Periodic cellular structures are widely used in engineering applications due to their lightweight, space-filling, and load-supporting nature. However, the configuration of cellular structures is generally fixed after they are initially built, and it is extremely difficult to change their structural properties—particularly their load-bearing capabilities—in a controllable fashion. Herein, it is shown that volumetric origami cells made of Tachi–Miura polyhedron (TMP) can exhibit in situ transition between flat-foldable and load-bearing states without modifying their predefined crease patterns or hitting the kinematically singular configuration. Theoretical analysis is conducted to study this mechanical bifurcation to establish the design principle, which is verified experimentally by fabricating self-folding TMP prototypes made of paper sheets and heat-shrinking films. The improvement of load-carrying capabilities by $10^5$ is demonstrated by switching the TMP from foldable to load-bearing configurations. These programmable structures can provide practical solutions in various engineering applications, such as deployable space structures, portable architectures for disaster relief, reconfigurable packing materials, and medical devices such as stents.

1. Introduction

Reconfigurable structures can transform their shapes without entailing the redesign of their architectures.\cite{11-14} Recently, origami has gained increasing attention from the scientific and engineering community, because various types of reconfigurable cellular structures can be constructed by simply folding surface materials. Examples include interleaved tube cellular structures,\cite{9,6} zipper-coupled tubes,\cite{6,8} waterbomb origami,\cite{7,8} and prismatic architectured materials based on snapology.\cite{8,9,10} These origami-based cellular structures exhibit great ability to transform their shape from one state to another. However, one of the major concerns of these structures is to solve the conflict between reconfigurable and load-bearing capabilities. Typically, reconfigurability needs structural flexibility, whereas load-bearing capability requires structural rigidity.

One way to achieve load-supporting property in origami is a self-blocking mechanism. For example, a stacked Miura-ori structure can significantly increase its maximum load capacity, once it reaches a densification stage.\cite{11,12} This mechanism harnesses internal contact between layers, which constrains further deformation of the entire structure. Although this mechanism brings a notable enhancement of structural stiffness in origami, such a structure inevitably hits the singular state (i.e., blocked state), from which its kinematic path becomes unpredictable. This makes the system impossible to be reconfigured back to the original state.

Previous studies have also reported non-locking mechanisms to achieve simultaneously reconfigurable and load-bearing capabilities\cite{5,6,9} by constructing highly overconstrained mechanisms. However, it remains a formidable challenge to switch freely between the reconfigurable and load-bearing modes. Another approach shows the potential of programmable structures switching between different cross-sections.\cite{13} This method utilizes the kinematic bifurcation singular state, leading to an exponential number of modes because of its combinational nature of the kinematics. The controlled actuation from such a kinematically singular state causes the uncertainty of the folding motion,\cite{14} making it theoretically impossible to switch reliably between modes.

This manuscript reports a new method of realizing a reliable switching between load-bearing capability and folding nature in origami-based cellular structures. Specifically, the proposed approach exploits the mechanical bifurcation of the Tachi–Miura polyhedron (TMP),\cite{15,19-17} which exhibits the in situ transition between two drastically different states: collapsible and load-bearing configurations (see Figure 1A for the conceptual
This behavior is attributed to pure kinematic motions of the TMP cells, which exhibit a single folding path but with multiple local minima in its dimensions. Depending on the configuration of the TMP unit cell, the structure can be folded into a completely flat (see the schematic illustration marked by (i) in Figure 1B), or a loading-carrying shape (see (ii) in Figure 1B, and the photograph for the corresponding paper prototype carrying approximately 17 times its own weight). It should be noted that this dual folding mechanism is based on a rigid origami motion, which means that all deformation takes place only along crease lines, without incurring elastic deformation or plastic buckling of planar facets. This is particularly important for engineering applications to construct a 3D architecture, because it would not necessitate curved or deformable facets. Moreover, the kinematic path depicted in the fold angle parameter space is regular between the states, meaning that the mechanism does not hit kinematic singularity, thus eliminating the uncertainty in the mode switch. Therefore, to actuate this structure, one only needs to control the folding angle of crease lines.

