016]. The most notable accomplishment is the development of mechanical metamaterials. One candidate for structural purposes is woodpile metamaterial consisting of slender cylindrical members stacked in a periodic architecture. Woodpile metamaterials can tailor dynamic performance by changing design parameters such as rod material, spacing and dimension [Kim and Yang, 2013]. Many studies have reported selective filtering of electromagnetic waves using woodpile metamaterials called photonic crystals [Liu and Zhang, 2011]. In mechanical system, modulation (usually attenuation) of acoustic waves such as noise and sonar can be realized with woodpile structures in air and underwater environment [Zhao et al., 2007; Delpero et al., 2016]. Recently, 3D woodpile metamaterials have been used for impact and blast mitigation [Kim et al., 2015, 2017]. Their results have demonstrated that local resonant vibrations of slender cylindrical members and nonlinear contact behaviors between members could modulate impact waves very efficiently without relying on material damping [Kim et al., 2013, 2017].
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2D and 3D woodpile metamaterials considered in previous studies [Zhao et al., 2007; Kim and Yang, 2014; Kim et al., 2015, 2017; Delpero et al., 2016] consisted of slender cylindrical members, which are arranged in specific 2D patterns or simply stacked in specific 3D patterns. Therefore, they could control the mechanical loadings along the designated direction only. For example, 3D woodpile metamaterials suggested by Kim’s group [Kim et al., 2015] could mitigate the compressive impacts along the stacking direction of cylindrical members. However, 3D woodpile metamaterials may be separated and cannot transfer any impacts if external loadings are applied along the opposite direction since cylindrical members are just stacked without any fastening mechanism. Therefore, 3D woodpile metamaterials cannot be used as structural components, which makes that the application of 3D woodpile metamaterials is limited to finding out physical phenomena of 3D woodpile metamaterials. For overcoming these problems and expanding the application of 3D woodpile metamaterials into structural components, we proposed a sandwich-structured woodpile metamaterial composed of 3D woodpile fastened between two rigid plates. Suggested structures are expected to have similar performances of the impact mitigation as well as be used as structural components since all members are restricted but local vibrations of cylindrical members and nonlinear contact behaviors between cylindrical members can be realized. We fabricated sandwich-structured woodpile metamaterials and measured transmitted impact under various geometrical configurations and boundary conditions. A simplified finite element model of sandwich-structured woodpile metamaterials was also constructed based on Hertzian contact law and simulated for impact tests. Computational results showed how impact waves propagated through sandwich-structured woodpile metamaterials. Finally, we investigated changes in dynamic response between 3D woodpile metamaterials and sandwich-structured woodpile metamaterials.

2. Experiments

Sandwich-structured woodpile metamaterials consisted of two plates, 3D woodpile structure and fasteners as shown in Fig. 1(a). Consistent with previous study [Kim et al., 2015], the 3D woodpile structure had $2 \times 2$ layups of cylindrical rods made of stainless steel and fastened mechanically between two stainless plates. To simplify experimental and numerical works, we substituted pre-load system made of spring plungers and force sensors for mechanical fasteners as shown in Fig. 1(b).

Test setup had four parts: sandwich-structured woodpile metamaterials, a striker to apply impact load, a force transducer for measuring transmitted impact and a pre-load system to mimic mechanical fasteners (Fig. 2(a)). Details on dimensions and material properties of sandwich-structured woodpile metamaterials are summarized in Table 1. The lower plate was positioned on a piezoelectric force transducer (PCB208C02, Piezoelectronics Inc., USA) while 3D woodpiles were stacked on the lower plate using a supporting fixture. The upper plate was located on 3D woodpiles. Pre-determined load was then applied to four corners of the upper plate to simulate...
mechanical fasteners. After detaching the supporting fixture, a stainless steel ball was dropped at the center of the upper plate from a height of 200 mm to generate impact waves to sandwich-structured woodpile metamaterials. Impact moment was detected using a PZT sensor at 20 mm apart from the center of the upper plate. Signals of the piezoelectric force transducer and PZT sensor were recorded via an

**Fig. 1.** (a) Configuration of suggested sandwich-structured woodpile metamaterials and (b) simplified model substituting the pre-load system for mechanical fasteners.

