Astr 323: Extragalactic Astronomy and Cosmology

Željko Ivezić
University of Washington, Spring Quarter 2014

Homework Problem Set 2

1) **Rotation Curves and Dark Matter** As we learned in Lecture 3, the measured rotational velocity of galaxies typically doesn't fall off with radius, R, as one would expect for an exponential (infinitely thin) disk (in the disk plane):

$$v_D(R) = v_o^D \sqrt{\frac{(R/R_c^D)^{1.3}}{1 + (R/R_c^D)^{2.3}}}, \quad (1)$$

where R_c^D is a characteristic disk length scale, and v_o^D is a characteristic disk velocity scale (v_o^D and R_c^D are related to each other through the disk mass).

An example of such a velocity curve is listed in file vrot.dat, available from the class web site. The first column lists R in kpc, the second column the rotational velocity in km/s, and the third column estimated measurement error for the rotational velocity in km/s.

The flat rotation curve can be explained by postulating a dark matter halo with density

$$\rho = \frac{\rho_o}{1 + (R/R_c^H)^2}, \quad (2)$$

leading to a velocity curve

$$v_H(R) = v_o^H \sqrt{1 - (R_c^H/R)\, \arctan{(R/R_c^H)}}, \quad (3)$$

where R_c^H is a characteristic halo length scale, and v_o^H is a characteristic halo velocity scale (v_o^H and R_c^H are related to each other through the central density ρ_o). Note that the total mass, $M = 4\pi \int \rho(R) R^2 dR$ is divergent for such a density profile.

The total expected velocity curve is then

$$v_T(R) = \sqrt{v_D^2(R) + v_H^2(R)}. \quad (4)$$

This problem will ask you to fit these expressions to provided data. While there are many excellent programs for multi-dimensional fitting, you don't need to, and are encouraged not to, use them. Instead, observe how the model curves change when you vary the free model parameters (i.e. R_c^D, v_o^D, R_c^H, and v_o^H). When asked about "the goodness of a fit", you need to compute the mean value of $(v_{model} - v_{data})^2/\sigma^2$, where σ stands for data errors (i.e. sum this for all 20 data points, and then divide by 20; it should really be divided by 19 but this is a small difference, which you will discuss in more detail in your statistics class). A fit is "good" if this quantity (called χ^2 (chi-squared) per degree of freedom) is of order unity. This is the same as saying that typical deviation of your model from the data is about 1 error bar. If you can't make your model go that close to your data (by varying free parameters), then you need to get yourself a new model.

1. Plot the data for $v_{rot}(R)$ vs. R (with error bars for velocity!); is the rotation curve flat? (1 point)

2. Determine the best possible fit using only $v_D(R)$ expression. What are R_c^D and v_o^D? Is the fit good? (1 point)

3. Determine the best possible fit using only $v_H(R)$ expression. What are R_c^H and v_o^H? Is the fit good? (1 point)

4. Now combine $v_D(R)$ and $v_H(R)$ into $v_T(R)$ and estimate the best fit parameters R_c^D, v_o^D, R_c^H, and v_o^H. Is the fit much better then when only disk or dark matter halo is used? (1 point)

5. Does this exercise convince you of the existence of dark matter (assuming the provided data are trustworthy)? Here you need to discuss/justify your answer, and not simply answer yes/no (1 point).

2) **Galaxy Luminosity Function** Our class website contains a link to a catalog of 6,669 galaxies (galaxiesDM.dat) from the SDSS "main" sample (flux-limited to $r < 17.8$) and from a tiny (by SDSS standards) $\sim$100 deg^2 of the sky. In addition to the u and r band magnitudes, position and redshift, the catalog also lists distance module in magnitudes, computed using the standard WMAP cosmology (it's ok if the latter doesn't make much sense – we'll talk about that later). You can compute the absolute r band magnitude for these galaxies using $M_r = r - DM$.

1. Plot M_r vs. redshift. Why is the data distribution limited to only one half of the diagram? (1 point)

2. Plot M_r histograms for three subsamples of galaxies with redshifts 0.03–0.05, 0.05–0.07 and 0.07–0.09, and renormalized to 1 for $M_r = -21$ (all three curves on the same plot). Does the luminosity function depend on redshift (i.e. are the three histograms similar)? (1 point)

3. Take all the galaxies from a narrow redshift slice 0.07-0.09 and plot M_r vs. $u - r$ color. Are they correlated? (1 point)

4. For galaxies from the 0.07-0.09 redshift range, compare the $u - r$ distribution for those with $-21 < M_r < -20.5$ and those with $M_r < -22$. Comment. (1 point)

5. Take all the galaxies from a narrow redshift slice 0.07-0.09 and use the $u - r$ color to split them into blue ($u - r < 2.22$) and red ($u - r > 2.22$). Plot and compare their M_r histograms. Are they same? Comment? (1 point)