2. Kinematics of the TMP Unit Cell

The TMP unit cell is a bellows-like origami structure as shown in Figure 2A. The structure consists of two sheets (see Figure 2B for the crease patterns for the two sheets). Each sheet is composed of two symmetric layers (i.e., \( N = 2 \)). The design of the TMP is defined by the three length parameters \( (l, m, d) \) as shown in Figure 2B and the angle \( (\alpha) \) between main (horizontal) and sub (inclined) crease lines. Based on these design parameters, the geometrical features of the TMP, such as the width \( (W) \) and breadth \( (B) \) of its cross-section, can be expressed as a function of a folding angle for the main crease lines \( (\theta_M) \) (see the 3D view in Figure 2A; also see Section S1, Supporting Information for more details).

To represent the folded state of the TMP, the folding ratio in terms of \( \theta_M \) is defined as

\[
\gamma = (\pi/2 - \theta_M)/(\pi/2)
\]  

(1)

Based on this definition, at \( \gamma = 0 \), the unit cell is folded into the flat state in the 1–3 axes, whereas \( \gamma = 1 \) indicates the flat state in the 1–2 plane. Figure 2C shows the folding motion of two TMPs as a function of \( \gamma \). Both have the same length parameters \( (l, m, d) = (4, 4, 3) \), but exhibit different \( \alpha \) values: 45° and 70°.
For $\alpha = 45^\circ$ case (the yellow-colored TMP in Figure 2C), the cross-section retains the convex polygon as $\gamma$ approaches one. However, $\alpha = 70^\circ$ case (the green-colored TMP in Figure 2C) shows the transformation from the convex to the concave cross-sectional shape.

To investigate how the loading capabilities between the two configurations differ, it is necessary to first analyze the breadth ($B$) change of the TMP unit cell as a function of $\gamma$ for the two different $\alpha$ angles, 45° and 70°, as shown in Figure 3A. Here, $B$ is normalized by the value at $\gamma = 1$. The case of $\alpha = 45^\circ$ shows that $B$ increases monotonically as $\gamma$ increases, and the structure takes its maximum value at $\gamma = 1$. In contrast, the TMP with $\alpha = 70^\circ$ exhibits the non-monotonic change of $B$, and it reaches the maximum breadth at the critical folding ratio, $\gamma_C = 0.28$.

It is this non-monotonic shape of $B$ that endows both collapsible and load-bearing capabilities of the TMP. That is, if one applies compression to the TMP along the two-axis (i.e., decreasing $B$), the structure will collapse when the initial posture of the TMP is positioned on the left side of the critical point (marked by [i] in Figure 3A). However, if the initial posture of the TMP is on the right side of the critical point (case [ii] in Figure 3A), the structure will deform in a way that its breadth is maintained kinematically, which leads to the load-bearing capability under assumption of rigid origami.

It is notable that the choice of this mechanical bifurcation between collapsible and load-bearing modes is determined by the initial posture of the TMP, without necessitating the manipulation of its crease patterns. Also, the inset illustrations in Figure 3A show that the cross-section of the collapsible mode represents a convex shape, whereas the load-bearing configuration exhibits a concave shape. The close relationship between the TMP's auxetic and load-bearing properties is discussed in Section S2, Supporting Information.

The next step is naturally how the TMP unit cell should be designed to exhibit the bifurcated folding motion described earlier. This design problem requires studying the critical transition behavior of the TMP in various length ratios ($d/m$) and crease angles ($\alpha$). For the sake of simplicity, $l = m$ is fixed. First, the existence of the critical folding ratio ($\gamma_C$) is examined by solving $d\theta/d\theta_M = 0$, which leads to

$$\cos \theta_C = \frac{1}{2} \left( -1 + \sqrt{1 - \frac{2(d/m)}{\tan \alpha}} \right),$$

where $\theta_C$ is an angle that can be calculated from $\theta_M$, and vice versa (see Section S1, Supporting Information for more details). Therefore, $\gamma_C$ can be determined from this $\theta_C$ (Equation (1)). If this process yields $\gamma_C$ in the reasonable range (i.e., $0 < \gamma_C < 1$), the TMP design exhibits the mechanical bifurcation with a controllable load-carrying capability. Otherwise, it will show only collapsible motions under loading.

