Astr 323: Extragalactic Astronomy and Cosmology

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Homework Problem Set 4

As we discussed it in class, a variety of recent observations such as CMB, SNe, and dynamical mass estimates can be interpreted by a single set of cosmological parameters (H_o=70 km/s/Mpc, $\Omega_k = 0$, $\Omega_r = 5 \times 10^{-5}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$ with $w = -1$), named the concordance model. These two homework problems explore some of the properties of this model, and compare it to a few other cosmological models.

In these problems you will have to perform numerical integration of a few functions. You can do this using standard tools, such as Mathematica, or you can write your own piece of code. If you choose the latter, within the context of these problems and required accuracy, numerical integrals can be turned into sums as:

$$\int_{x_1}^{x_2} f(x)\mathrm{d}x \approx \Delta \sum_{i=0}^{N} f(x_i), \tag{1}$$

where it is assumed that $\Delta << (x_2 - x_1)$, and x_i are a grid of N values of x, increasing from x_1 to x_2, with constant spacing Δ. For example, you can safely use $\Delta = 0.01$ for redshift integrations below to achieve an accuracy of a few percent.

1) **The Concordance Cosmological Model**

a) Compute $E(z)$ (see eq.54 from Lecture 7, and note that the equation gives *the square of E(z)*) for the concordance model in the 0–20 redshift range. Then compute separately the contribution of each individual component (matter, radiation and dark energy) and divide by the overall $E(z)$ to get relative contributions. Plot these contributions as functions of z on the same graph and use it to answer the following questions: At what redshift does the dark energy contribution become larger than the radiation contribution (1/2 point)? At what redshift does the dark energy contribution become larger than the matter contribution (1/2 point)?

b) The deceleration of a flat ($\Omega_k = 0$) universe can be computed as

$$q(z) = -\frac{\ddot{R}\,R}{\dot{R}^2} = \frac{Q(z)}{2\,E^2(z)}, \tag{2}$$

where

$$Q(z) = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda[1 + 3w(z)] \tag{3}$$

Here $\dot{R} = \mathrm{d}R/\mathrm{d}t$ and $\ddot{R} = \mathrm{d}\dot{R}/\mathrm{d}t$. Note that with this definition of deceleration q, an accelerating universe has a *negative* deceleration. Compute and plot $q(z)$ for the concordance model. At what redshift did the universe start accelerating (1 point)?

c) Analogously to problem a), compute and plot individual contributions to $q(z)$. Is the current overall acceleration of the universe more similar in value to the radiation contribution or the matter contribution (1 point)?

d) Using eq.58 from Lecture 7, compute the lookback time for the concordance model and $z < 20$. Plot the lookback time as a function of redshift. At what redshift was the lookback time 6.85 Gyr, or half the present age of the universe (2 points)?

2) **The Comparison of Different Cosmological Models**

In this problem you will compare the predictions of the concordance model and a few other cosmological toy models in the 0–3 redshift range. For all these models assume $\Omega_k = 0$, $\Omega_r = 5 \times 10^{-5}$ and H_o=70 km/s/Mpc. Other model properties are

Ω_m	Ω_Λ	w	Comment
0.30	0.70	-1.0	the concordance model
1.00	0.00	0.0	flat, matter-only
0.95	0.05	-1.0	flat, matter-dominated
0.05	0.95	-1.0	flat, DE-dominated
0.30	0.70	-0.6	weird dark energy instead of Λ

Table 1: Cosmological Models

a) Compare $q(z)$ for these models - does any of these models produce acceleration (i.e. $q < 0$) and for which redshift range? (1 point)

b) Compute distance module, DM, for these models (see eq. 66 from Lecture 7), and subtract DM for the concordance model from DM for other models. Plot these DM differences vs. z. Which cosmological models predict larger DM (i.e. fainter SNe) than in the concordance model? (2 points)

c) Plot the angular size of a 10 kpc large galactic disk as a function of redshift for these five models (see eq. 61 in Lecture 7). Which model predicts the largest angular size at $z = 1$? (2 points)