Astr 323: Extragalactic Astronomy and Cosmology

Spring Quarter 2014, University of Washington, Željko Ivezić

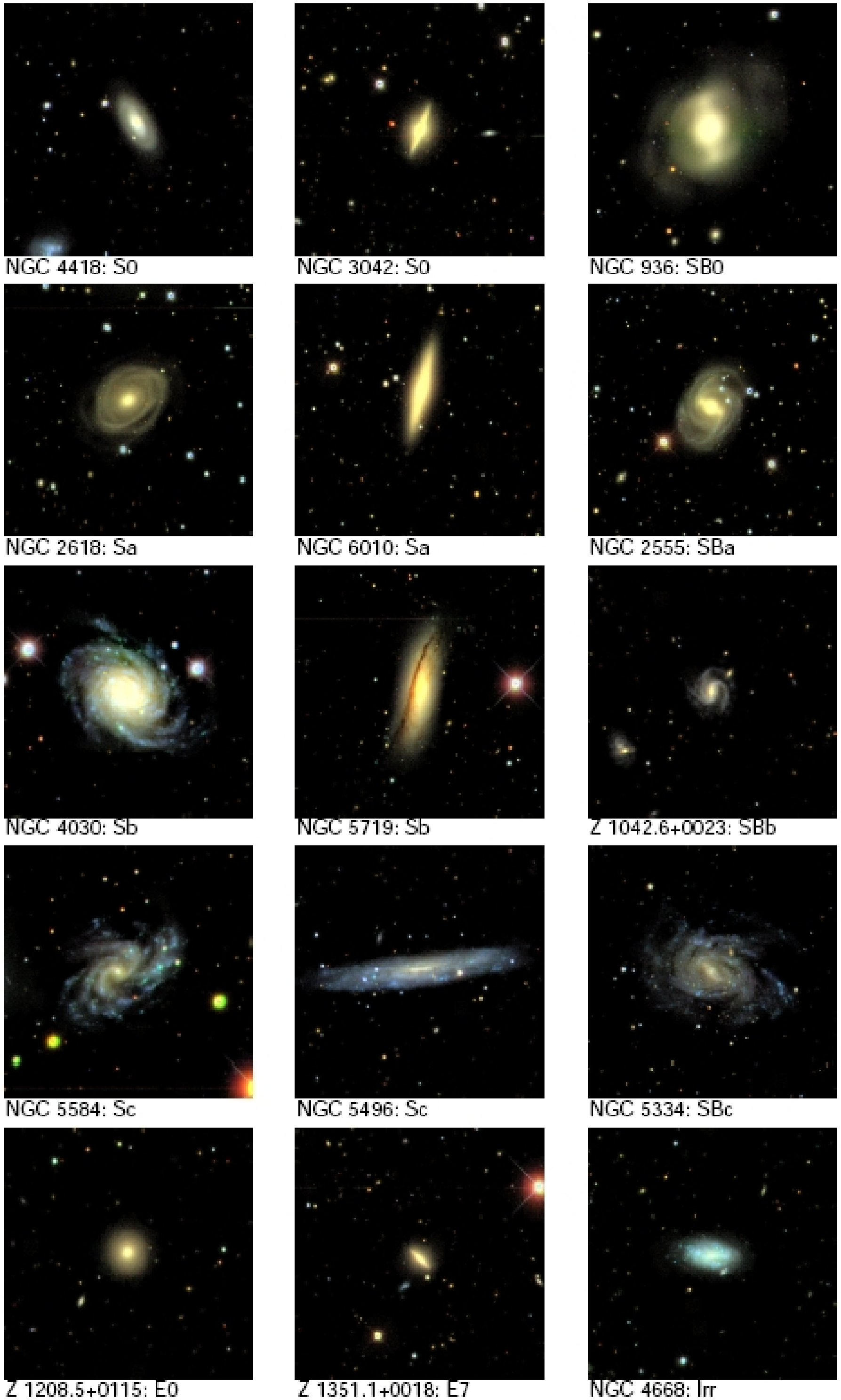
Lecture 3:

Galaxies

Outline

- A Little Bit of History
- Galaxy Types, Properties and Classification
- Spiral Galaxies
- Elliptical Galaxies

Hubble's Classification of Galaxies



5 arcmin x 5 arcmin true-color g-r-i images from SDSS comissioning data.
Galaxy classification is from SIMBAD.

Galaxies

- Galaxies are (mostly) made of stars (also gas, dust, active galactic nuclei – AGN); hence have similar (but not identical!) color distributions
- They come in various shapes and forms (spiral vs. ellipticals; aka exponential vs. de Vaucouleurs profiles)
- Some host AGNs, some have high star-formation rates, some are very unusual (dwarf galaxies, mergers, etc.)
- We are interested in various distribution functions (e.g. for luminosity, colors, mass, age, metallicity, size, etc.) – the hope is to figure out how galaxies formed and evolved
- Nearest neighbors: the Andromeda galaxy (M31), Large and Small Magellanic Clouds, the Sgr Dwarf (may be more)

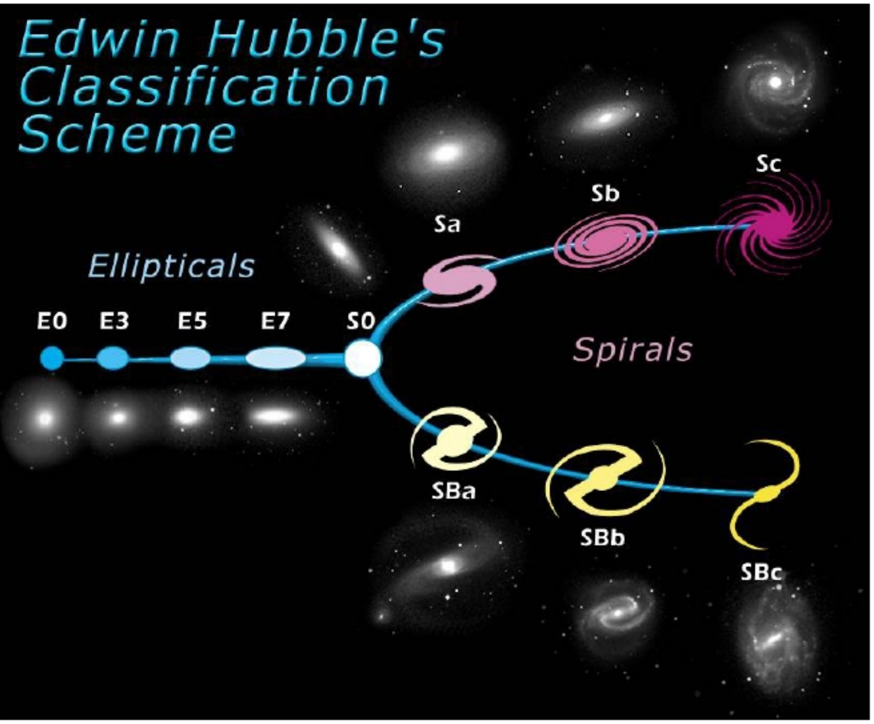
Most important historical breakthroughs in galaxy research

- Around 1610: Galileo Galilei resolves the Milky Way into individual stars
- Around 1750: Immanuel Kant developes the idea of "island universes" – different galaxies just like our own
- Around 1850: William Parsons discovers spiral structure and proposes that some galaxies rotate
- 1923/24: Edwin Hubble resolves M31 and M33 into individual stars – confirms that they are galaxies just like our own
- 1929: Edwin Hubble discoveres the expansion of the Universe

- 1933: Fritz Zwicky claims the existence of “dark matter” based on observed speeds of cluster galaxies (*nobody believes him!* – for a rap song about this see astro-ph/9610003)

- 1970-1980: Vera Rubin’s work on rotation curves of spiral galaxies – dark matter idea becomes widely accepted

Hubble's Morphological Classification



- Broadly, galaxies can be divided into ellipticals, spirals, and irregulars
- Broadly, spirals are divided into normal and barred (similar frequencies): S and SB
- The subclassification (a, b, or c) refers both to the size of the nucleus and the tightness of the spiral arms. For example, the nucleus of an Sc galaxy is smaller than in an Sa galaxy, and the arms of the Sc are wrapped more loosely.
- The number and how tightly the spiral arms are wound are well correlated with other, large scale properties of the galaxies, such as the luminosity of the bulge relative to the disk and the amount of gas in the galaxy. This suggests that there are global physical processes involved in spiral arms.

Overview of classification systems:

System	Principal criteria	Symbols	Examples
Hubble-Sandage (Sandage (1961-1995))	barrishness; openess of arms/disk-bulge ratio; degree of resolution of arms into stars	E, S0, S, SB, Irr a, b, c	M87=E1 M31=Sb M101=Sc LMC=Irr I
De Vaucouleurs (de Vaucouleurs (1959))	barrishness; openess of arms/disk-bulge ratio; rings or s shapes	E, S0, S, SA, SB, I a, b, c, d, m (r), (s)	M87=E1P M31=SA(s)b M101=SAB(rs)cd LMC=SB(s)c
Yerkes (Morgan (1958-1970))	central concentration of light; barishness/smoothness	k, g, f, a E, R, D, S, B, I	M87=kE1 M31=kS5 M101=fS1 LMC= afI2
DDO (van den Bergh (1960-1976))	richness of disk in young stars; barrishness; central concentration of light; quality and length of arms	E, S0, A, S, Ir B a, b, c I, II,. . . , V	M87=E1 M31=Sb I-II M101=Sc I LMC=Ir III-IV

- Primary classification criteria of commonly used Hubble-Sandage system:
  - Bulge-to-disk ratio (S0/Sa: 5 to 0.3, Sb: 1 to 0.1, Sc/Irr: 0.2 to 0)
  - Opening angle of spiral arms (Sa: 0 to 10, Sb: 5 to 20, Sc: 10 to 30 degrees)
  - Bars

- Physical parameters varying along the Hubble-Sandage system:
  - Stellar mass M increases from irregulars ($10^8 M_\odot$) to ellipticals ($10^{12} M_\odot$)
  - Specific Angular Momentum J/M of baryons increases from ellipticals to spirals
  - Mean age increases from irregulars through spirals to ellipticals (B-V increases from 0.3 to 1.0, mass-to-light M/L_B ratio increases from about 2 to 10)
  - Mean stellar density of spheroids increases with decreasing spheroid luminosity
  - Mean surface brightness of disks increases with luminosity
  - cold gas content increases along Hubble sequence (fraction of baryonic mass: 0 in E/S0, 0.1 to 0.3 in Sa to Sc, up to 0.9 in Irr)
  - hot gas content only significant in massive E (few percent of baryonic mass)

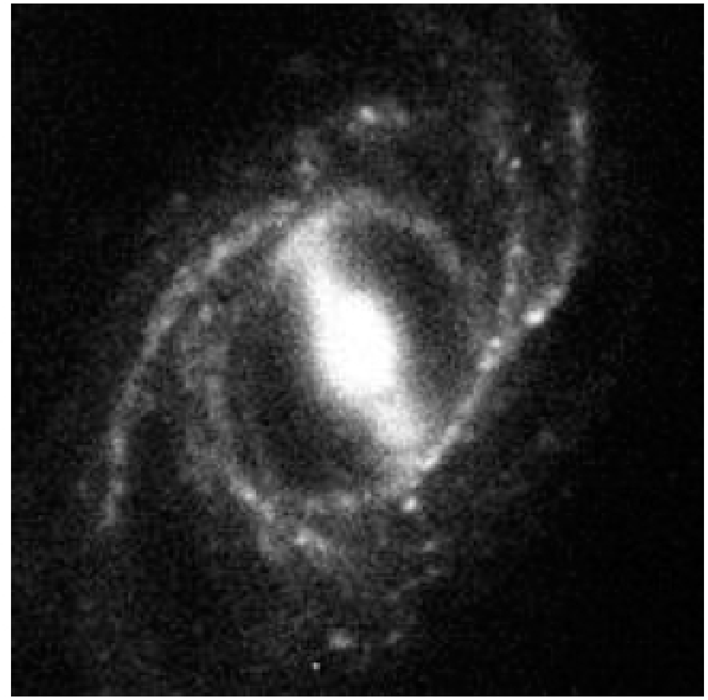
Spiral (Sa) Galaxies:

NGC 3223: Sa-galaxy

M 104 (Sombrero), Sa-galaxy
(P.Barthel, VLT)

Barred-Spiral (SBb) Galaxies:

M 95: SBb-galaxy



NGC 2523: SBb-galaxy

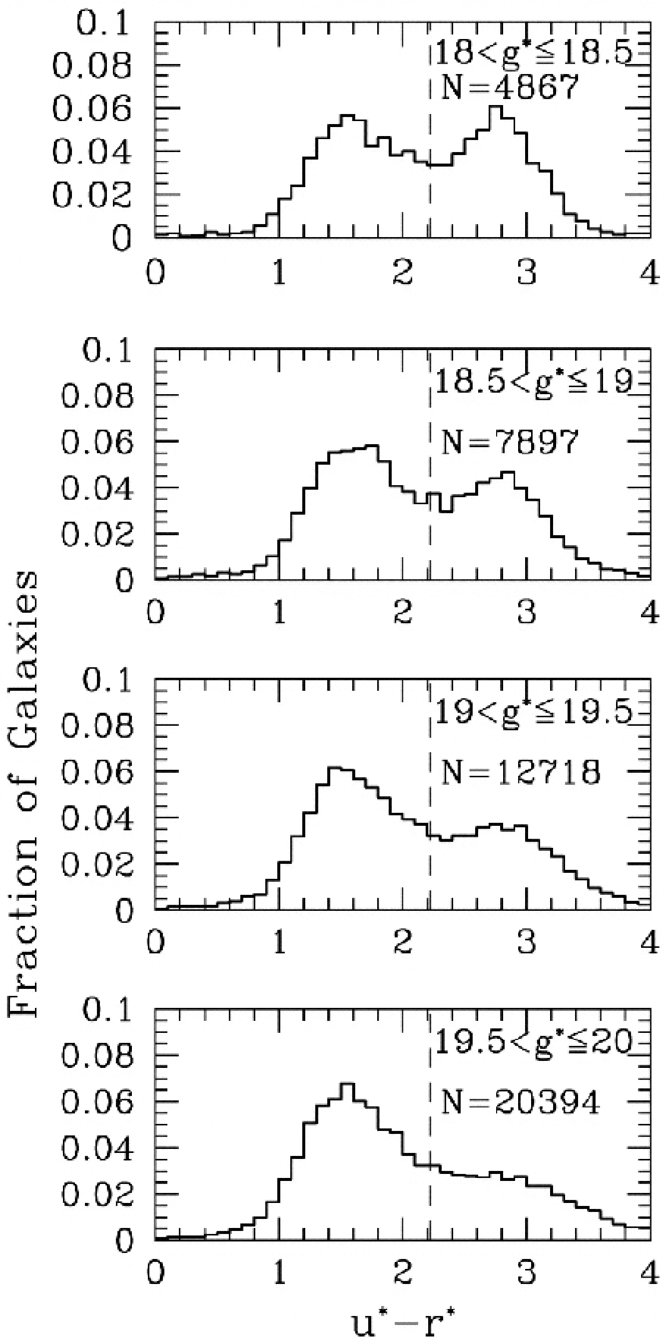
Irregular (Irr) Galaxies:

LMC: Irr-galaxy

SMC: Irr-galaxy

Galaxy types that didn't make it into the Hubble-Sandage system

- dwarf galaxies:
  dE: dwarf ellipticals or dwarf spheroidals, similar to E but of low luminosity and low surface brightness
  BCD: Blue Compact Dwarfs, concentrated starburst, faint old stellar components
- cD-galaxies:
  Yerkes classification for "extra (c) large and diffuse (D)" galaxies, found in the centers of clusters and groups
- low surface brightness (LSB) galaxies: mostly luminous but very low surface brightness disk galaxies
- active galaxies: radio galaxies; galaxies with unusual nuclear emission lines and/or extreme nuclear luminosity (QuasiStellarObjects-QSOs, Seyfert galaxies) and/or with powerful non-thermal radio emission (radio galaxies, quasars)
- interacting, merging and starbursting galaxies (IRAS mergers, ULIRGs, i.e. Ultra-Luminous Infra-Red Galaxies)



Colors are correlated with morphology

- Galaxies have bi-modal color distribution (e.g. SDSS $u - r$ color – the ratio of the UV and red fluxes)
- Colors correlated with shapes and profiles: blue galaxies tend to be spiral and red galaxies tend to be elliptical (there are deviants such as e.g. "anemic spirals")
- A good parametrization for shapes (i.e. intensity vs. radius R) is **the Sersic index n:** $I(R) \propto \exp(-(R/R_e)^{1/n})$
- $n = 1$: exponential profile
- $n = 4$: de Vaucouleurs profile

Note the bulge contribution!