![Figure 2. Geometry of the TMP. A) The TMP unit cell is shown in the 3D view (Left) and the front view (Right). B) The crease patterns of the TMP unit cells are described. The red and blue lines indicate the mountain and valley creases, respectively. The gray area is the adhesive region to connect these two sheets. C) The TMP structure can be folded from the initial flat state to the other flat configuration (from left to right). The design parameters used are $(l, m, d) = (4, 4, 3)$, $N = 7$, and $\alpha = 45^\circ$ (yellow) / 70° (green).](image-url)
Based on Equation (2), various configurations of the TMP are examined in the design space as shown in Figure 3B. Here, the color intensity represents the value of $\gamma_c$ between 0 and 1, such that the colored area denotes the design space where the critical transition takes place during the folding motions of the TMP. The white area shows the design space with $\gamma_c$ outside the reasonable range. In this case, the TMP will present only a collapsible folding motion, no matter where the initial posture is set. For example, if $d/m = 0.75$ is selected (dashed vertical line in Figure 3B), the TMP exhibits collapsible motions at $\alpha = 45^\circ$, showing the convex cross-sectional shape during folding (see the inset for the cross-sectional shape at $\gamma = 1$). As $\alpha$ increases, the TMP starts to change its cross-sectional shape from the convex to concave geometry. At $\alpha = 70^\circ$, the TMP takes the concave cross-sectional shape, enabling both collapsible and load-bearing behavior. This finding is consistent with the folding and load-carrying motions of the TMP shown in Figure 2A,C.

The boundary between the colored and white zones can be calculated numerically by solving Equation (2). The result is shown in the solid black curve in Figure 3B. In the case of $d/m = 0.75$, the boundary is formed at $\alpha = 57^\circ$ (see the crossing between the black curve and the vertical dashed line in Figure 3B). Note in passing that if an infinite chain of TMP unit cells stacked in the two-axis is considered, this boundary approaches $\alpha = 45^\circ$ regardless of the length ratio (see Section S3, Supporting Information for more details). It is also found that not all parameters considered in this design space produce a realistic TMP structure. In Figure 3B, the gray-colored area represents a forbidden design space, where the collision between the side facets happens (see the upper right inset in Figure 3B). Mathematically, this self-intersection can be avoided if $2l - d \cot(\alpha) + 2m \cos(2\alpha) > 0$ and this boundary (upper black curve) is shown in Figure 3B.

### 3. Experimental Verification of the Single TMP Analysis

#### 3.1. Implementation of Self-Folding Creases

To verify our design principle, TMP prototypes were made of paper sheets and conduct compression tests on single TMP unit cells. The key point is to observe two highly distinctive folding motions: collapsible and load-bearing behavior below and above the critical folding ratio ($\gamma_c$), respectively. To fine-tune the initial posture of TMP prototypes around this critical point, a simple self-folding mechanism based on heat-shrink polyvinyl chloride (PVC) films was introduced. Figure 4A shows the layout of our self-folding hinge consisting of five layers, including the PVC film (see Materials and Methods, and also Movie S1, Supporting Information). In this design, there is a gap (denoted by $L_{gap}$ in Figure 4A) between the pairs of inserted layers (with arm length $L_1$). When the PVC film was heated using a heat gun in this study, the contraction of the PVC film generated the bending moment, eventually closing this gap. Figure 4B shows the snapshots of the self-folding process, when $L_{gap} = 2$ and $L_1 = 18$ mm (Movie S1, Supporting Information).

In the self-folding process, the final folded crease angle ($\Theta$ as shown in the inset of Figure 4C) is governed by the ratio of $L_{gap}/L_1$. If $L_{gap}/L_1$ is smaller, the crease will be bent less (i.e., $\Theta$ closer to 180°), whereas the larger $L_{gap}/L_1$ will induce a more drastic bending (i.e., smaller $\Theta$). Figure 4C shows $\Theta$ as a function of $L_{gap}/L_1$ for various prototypes. Here, the red markers represent the experimental measurements from three prototypes per designated $L_{gap}/L_1$ value, and the black curve is the prediction from our geometrical model (see Section S4, Supporting Information for more details). Figure 4D shows the digital images of the three different crease angles achieved by imposing $L_{gap}/L_1 = 0.11, 0.33,$ and 0.66.