**Fig. 2.** (a) Schematic configuration, (b) real image of the test setup and (c) enlarged view of sandwich-structured metamaterials for investigating the impact mitigation.
Table 1. Dimensions and material properties of components.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Plate</th>
<th>Cylinder</th>
<th>Striker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>Stainless steel</td>
<td>Stainless steel</td>
<td>Steel</td>
</tr>
<tr>
<td>$E$ (GPa)</td>
<td>200</td>
<td>200</td>
<td>210</td>
</tr>
<tr>
<td>$v$</td>
<td>0.29</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>7800</td>
<td>7800</td>
<td>7860</td>
</tr>
</tbody>
</table>

oscilloscope (MDO3022, Tektronix Electronic Components, USA) for 2 ms with a sampling rate of 100 MHz.

We conducted impact tests with respect to the number of layers. Transmitted impact forces were measured using the force transducer. They were rearranged by
synchronizing each signal by impact moment detected using a PZT sensor. Based on the method reported in the reference [Kim et al., 2015], we only used the leading portion of transmitted force signals to eliminate other factors such as reflected waves of subsequent force signals. We repeated impact tests of 3D woodpile metamaterials and compared results of our system to reference data.

In addition, we considered the effect of pre-load equal to the fastening force shown in Fig. 1(a) on impact mitigation. We repeated each test more than five times for statistical analysis of results.

3. Numerical Models

It is known that 3D model must have numerous contacts, making element size of contact very fine. In addition, computational time for 3D model is very long. Therefore, we modeled sandwich-structured woodpile metamaterials with 1D and 2D elements for simplification of finite element models. Keeping consistency with previous study [Kim et al., 2015], we used same element types and sizes for cylindrical members, plates and nonlinear springs as well as same contact models. 200 beam elements with 0.5 mm size were used for cylindrical rods, 38,656 shell elements with 0.5 mm of global element size (minimum 0.18 mm for contacts with cylindrical members and loading areas) for the plate, one mass element for striker. Each element is connected using nonlinear spring elements whose behaviors are governed by Hertzian contact [Kim et al., 2015]. In this model, we used three different mechanical contact models: between the striker and upper plate, between the plate and cylindrical rods, and between cylindrical rods. Mechanical contact between two elastic bodies with curved surfaces can be presented by the following equation [Briscoe, 1986; Popov, 2010]:

\[ F = \frac{4}{3} E^* R^{1/2} \delta^{3/2}, \]  

where \( F \), \( R \), \( \delta \) represent normal force, effective radius and displacement, respectively. \( R \) and \( E^* \) can be calculated by the following equation:

\[ \frac{1}{E^*} = \frac{1 - v_2^2}{E_1} + \frac{1 - v_1^2}{E_2} \quad \text{and} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \]  

Thus, we derived contact behavior between the striker and the upper plate by modification of Eq. (3.1) with the same material properties (\( E_1 = E_2 = E \) and \( v_1 = v_2 = v \)) and infinite radius of the plate (\( R_1 = R_{\text{striker}}, R_2 = \infty \)) using the following equation:

\[ F = \frac{2}{3} \frac{E}{1 - v^2} R_{\text{striker}}^{1/2} \delta^{3/2}. \]  

Similarly, modification under the same material properties (\( E_1 = E_2 = E \) and \( v_1 = v_2 = v \)) and same radii of the rods (\( R_1 = R_2 = R_{\text{rod}} \)) was done for mechanical
contacts between cylindrical rods as follows:

\[ F = \frac{2}{3} E \left( \frac{R_{\text{rod}}}{2} \right)^{1/2} \delta^{3/2}. \]  

(3.4)

For mechanical contacts between plates and cylindrical rods, we modified Eq. (3.1) with the same material properties \((E_1 = E_2 = E \text{ and } v_1 = v_2 = v)\) and infinite radius of the plate \((R_1 = R_{\text{rod}}, R_2 = \infty)\) as shown below

\[ F = \frac{2}{3} E \left( \frac{R_{\text{rod}}}{2} \right)^{1/2} \delta^{3/2}. \]  

(3.5)

Mechanical behaviors of each contact (Eqs. (3.3)–(3.5)) were implemented as force–displacement curves of nonlinear springs with tabular format into commercial finite element software (Abaqus 6.13, Dassault Systèmes Simulia Corp., USA). Clamped boundary condition was assigned at the sensor area on the lower face of the lower plate. Lateral deformation of cylindrical rods was constrained. Only vertical motion of the striker (point mass element) was allowed.