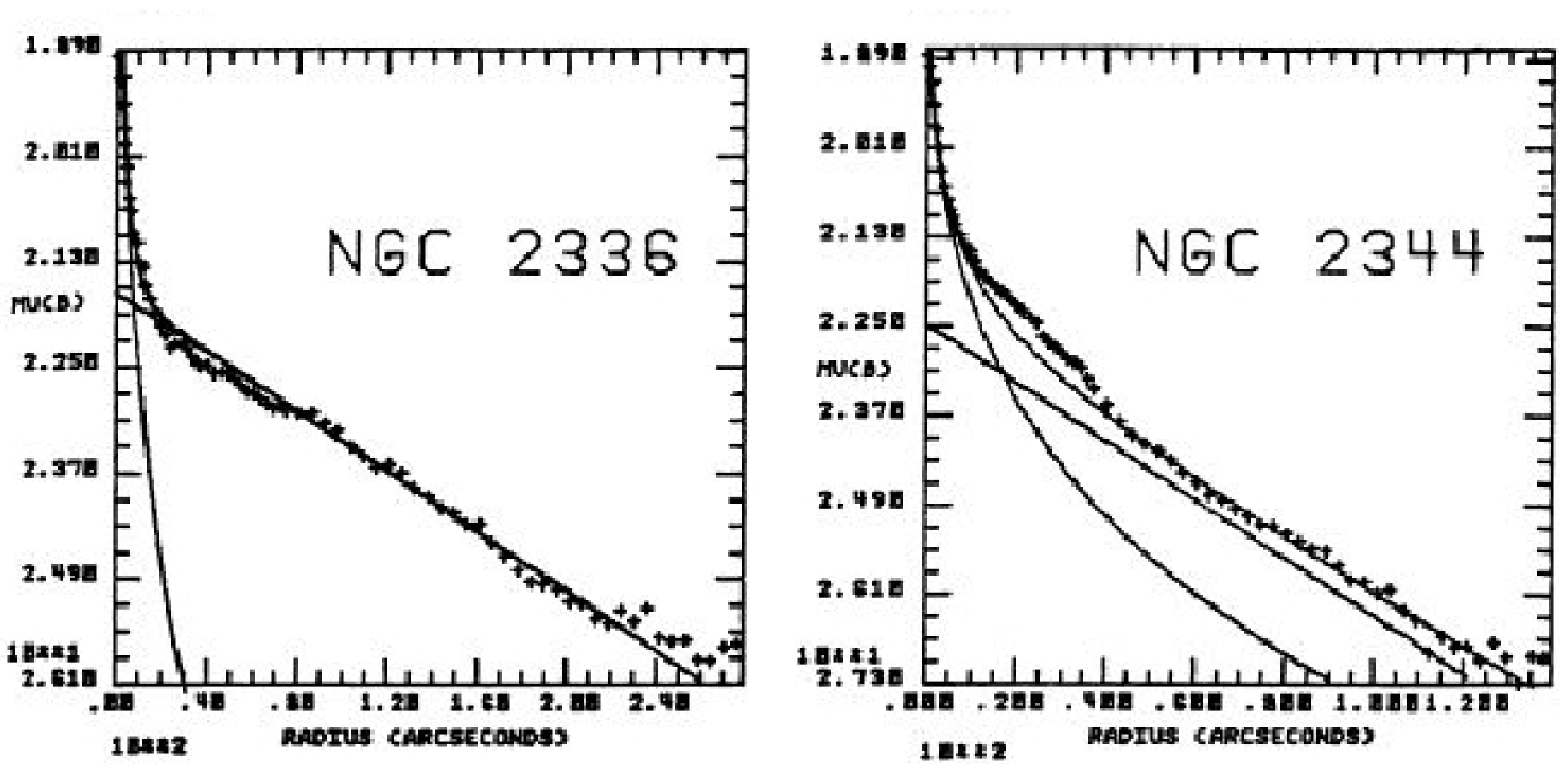


FIG. 6.—Elliptically averaged profiles for the 26 spiral galaxies. For NGC 4594 the north half of the minor axis is shown. Where an iterative decomposition was possible, disk, bulge, and total profiles are shown. In all other cases, only the bulge fit is shown.

The light intensity distribution as a function of (elliptical) radius

Astronomers usually express brightness on a logarithmic (magnitude) scale (well, at least optical astronomers do):

$$\log I(R) = a - b\,R^{1/n} \tag{1}$$

Given an image of a galaxy (i.e. I as a function of R), one can determine a, b and n. Sometimes, n is fixed as a function of galaxy's morphology, and only a and b are fit to the data.

Sometimes, de Vaucouleurs profile is expressed as

$$I(R) = I_o\,10^{-3.33\,[(\frac{R}{R_{1/2}})^{1/4} - 1]} \tag{2}$$

or for surface brightness (e.g. mag/arcsec^2)

$$\mu = -2.5\,\log\,[I(R)] = \mu_o + 8.33\,[(\frac{R}{R_{1/2}})^{1/4} - 1] \tag{3}$$

The light intensity distribution as a function of (elliptical) radius

Integral of $I(R)$ over the entire galaxy gives flux

$$F = 2\pi \int_0^\infty I(R)\, R dR \tag{4}$$

This assumes that the profile was averaged in elliptical annuli. In general,

$$F = \int_0^\infty \int_0^{2\pi} I(R,\phi)\, R dR d\phi \tag{5}$$

Here, $I(R)$, and this F is measured at some wavelength (and in some band). Integration over all wavelengths gives *bolometric* flux.

If flux F is multiplied by $4\pi D^2$, where D is distance, one gets *luminosity*. Beware of units!

The light intensity distribution as a function of (elliptical) radius

Freeman's law: when $\mu(R)$ is extrapolated to the center of the galaxy (R=0, and excluding the bulge contribution), one gets a similar answer (to within 40-50%) for all spiral galaxies!

This all (i.e. exp. profile and Freeman's law) applies to disks of spiral galaxies. What about (luminous) halo?

The answer depends on tracer; for the Milky Way

- globular clusters: $I(R) \propto R^{-3.5}$

- RR Lyrae: $I(R) \propto R^{-3.0}$

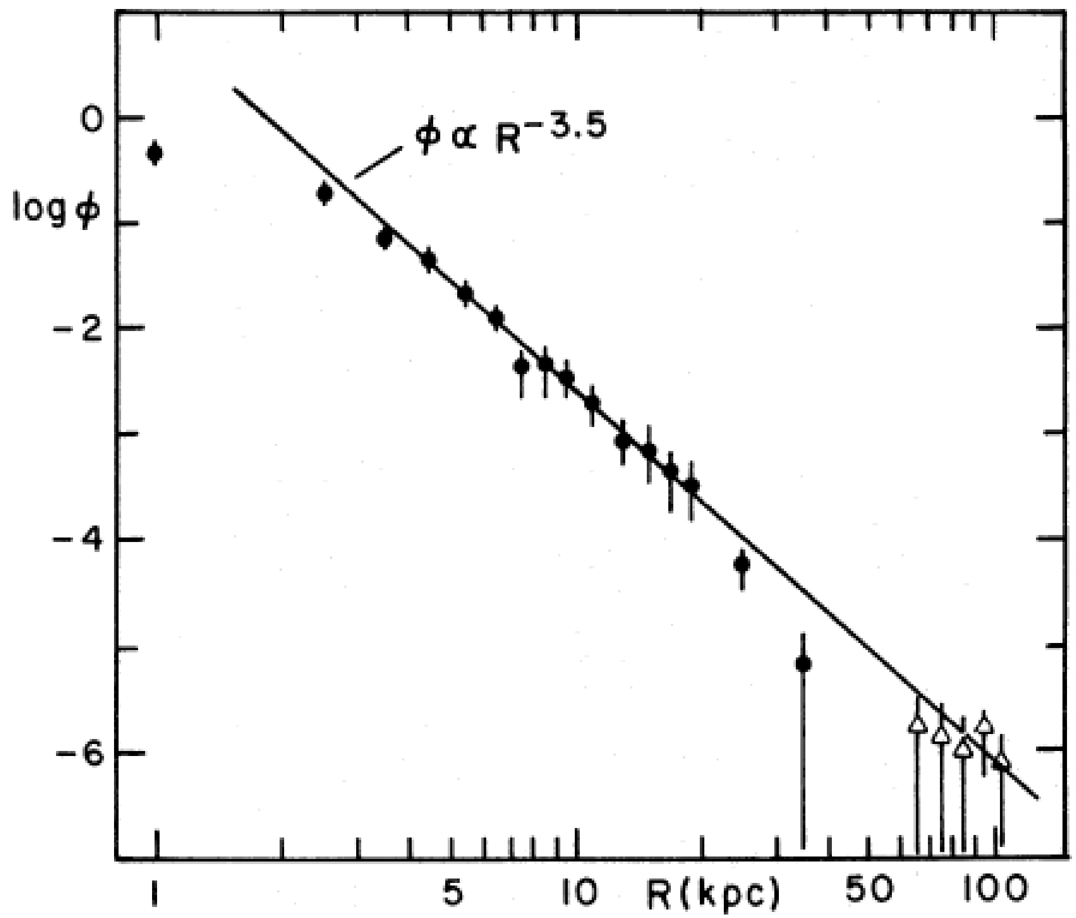
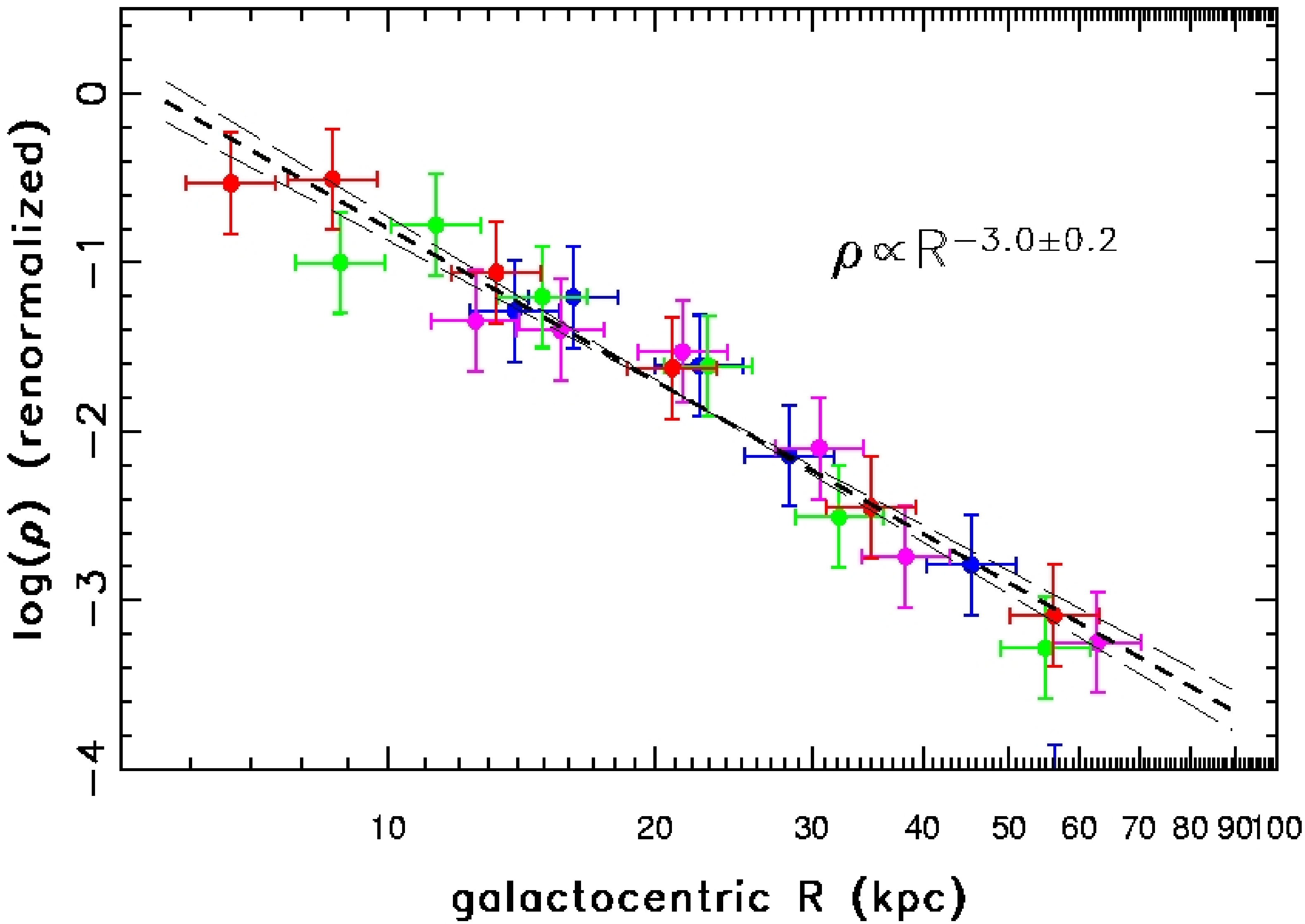
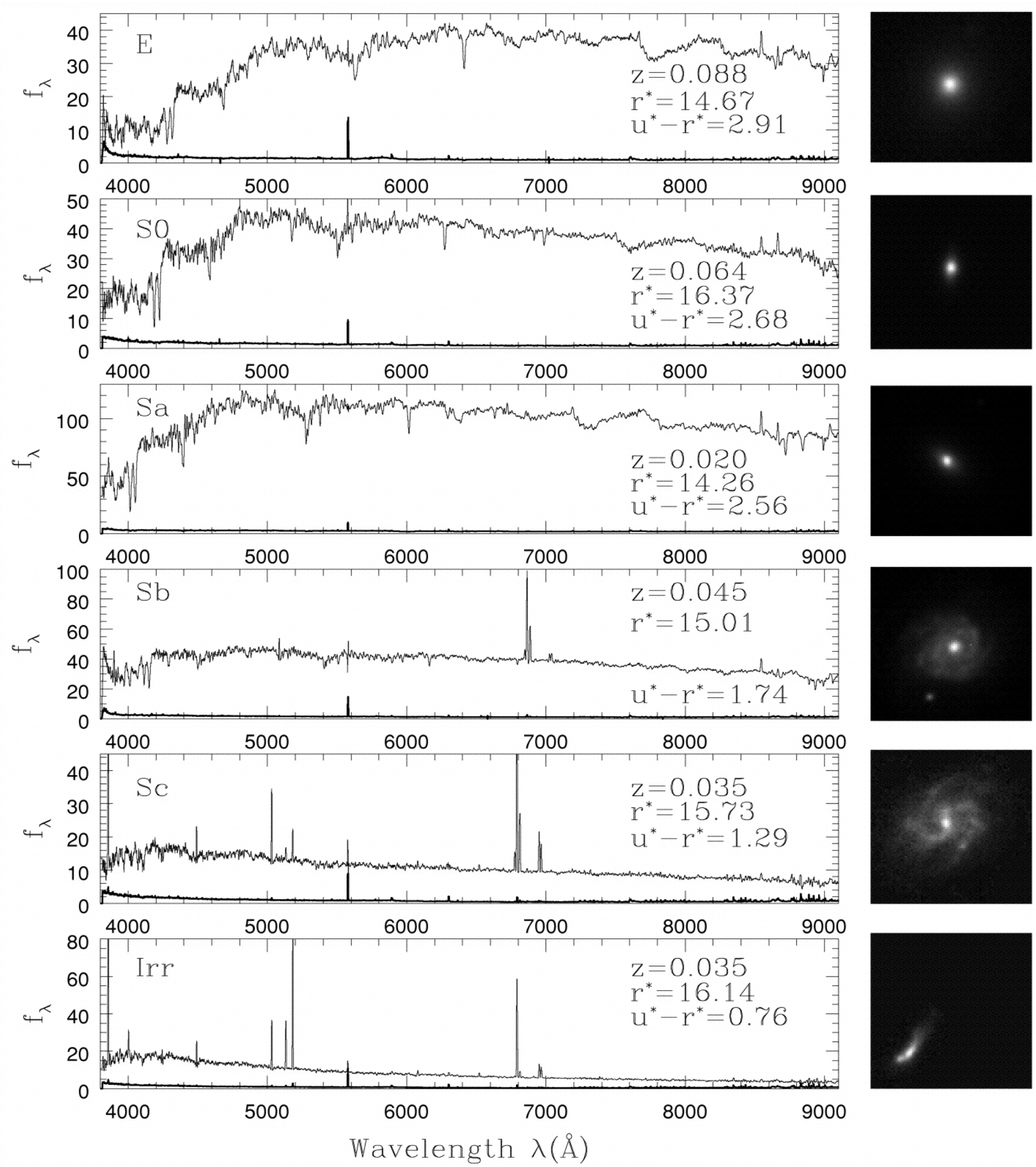


FIG. 2.—The number of clusters per cubic kiloparsec (ϕ) is plotted against galactocentric distance (R). The solid circles represent the clusters with $|Z| < 20$ kpc; the open triangles represent the clusters with $|Z| > 37$ kpc. There are no clusters in the zone $33 < R < 60$ kpc.

Galactic number density distribution of globular clusters from Zinn (1985)

RR Lyrae from Ivezić et al. (2003)





- Spectra are correlated with morphology
- Galaxies with weak blue flux tend to be ellipticals (consistent with conclusions based on colors, of course)
- Galaxies with emission lines tend to be spiral galaxies (though not all)
- Both AGNs and star-forming galaxies show emission lines: How do we separate AGNs from star-forming galaxies? Using the emission line ratios (which are also correlated with colors and shapes)

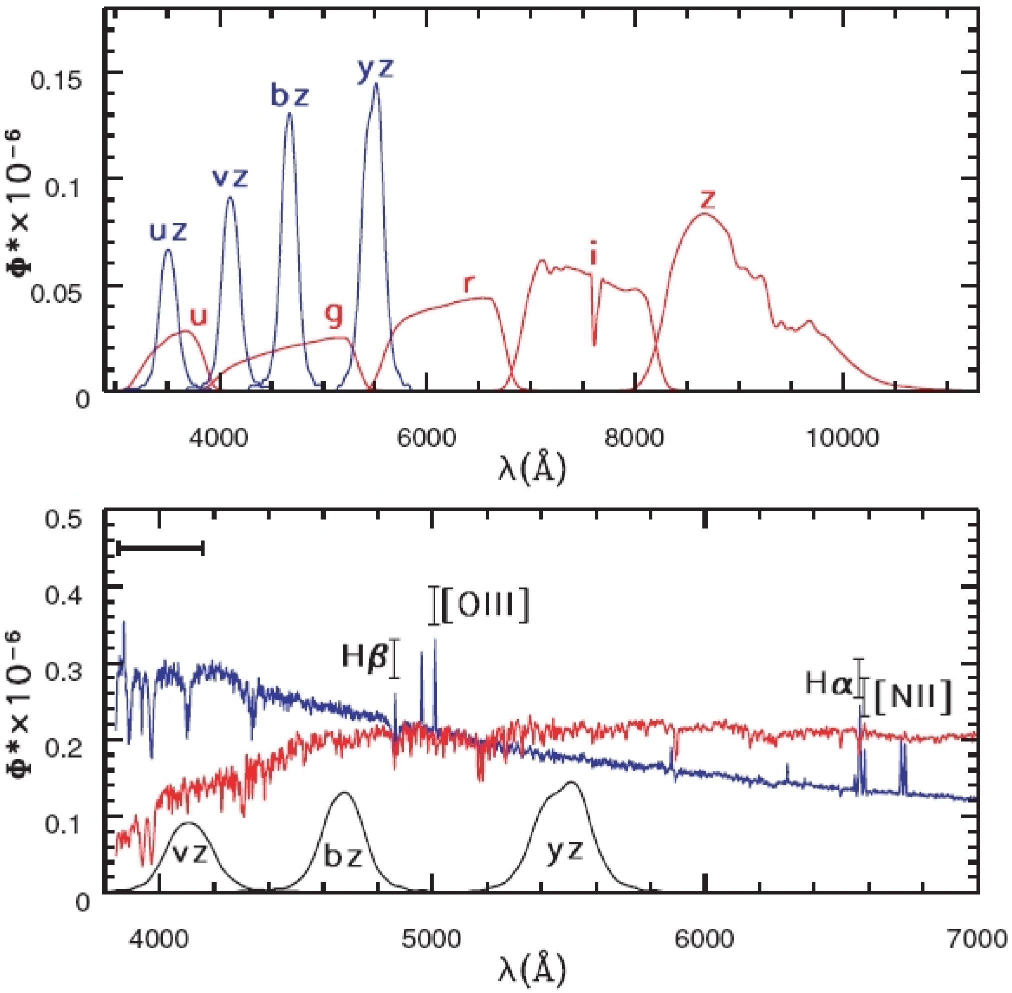


Figure 1. The top panel shows renormalized (see equation 3) filter transmission curves for the SDSS photometric system (*ugriz*), and for the Strömgren photometric system. The bottom panel emphasizes the 3800–7000 Å region. The two spectra are typical for blue and red galaxies, and the four labelled emission lines are used to separate star-forming from AGN galaxies. The horizontal bar in the top left-hand corner marks the wavelength region used in the analysis by Kauffmann et al. (2003).