---

**Figure 3.** Kinematic analysis on the TMP unit cell. A) The breadth ($B$) is examined as a function of the folding ratio ($\gamma$) for the two different $\alpha$ values: 45° (yellow) and 70° (green). The inset shows the cross-sectional geometry change for each folding stage. For $\alpha = 70^\circ$, the structure takes its maximum value of $B$ at the critical folding ratio ($\gamma_c = 0.28$). B) This critical folding ratio can be tuned by altering the length ratio ($d/m$) and crease angle ($\alpha$). The lower black curve is the boundary between the configuration with/without the critical transition. The gray area indicates the invalid design parameters due to the self-intersection of TMP facets. The red arrow in the upper inset points to the collision between the side parts.
3.2. Compression Tests on TMP Unit Cells

For the fabrication of TMP cells, the following geometrical parameters were used: (l, m, d) = (40, 40, 30) mm, N = 4, and \(\alpha = 70^\circ\), which resulted in \(\gamma_C = 0.28\) according to the established design guideline (Figure 3B). Using these design parameters, two types of TMP prototypes with distinctive initial folded state (\(\gamma_0\)) were fabricated: one with \(\gamma_0 < \gamma_C\) for the collapsible behavior, and the other with \(\gamma_0 > \gamma_C\) for the load-bearing feature. We achieved the change of this initial folded state by leveraging the self-folding mechanism described in the previous section. Specifically, \(L_{\text{gap}}/L_1\) was manipulated to obtain (i) \(\gamma_0 = 0.21\) for collapsible behavior (\(L_{\text{gap}}/L_1 = 0.11\)) and (ii) \(\gamma_0 = 0.37\) for load-bearing feature (\(L_{\text{gap}}/L_1 = 0.33\)). The positions of these initial folded states with respect to the critical point are shown in Figure 5A along with their cross-sectional geometries.

As a next step, compression tests on these two different prototypes were conducted. Figure 5C shows the force (\(F_1\)) and displacement (\(u_1\)) relationships for (i) the collapsible (blue) and (ii) load-bearing (red) configurations. Here, dashed curves with bands represent experimental measurements with standard deviations, and solid curves show the analytical prediction based on the rigid origami model. In the analytical model, the crease lines in the TMP were modeled as a hinge with a torsion spring (see Section S5, Supporting Information for more details). Force was normalized by \(k_0\), \(d\), and the total length of the main crease line (\(L_M\); see Section S5, Supporting Information for details). Similarly, the displacement was non-dimensionalized by the maximum TMP height at the critical point (i.e., \(B_i\), as indicated in Figure 3A).

Figure 5C clearly shows two drastically different behaviors between the load-bearing and collapsible configurations. The experimental results corroborate our analytical prediction. In the load-bearing configuration, however, the deviation between experimental and predicted results increased toward the end of compression. This was due to the limitation of our rigid origami model, in which it was assumed that all surfaces maintained their shapes without deformation during folding/unfolding motions. However, if the structure approached \(\gamma = 0\) or 1, the facet deformation was inevitable because of the effect of material thickness and the friction between the prototype and the stage of the testing setup (see Figure 5D, and Movie S3 and S4, Supporting Information for compression of the collapsible and load-bearing configurations). For the collapsible case, the initial increase of force was greater than the prediction, and this was attributed to the resistance of PVC films in the process of unfolding. Despite the effect of friction and surface deformation, the collapsible configuration needed much smaller force to be folded into a flat stage (i.e., \(\gamma = 0\)), compared to the maximum force sustained in the load-bearing stage (i.e., toward the direction of \(\gamma = 1\)).

To further investigate the drastic contrast of the TMP’s load-bearing capability in relation to design parameters and initial
conditions, we used the rigid origami model and examined how much force was required to compress the TMP to the completely collapsed or load-bearing configurations. In this analysis, if $\gamma_0$ was below (above) $\gamma_C$, the maximum force required for the folding between $\gamma_0$ and $\gamma = 0$ ($\gamma = 1$) was calculated. Figure 5E shows the surface map of the maximum force required as a function of $\gamma_0$ and $\alpha$ in case $l/m = 1$ and $d/m = 0.75$. The black solid curve in this figure represents the boundary between the collapsible and load-bearing configurations, which is calculated from Equation (2). The surface map clearly shows the two distinctive zones, representing collapsible and load-bearing configurations. The two fabricated prototypes are indicated by the circular markers (i) and (ii) in Figure 5E. By crossing the boundary from (i) to (ii), the structure exhibited a significant increase of the maximum support force by two orders of magnitude. Ideally the maximum force became an infinity in the load-bearing stage as the TMP approached $\gamma = 1$. In experiments, however, surface deformation and buckling took place before reaching such an extremely high force.