Finite element analysis consisted of two steps: a static analysis for the pre-load and a dynamic analysis for the impact. Fastening forces were applied on the four mechanical fastening locations shown in Fig. 3 as pre-load in static analysis. In dynamic analysis, the initial velocity of 1.98 m/s to the striker was assigned rather than drop at height of 200 mm. We obtained transmitted impact force signals using a reaction force of the clamped area with respect to the number of woodpile layups and the amount of the pre-load. Contact forces between members of sandwich-structured woodpile metamaterials were also analyzed to investigate how impact waves propagated through suggested metamaterials.

![Fig. 3. Finite element model for the analysis of the impact mitigation of sandwich-structured woodpile metamaterials.](image-url)
4. Results and Discussion

4.1. Impact mitigation of sandwich-structured woodpile metamaterials

Figure 4 shows surface maps of transmitted forces in time-space domain for 3D woodpile metamaterials. Vertical axis represents time after impact while horizontal axis depicts the number of stacked woodpile layers. When the rod spacing was 28 mm (Fig. 4(b)), transmitted forces kept the waveform shape while the impact wave propagated to the last layer even though their magnitude was slightly decreased due to material damping. However, scattered waveforms were observed in 56 mm rod spacing (Fig. 4(d)), leading to high attenuation of incident impact. Tendencies of impact transmission of 3D woodpile metamaterials shown in Figs. 4(b) and 4(d) were almost same as previous results [Kim et al., 2015] except for the magnitude of...
transmitted forces since we used slightly smaller striker for impact tests. Therefore, we proved that our experiments were well organized according to previous study [Kim et al., 2015].

Impact waves propagated through sandwich-structured woodpile metamaterials with rod spacing of 28 mm and 56 mm are shown in Fig. 5. Impact wave propagations obtained experimentally (Figs. 5(c) and 5(d)) and numerically (Figs. 5(e) and 5(f)) had similar tendency. Clear sinusoidal waves transmitted to the end of sandwich-structured woodpile metamaterials. However, their magnitudes decreased fast with less scattering when the rod spacing was 28 mm as shown in Figs. 5(c) and 5(e). If rod spacing was 56 mm, scattered waveforms and clear waveforms were

Fig. 5. Sandwich-structured woodpile metamaterial models with (a) 28 mm and (b) 56 mm spacing and image maps of transmitted impact force obtained (c), (d) experimentally and (e), (f) numerically.
observed simultaneously with respect to the number of layers. However, scattered waveforms could not attenuate the impact force. Most energy was transferred to the adjacent layer since the highest transmitted force of each layer was all but consistent as shown in Figs. 5(d) and 5(f). Small discrepancies in the magnitude of transmitted impact force obtained experimentally and numerically were caused by the differences of boundary conditions of experiments and finite element analyses and the damping effects, which are not considered in finite element analyses [Kim et al., 2015]. Figure 6 represents incident impact and transmitted forces with respect to rod spacing when the number of woodpile layers was 15. Incident impact forces of sandwich-structured woodpile metamaterials with rod spacing of 28 and 56 mm were nearly the same (1850N). However, transmitted force with rod spacing 28mm (250N) was much lower than that with rod spacing of 56 mm (485N).

Fig. 6. Transmitted forces of sandwich-structured woodpile metamaterials with respect to the number of layers (Solid line: 28 mm spacing, dashed line: 56 mm spacing).
Given that transmitted force of 3D woodpile metamaterials with rod spacing of 28 mm was much higher than that with rod spacing of 56 mm according to previous research [Kim et al., 2015], the impact mitigation of sandwich-structured woodpile metamaterials was entirely opposite to that of 3D woodpile metamaterials.

4.2. Local resonance effects on the dynamic responses of sandwich-structure woodpile metamaterials

Finite element analyses revealed small deformation of the upper plate but relatively large deflections of woodpiles with 1st bending mode for sandwich-structured woodpile metamaterials with rod spacing 28 mm as shown in Fig. 7(a). Therefore, the upper plate gave whole impact energy to the 1st woodpile layer which made the contact force felt between the upper plate and the 1st woodpile layer much...
like incident impact. However, the 1st bending mode of woodpiles could contribute sufficiently to coupling between propagated waves and local resonances. Thus, the transmitted force with 15 woodpile layers was much lower (about 13.5%) than the incident impact.

On the other hand, large deformation of the upper plate was observed (Fig. 7(b)) when the rod spacing was 56 mm. This led to relatively large absorption of the transferred energy. Transmitted force to the 1st woodpile layer (400 N) was much smaller than the incident impact. However, we observed small deflections of woodpiles with the 3rd bending mode (Fig. 7(b)) which could not interact with propagated waves due to relatively higher frequency. Therefore, a considerable amount of incident impact (about 26.2%) was transmitted to the 15th woodpile layer.

At this point, we noted that resonant modes of woodpiles for 3D woodpile metamaterials with rod spacing of 28 and 56 mm were the 3rd and the 1st bending mode, respectively, according to previous research [Kim et al., 2013]. We can explain the difference between impact propagations of sandwich-structured woodpile metamaterials and 3D woodpile metamaterials by relative motion (mechanical behaviors) of the upper plate and woodpiles. The upper plate of sandwich-structured woodpile metamaterials constrained the deformation of the 1st woodpile layer since the entire 1st woodpile layer was in contact with the upper plate along the longitudinal direction. Therefore, impact waves applied to the upper plate were transferred to the 2nd woodpile layer via the 1st woodpile layer very fast when the rod spacing was 28 mm since the upper plate had a small deformation. Due to the relatively long overhang length (Fig. 4(a)), the end position of the 2nd woodpile layer could not catch up with the downward movement of contact points between the 1st and the 2nd woodpile layer, making the 2nd woodpile layer deform like the 1st bending mode of woodpile members (Fig. 4(a)). However, at rod spacing of 56 mm, both the center and the end followed downward movement of contact points slightly later (Fig. 4(b)) since the overhang length was similar to rod spacing (Fig. 4(b)). This implied that plates and fasteners (i.e., pre-load) of sandwich-structured woodpile metamaterials made their propagation characteristics of impact waves totally different from those of 3D woodpile metamaterials by changing the bending mode of each woodpile.

As mentioned above, the deformation of the upper plate was changed as the rod spacing. Thus, the apparent stiffness of sandwich-structured woodpile metamaterials can be calculated as the ratio of the impact force and displacement (i.e., spring constant). Since the displacements of the upper plate with rod spacing of 28 mm and 56 mm were 24.01 µm and 27.69 µm, respectively, the apparent spring constants were 63.5 kN/m mm and 54.9 kN/mm. This means that the sandwich-structured woodpile metamaterial with rod spacing of 28 mm had higher stiffness as well as higher damping rather than that of 56 mm. Therefore, we can improve the stiffness and damping of sandwich-structured woodpile metamaterials simultaneously by changing rod spacing.
4.3. **Nonlinear contact effects on the dynamic responses of sandwich-structured woodpile metamaterials**

Figure 8 shows calculated results of impact mitigations of sandwich-structured woodpile metamaterials with nonlinear contacts (Fig. 8(a)) and linear springs (Figs. 8(b)–8(e)) as depicted in Table 2 when the rod spacing was 28 mm. As can be expected, transmitted forces and wave propagation speed increased as the linear spring constants. Remarkable thing is that the sandwich-structured woodpile metamaterial with nonlinear contacts had lower transmitted force and wave propagation speed even though spring constants of nonlinear contacts varied nonlinearly as depicted in Eqs. (4) and (5), which included the linear spring constants considered in this work. This result was more obvious as shown in Fig. 8(f) by comparing transmitted forces of leading waves in Figs. 8(a)–8(e).

![Image maps of transmitted impact force of sandwich-structured woodpile metamaterial with 28 mm spacing considering (a) nonlinear contacts and (b)–(e) linear contacts with different spring constants and (f) comparison of transmitted impact forces with 15 layups.](image-url)
Table 2. Considered linear spring constant sets for investigating nonlinear contact effects.

<table>
<thead>
<tr>
<th>ID</th>
<th>Contact force (N)</th>
<th>Spring constant of cylinder (kN/m)</th>
<th>Spring constant of cylinder on plate (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>250</td>
<td>23,500</td>
<td>42,900</td>
</tr>
<tr>
<td>B</td>
<td>500</td>
<td>29,500</td>
<td>42,900</td>
</tr>
<tr>
<td>C</td>
<td>700</td>
<td>33,000</td>
<td>42,900</td>
</tr>
<tr>
<td>D</td>
<td>1000</td>
<td>37,000</td>
<td>42,900</td>
</tr>
</tbody>
</table>

Frequency spectra of transmitted forces shown in Fig. 9 were compared for investigating nonlinear contact effects [Kim and Yang, 2014]. Frequency spectra in the unit of dB as shown in Fig. 9 means the relative amount of transmitted energies or forces of each frequency components. So it is easily expected that impact loadings with low-frequency spectra cannot propagate through mechanical systems. For example, the low-frequency spectrum was observed between 1.3 kHz and 4.9 kHz in ‘Non’ of Fig. 9 so the incident loading with this frequency region, called as low-energy band or band-gap, are mitigated such as a band-stop filter.

For linear spring cases (A–D of Fig. 9), cutoff frequencies of low-energy bands (blue areas) for sandwich-structured woodpile metamaterials increased as contact spring constants increased but width of low-energy bands decreased. For example, cutoff frequency and width of 1st low-energy band for case A were 1.37 kHz and 3.0 kHz but those for case B were 1.49 kHz and 2.8 kHz. For the nonlinear contact case (Non of Fig. 9), 1st low-energy band had 1.30 kHz of cutoff frequency and 3.6 kHz of width. Noticeable result is that the band-gap of the nonlinear contact case (Non of Fig. 9) had lower cutoff frequency but was wider than considered linear spring cases. Low cutoff frequency and large width of low-energy bands mean more incident impact energy can be absorbed in low-frequency region. Therefore, nonlinear contacts between cylindrical members of sandwich-structured woodpile metamaterials made a great role on the impact mitigation [Kim et al., 2015].
Comparing the frequency spectra of 3D and sandwich-structured woodpile metamaterials gave the apparent difference of dynamic behaviors as shown in 3D and Non of Fig. 9. 3D woodpile metamaterials had 1st low-energy band from 1.3 kHz to 2.6 kHz and 2nd from 4.6 kHz to 5.1 kHz, which were included in 1st low-energy band of sandwich-structured woodpile metamaterials. As mentioned in the previous result [Kim et al. 2015], the frequency spectra of 3D woodpile metamaterials came from nonlinear contacts between cylindrical members. Therefore, sandwich-structured woodpile metamaterials had wider 1st low-energy band since the two plates changed the bending mode of cylindrical members.

5. Conclusions

Results of this study suggested that sandwich-structured woodpile metamaterials could be used in structural application to overcome the limitation of 3D woodpile metamaterials. We compared impact mitigation characteristics of sandwich-structured woodpile metamaterials to those of 3D woodpile metamaterials experimentally. Different from 3D woodpile metamaterials, transmitted forces of sandwich-structured woodpile metamaterials with smaller rod spacing were much lower even though they had less scattered waveforms. Finite element analyses revealed that the upper plate and the 1st woodpile layer constrained each other. This made woodpiles deform with the 1st bending mode at rod spacing of 28 mm and the 3rd bending mode at rod spacing of 56 mm. Since the 1st bending mode with relatively low-frequency could contribute to coupling between transmitted forces and local resonances, the transmitted force through sandwich-structured woodpile metamaterial with smaller rod spacing (28 mm) was much lower. Moreover, the displacement of sandwich-structured woodpile metamaterials at rod spacing of 28 mm
was lower than that at rod spacing of 56 mm, which explained smaller rod spacing gave higher stiffness as well as higher damping. In addition, we compared the frequency spectra of sandwich-structured woodpile metamaterials with nonlinear contacts to those without nonlinear contacts, which had elements connected linear springs artificially. Nonlinear contacts had lower cutoff frequency of low-energy bands with larger width than linear cases, which gave higher impact mitigation characteristics.

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References


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