- Star formation vs AGN
- It is not easy to distinguish galaxies with active star formation from those that harbour an AGN (active galactic nuclues; a.k.a. black hole with an accretion disk)
- We can use the emission line strength to separate them: H_α, H_β, $[NII]$, $[OIII]$
- Physical origin: AGN have power-law spectra, so they have more UV photons than even the hottest stars; as a result, for a given $[OIII]/H_\beta$ ratio, AGNs have larger $[NII]/H_\alpha$ ratio than star-forming galaxies

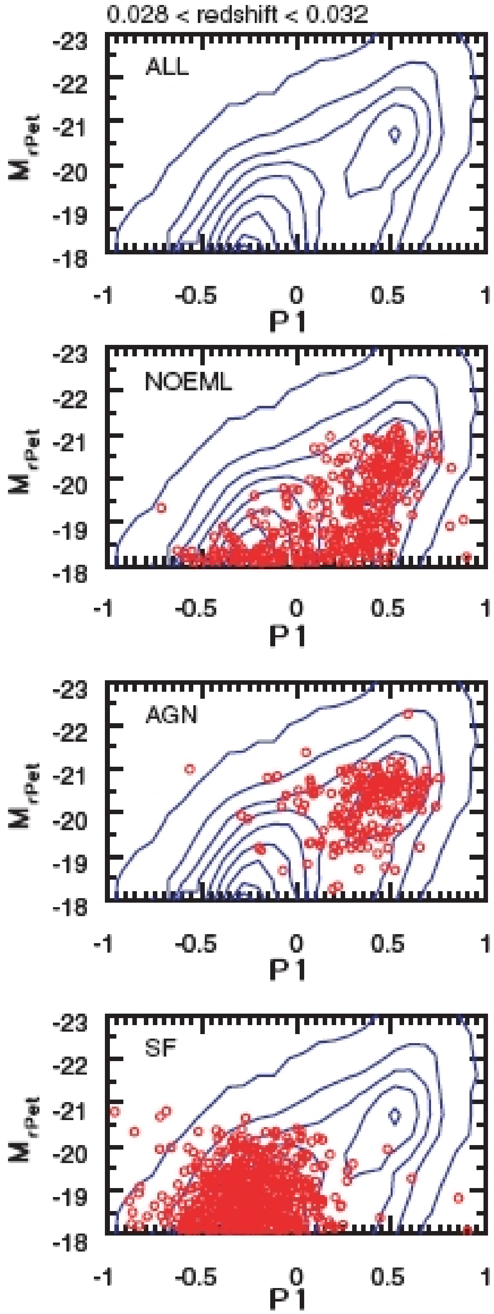
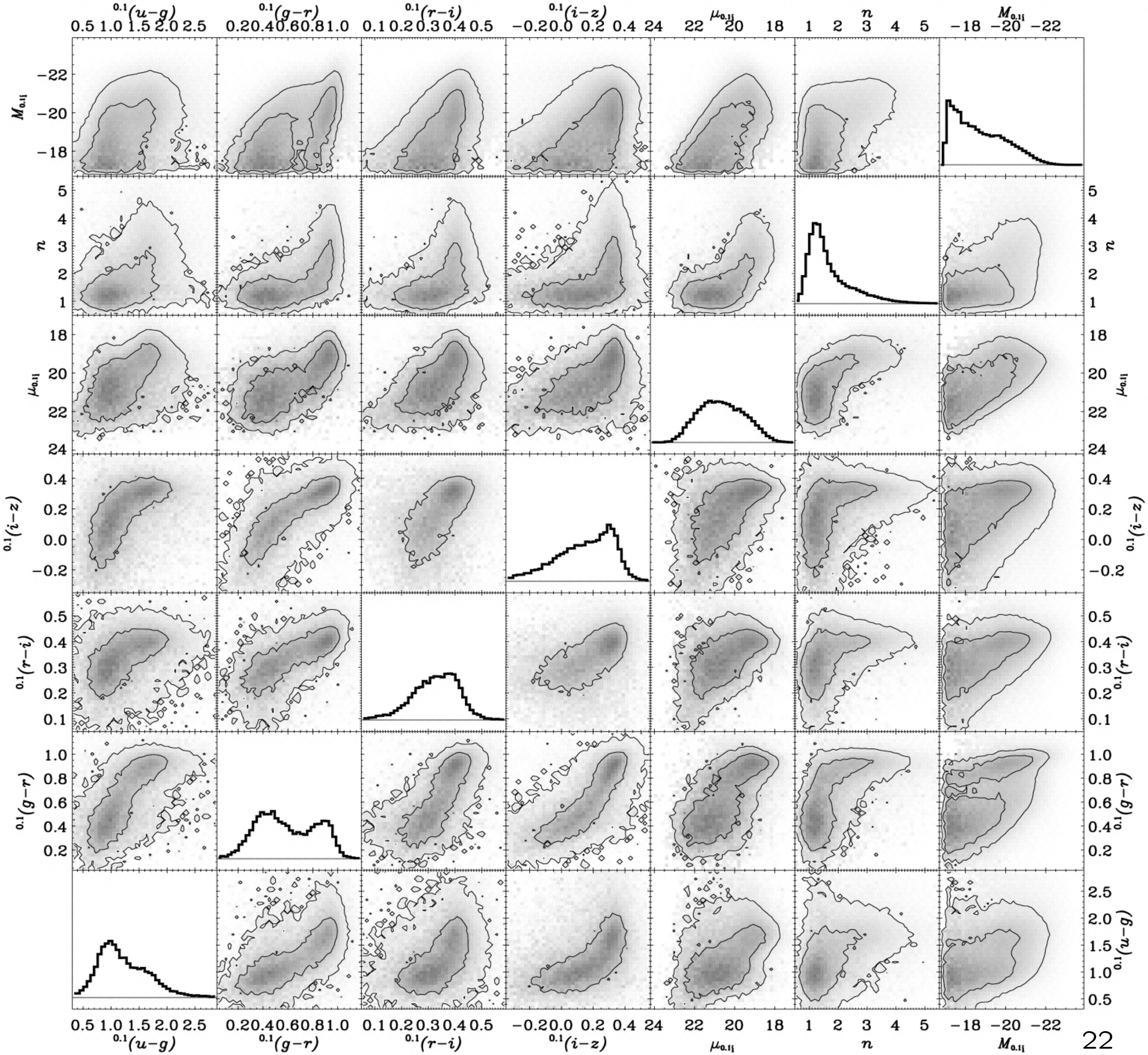
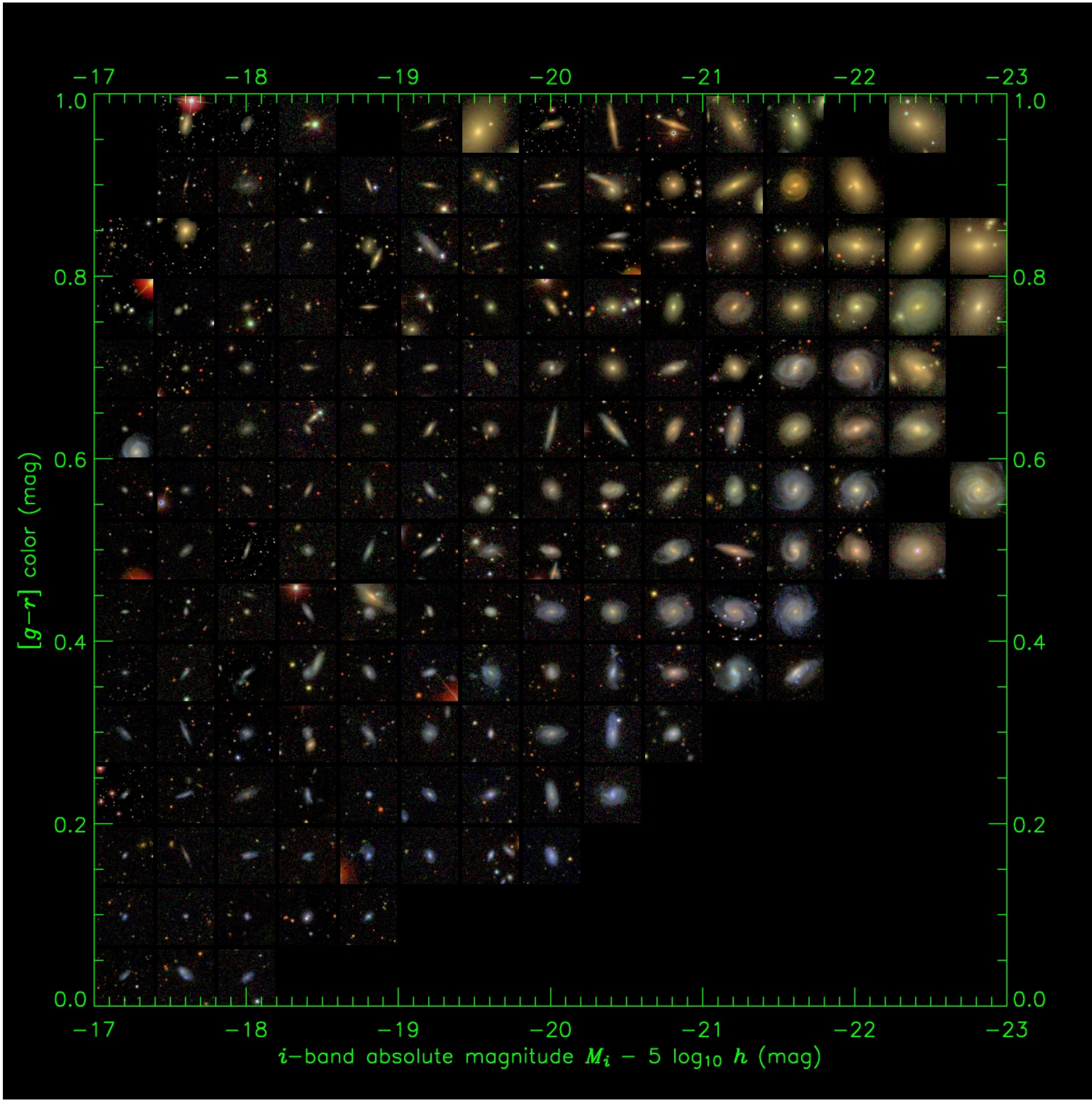


Figure 21. The colour–absolute magnitude diagram for SDSS main galaxies. The top panel shows the absolute r-band magnitude as a function of the $P1$ rest-frame Strömgren colour for galaxies with $0.028 < z < 0.032$. The remaining panels compare the distribution of all galaxies (contours) to the distributions of three subsamples (symbols) selected using emission lines: galaxies without emission lines in the upper middle panel, AGN galaxies in the lower middle panel, and star-forming galaxies in the bottom panel.

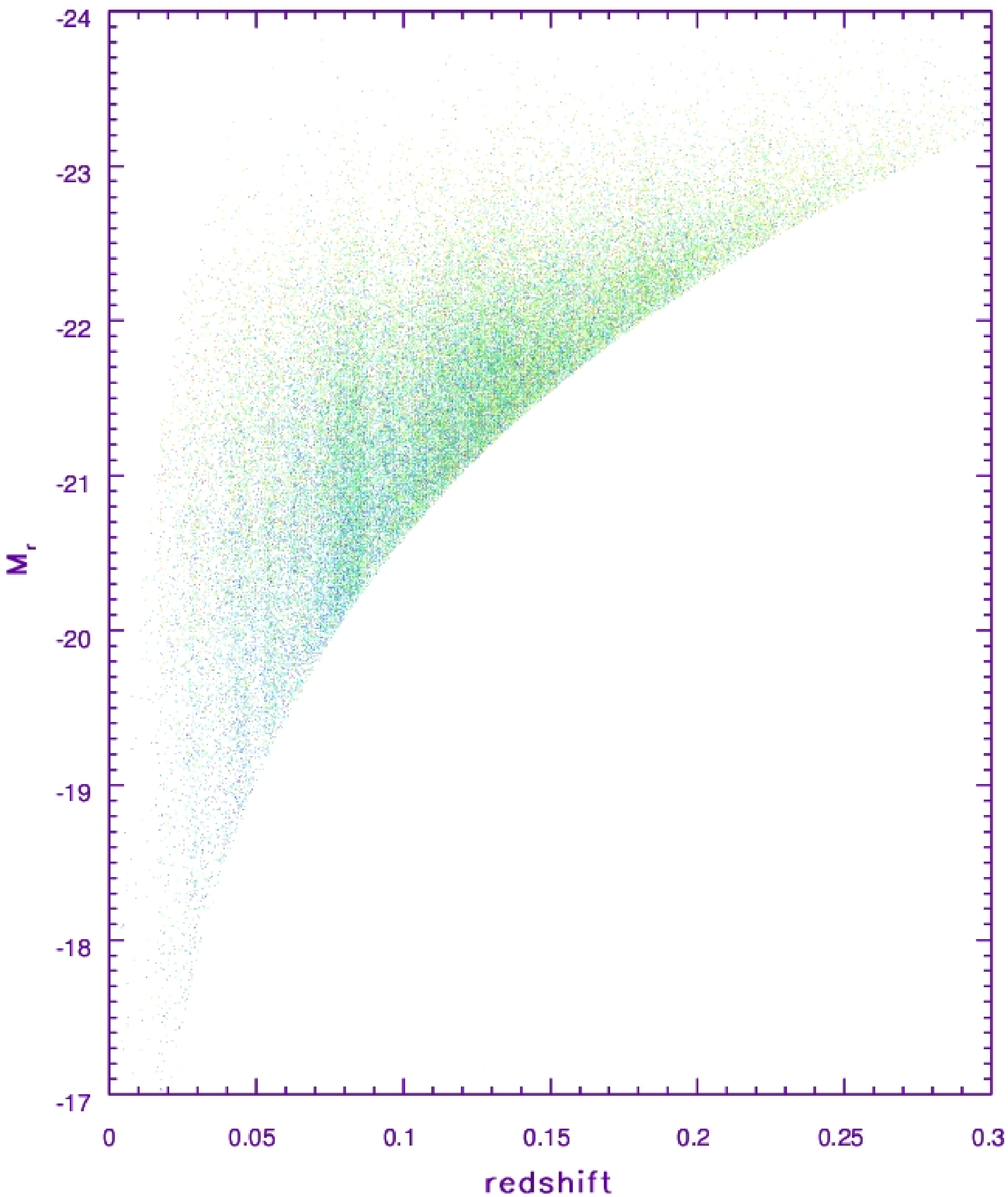
- Correlations of Galaxy Parameters
- Many physical parameters are correlated with each other; for example, the luminosity, concentration of the light profile, and spectral line strengths are correlated with colors
- In the color-color space, galaxies form a very thin locus: the SEDs of galaxies are nearly one-dimensional family (at the level of $\sim$0.02 mag)
- Next page: SDSS sample from Blanton et al. (2003); the quantities are $u-g$, $g-r$, $r-i$ and $i-z$ colors, surface brightness, Sersic index, and absolute magnitude in the r band; the grayscale plots show galaxy distribution in 2D diagrams, together with distributions of each individual quantity (histograms).



$^{0.1}(u-g)$
$^{0.1}(g-r)$
$^{0.1}(r-i)$
$^{0.1}(i-z)$
$\mu_{0.1i}$
n
$M_{0.1i}$



−17 −18 −19 −20 −21 −22 −23
1.0 0.8 0.6 0.4 0.2 0.0
[g−r] color (mag)
i−band absolute magnitude $M_i - 5 \log_{10} h$ (mag)



Luminosity Function

- Luminosity Function is the distribution in the luminosity–position plane; how many galaxies per unit interval in luminosity and unit volume (or redshift): $\Psi(M, z)$
- Imagine a tiny area with the widths ΔMr and Δz centered at some Mr and z in the plot to the left: count the number of galaxies, divide by $\Delta Mr \Delta z$, and correct for the fraction of sky covered by your survey: this gives you $\Psi(M, z)$.

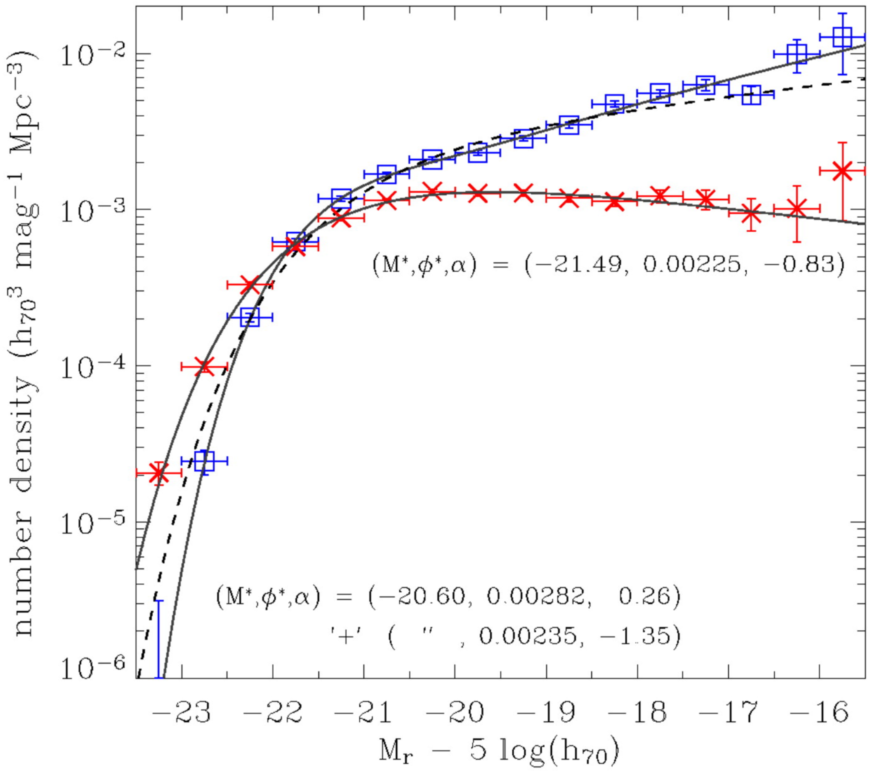
Luminosity Function

- Luminosity Function is the distribution in the luminosity–position plane; how many galaxies per unit interval in luminosity and unit volume: $\Psi(M, z)$

- Often, this is a separable function: $\Psi(M, z) = \Phi(M)\, n(z)$, where $\Phi(M)$ is the absolute magnitude (i.e. luminosity) distribution, and $n(z)$ is the number volume density.