Now we demonstrate the in situ transition between the two modes in cycles and measure the contrast of the support force experimentally. TMP prototypes made of seven layers ($N = 7$) were prepared and the (one-way) self-folding creases were removed for the repeatable in situ transition. The initial conditions between collapsible and load-bearing states were changed manually, and the force applied to the prototypes for each loading cycle was measured (see Movie S5, Supporting Information for the in situ transformation of the paper prototype used for this cyclic loading test). Figure 5F shows the experimental results with the insets showing the target configurations. Here, the measured force was normalized by the force required to compress the collapsible configuration ($\gamma_0 = 0.21$) to a flat stage in the first cycle. The experimental results clearly showed drastic contrast of load-carrying capabilities between the two stages (blue and red for collapsible and load-carrying configurations). Quantitatively, even though the prototypes were made of paper sheets and were loaded up to the repeatable elastic regime, the TMP increased the maximum support force by two orders of magnitude, simply by changing the initial posture from the collapsible to the load-bearing state. This verifies the capability of the TMP to achieve both reconﬁgurable and load-carrying capabilities in a controllable manner.
4. Design/Fabrication of TMP-Based Cellular Structures

Finally, we explore the design and fabrication of space-filling tessellations using TMP cells as a building block (Figure 6A). Such a cellular structure derives its unique characteristics from the comprising TMP unit cells. A physical prototype consisting of eight TMP unit cells was fabricated by employing the same design parameters used in the previous section (Figure 6B). This cellular assembly also shows the in situ transition between collapsible and load-bearing behaviors (Movie S6, Supporting Information). Based on this in situ transition, one of the potential applications of the TMP cellular structure is a deployable bridge for disaster relief. Figure 6C shows the conceptual illustration of the deployment of the TMP-based bridge-like structure, which can be folded flat for stowing and can be deployed for load-carrying purposes (see Movie S7, Supporting Information for its deployment motion). Similarly, TMP architectures can be used for constructing a deployable-stiff space structure for space habitats (see Figure S7, Supporting Information). While this study focused on paper-based TMP prototypes, the design principle can be applied to different fabrication materials and approaches. For example, a TMP structure made of rigid acrylic panels can be constructed (see Figure S8, Supporting Information for its geometrical configurations and Movie S8, Supporting Information for its folding motion). On the contrary, soft TMP structures can also be fabricated using additive manufacturing techniques. We explore the feasibility by printing TMP-based soft cellular structures made of thermoplastic polyurethane elastomer (Figure S9, Supporting Information). Using this fabrication approach, it is shown that two seemingly identical TMP cells can present drastically different load-carrying behaviors (Movie S9, Supporting Information). These additive manufacturing techniques can potentially enhance fabrication accuracy, ease of manufacturing processes, and open new applications for TMP structures.

5. Conclusions

We analyzed and demonstrated highly versatile folding behavior of the TMP by employing rigid origami model and paper-based prototypes with self-folding creases. This volumetric origami can exhibit two drastically different configurations, collapsible and load-bearing ones, without modifying the predefined crease patterns. This was achieved by leveraging a mechanical bifurcation intrinsic in TMP, which enables in situ transition between the collapsible and load-bearing states in an efficient and controllable manner. For experimental demonstrations, a self-folding technique based on heat-shrinking films was employed. Although this mechanism supports one-way actuation only, we envision that the TMP cellular structures can also transform their shapes repeatedly by using two-way reversible actuation methods such as shape memory alloys and electroactive polymer actuators. This is possible because the kinematic nature of TMP allows the switching without hitting a singular state. We envision that the TMP architectures can be employed to a wide range of engineering applications, such as a portable bridge for disaster relief, acoustic waveguides, deployable space habitat, and medical devices.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Acknowledgements

We are grateful for the support from the National Science Foundation (CAREER-1553202) and the Washington Research Foundation.

Conflict of Interest

The authors declare no conflict of interest.
Keywords
load-bearing capabilities, rigid origami, self-folding

Received: May 13, 2019
Revised: July 26, 2019
Published online: