ASTR 509: Physical Foundations of Astrophysics III: Stellar Dynamics

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Midterm Exam

Each of the three problems is worth 10 points. You have 60 minutes.

Problem 1

a) Describe the equations of motion and essential characteristics of the motion in a spherical potential, $\Phi(R)$, and in an axisymmetric potential $\Phi(R, z)$.

(hint: in cylindrical coordinates $a_R = \ddot{R} - R\dot{\phi}^2$, $a_\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi}$)

b) Sketch the zero-velocity curve for a star that is at $(x, y, z) = (R_c, 0, 0)$ for $t = 0$, with a velocity $(v_x, v_y, v_z) = (\sqrt{3}v_o, 0, 0)$, in the potential

$$\Phi(R, z) = \frac{v_o^2}{2}\left[(\frac{R}{R_c})^2 + \frac{|z|}{H}\right] - \Phi_o \tag{1}$$

c) Sketch the surface of section (i.e. $\dot{R}$ vs. R diagram for $z = 0$) for this star.

Problem 2

The epicycle approximation for a body in nearly-circular orbit is

$$R \approx R_g + X\cos(\kappa t + \Psi) \tag{2}$$

and

$$\phi = \phi_o + \Omega_g t - \frac{2\Omega_g X}{\kappa R_g}\sin(\kappa t + \Psi), \tag{3}$$

where

$$\kappa^2 = (R\frac{\mathrm{d}\Omega^2}{\mathrm{d}R} + 4\Omega^2)_{R=R_g} \tag{4}$$

and

$$\Omega^2 = \frac{1}{R}(\frac{\partial\Phi}{\partial R})_{(R,z=0)} \tag{5}$$

The radius of the circular orbit, R_g, is defined as the minimum of the effective potential

$$\Phi_{\text{eff}} = \Phi + \frac{L_z^2}{2R^2}. \tag{6}$$

a) How accurate is the epicycle approximation for motion in a harmonic potential $\Phi(R,z) = \alpha R^2 + \beta z^2$? Why?

b) A star is in a Keplerian orbit described by $\phi(t=0) = 0$ and

$$R = \frac{p}{1 + \epsilon\cos(\phi)}. \tag{7}$$

Use the epicycle approximation to determine $v_R(t) = \dot{R}$ (hint: determine R_g and X by requiring perfect agreement between the exact and approximate solutions for $\phi = 0$ and $\phi = \Pi$. Sketch the surface of section using this result.

c) Use the epicycle approximation to find out whether closed orbits are possible in a Plummer potential

$$\Phi = -\frac{\mathrm{GM}}{(R^2 + b^2)^{1/2}} \tag{8}$$

for $b \neq 0$.

Problem 3

Given a logarithmic potential

$$\Phi(R, z) = -\frac{v_o^2}{2} \ln(\frac{R_c^2 + R^2 + \alpha z^2}{R_c^2}) - v_o^2 \tag{9}$$

a) Sketch the behavior of the circular velocity as a function of R/R_c.

b) Are non-circular orbits possible in this potential? (hint: use the epicycle approximation)

c) A particle is released at $t = 0$ from $(x, y, z) = (R_c, 0, 0)$ with a velocity $(v_x, v_y, v_z) = (0, v_1, 0)$. Sketch the orbits for $v_1 = v_0/2$, $\sqrt{2}v_0/2$, v_o, and $\sqrt{2}v_o$.