- Luminosity is a product of flux and distance squared (ignore cosmological effects for simplicity): $L = 4\pi D^2 F$

- The samples are usually *flux-limited* (meaning: all sources brighter than some flux limit are detected) – the minimum detectable luminosity depends on distance: $L > 4\pi D^2 F_{min}$, or for absolute magnitude $M < M_{max}(D)$ (c.f. the first plot)



The dependence of LF on galaxy type

- The comparison of LFs for blue and red galaxies (from Baldry et al. 2004, ApJ, 600, 681-694)
- The red distribution has a more luminous characteristic magnitude and a shallower faint-end slope, compared to the blue distribution
- The transition between the two types corresponds to stellar mass of $\sim 3 \times 10^{10}$ $M_\odot$
- The differences between the two LFs are consistent with the red distribution being formed from major galaxy mergers.

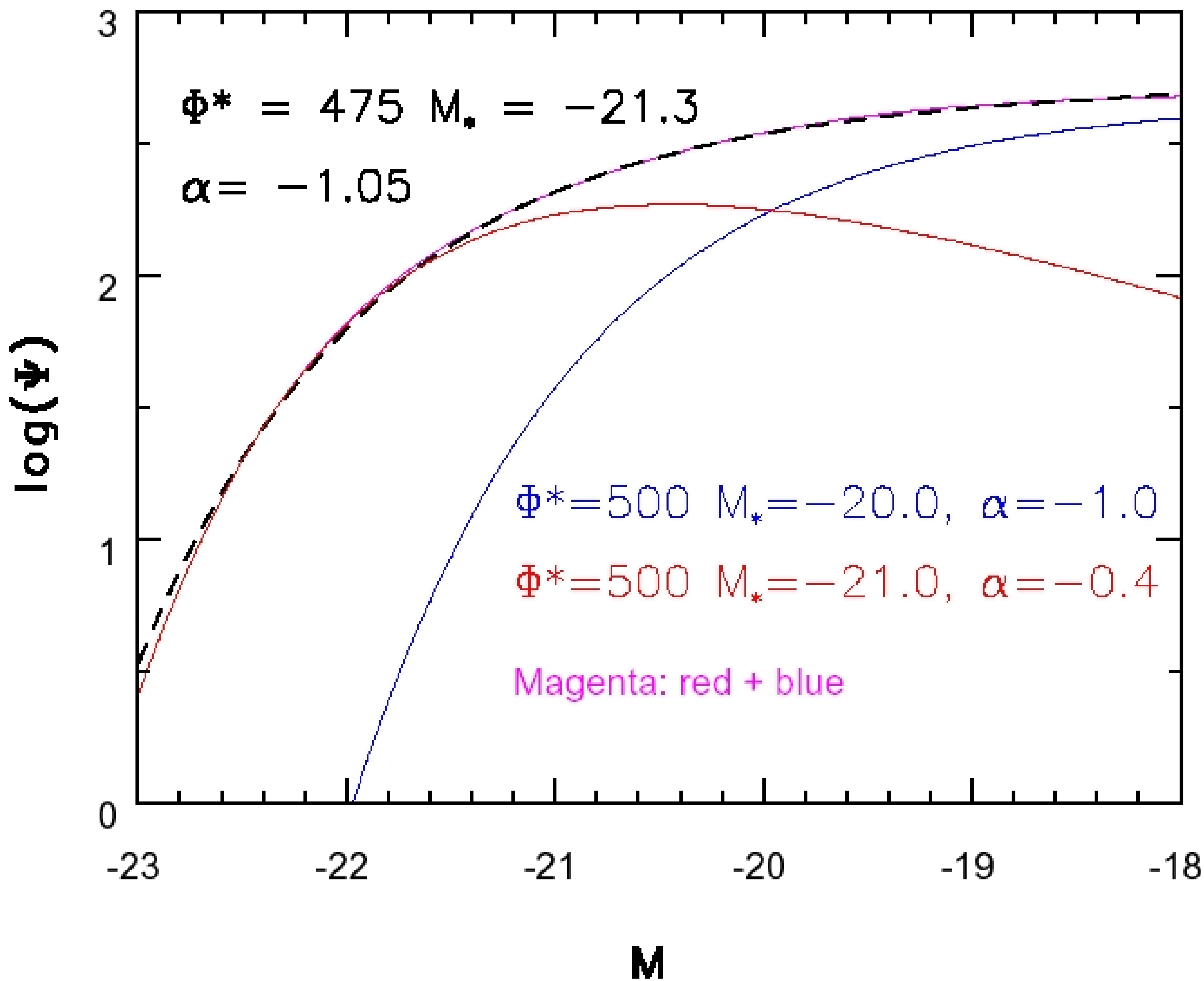
Schechter Function

Galaxy luminosity distribution resembles a power-law, with an exponential cutoff. This distribution is usually modeled by Schechter function:

$$\Phi(L) = \Phi^* \left(\frac{L}{L_*}\right)^{\alpha} \mathrm{e}^{-L/L_*} \tag{6}$$

Or using absolute magnitudes:

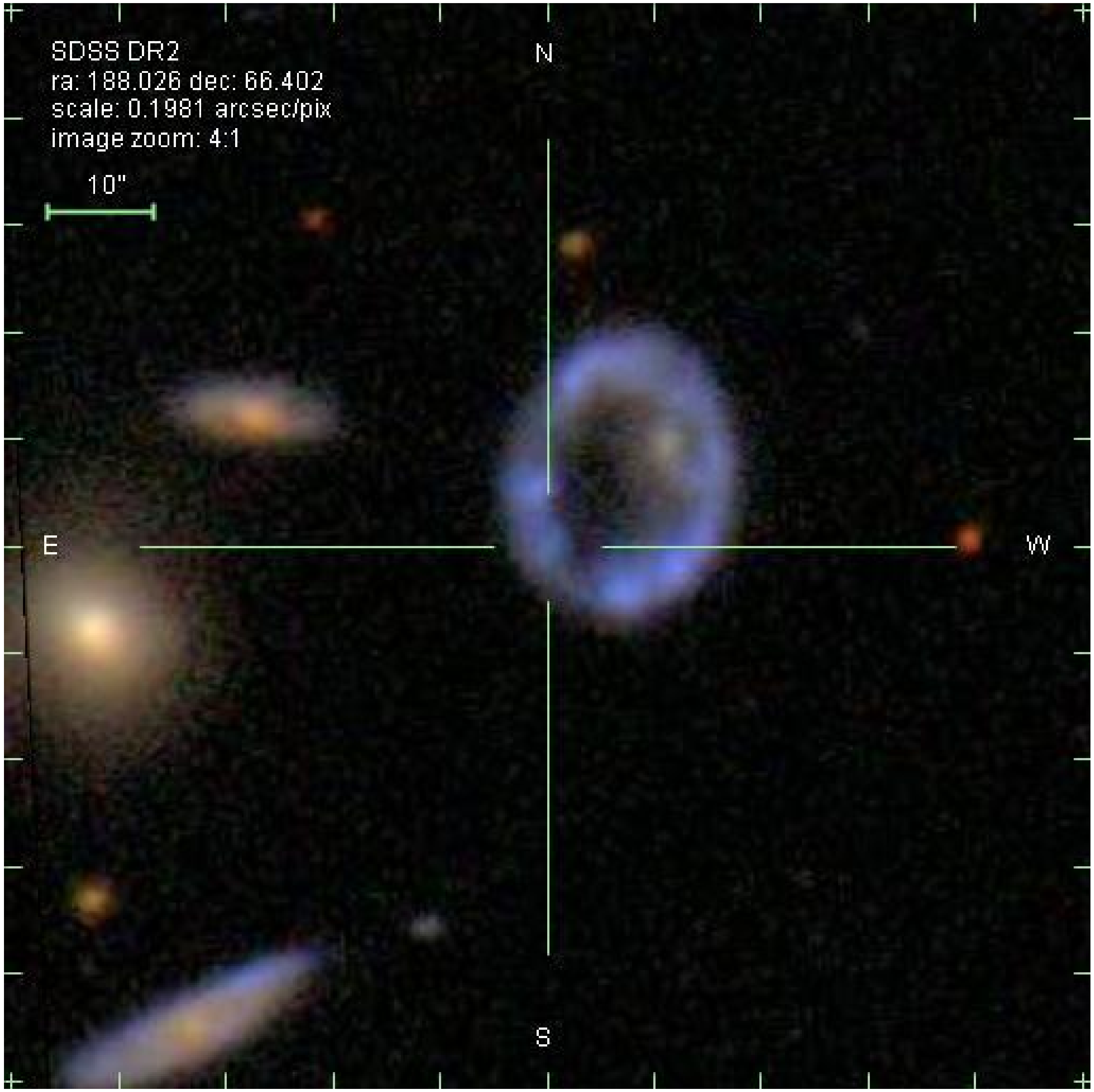
$$\Phi(Mr) = 0.4\Phi^* \, \mathrm{e}^{-0.4(\alpha+1)(Mr-M^*)} \, \mathrm{e}^{-\mathrm{e}^{-0.4(Mr-M^*)}} \tag{7}$$


Φ* = 475 M* = −21.3
α= −1.05
Φ*=500 M*=−20.0, α=−1.0
Φ*=500 M*=−21.0, α=−0.4
Magenta: red + blue
log(Ψ)
M
3
2
1
0
-23
-22
-21
-20
-19
-18

Spiral Galaxies

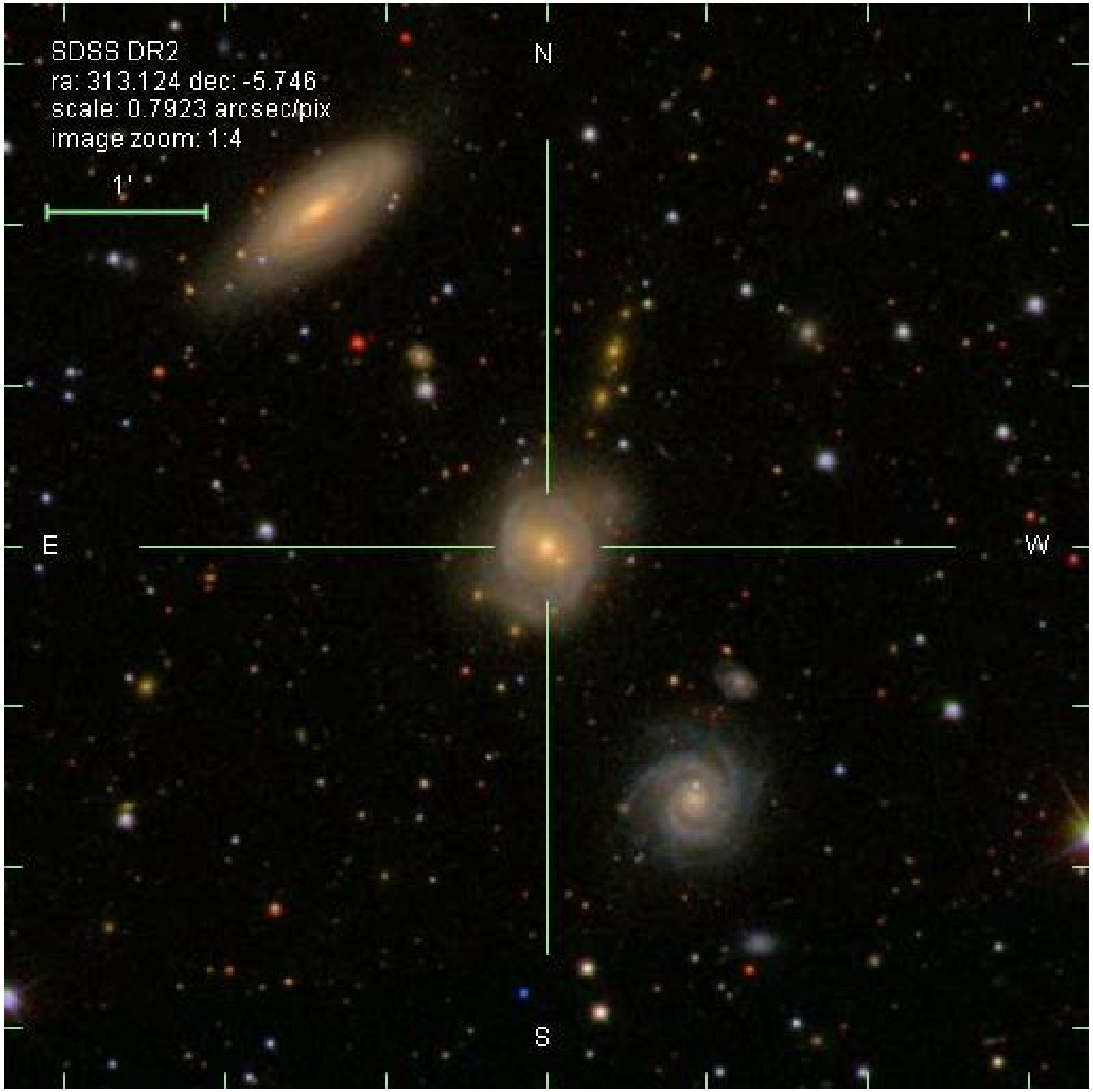
1. Spiral Arms and Disk Instabilities

2. Dynamics of Spiral Galaxies

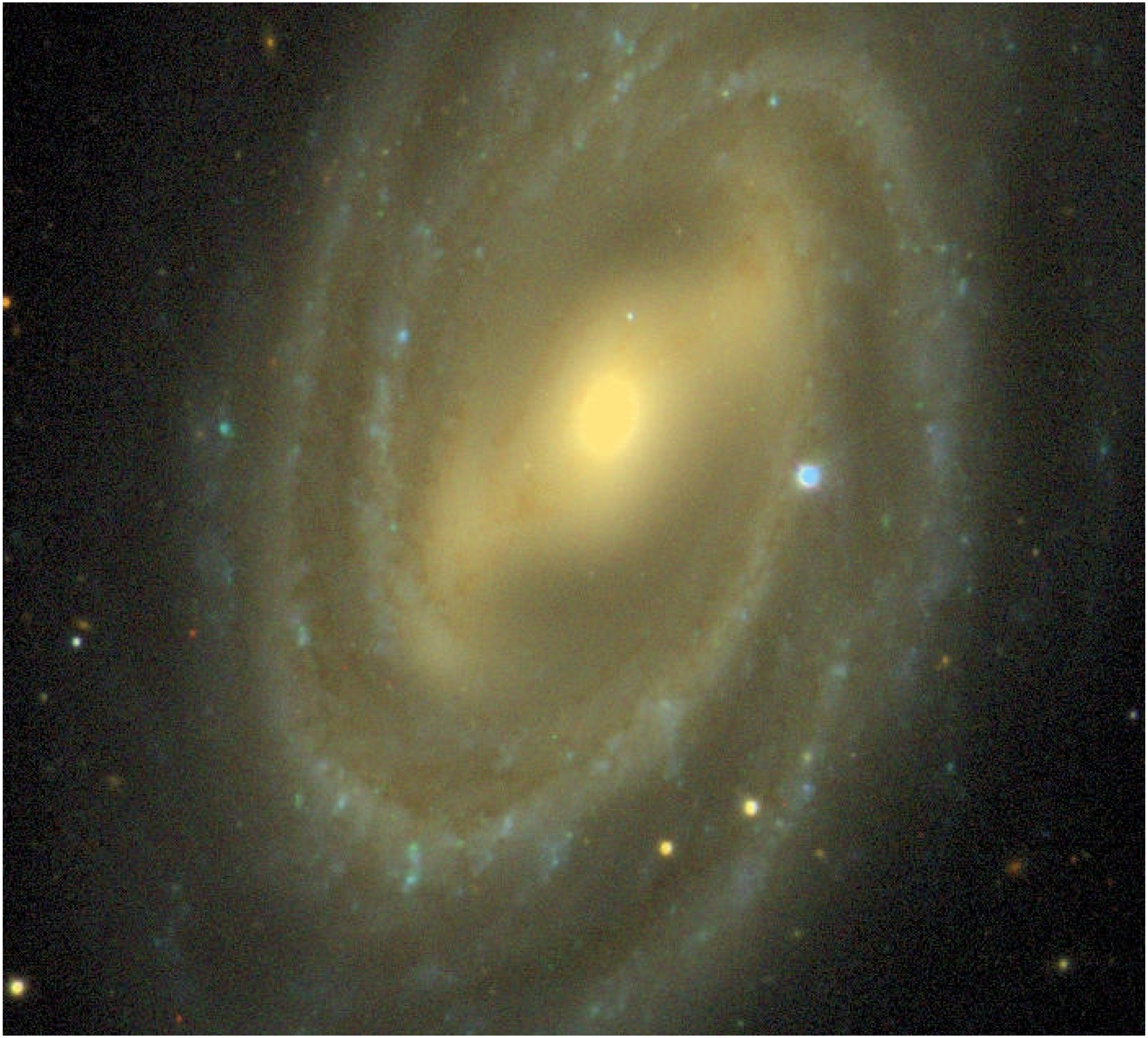


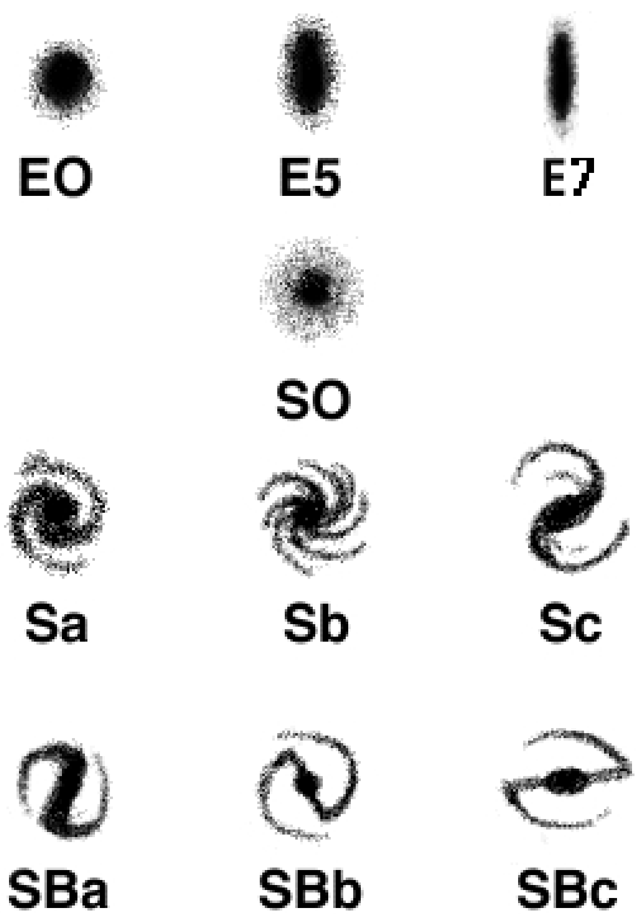
Disk instabilities come in many shapes and forms! The spiral structure is arguably the most beautiful disk instability.

Lord Rosse in 1845 “discovered” spiral structure in M51 (this is an HST image of M51)



Not all spirals are alike!





Hubble's Morphological Classification

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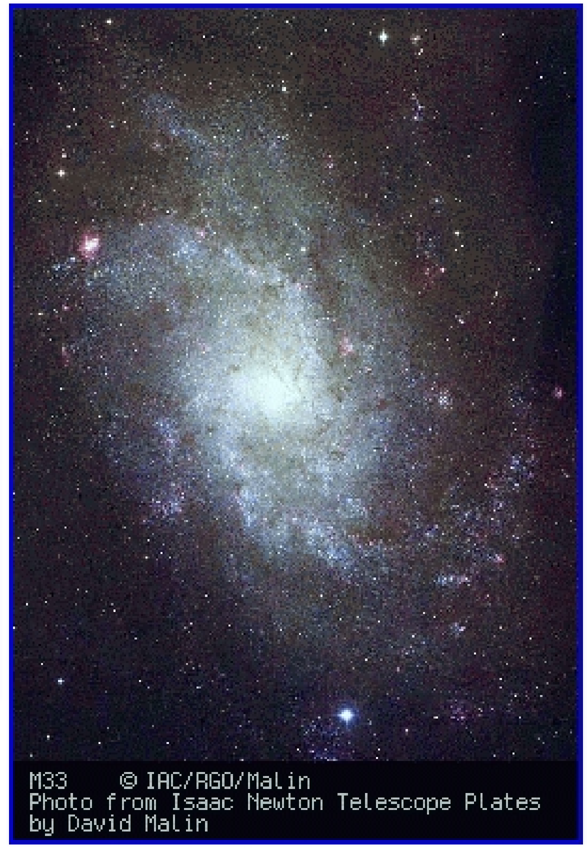
- The number and how tightly the spiral arms are wound are well correlated with other, large scale properties of the galaxies, such as the luminosity of the bulge relative to the disk and the amount of gas in the galaxy. This suggests that there are global physical processes involved in spiral arms.

A ``grand-design''spiral seen face-on.



A flocculent spiral with ragged spiral arms.

In addition to Hubble's classification, there are different types of spiral structure: grand design spirals, with clearly outlined and well organised globally correlated spiral structure, and flocculent (fluffy) spirals with many small short globally uncorrelated spiral arms

Theories of Spiral Structure

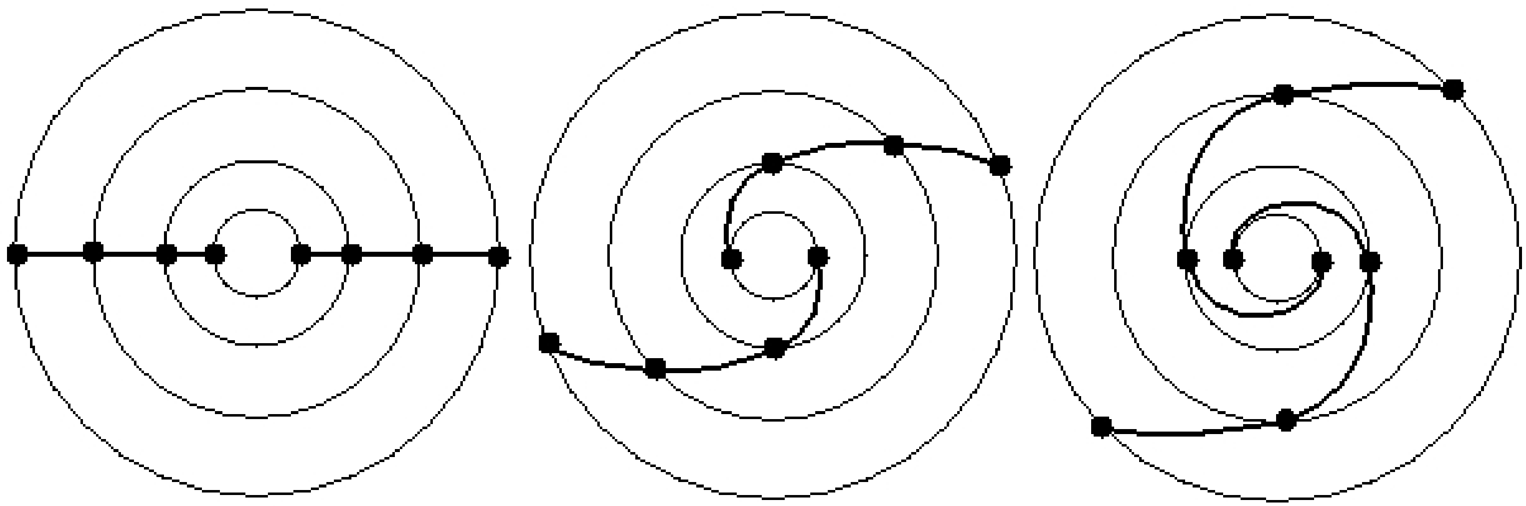
Despite 50 years of work, spirals are not very well understood. It seems clear now that the spiral structure of galaxies is a complex problem without any unique and tidy answer.

Differential rotation clearly plays a central role, as well as global instabilities, stochastic spirals, and the shocks patterns that can arise in shearing gas disks when forced by bars.

There are (at least) two popular theories, one of which is more commonly used to explain grand design spirals, the other for flocculent spirals.

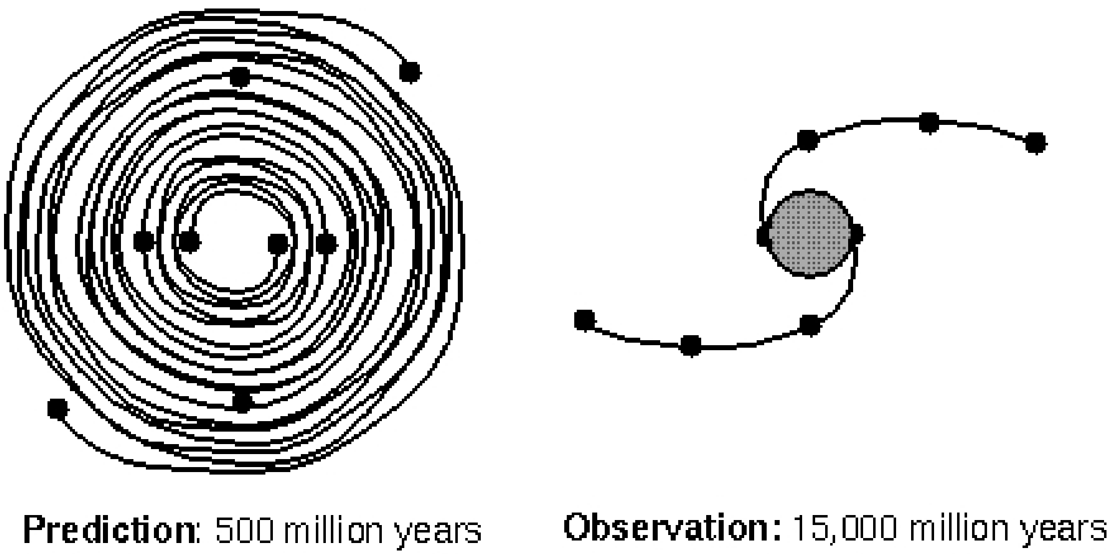
But before proceeding: **winding problem** (Lindblad)

Winding problem



Differential rotation: stars near the center take less time to orbit the center than those farther from the center. Differential rotation can create a spiral pattern in the disk in a short time.

Winding problem



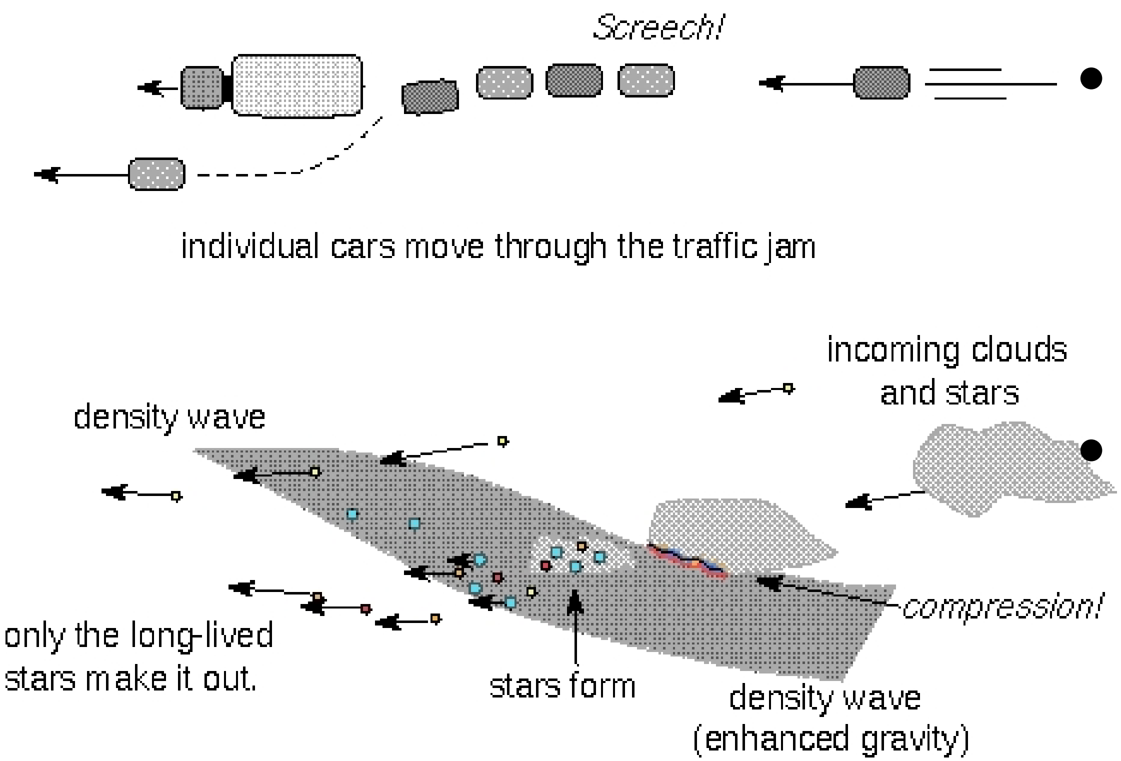
The problem: most spiral galaxies would be tightly wound by now, which is inconsistent with observations.

Spiral arms cannot be a static structure (i.e. at different times, arms must be made of different stars)

Density Wave theory

C.C. Lin & F. Shu (1964-66)

- This is the preferred model for grand design spirals.
- The spiral arms are over-dense regions which move around at a different speed than star: stars thus move in and out of the spiral arm
- How these density waves are set up is unclear, but it may have to do with interactions. Once they are set up, they must last for a long enough time to be consistent with the observed number of spiral galaxies



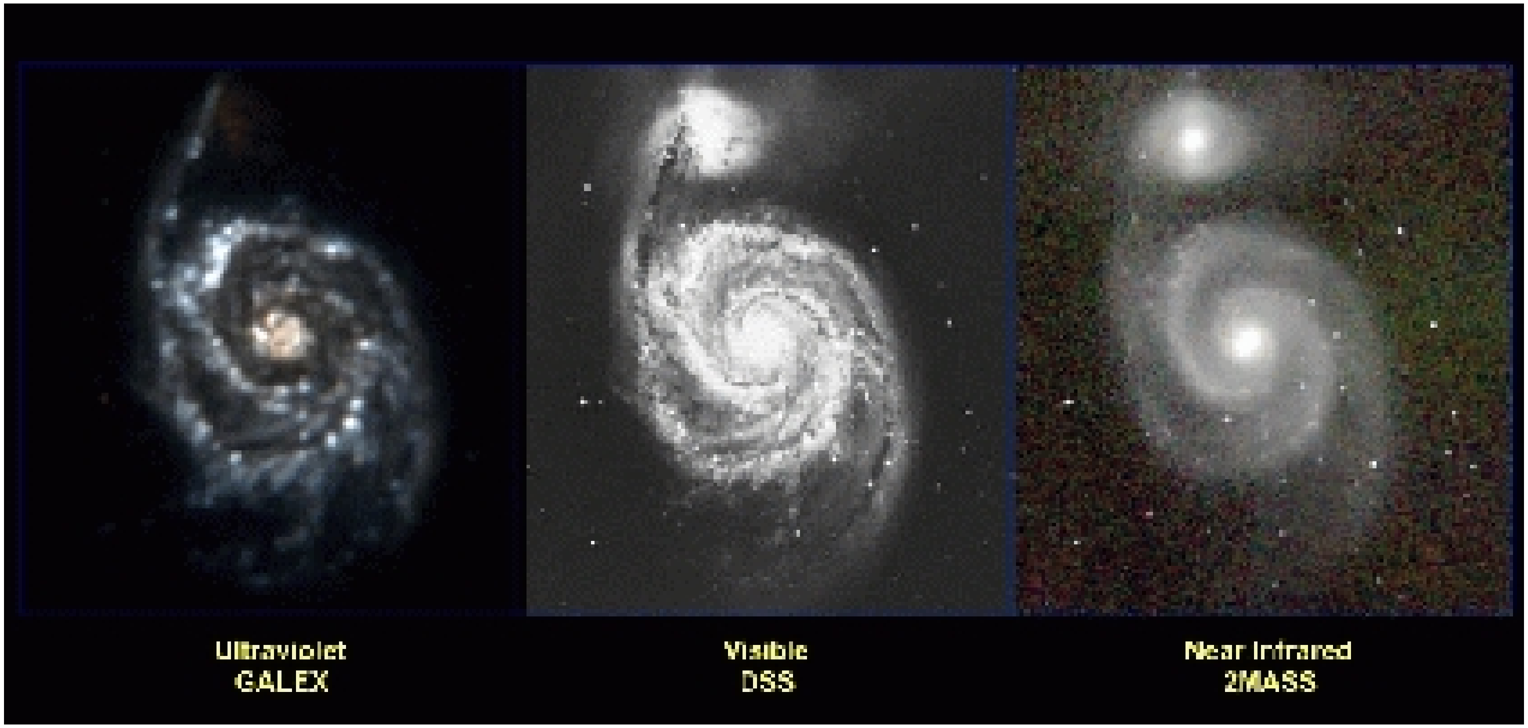
Spiral density waves are like traffic jams. Clouds and stars speed up to the density wave (are accelerated toward it) and are tugged backward as they leave, so they accumulate in the density wave (like cars bunching up behind a slower-moving vehicle). Clouds compress and form stars in the density wave, but only the fainter stars live long enough to make it out of the wave.

Stochastic Self-Propagative Star Formation

- This model probably cannot explain grand design sprials, but it may account for flocculent spiral structure.

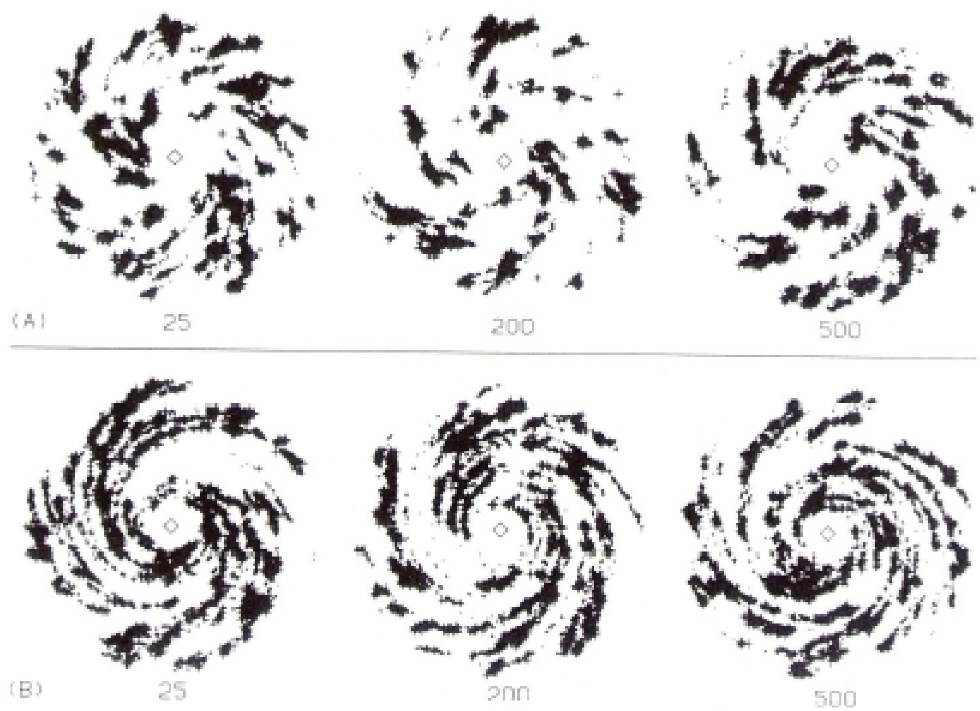
- Ongoing star formation triggers star formation in areas adjacent to it. As the galaxy rotates, differential rotation leads to the appearance of a spiral pattern.

Spiral arms are made of short-lived massive blue stars!



Note that the smaller galaxy (NGC 5195) is not visible in GALEX image (left)

The spiral structure is associated with (short-lived) hot stars.



Numerical simulations of stochastic self-propagating star formation in spirals (Gerola & Seiden 1978, ApJ 223, 129). Originally roundish star formation regions get stretched out in differential rotation. Numbers give the time in units of 15 Mio. years, the upper panel is for the rotation curve of M101, the lower for M81.

The Sombrero Galaxy (VLT ANTU + FORS1)

ESO PR Photo 07a/00 (22 February 2000)

Disks contain a lot of dust! Spiral arms are almost exclusively seen in disks with a lot of gas and dust, unlike bars which are often seen in galaxies without ISM. Bars are not a wave of star formation – they are orbital features.

To remember:

- Spiral arms are not static structure (winding problem)
- Not all spirals are alike: more than one pattern
- The appearance dominated by young luminous blue stars, but the overall density of *all* stars is elevated by 10-20% in spiral arms

Rotation of Stars in the Disks of Spiral Galaxies

- Most stars in spiral galaxies are concentrated in fairly thin disks

- Stars move around the galaxy center – described by the rotation (circular velocity) curve $v_c(R)$

- The shape of rotation curve depends on the distribution of enclosed mass – e.g. for a point mass $v_c(R) \propto 1/\sqrt{R}$

- In general, $v_c(R) = R\, d\Phi(R)/dR$, where Φ is the gravitational potential (Φ follows from the mass density profile via Poisson equation)

- We know the disk light intensity profile; we can assume that mass is following light and predict $v_c(R)$ for an exponential disk; but...

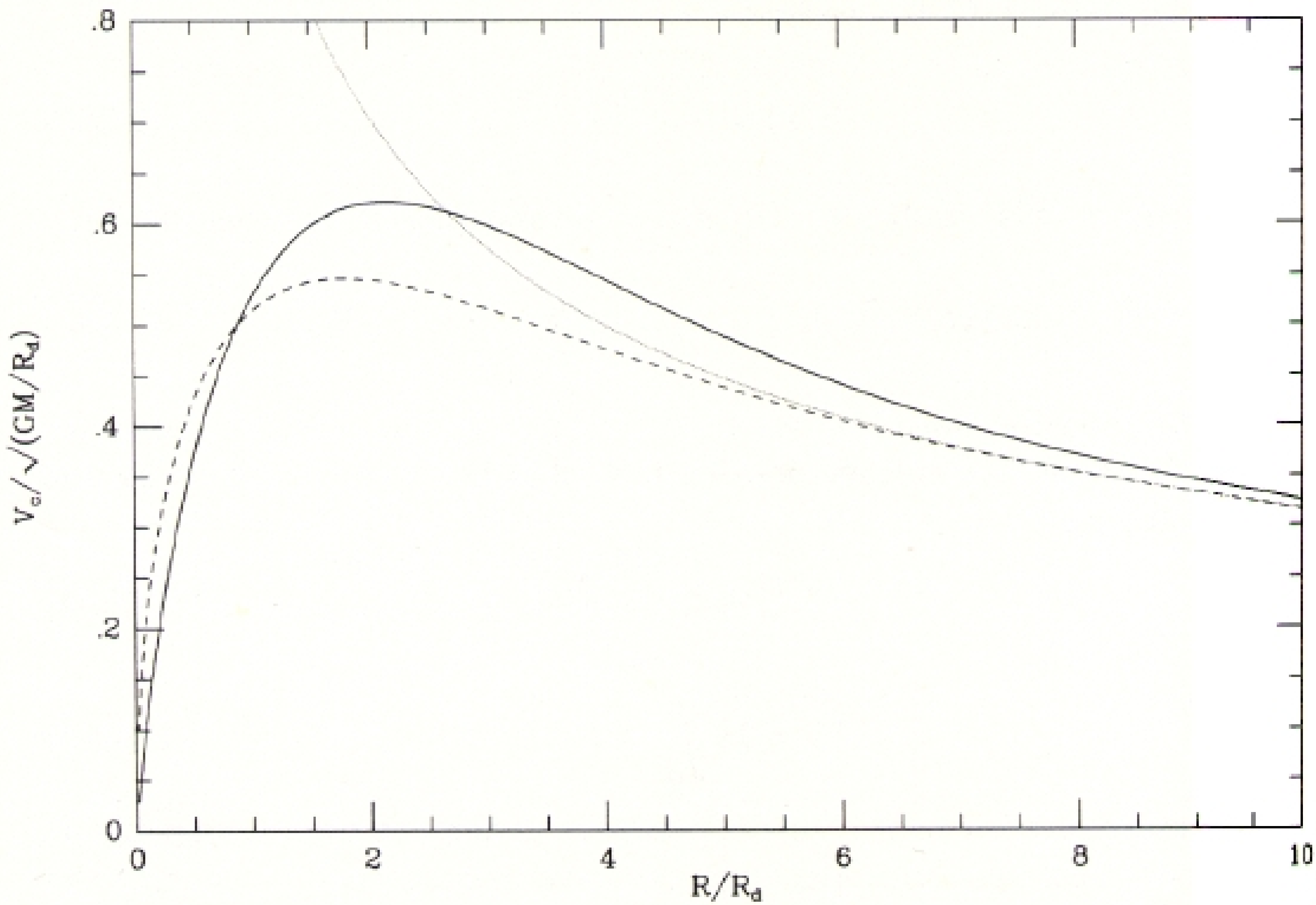


Figure 2-17. The circular-speed curves of: an exponential disk (full curve); a point with the same total mass (dotted curve); the spherical body for which $M(r)$ is given by equation (2-170) (dashed curve).

Rotation of Stars in the Disks of Spiral Galaxies

The prediction for rotation curve in an infinitely thin exponential disk (the previous slide) involves (somewhat) complicated Bessel functions. A much simpler, but still decent approximation is

$$v_c(R) = 0.876\sqrt{\frac{GM}{R_e}}\sqrt{\frac{r^{1.3}}{1+r^{2.3}}} \tag{8}$$

where R_e is the scale length ($I(R) \propto \exp(-R/R_e)$), and $r = 0.533R/R_e$.

Note that for $R >> R_e$, $v_c(R) \propto 1/\sqrt{R}$ (disk "looks" like a point mass)

FYI: if M is measured in solar masses ($M_\odot$), R in pc, v_c in km/s, then the gravitational constant is $G = 233$

What do we get from observations?

Measurements of the Rotation Curve

The circular speed can be determined as a function of radius by measuring the redshift of emission lines of the gas contained in the disk:

Hot stars ionize gas: hydrogen emission lines (e.g. H_α) in the optical

Neutral atomic hydrogen gas: hyperfine structure transition (due to flip in electron spin) leads to 21 cm radio line

H_α measurements

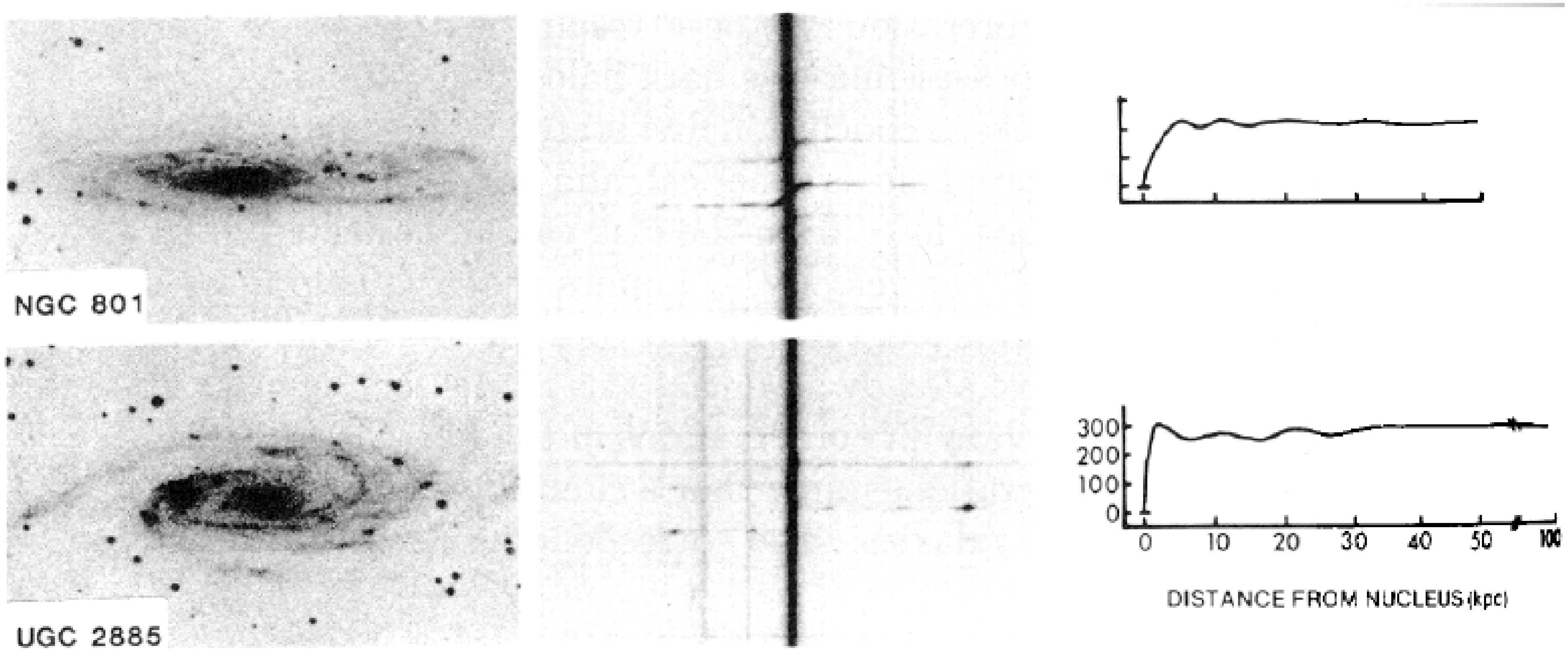


Figure 10-1. Photographs, spectra, and rotation curves for five Sc galaxies, arranged in order of increasing luminosity from top to bottom. The top three images are television pictures, in which the spectrograph slit appears as a dark line crossing the center of the galaxy. The vertical line in each spectrum is continuum emission from the nucleus. The distance scales are based on a Hubble constant $h = 0.5$. Reproduced from Rubin (1983), by permission of *Science*.

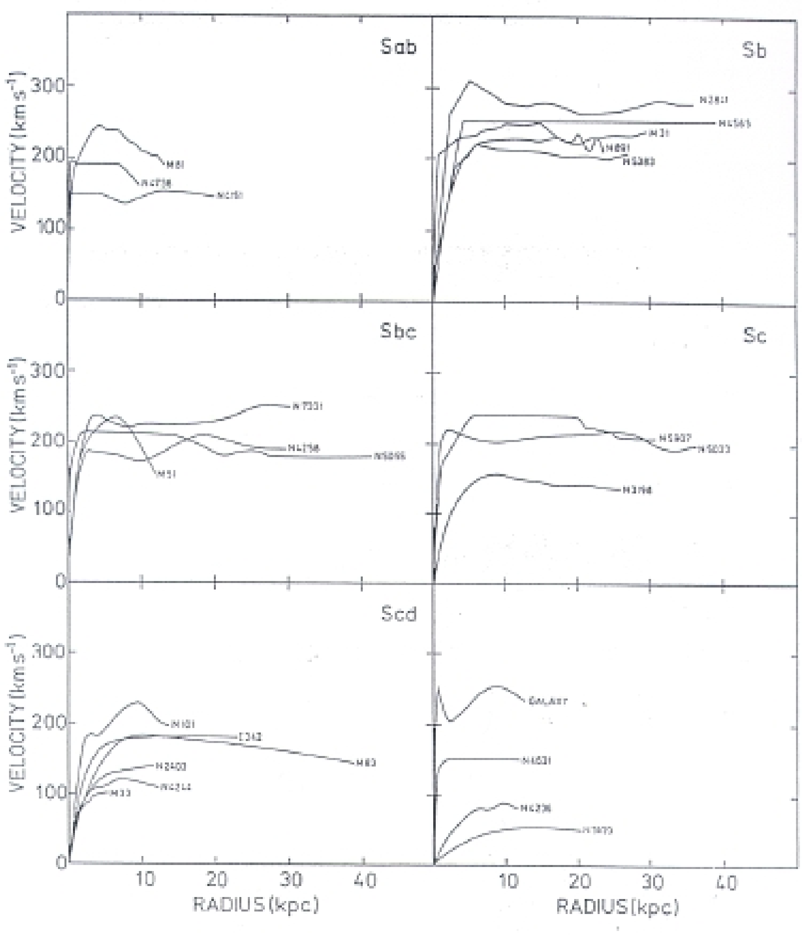


FIGURE 1. Rotation curves of 25 galaxies of various Hubble types.

From A. Bosma's PhD Thesis (1978).

"Flat" rotation curves

- The measurements show that rotation curves are "flat" – they are not approaching the $v_c(R) \propto 1/\sqrt{R}$ behavior expected in the outer parts of disks
- Therefore, there must be an invisible galaxy component that is capable of producing gravitational force
- Earlier (1930's) suggested by Fritz Zwicky, became an accepted view after Rubin's work
- While, in principle, this discrepancy could also be due to a different gravitational law (i.e. force that is not $\propto 1/R^2$), the modern data, including cosmic microwave background measurements, suggest that indeed that is a "dark matter" component contributing ∼5 more gravitational force than stars and gas combined!

A simple analytical model for dark matter halos

$$\rho = \rho_0 \frac{a^2}{r^2 + a^2} \qquad \left(r \gg a \rightarrow \rho \sim r^{-2}\right)$$

$$M(r) = \int_0^r 4\pi\rho r^2 dr \qquad \text{(Bronstein No. 65)}$$

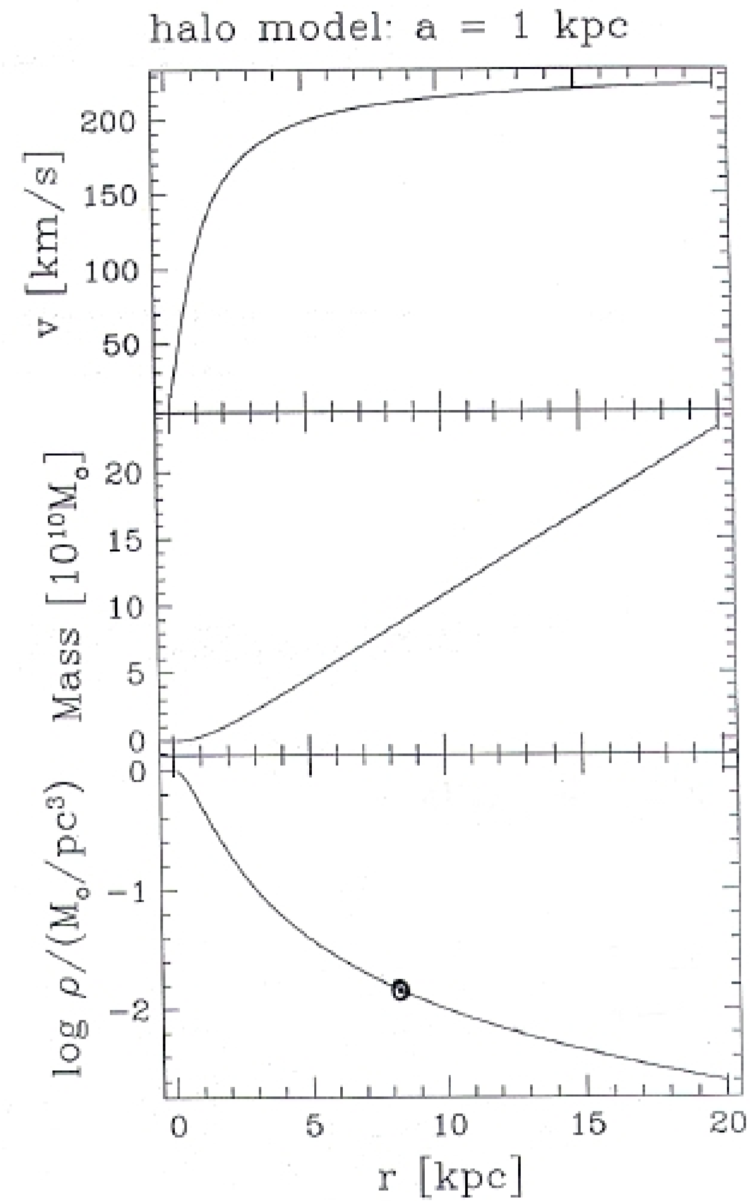
$$= 4\pi\rho_0 a^3 \left(\frac{r}{a} - \arctan\frac{r}{a}\right)$$

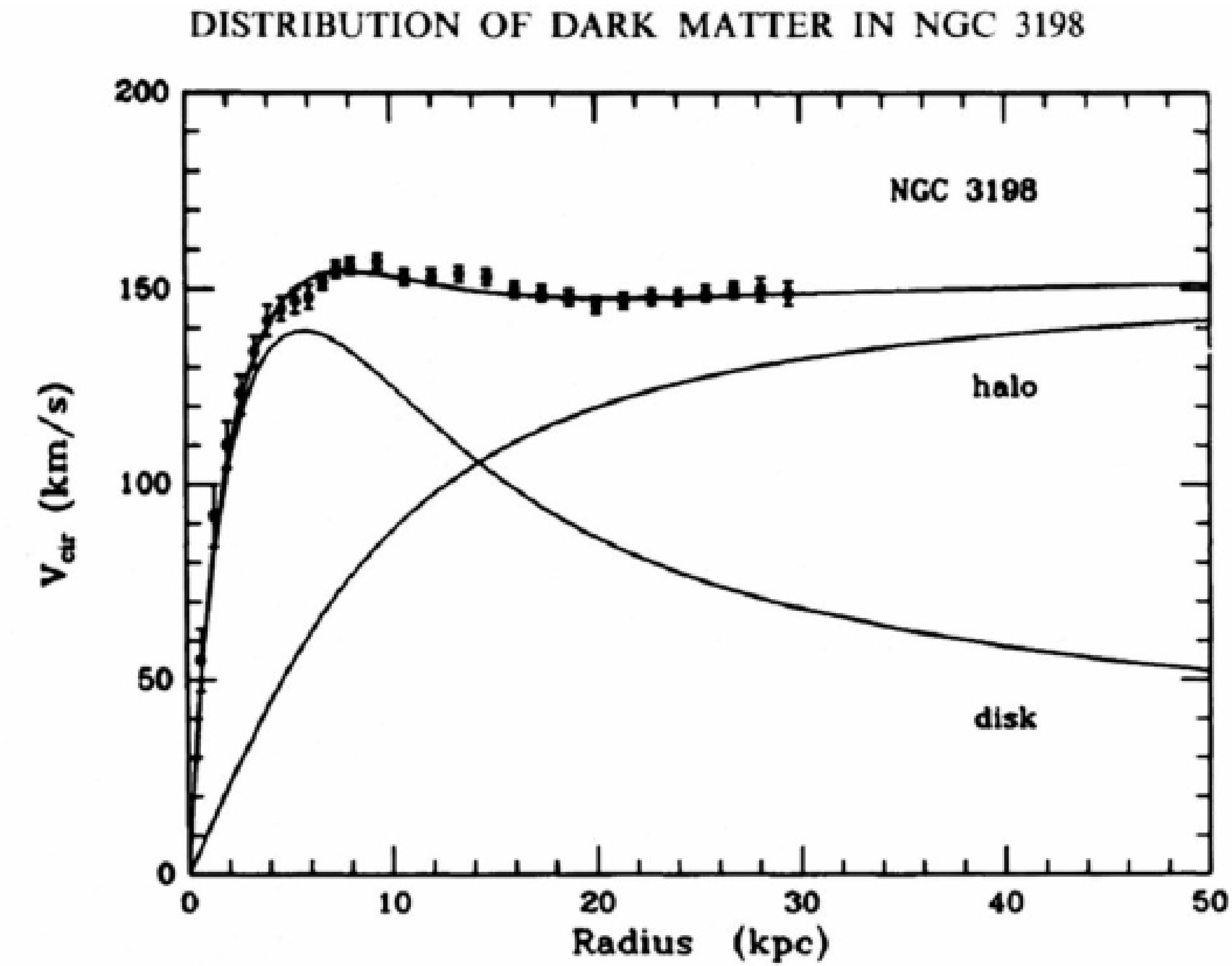
$$\text{circular velocity :} \quad v_c(r) = \sqrt{\frac{GM(r)}{r}}$$

$$v_c(r) = \left[4\pi G\rho_0 a^2 \left(1 - \frac{a}{r}\arctan\frac{r}{a}\right)\right]^{\frac{1}{2}}$$

$$r \gg a : \quad v_c \rightarrow \sqrt{4\pi G\rho_0 a^2} \simeq \text{const}$$

$$r \ll a : \quad v_c \rightarrow \sqrt{\frac{4\pi G\rho_0 a^2}{3}} \cdot \frac{r}{a}$$





see: van Albada et al. ApJ 295, 305 (1985)

$$\Phi = \Phi_{halo} + \Phi_{disc} \Rightarrow v_c^2 = v_{c,halo}^2 + v_{c,disc}^2 \; (v_c^2 = r\frac{\partial\Phi}{\partial r})$$

The Tully-Fisher Relation

The (maximum, the flat part value) rotation velocity is related to galaxy's luminosity:

$$M_B = A \log v_c + B \tag{9}$$

where $A \sim -10$ and $B \sim 3$ depend slightly on galaxy's morphological type.

Another way of expressing the same correlation

$$L \propto {v_c}^{-0.4A} \propto v_c^4 \tag{10}$$

Why? From the virial theorem, $v^2 \propto M/R$. Also, $L \propto IR^2$, and hence

$$L \propto (\frac{M}{L})^{-2} I^{-1} v^4 \tag{11}$$

Since I ~const. (Freeman's law), the Tully-Fisher relation implies that $(\frac{M}{L}) \sim$ const. for spiral galaxies (~30 in the B band, and in solar units)

The Virial Theorem

III. THE VIRIAL THEOREM APPLIED TO CLUSTERS OF NEBULAE

If the total masses of clusters of nebulae were known, the average masses of cluster nebulae could immediately be determined from counts of nebulae in these clusters, provided internebular material is of the same density inside and outside of clusters.

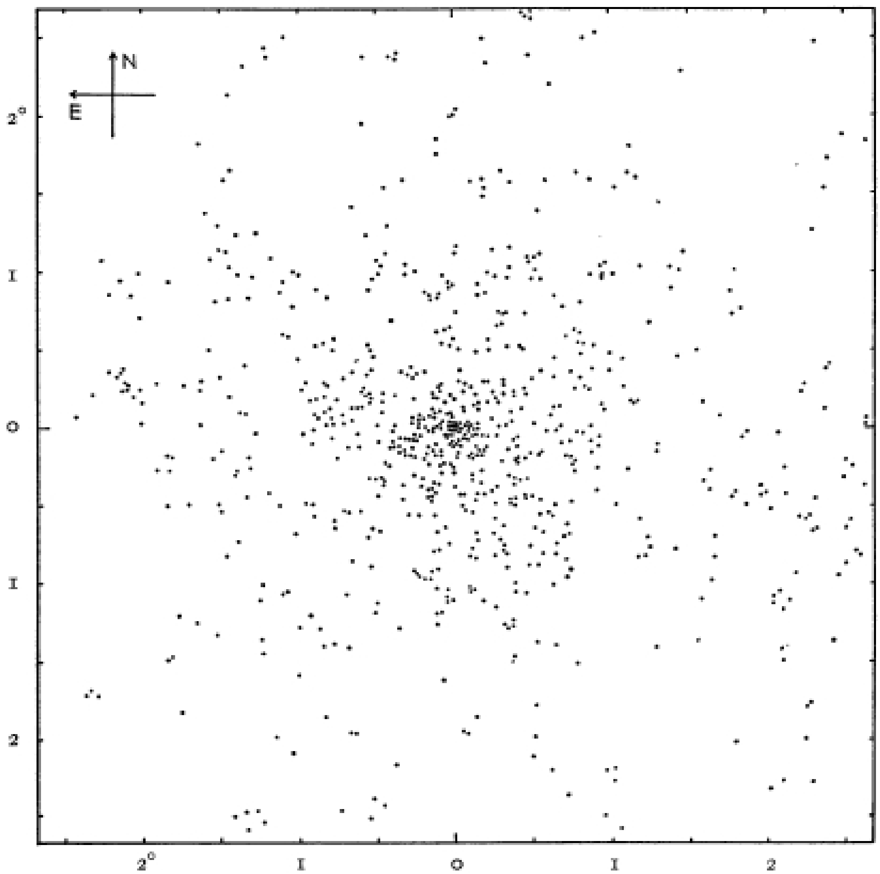
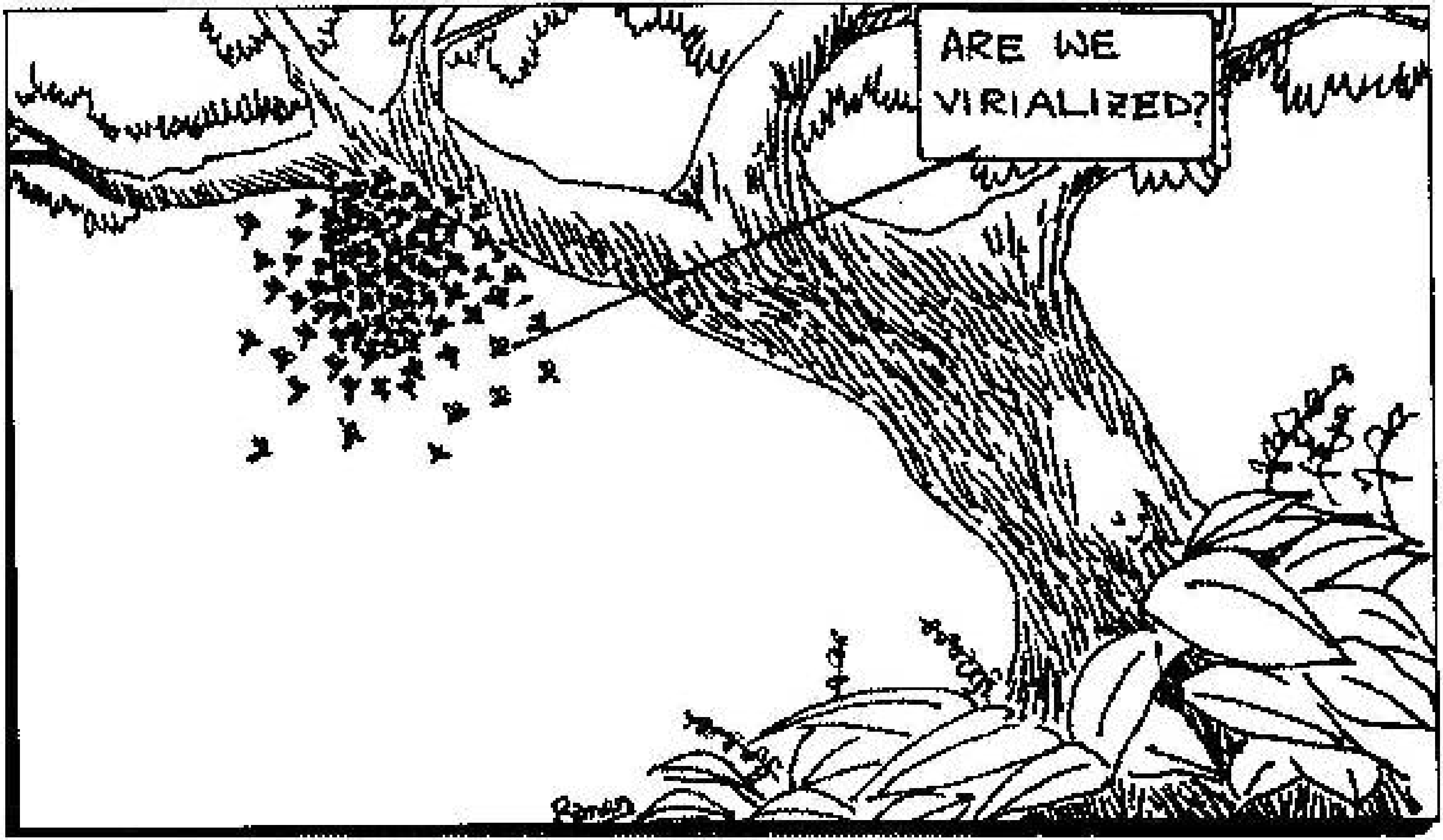


FIG. 3.—The Coma cluster of nebulae

As a first approximation, it is probably legitimate to assume that clusters of nebulae such as the Coma cluster (see Fig. 3) are mechanically stationary systems. With this assumption, the virial theorem of classical mechanics gives the total mass of a cluster in terms of the average square of the velocities of the individual nebulae which constitute this cluster.[5] But even if we drop the assumption that clus-

- In a system of N particles, gravitational forces tend to pull the system together and the stellar velocities tend to make it fly apart. It is possible to relate kinetic and potential energy of a system through the change of its moment of intertia
- In a steady-state system, these tendencies are balanced, which is expressed quantitatively through the the Virial Theorem.
- A system that is not in balance will tend to move towards its virialized state. (virialized ≈ relaxed ≈ equilibrium)



Cartoon courtesy of and ©1999 by B. Nath.

The Virial Theorem(s)

- The Scalar Virial Theorem tells us that the *average* kinetic and potential energy must be in balance.

- The tensor Virial Theorem tells us that the kinetic and potential energy must be in balance in each separate direction.

- The scalar virial theorem is useful for estimating global *average* properties, such as total mass, escape velocity and relaxation time, while the tensor virial theorem is useful for relating shapes of systems to their kinematics, e.g. the flatness of elliptical galaxies to their rotational speed.

The Virial Theorem

Zwicki's derivation: (Ap. J. 1937, 86, 217)

$$m_i \frac{d^2 \vec{x}_i}{dt^2} = \vec{F}_i \tag{12}$$

where $\vec{F}_i$ is the total forces on galaxy i.

Scalar multiplication with $\vec{x}_i$ gives:

$$\frac{1}{2}\frac{d^2}{dt^2}(m_i x_i^2) = \vec{x}_i \cdot \vec{F}_i + m_i \left(\frac{d\vec{x}_i}{dt}\right)^2 \tag{13}$$

(summing over all system particles is implied). The term on the left side represents the change of the momentum of inertia, the second term on the right side is related to kinetic energy, and the first term on the right side is called *virial*.

The Virial Theorem

It can be shown (the so-called Euler theorem from classical mechanics) that for $\Phi \propto 1/r$

$$\sum \vec{x}_i \cdot \vec{F}_i = \sum \vec{x}_i \cdot \nabla \Phi = -\Phi \tag{14}$$

That is, the virial is related to potential energy of the system (true for any *homogeneous* function of the order k such that $\Phi(\lambda x) = \lambda^k \Phi(x)$ – the virial is equal to $k\Phi$).

In a steady state,

$$\frac{1}{2}\frac{d^2}{dt^2}(m_i x_i^2) = 0, \tag{15}$$

and, for a self-gravitating system in steady state

$$2K + \Phi = 0 \tag{16}$$

where $K = M < v^2 > /2$ is the kinetic energy. Thus,

$$E = K + \Phi = -K = \frac{1}{2}\Phi \tag{17}$$

The Scalar Virial Theorem: Applications

- If a system collapses from infinity, half of the potential energy will end up in kinetic energy, and the other half will be disposed of! From the measurement of the circular velocity and the mass of Milky Way (which constrain the kinetic energy), we conclude that during their formation, galaxies radiate away about 3×10^{-7} of their rest-mass energy.

- For a virialized spherical system, $M = 2R\sigma^2/G$. We can estimate total mass from the size and velocity dispersion. E.g. for a cluster with σ=12 km/s, and R=3 pc, we get $M = 2 \times 10^5$ $M_\odot$.

Elliptical Galaxies

You've seen one, you've seen them all! Not true.

- Giant luminous ellipticals, cD: the largest (1 Mpc!) and most luminous galaxies, very large mass-to-light ratios (lots of dark matter), masses $10^{13} - 10^{14}$ $M_\odot$

- Normal ellipticals: most numerous, masses $10^{8} - 10^{13}$ $M_\odot$

- Dwarf ellipticals: masses $10^{7} - 10^{9}$ $M_\odot$ – fundamentally different from all other ellipticals by having low surface brightness and lower metallicity

- Dwarf spheroidals: masses $10^{7} - 10^{8}$ $M_\odot$, the low-mass end of normal ellipticals

- Blue compact dwarf galaxies: masses $\sim 10^9$ $M_\odot$, similar to dwarf ellipticals but unusually blue colors – indicates ongoung star formation (yes, they do have lots of gas); very low mass-to-light ratios

The Faber-Jackson Relation

Remember the Tully-Fisher Relation for spiral galaxies?

$$L \propto v_c^4 \tag{18}$$

Here v_c is the rotational velocity.

Do we have an analogous relation for elliptical galaxies?

Unlike spiral galaxies, elliptical galaxies don't rotate – use the velocity dispersion, σ, instead.

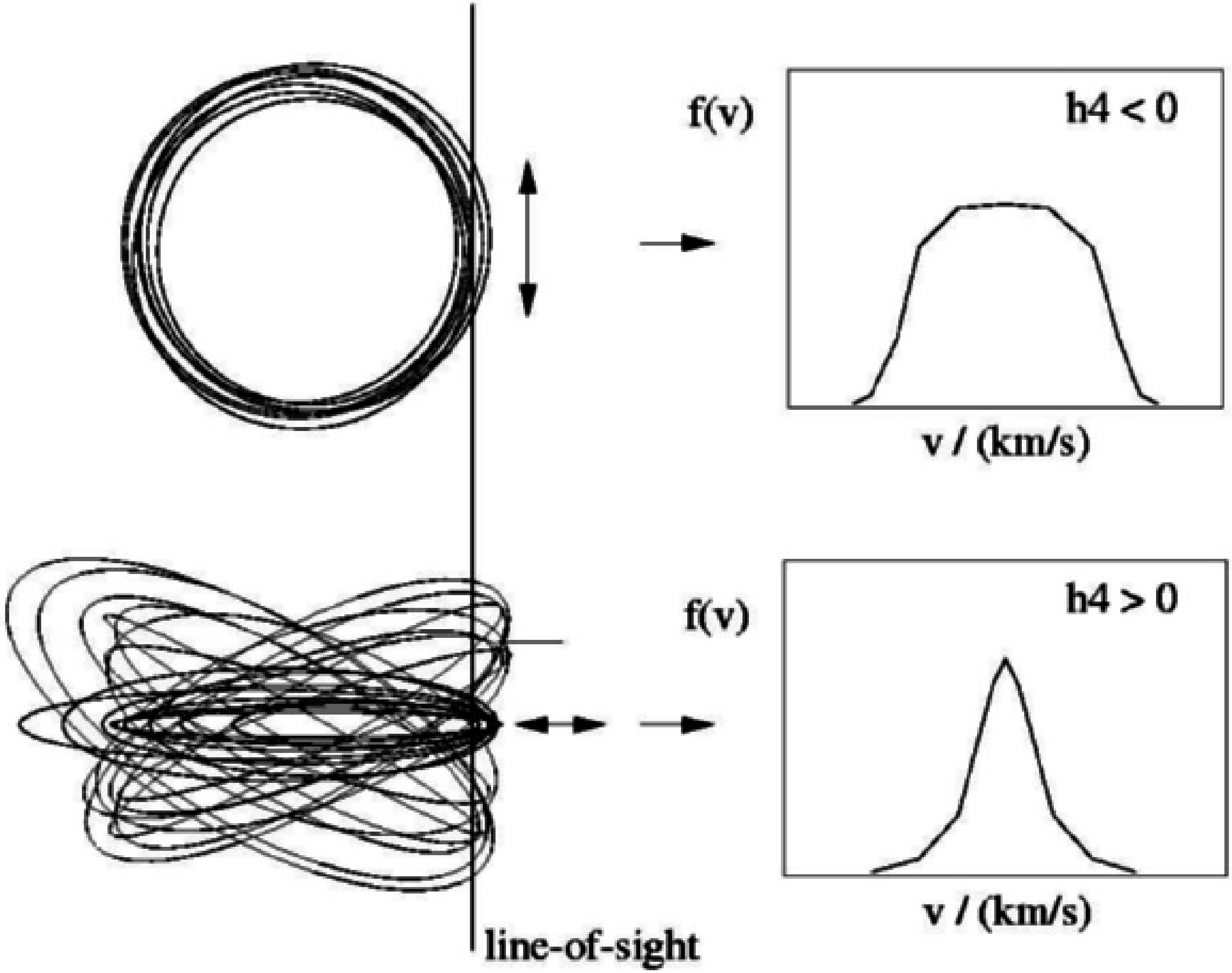
The Faber-Jackson Relation:

$$L \propto \sigma^4 \tag{19}$$

Actually, the exponent varies from 3 to 5, depending on sample and band.

The scatter in the FJ relation is decreased by adding another physical parameter – the fundamental plane:

$$L \propto \sigma^{2.65} r_e^{0.65} \tag{20}$$



The velocity dispersion is the width of the velocity distribution.

Until the late 1970s, it was believed that elliptical galaxies are simple systems: gas-free, disk-free, rotationally flattened ellipsoids of very old stars. In the last 20 years, most of these assumptions turned out to be wrong or only crude approximations:

- Massive ellipticals are not flattened by rotation, but are anisotropic.
- Ellipticals do have an interstellar medium, but it is hot $T > 10^6$K.
- A significant fraction of ellipticals exhibits kinematic peculiarities (like conter-rotating cores) which point to a 'violent' formation process.
- Ellipticals frequently contain faint stellar disks.
- Low mass ellipticals seem to contain intermediate age stars.
- All ellipticals and bulges seem to contain supermassive black holes amounting to about 0.2% of their mass.

Until today, ellipticals are characterized by their apparent flattening (despite it is dependent on projection):

E0 ... E7, where the number corresponds to $10 \cdot (1 - b/a)$

with b being the projected short axis and a the projected long axis.

The Light Profiles of Elliptical Galaxies

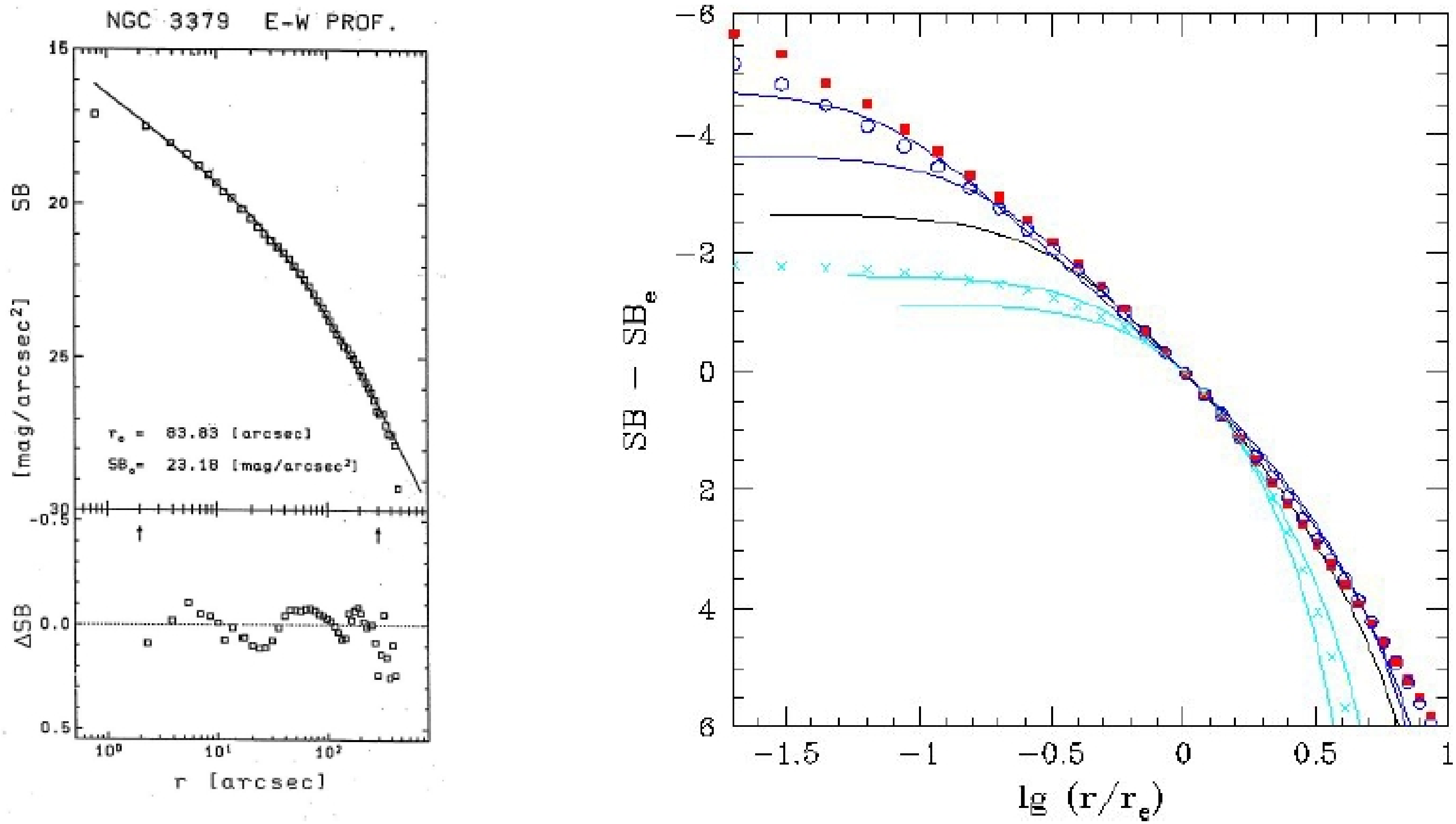
Remember the Sersic profile from Lecture 2: $I(R) \propto \exp(-(R/R_e)^{1/n})$?
For elliptical galaxies $n = 4$ – de Vaucouleurs profile:

$$I(R) = I_o \, 10^{-3.33\,[(\frac{R}{R_{1/2}})^{1/4} - 1]} \tag{21}$$

Another commonly used profiles are King models (isothermal sphere) and Jaffe's spheres. The latter has almost identical light profile as de Vaucouleurs profile, but the density law and gravitational potential are analytic:

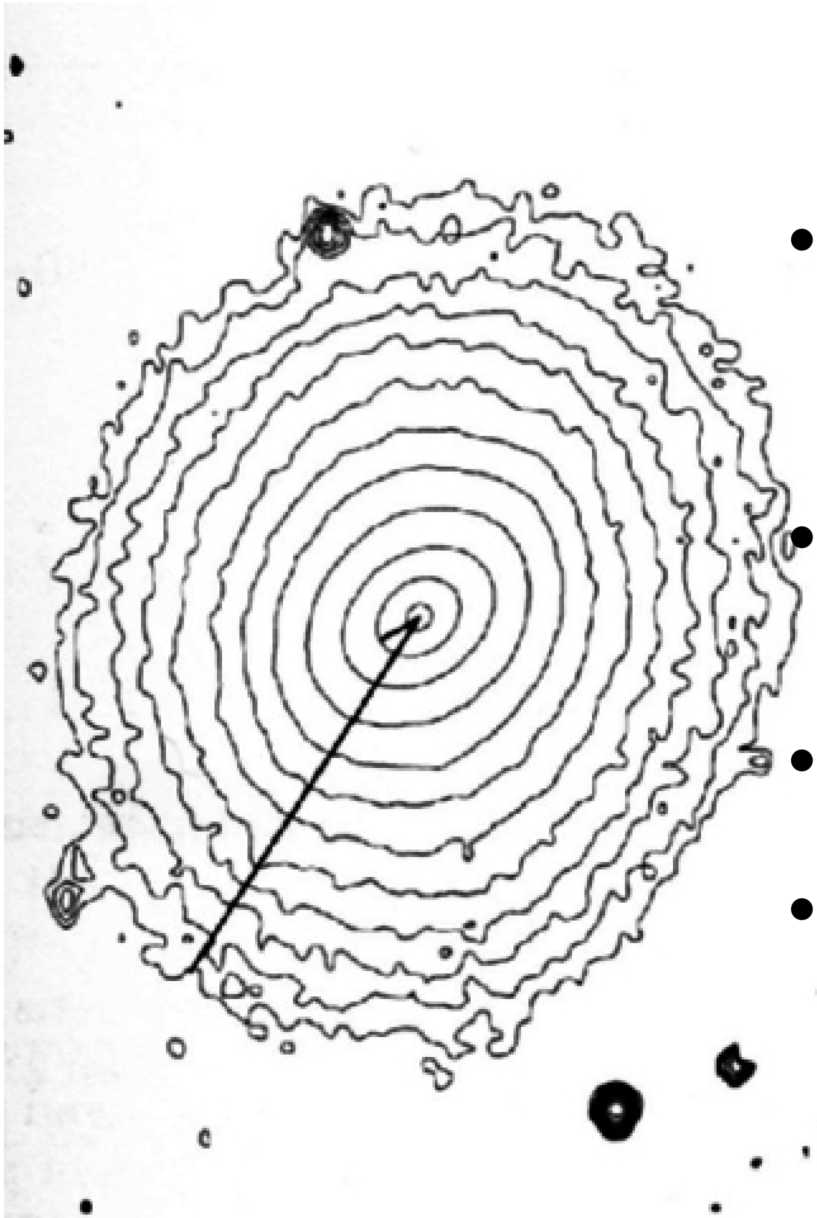
$$\rho_L(r) = \frac{L}{4\pi r_o^3} \left(\frac{r_o}{r}\right)^2 \frac{1}{(1 + r_o/r)^2} \tag{22}$$

$$\Phi(r) = \frac{GL}{r_o} \left(\frac{M}{L}\right) \ln\left(\frac{1}{1 + r_o/r}\right) \tag{23}$$



Left: Jaffe-fit to the prototypical elliptical NGC 3379 from Surma, Seifert, Bender (1992) *A&A*. **Right:** Comparison of different SB-profiles of galaxies. Red squares: Jaffe model, blue circles: $r^{1/4}$-model, cyan crosses: exponential, lines are King models of different binding energy.

True shapes of elliptical galaxies



- The classification of elliptical galaxies (E0–E7)is based on **apparent** flattening: are the true shapes bi-axial (as expected for rotation), or triaxial (as expected for randomly distributed orbits)?
- Elliptical galaxies are modestly triaxial – a:b:c $\sim$ 1:0.95:0.7 (nearly *oblate*, a=b>c, like an UFO, as opposed to *prolate*, a>b=c, like a football)
- We know that because of the effect called "isophote twist", which doesn't happen for bi-axial shapes, only for triaxial
- Therefore, (most) elliptical galaxies are **NOT** supported by rotation
- Isophotes are not exactly elliptical: *boxy* vs. *disky*. The latter can be explained as a superposition of an elliptical bulge on a faint edge-on disk.

FIGURE 3. — Distribution of the ellipticity classes for all observed elliptical galaxies.

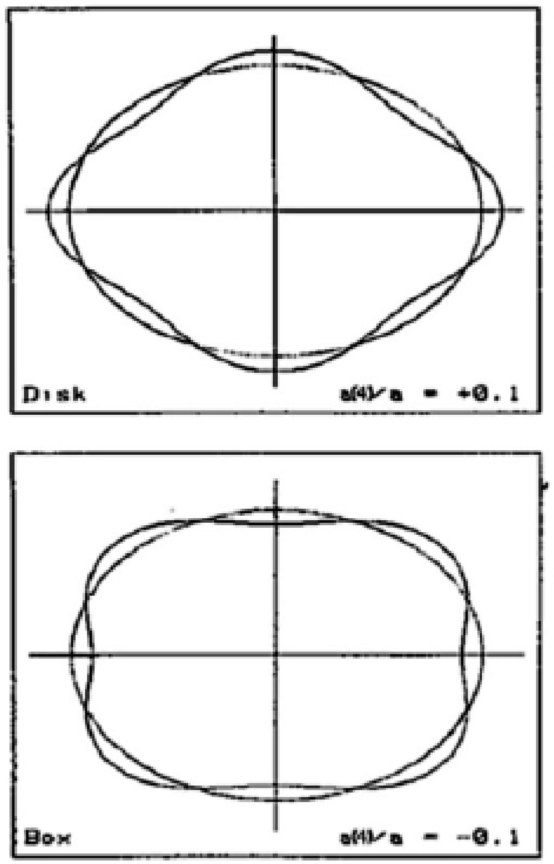


FIGURE 5. — Schematic drawing illustrating isophotes with $a(4)/a = +0.1$ and $a(4)/a = -0.1$.

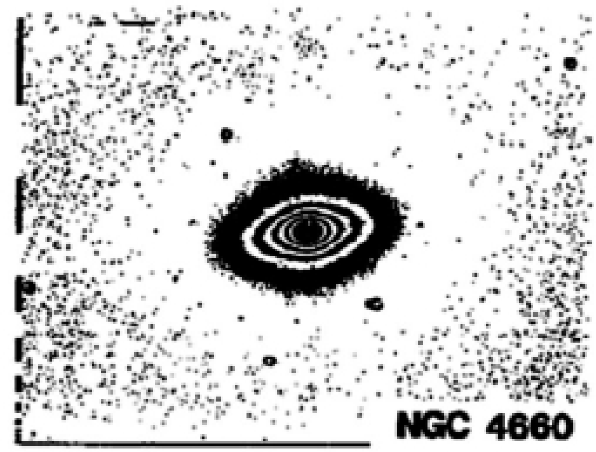


FIGURE 6. — R-image of NGC 4660, an elliptical galaxy with a disk-component in the isophotes ($a(4)/a \sim +0.03$).

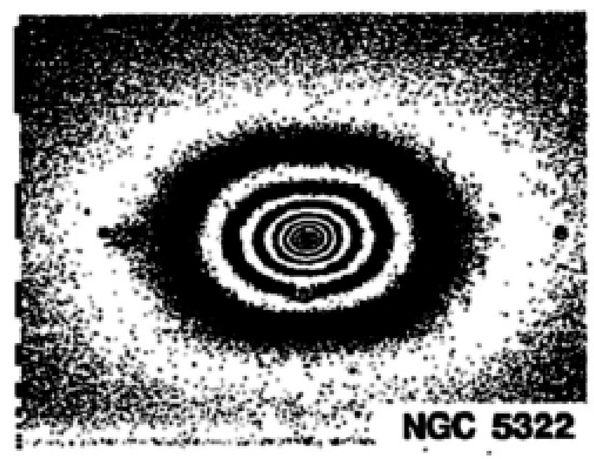
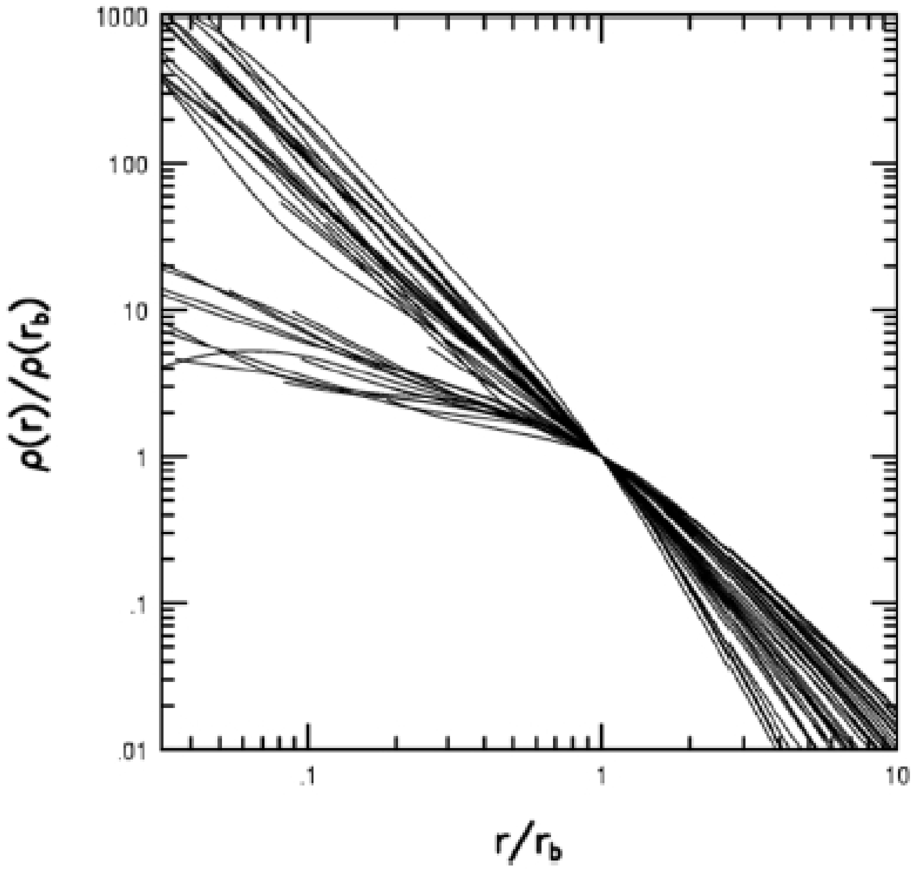
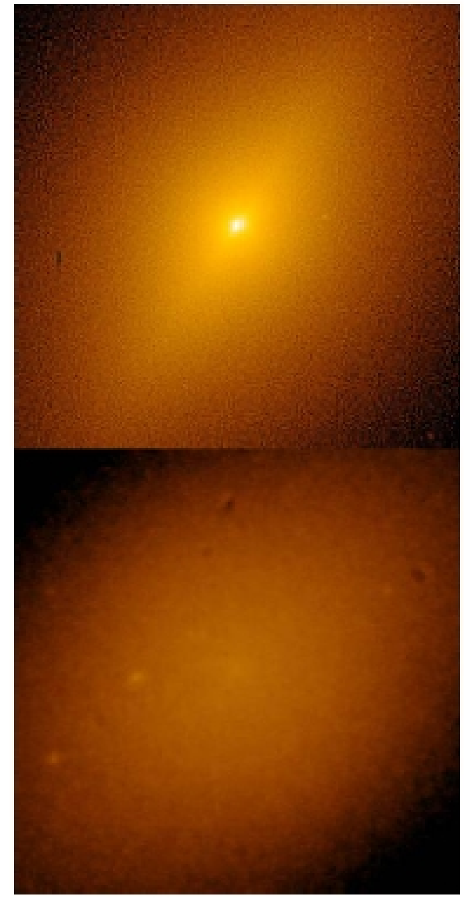


FIGURE 7. — R-image of NGC 5322, an elliptical galaxy with box-shaped isophotes ($a(4)/a \sim -0.01$).

Examples for boxy and disky isophotes from Bender et al. 1988

The central regions of elliptical galaxies include two types of profiles: **cuspy cores** and **power-law cores**





Dark Matter in Elliptical Galaxies

In the central part of elliptical galaxies there is no evidence for dark matter (mass-to-light ratio ~5–10 in solar units – typical for old stellar populations). In the outer parts it is harder to find such evidence because there is no gas on circular orbit as is the case for spiral galaxies.

Nevertheless, there are several methods that indicate the presence of dark matter in elliptical galaxies:

1. Analysis of stellar kinematics (detailed models of motion in gravitational potential)

2. Gravitational lensing (later in this class)

3. X-ray halos (application of virial theorem)