Astr 323: Extragalactic Astronomy and Cosmology

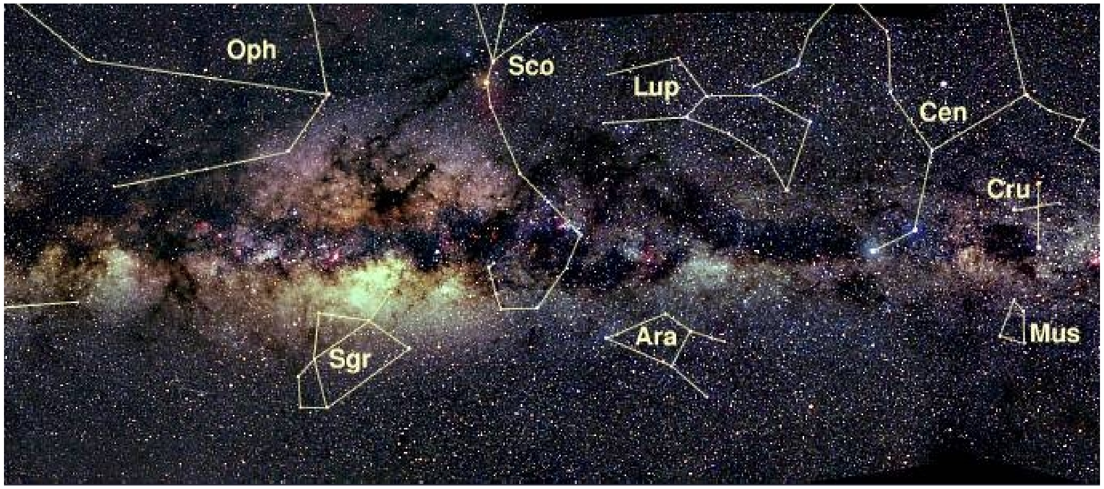
Spring Quarter 2014, University of Washington, Željko Ivezić

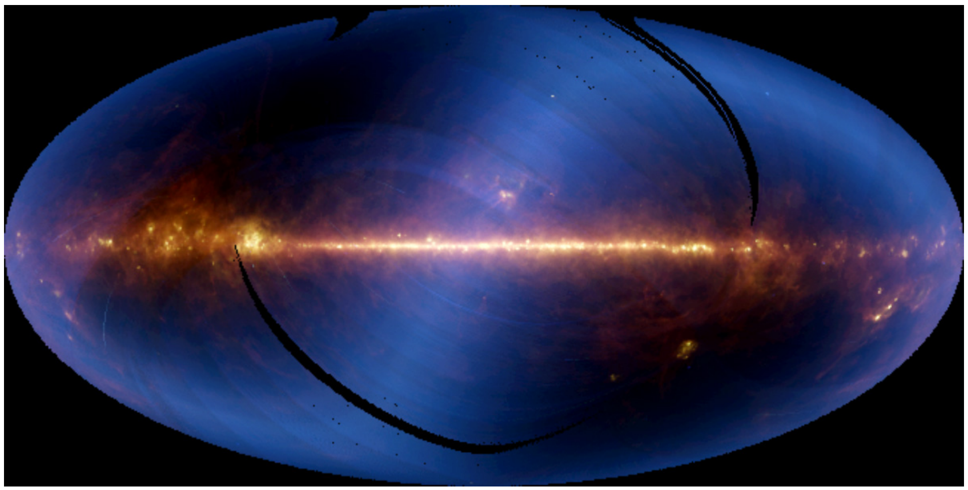
Lecture 2:

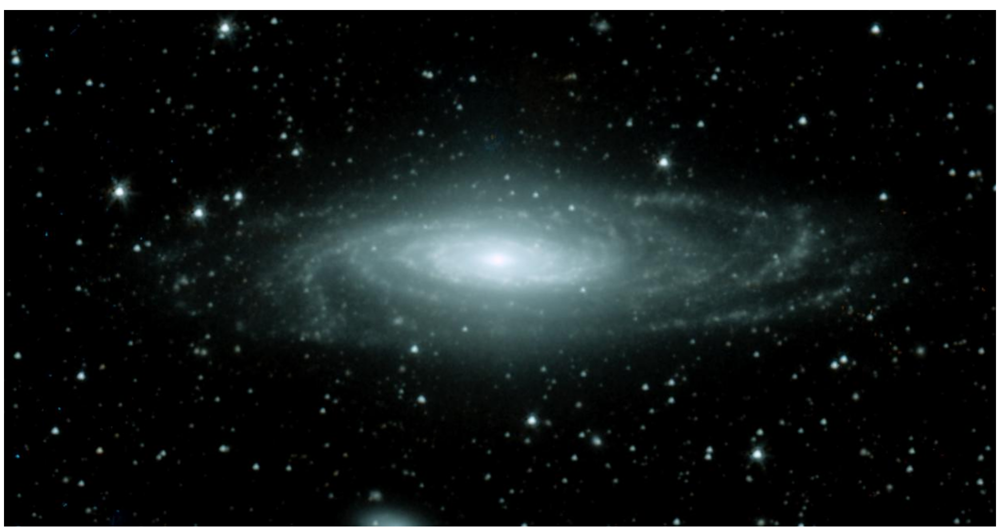
The Milky Way

Outline

- Spatial distribution of stars: disk, halo, bulge
- Spatial distribution of stars: substructure
- Galactic center: evidence for a black hole
- Interstellar medium: gas and dust
- Examples of ongoing research: dissecting Galaxy with SDSS
- Stellar kinematics: rotation and random motions





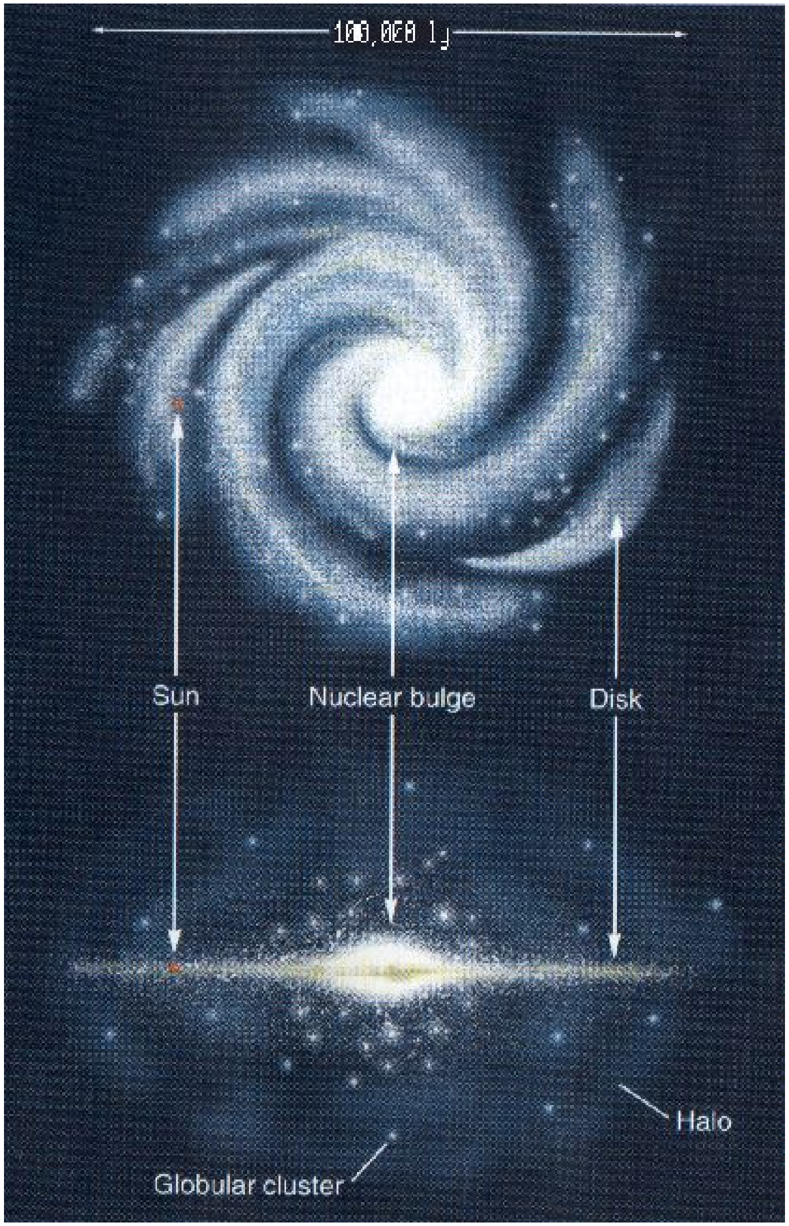


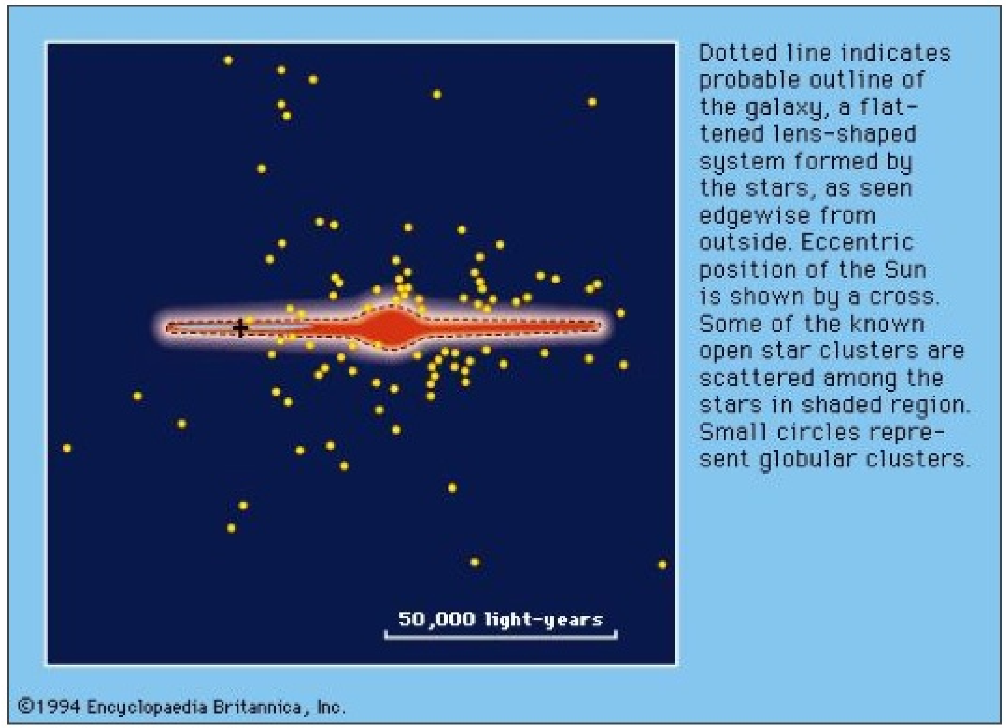
Introduction

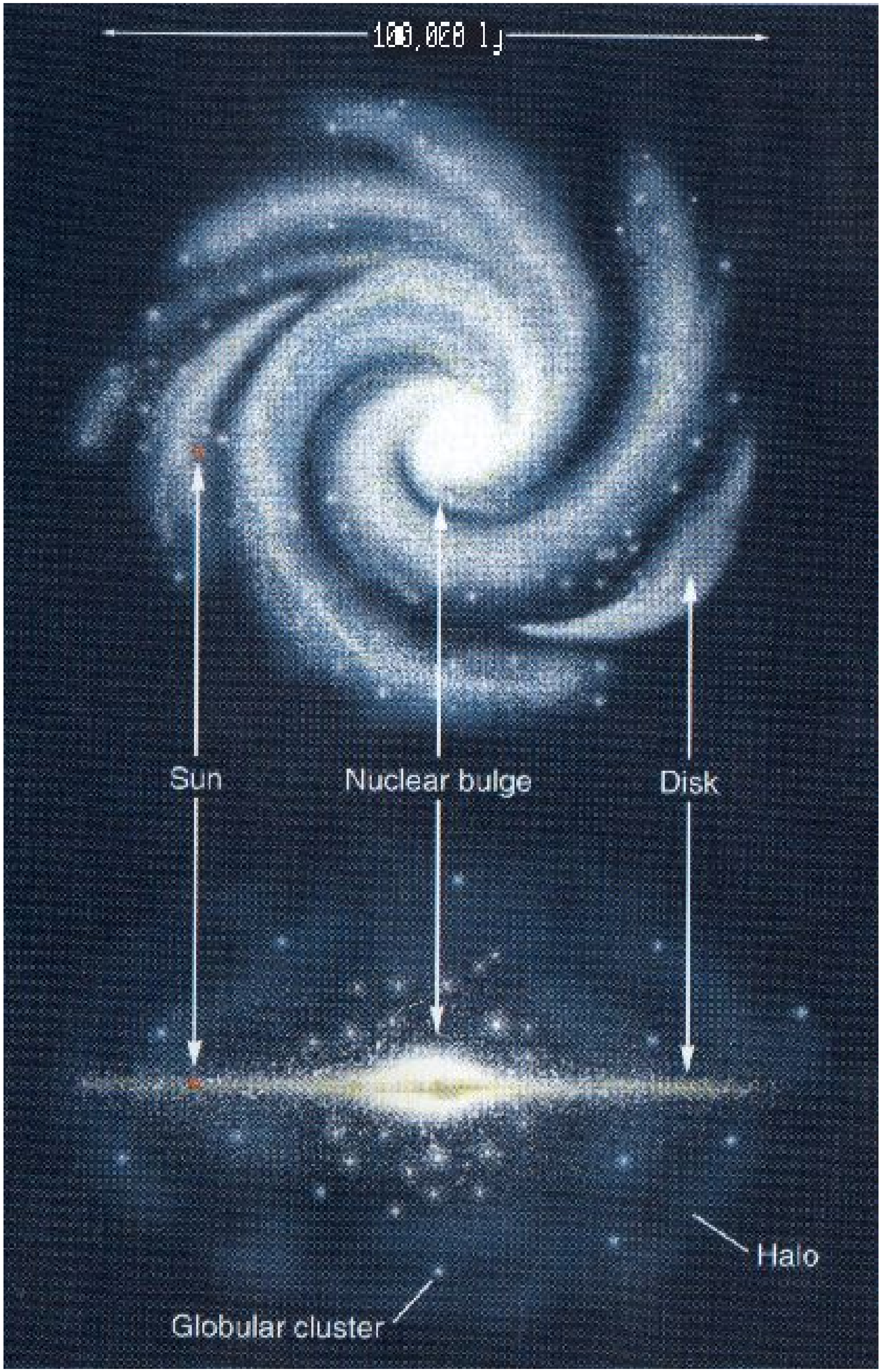
- Top left: 30° by 10° (optical) view towards the Galactic center (from Axel Mellinger)
- Middle left: The all-sky view by the Infrared Astronomical Satellite
- Bottom left: a spiral galaxy (NGC 7331) similar to the Milky Way
- Conclusion: the density of stars on the sky varies greatly because we are observing from inside a disk of stars
- We live in a a spiral galaxy the same conclusion supported by the motions of stars and the presence of abundant interstellar medium (more later)

The position of the Galactic center

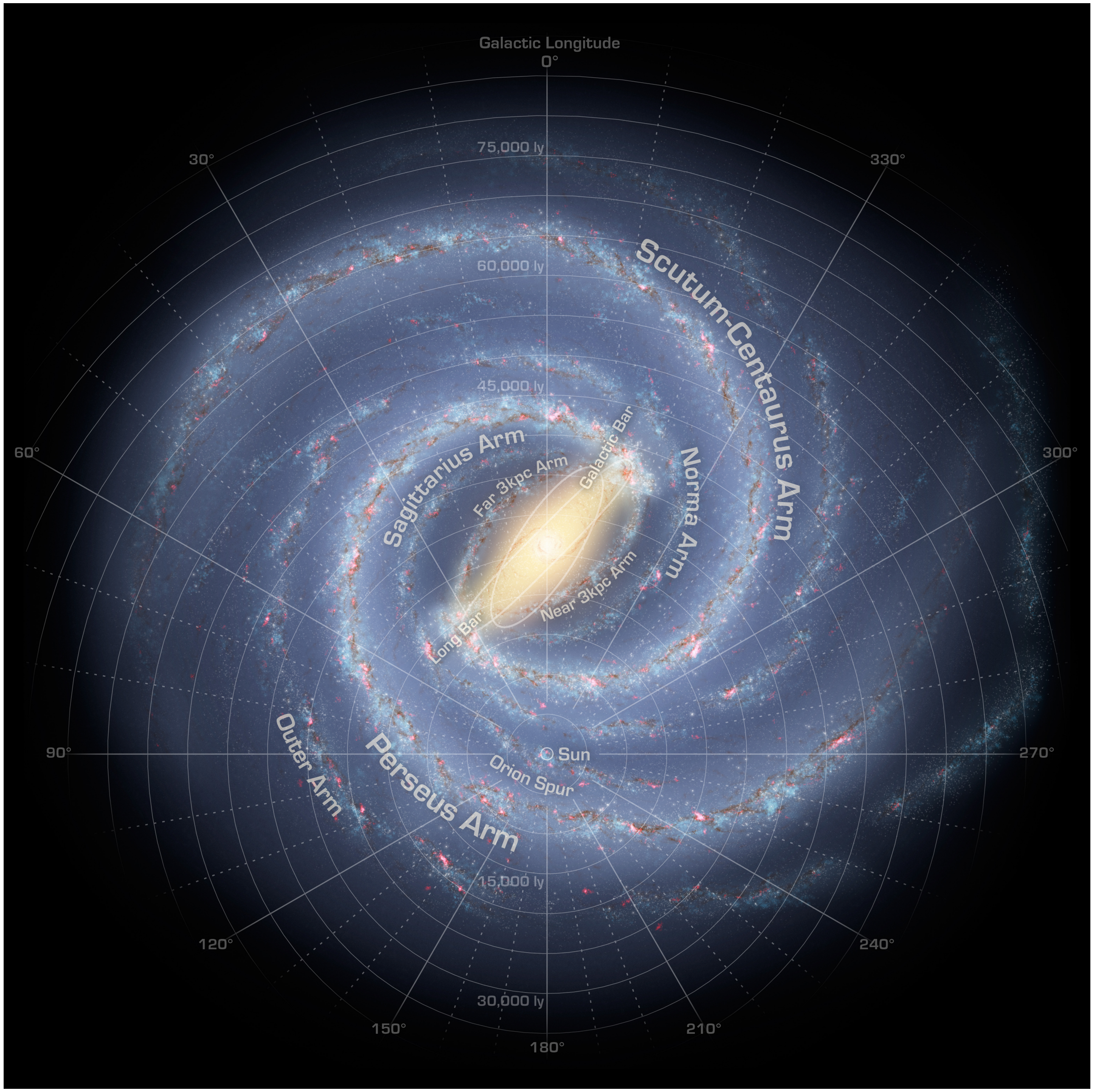
- Shapley used the distribution of globular clusters to demonstrate that the Sun is not in the center of the Milky Way (8 kpc)





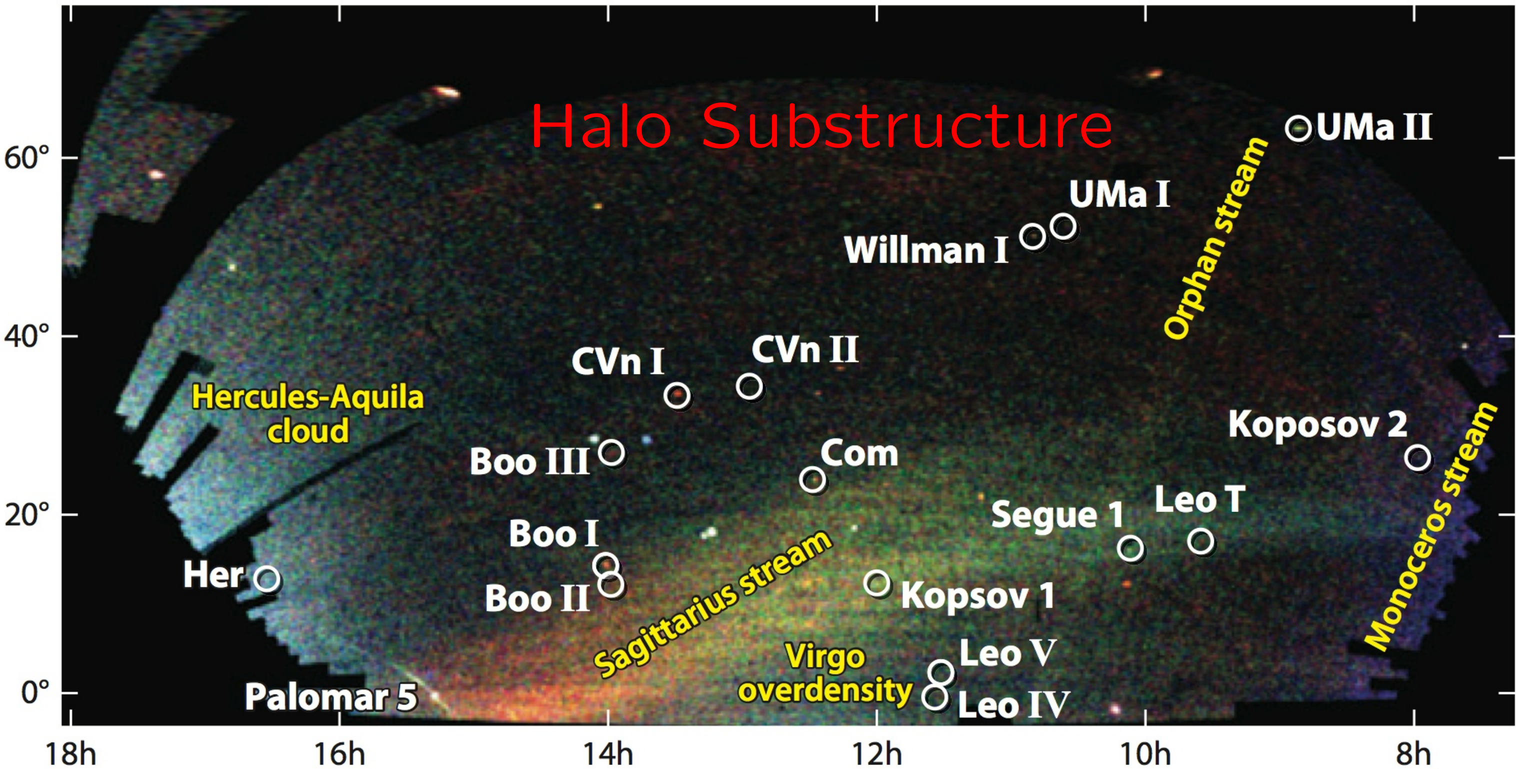


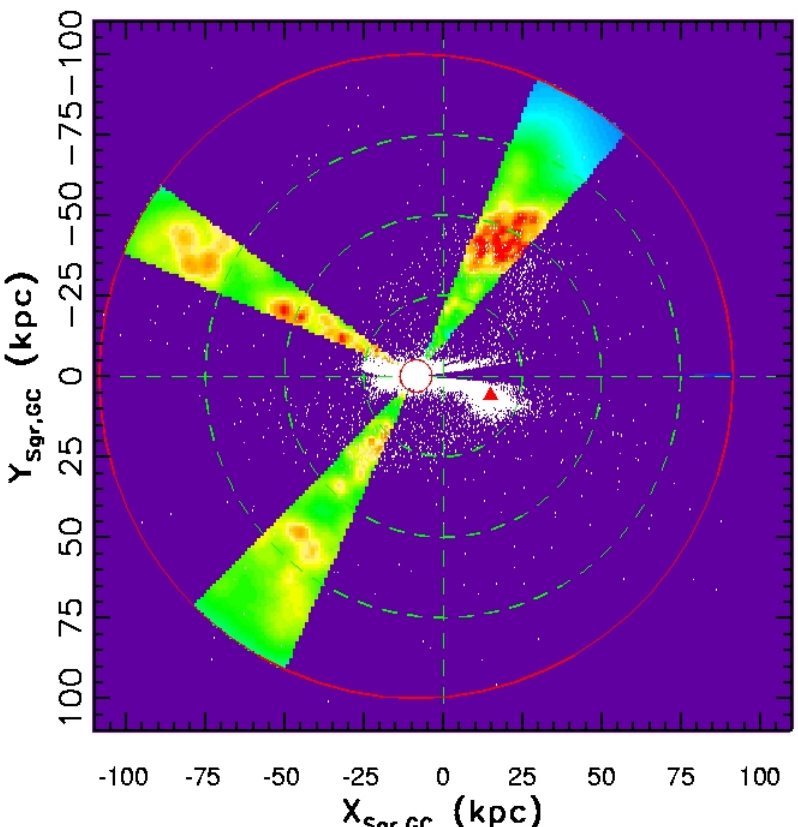
GALACTIC DISK	GALACTIC HALO	GALACTIC BULGE
Highly flattened	Roughly spherical—mildly flattened	Somewhat flattened and elongated in the plane of the disk ("football shaped")
Contains both young and old stars	Contains old stars only	Contains both young and old stars; more old stars at greater distances from the center
Contains gas and dust	Contains no gas and dust	Contains gas and dust, especially in the inner regions
Site of ongoing star formation	No star formation during the last 10 billion years	Ongoing star formation in the inner regions
Gas and stars move in circular orbits in the Galactic plane	Stars have random orbits in three dimensions	Stars have largely random orbits but with some net rotation about the Galactic center
Spiral arms	No obvious substructure	Ring of gas and dust near center; Galactic



Revised Spiral Arms

- The stellar bar was discovered in 1990s based on IRAS data
- It was believed that the Galaxy has four spiral arms: the Scutum-Centaurus, Perseus, Sagittarius and Norma
- The stellar counts from Spitzer galactic plane survey (Benjamin et al. 2008) strongly suggest that there are only two major arms, the Scutum-Centaurus and Perseus arms, as is common for barred galaxies





- The table on the previous page is wrong: most recent data clearly show that **halo has rich substructure**
- Top left: the counts of SDSS stars color-coded by distance (red: $\sim$10 kpc, blue: several kpc)
- Bottom left: the distribution of SDSS RR Lyrae stars and 2MASS red giants

The Virgo Overdensity: the latest news

- "...the Virgo Overdensity ... is best explained by a minor merger." (Bonaca et al. 2012, AJ 143, 105),
- "... a tri-axial dark matter halo is favored and we exclude a prolate shape." (Casey, Keller & Da Costa 2012, AJ 143, 88; figure below is from this paper)

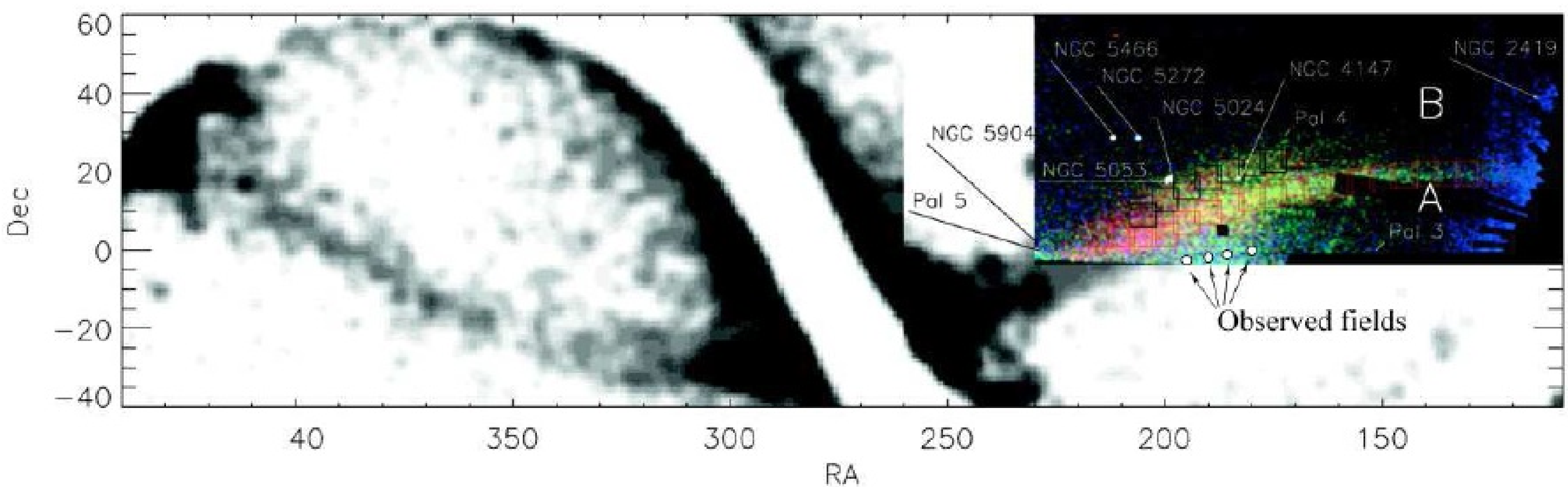
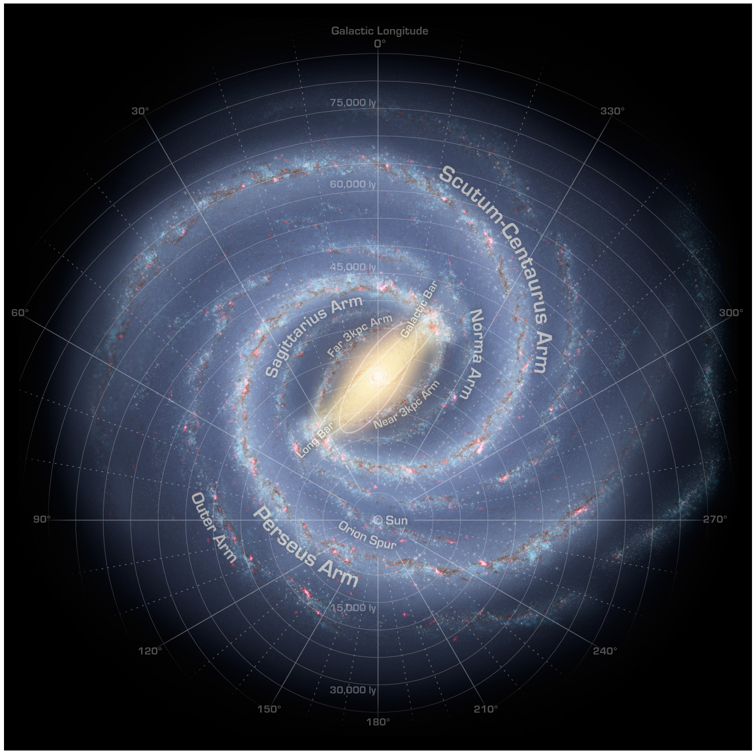
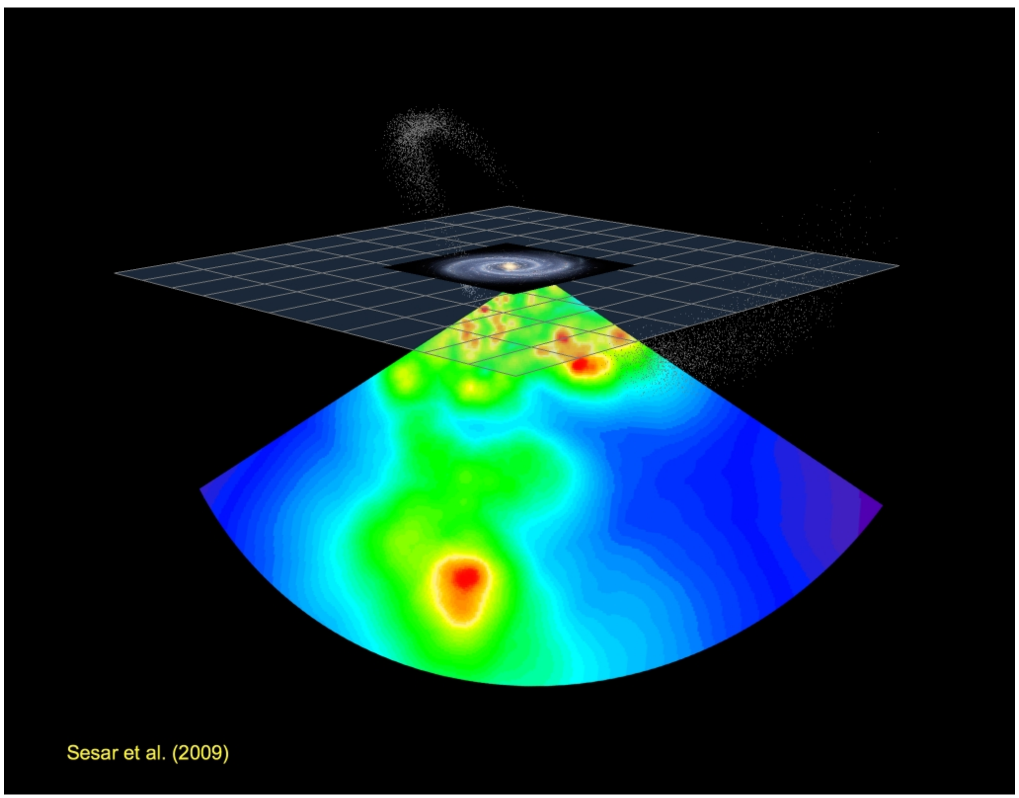


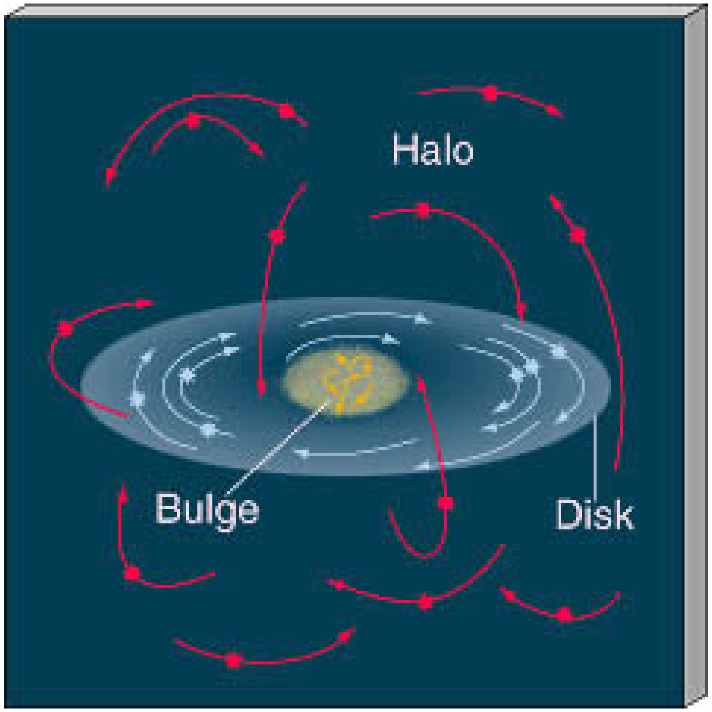
Fig. 5.— Observed fields are outlined upon a panoramic view of the Sgr stream, to demonstrate our field locations in context with the Sgr stream. This plot is an adaptation of Figure 2 in Belokurov et al. (2006), which uses the 2MASS M-giant sample of Majewski et al. (2003).

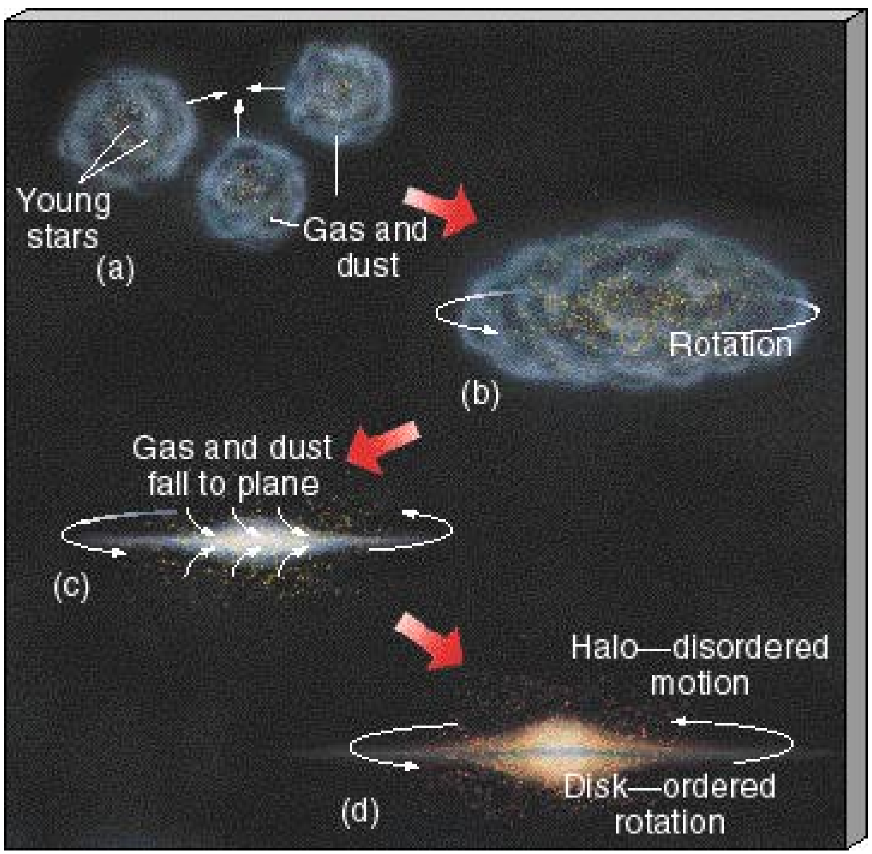




Outer halo studies: RR Lyrae (from SDSS)

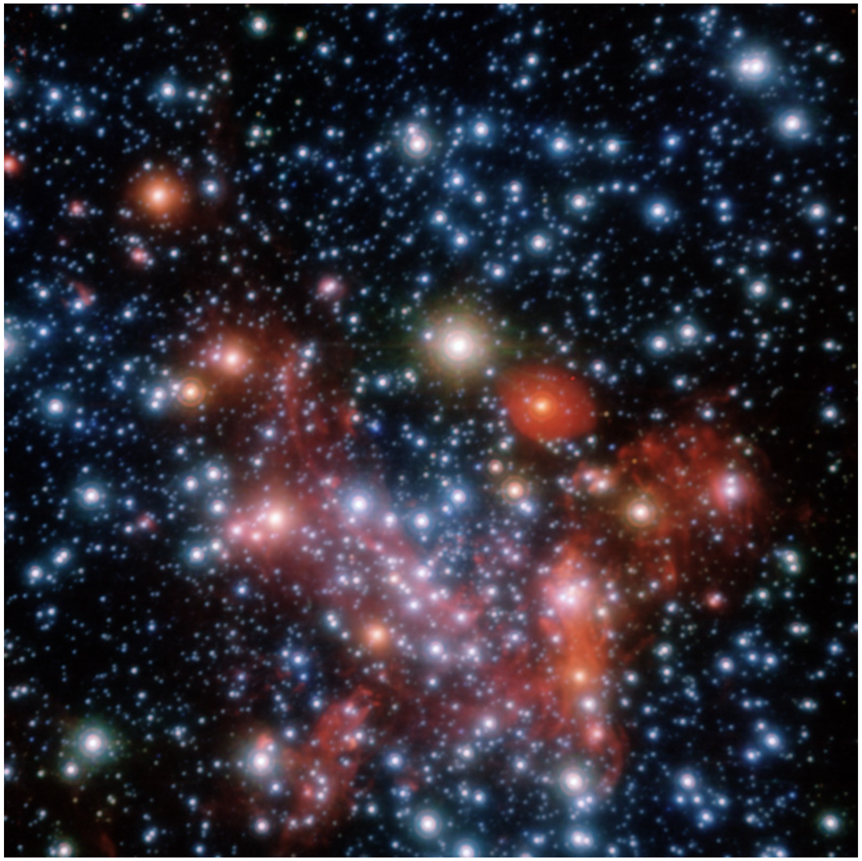
- **Top left:** the disk structure (artist's conception based on the Spitzer and other surveys of the Galactic plane)
- **Bottom left:** the halo density (multiplied by R^3; yellow and red are overdensities relative to mean $\rho(R) \propto R^{-3}$ density) as traced by RR Lyrae from SDSS Stripe 82 (Sesar et al. 2010ab, ApJ 708, 717; ApJ 717, 133), compared in scale to the top panel
- Conclusions: the spatial distribution of halo stars is highly inhomogeneous (clumpy); when averaged, the stellar volume density decreases as $\rho(R) \propto R^{-3}$ out to $\sim$30 kpc, and then becomes steeper.





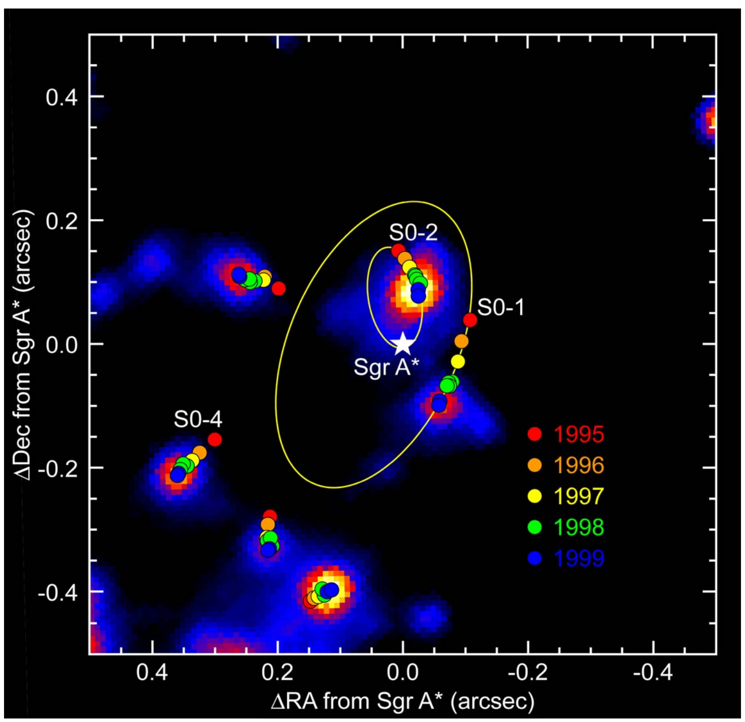
Kinematics

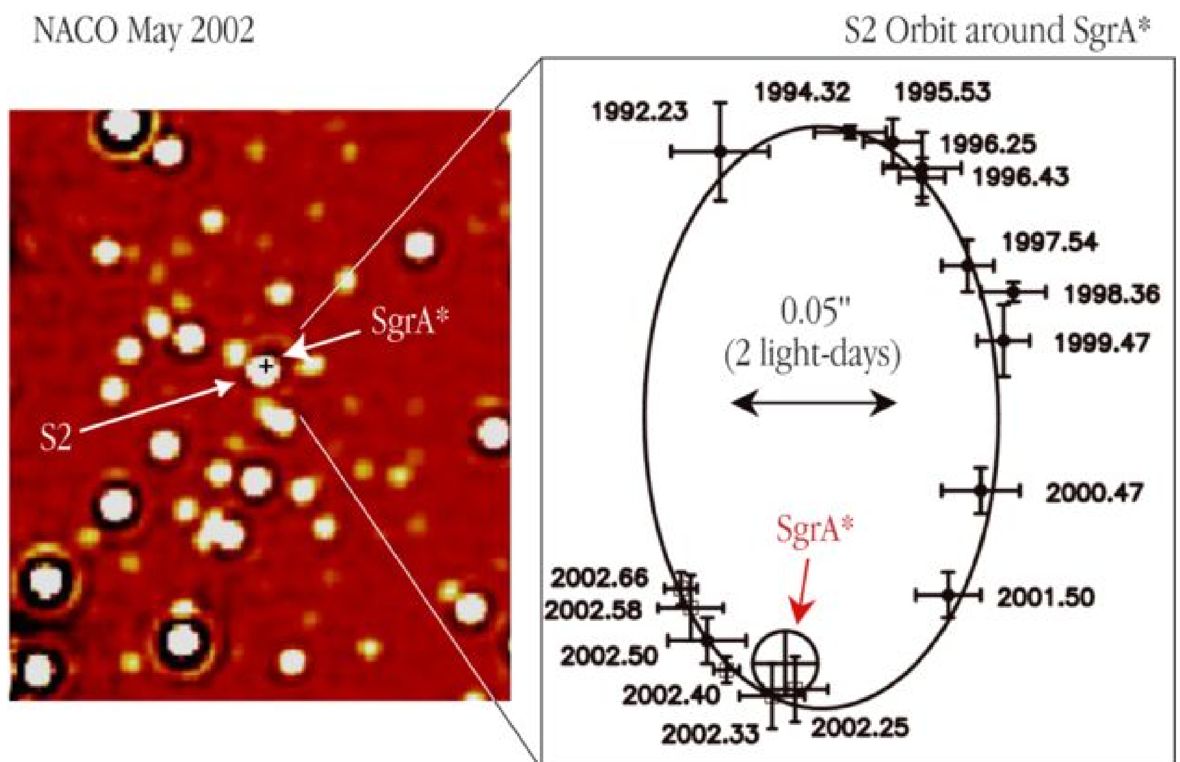
- Stars move in a gravitational potential
- Two types of motion: disk stars **rotate** around the center, while halo stars are on randomly distributed elliptical orbits
- The motion of stars was set during the formation period
- The details are governed by the laws of physics: conservation of energy and conservation of angular momentum!
- As the cloud collapses, its rotation speed must increase. As it spins faster, it must flatten.



Black Hole in the Galactic Center

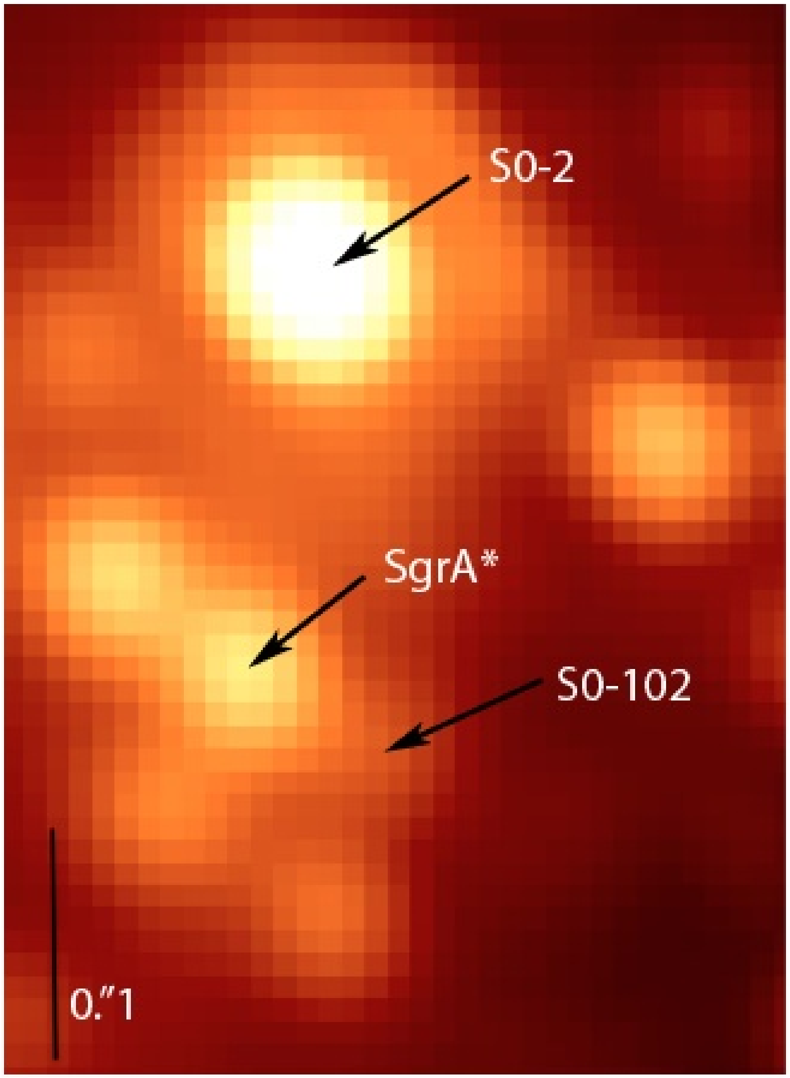
- Stars move in a gravitational potential: a large mass (a few $10^6\ M_\odot$) confined to small space (0.1-0.2 AU) is required to explain about ~30 **observed** orbits
- Two teams: UCLA team led by Andrea Ghez, and European team led by Reinhard Genzel





Black Hole in the Galactic Center

- Meyer et al. (2012, Science 338, 84): after 17 years of imaging the galactic center at the highest angular resolution possible today: two stars with full phase coverage and periods of less than 20 years.



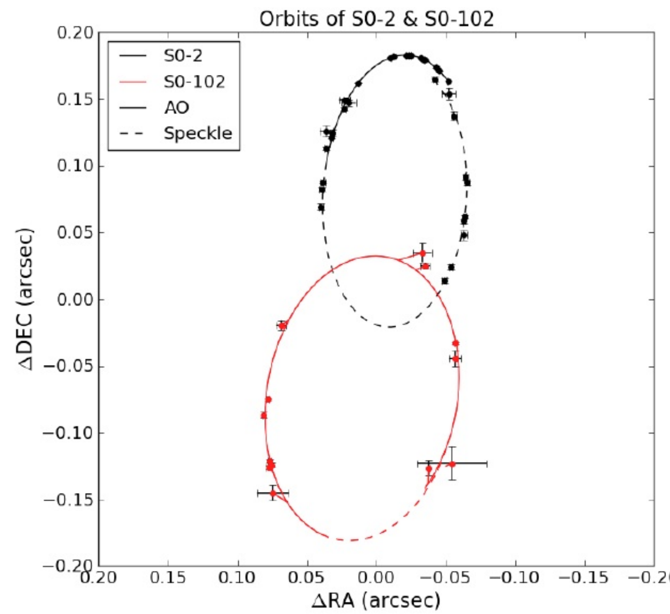
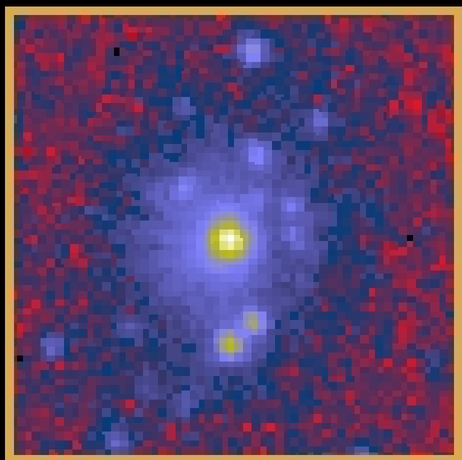
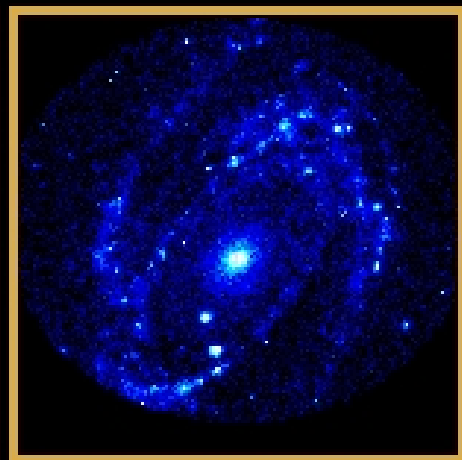


Figure 1 A Keck/NIRC2 adaptive optics image from May 2010 showing the short-period star S0-102, which is besides S0-2 the only star with full orbital phase coverage, and the electromagnetic counterpart of the black hole, Sgr A*. The image was taken at a wavelength of 2.12 μm and shows the challenge of detecting S0-102, which is 16 times fainter than S0-2 and lies in this crowded region.

M81 – Spiral Galaxy (Type Sb)

Distance: 12,000,000 light-years (3.7 Mpc) Image Size = 14 x 14 arcmin Visual Magnitude = 6


X-Ray: ROSAT


Ultraviolet: ASTRO-1

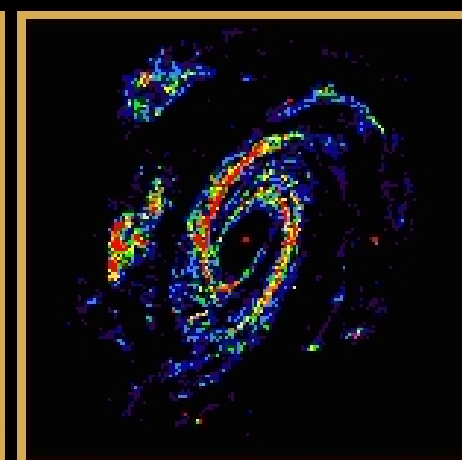

Visible: DSS

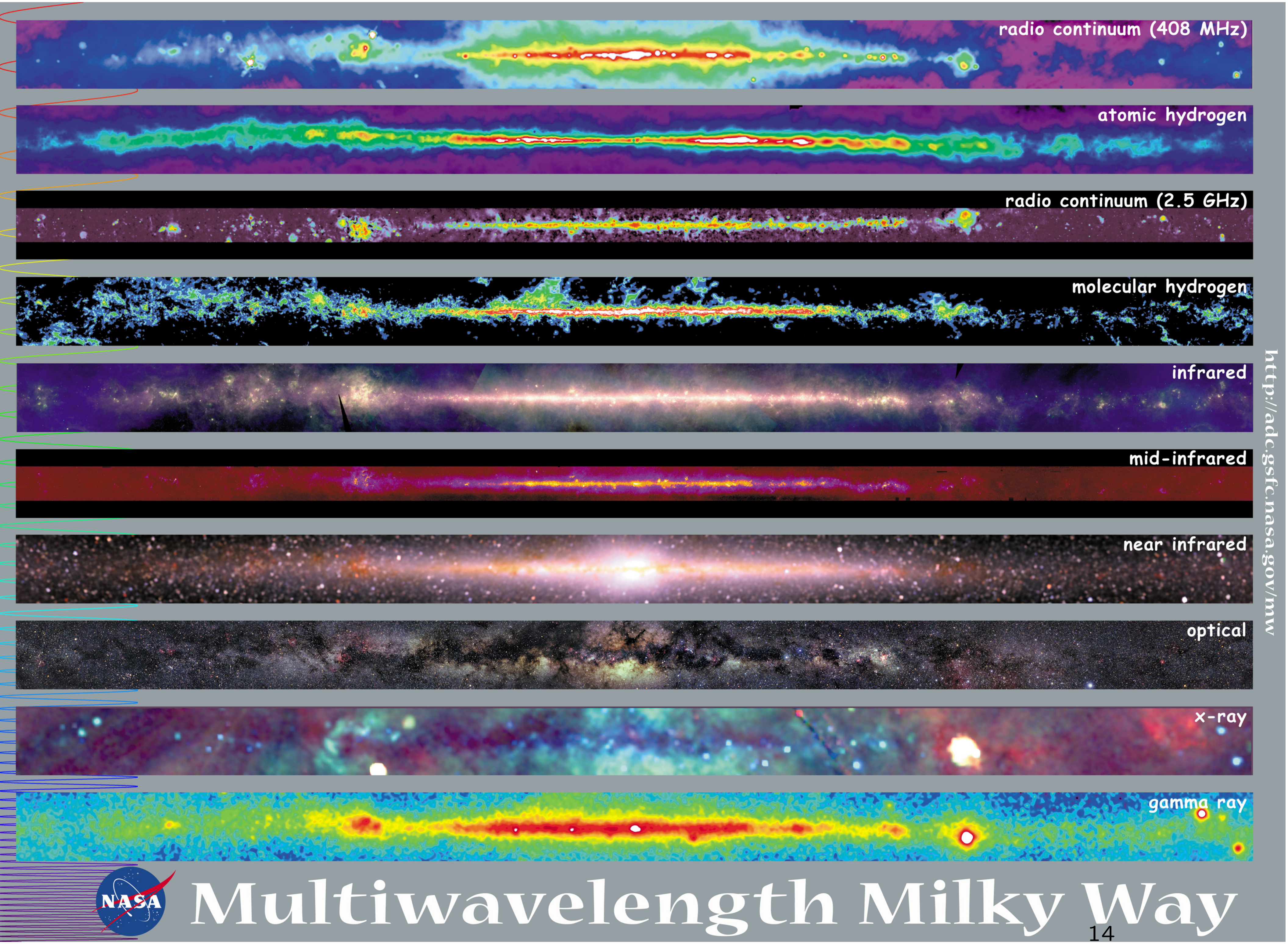

Visible: R. Gendler


Near-Infrared: Spitzer


Mid-Infrared: Spitzer


Far-Infrared: Spitzer


Radio: VLA


radio continuum (408 MHz)
atomic hydrogen
radio continuum (2.5 GHz)
molecular hydrogen
infrared
mid-infrared
near infrared
optical
x-ray
gamma ray
http://adc.gsfc.nasa.gov/mw
Multiwavelength Milky Way

Stars form from gas in galaxies

"Interstellar Medium" = "ISM"

- Hot ionized Gas
- Neutral Atomic Gas
- Cold Molecular Gas
- Dust

What these phases are called:

- Hot ionized Gas
- Neutral Atomic Gas
- Cold Molecular Gas
- Dust

"HII" = "H two"

"HI" = "H one"

"H_2"

"Dust"

Nomenclature: "ElementI" = unionized Element
"ElementII" = singly ionized Element
"ElementIII" = doubly ionized Element…etc

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What fraction is in each phase?

- Hot ionized Gas
- Neutral Atomic Gas
- Cold Molecular Gas
- Dust

<15%

~65%

~20%

<5%

Typical Densities

- Hot ionized Gas
- Neutral Atomic Gas
- Cold Molecular Gas
- Dust

Low
(<0.5 atoms/cm^3)

Medium
(1-10 atoms/cm^3)

High
(10^2-10^5 atoms/cm^3)

Very High
(solid)

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Typical Temperatures

- Hot ionized Gas
- Neutral Atomic Gas
- Cold Molecular Gas
- Dust

Hot
(>10^4-10^7 K)

Medium
(100 -10^4 K)

Cold
(<50K)

Medium-cold
(<100K)

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Detected How?

- Hot ionized Gas
- Neutral Atomic Gas
- Cold Molecular Gas
- Dust

Hα emission line (6563Å)
X-Rays (if T>10^6K)

21cm emission line
(hyperfine splitting of H ground state)

CO rotational emission line
(mm wavelengths)

Thermal (Black-body) radiation at far-infrared wavelengths

Distributed How?

- Hot ionized Gas
- Neutral Atomic Gas
- Cold Molecular Gas
- Dust

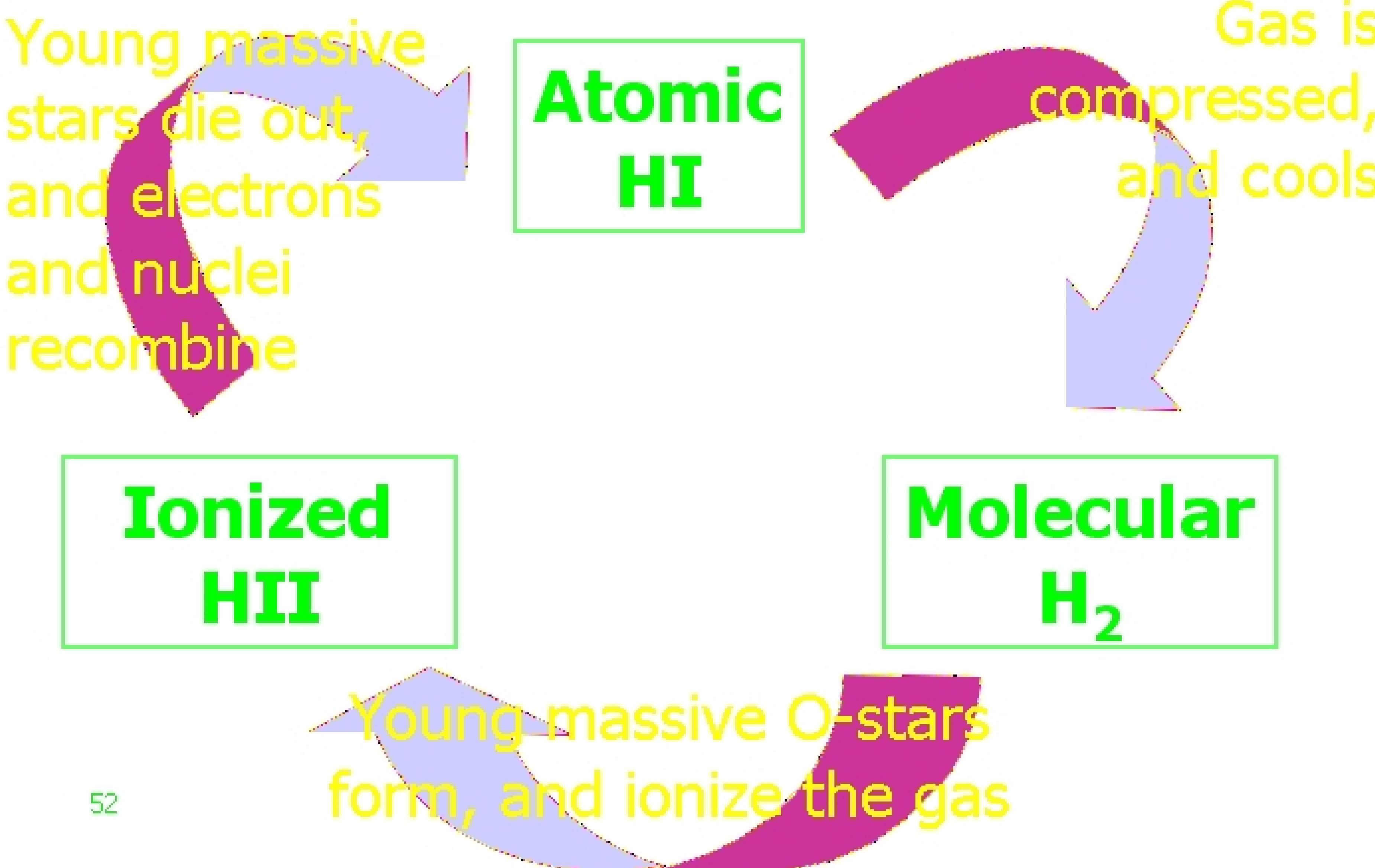
Halos of Galaxies

Galaxy Midplane, out to large radii beyond stars

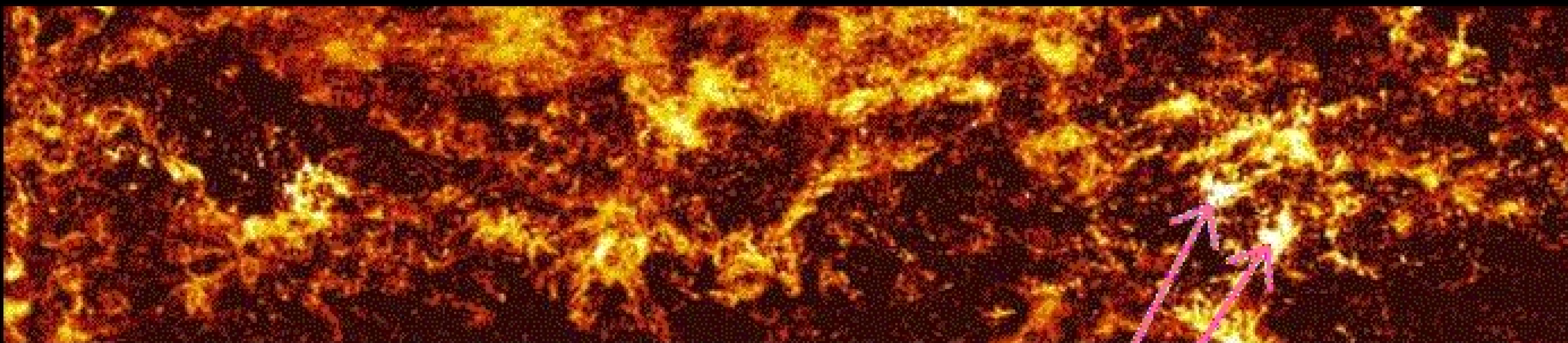
Galaxy Midplane, concentrated in spiral arms

Tracks the distribution of gas

How are the three phases of gas inter-related?



Molecular gas is clumpy on small scales.

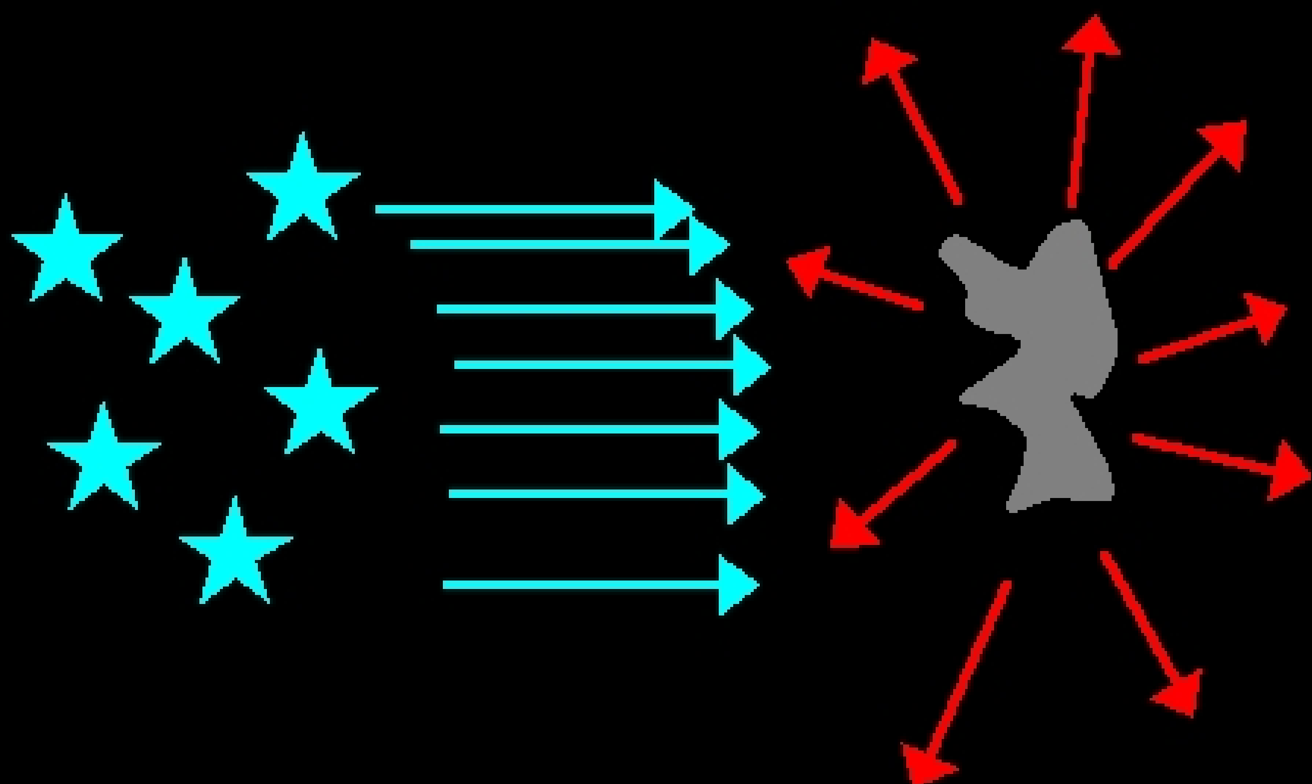


(View of the outskirts, away from the center)

"Molecular Clouds"

This is why stars form in clusters!

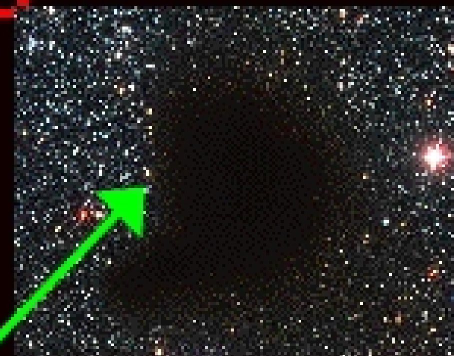
The amount of dust can be measured using light that has been **reprocessed** into the infrared.

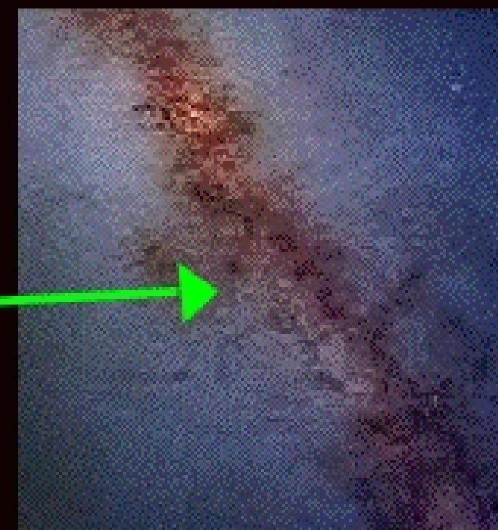


UV & optical light is absorbed by dust...

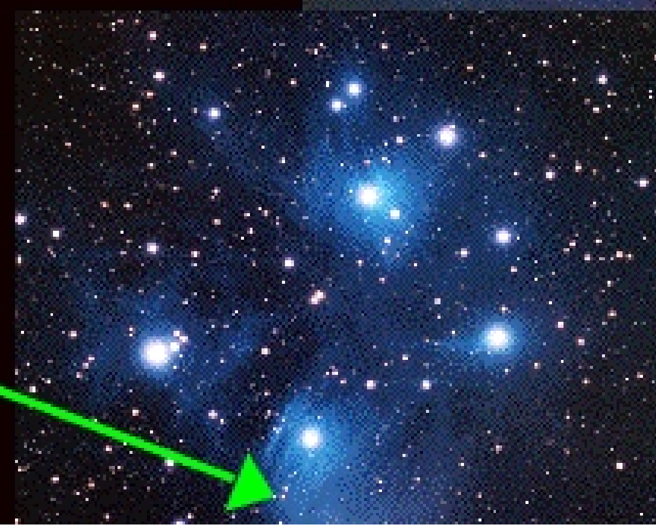
...which heats up to 10-100K and radiates like a greybody at 10-300μm

Dust plays many important roles in galaxies

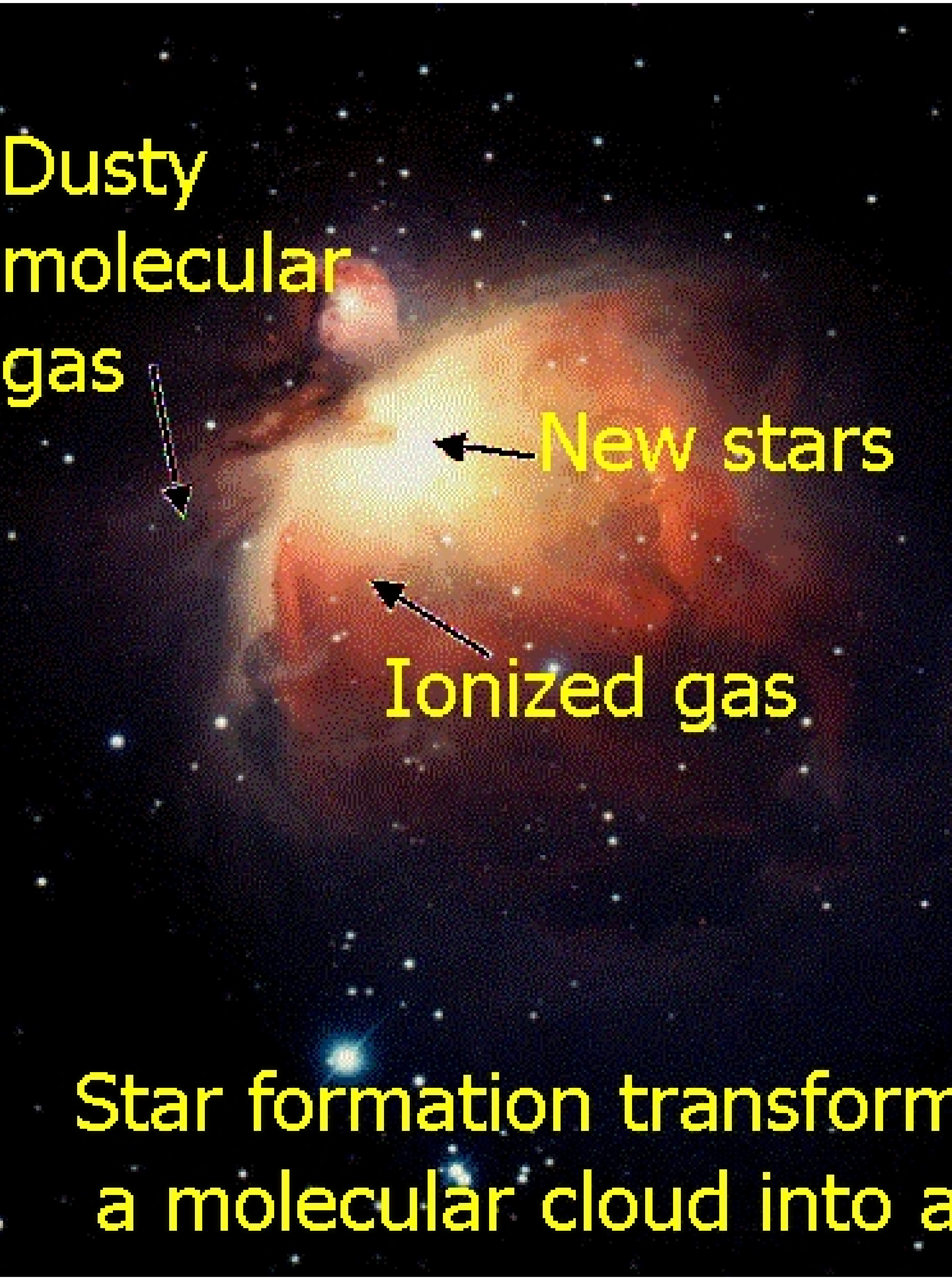




1. Extinction/Attenuation
2. Reddening
3. Reprocessing UV/optical light into the infrared
4. Scatters light.
5. Locks up metals



The Orion Nebula



- Hot young O & B stars heat the surrounding gas, ionizing it.

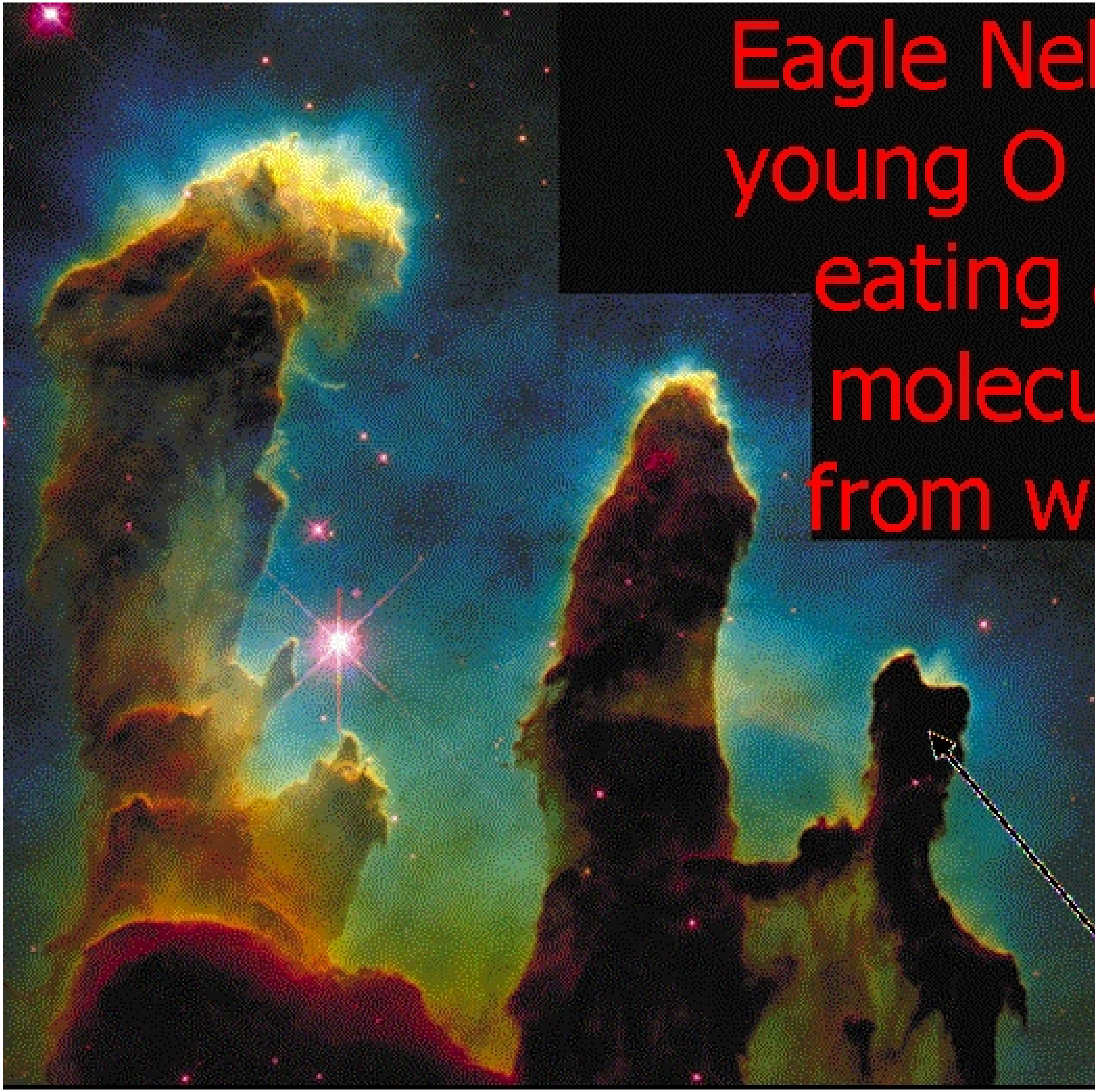
$$H_2 \rightarrow HII$$

Molecular H   Ionized H

Star formation transforms a molecular cloud into an

"HII Region"

Eagle Nebula: Hot young O & B stars eating away the molecular cloud from which they formed!



Gas is still molecular in the columns…

Optical Surveys

- Hipparcos: 3,000 stars visible by naked eye

- and many others...

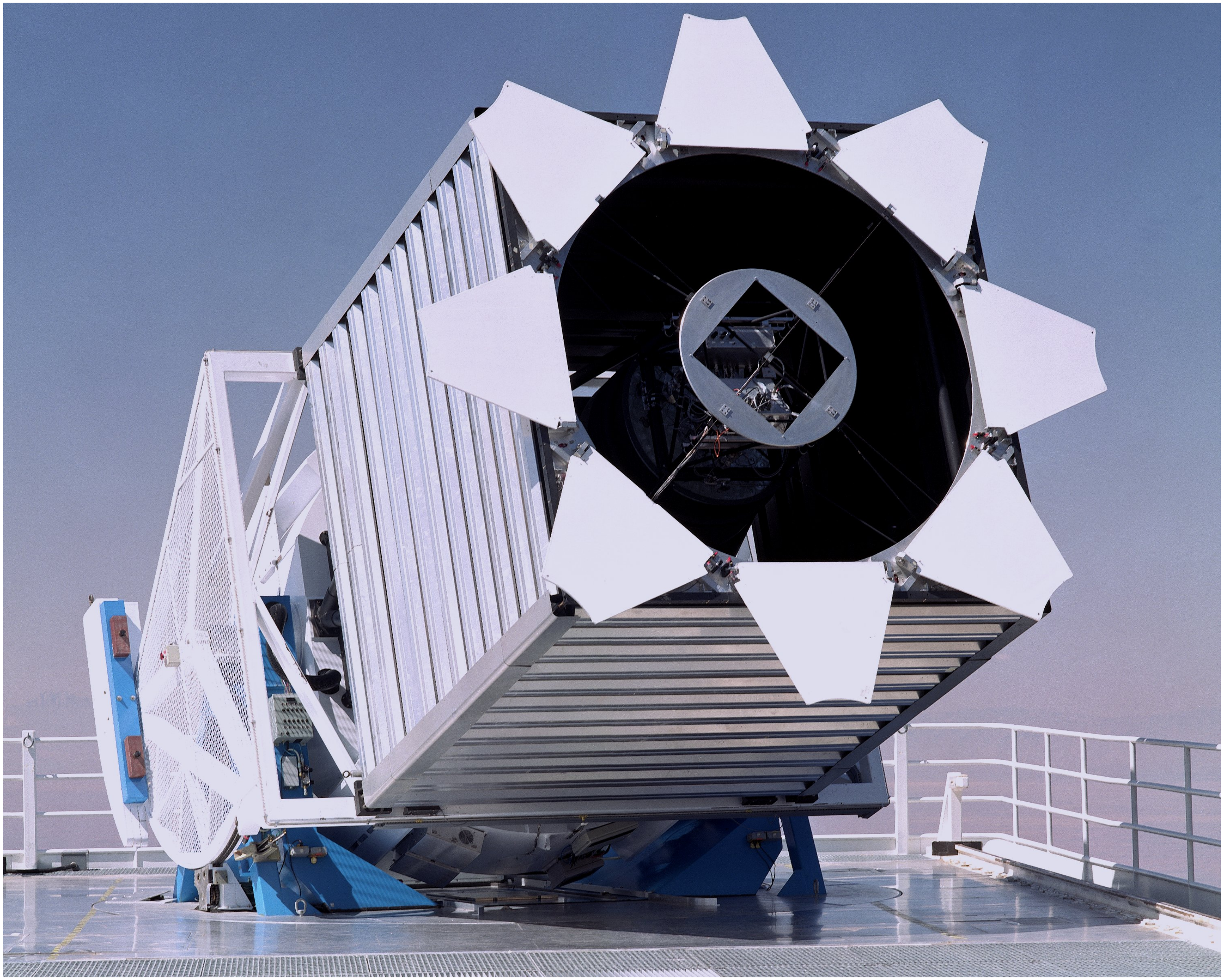
- Palomar Observatory Sky Survey: (first 1950-57, second 1985-1999) photographic, nearly all-sky, two bands, $m<20.5$, astrometric accuracy $\sim$0.5 arcsec, photometric accuracy 0.2-0.4 mag (both very non-Gaussian), USNO-B catalog: 10^9 sources

- SDSS: digital, 1/4 sky, 5 bands, $m<22.5$, astrometric accuracy $<$0.1 arcsec absolute, $\sim$0.02 arcsec relative, photometric accuracy 0.02 mag (both nearly Gaussian), several 10^8 sources

Sloan Digital Sky Survey (I and II)

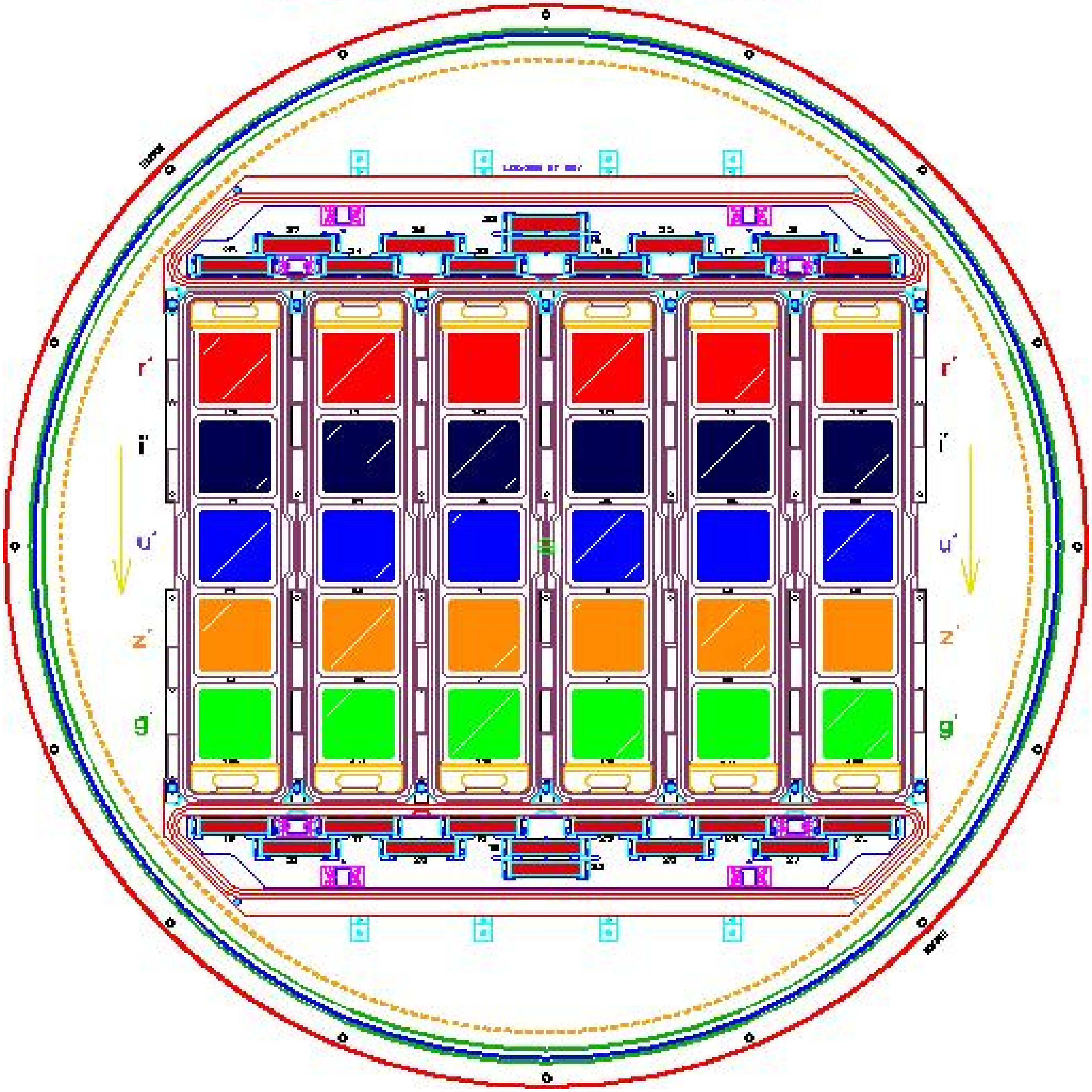
- Imaging Survey
  - 10,000 deg^2 (1/4 of the full sky)
  - 5 bands (ugriz: UV-IR), 0.02 mag photometric accuracy
  - $<$ 0.1 arcsec astrometric accuracy
  - 100,000,000 stars and 100,000,000 galaxies

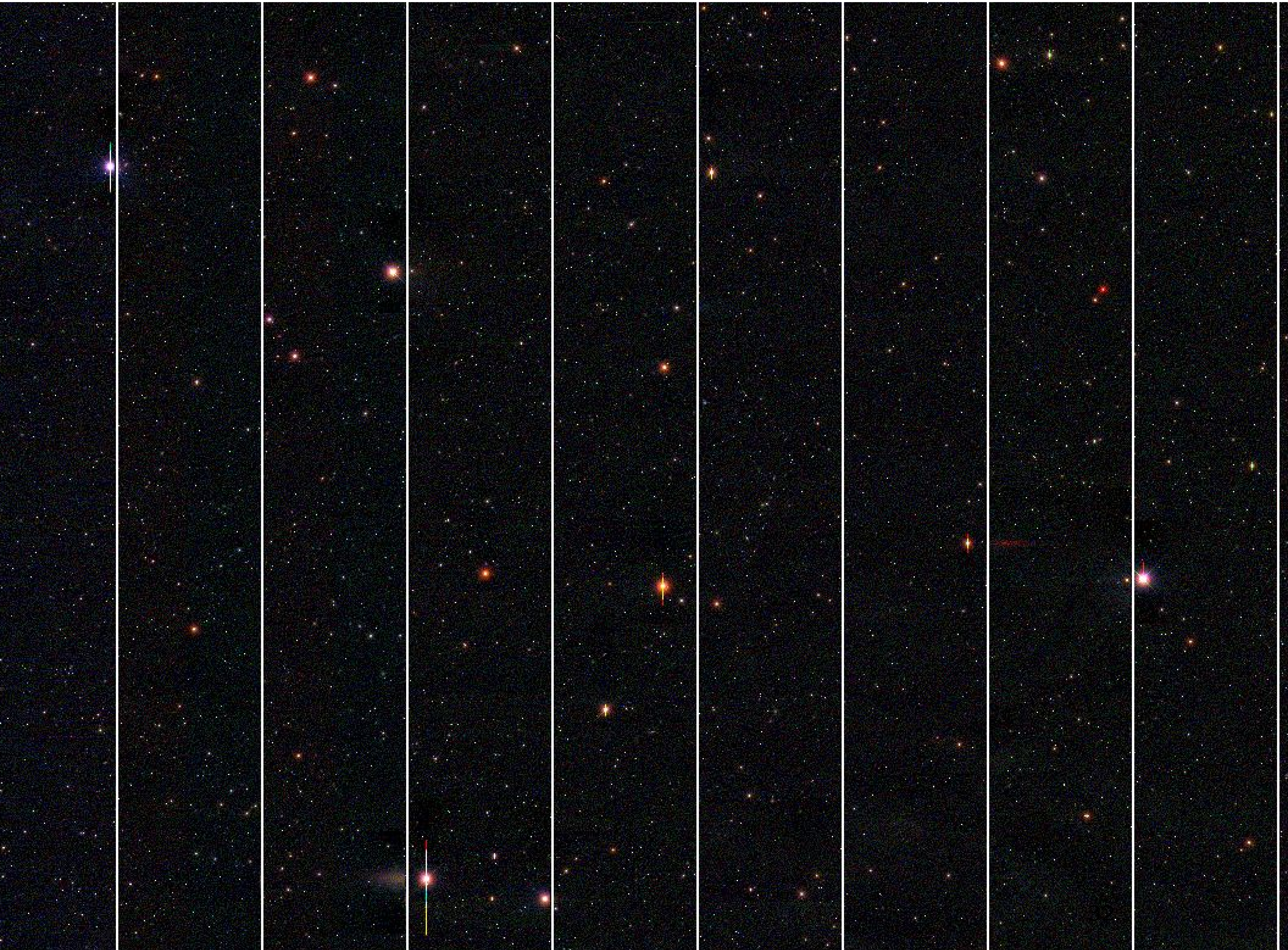
- Spectroscopic Survey
  - 1,000,000 galaxies
  - 100,000 quasars
  - 100,000 stars

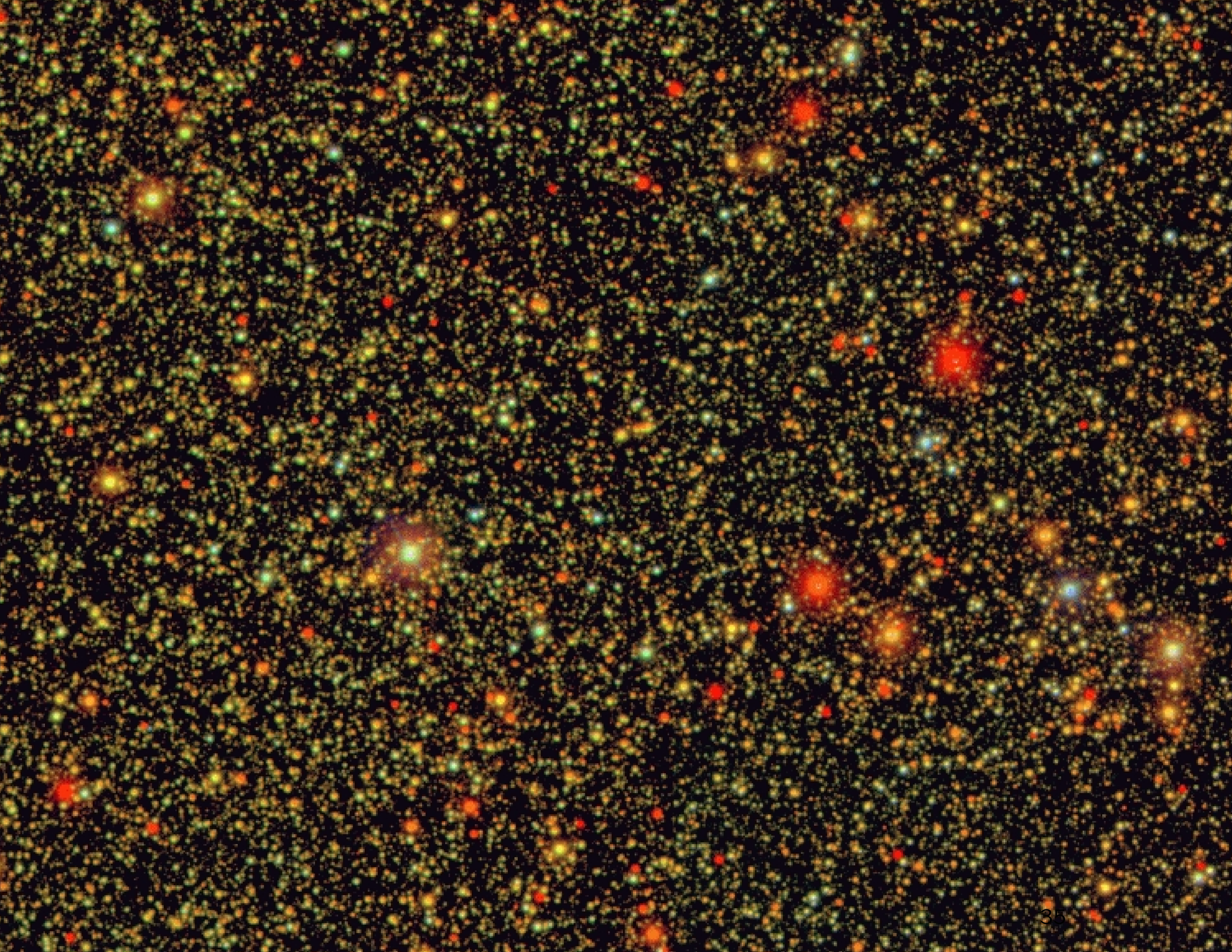


SDSS Telescope

SDSS CAMERA







SDSS and the Milky Way

- Advantages for studying the Milky Way structure
  - Accurate photometry: photometric distance estimates
  - Numerous stars: small random errors for number density
  - Large area and faint limit: good volume coverage

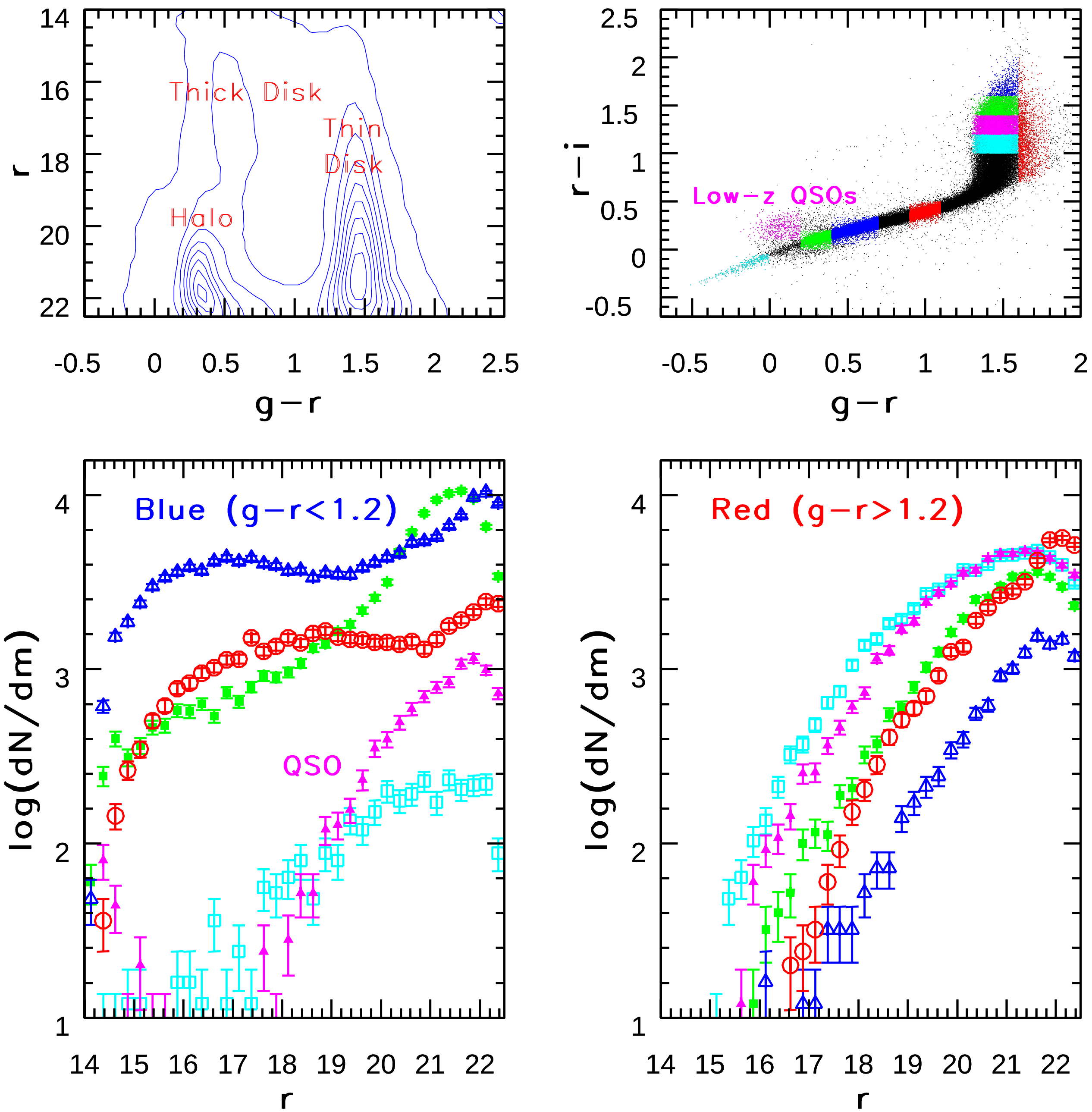
- Traditional approach: assume initial mass function, fold with models for stellar evolution; assume mass-luminosity relation; assume some parametrization for the number density distribution; vary (numerous) free parameters until the observed and model counts agree. Uniqueness? Validity of all assumptions?

- SDSS: adopt color-luminosity relation, estimate distance to each star, bin the stars in XYZ space and directly compute the stellar number density

Stellar Counts

There is a lot of information about the Milky Way structure (and stellar initial mass function, and stellar evolution) in SDSS imaging data.

9 epochs, unresolved, n=216830, psf mags, area=60 deg^2



Stellar Counts

There is a lot of information about the Milky Way structure (and stellar initial mass function, and stellar evolution) in SDSS imaging data.

How can we extract and interpret this information? What is the meaning of local maxima in the differential counts for some (but not all) color cuts?

Computing Differential Stellar Counts $n(m)$

1. $n(m) = dN/dm = dN/dV \, dV/dm$,
   $dN/dV = \rho(l, b, D)$ (ρ constrains Galactic Model)

2. For a pencil beam: $dV = \Delta\Omega \, D^2 \, dD$

3. $D = 10\text{pc} \, 10^{0.2(m-M)}$, $dD/dm = 0.2 \, ln(10) \, D(m)$

4. $n(m) = \rho(l, b, m) \, 0.2 \, \Delta\Omega \, ln(10) \, (10 \, pc)^3 \, 10^{-0.6M} \, 10^{0.6m}$

$$n(m) \propto \rho(l, b, m) \, 10^{0.6m}$$

Examples for $n(m) \propto \rho(l,b,m)\, 10^{0.6m}$

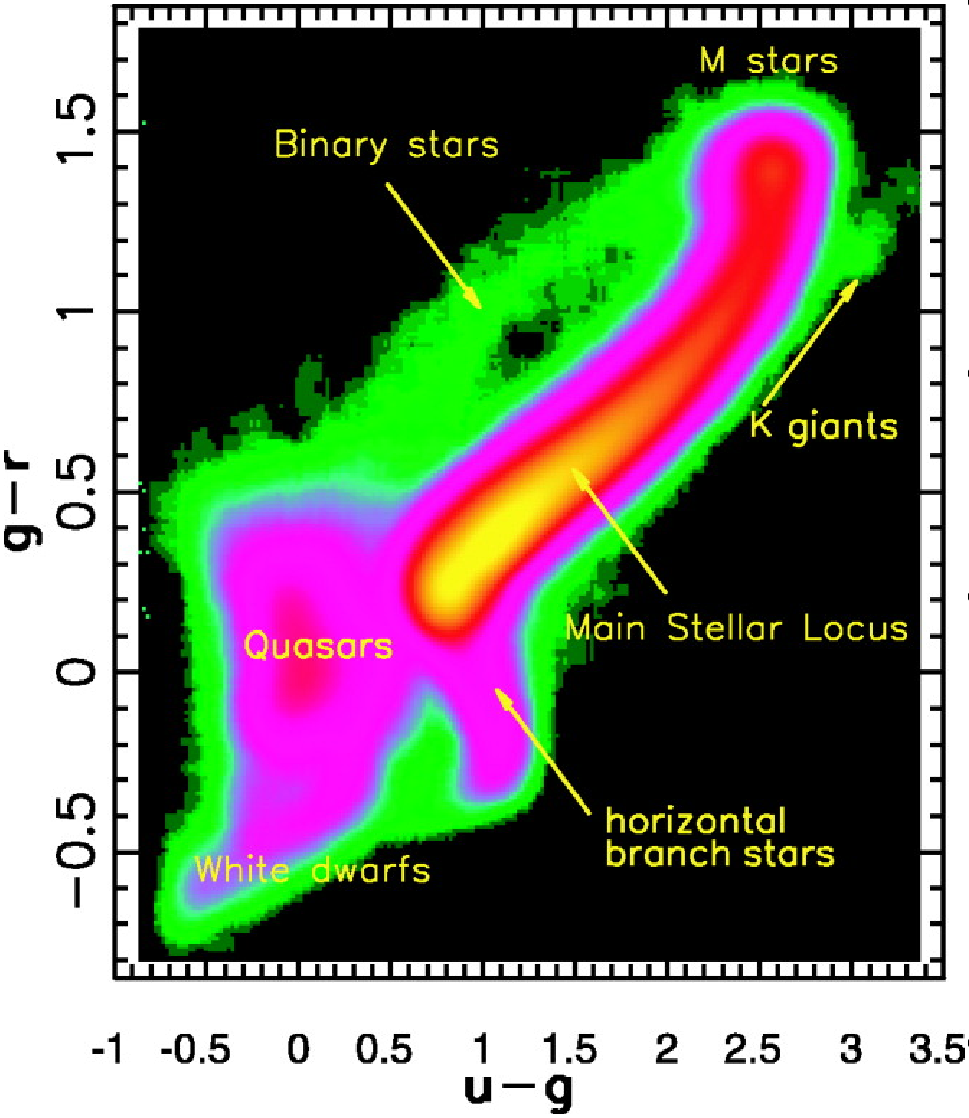
- Power-law: $\rho(l,b,D) \propto D^{-n}$

$$n(m) \propto 10^{k\,m},\ k = 0.6 - 0.2\,n$$

  - Euclidian counts (n=0): $n(m) \propto 10^{0.6\,m}$,
  - Halo counts (n=3): $n(m) = const.$

- Exponential disk: $\rho(l,b,D) \propto e^{-D/H}$
at a distance $D = k\,H$, $n(m)$ has a local slope corresponding to a power-law with $n = k$. Hence, for $D = 3\,H$, the differential counts for exponential density distribution have a local maximum!

SDSS Color-color diagrams

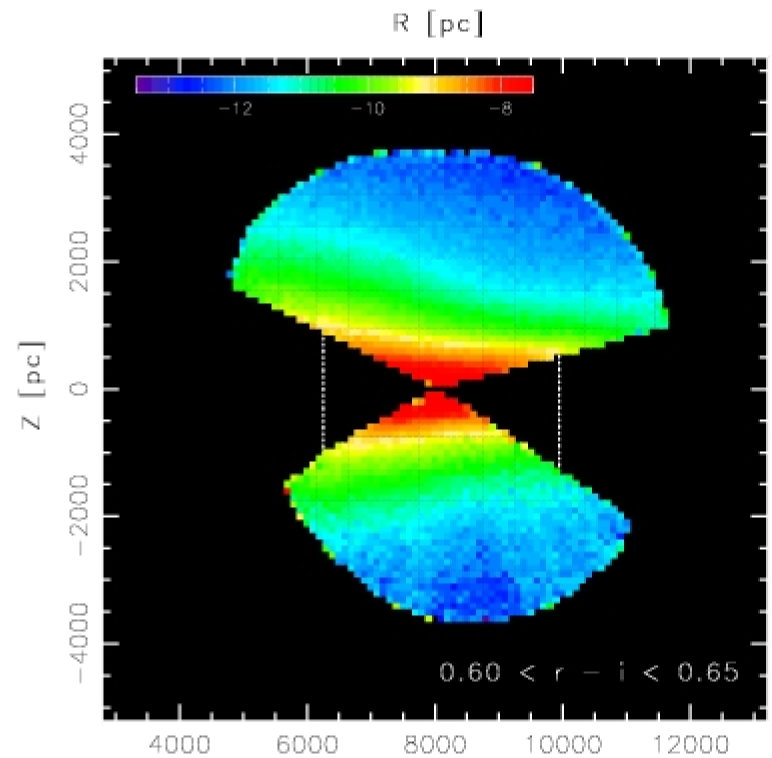


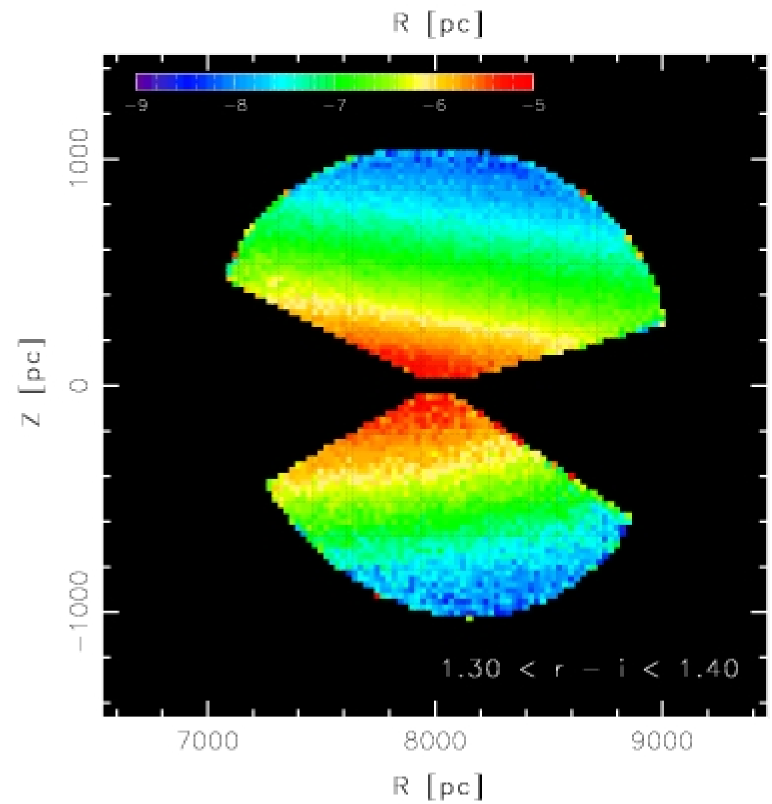
Smolčić et al. (2004)

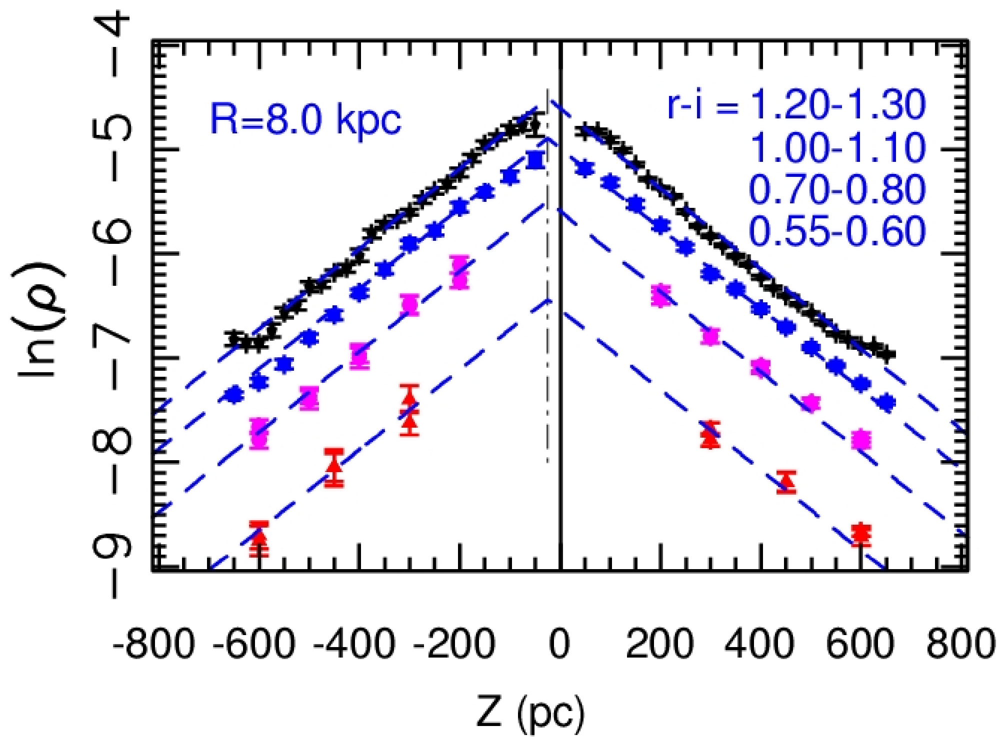
- Wide wavelength coverage of SDSS bandpasses, together with accurate and robust photometry, encodes a large amount of information
- Stars on the main stellar locus are dominated (∼98%) by main sequence stars
- The position of main sequence stars on the locus is controlled by their spectral type/effective temperature/luminosity, and thus can be used to estimate distance: photometric parallax method
- A short review of results from Jurić et al. (2008, Astrophysical Journal 673, 864.)

Local maps: thin disk

- Red(ish) stars have small luminosity: sampled to a few kpc
- The maps are roughly consistent with an exponential disk out to ∼1 kpc: the lines of constant density are straight lines
- The slope of these lines is given by the ratio of exponential scale height and scale length



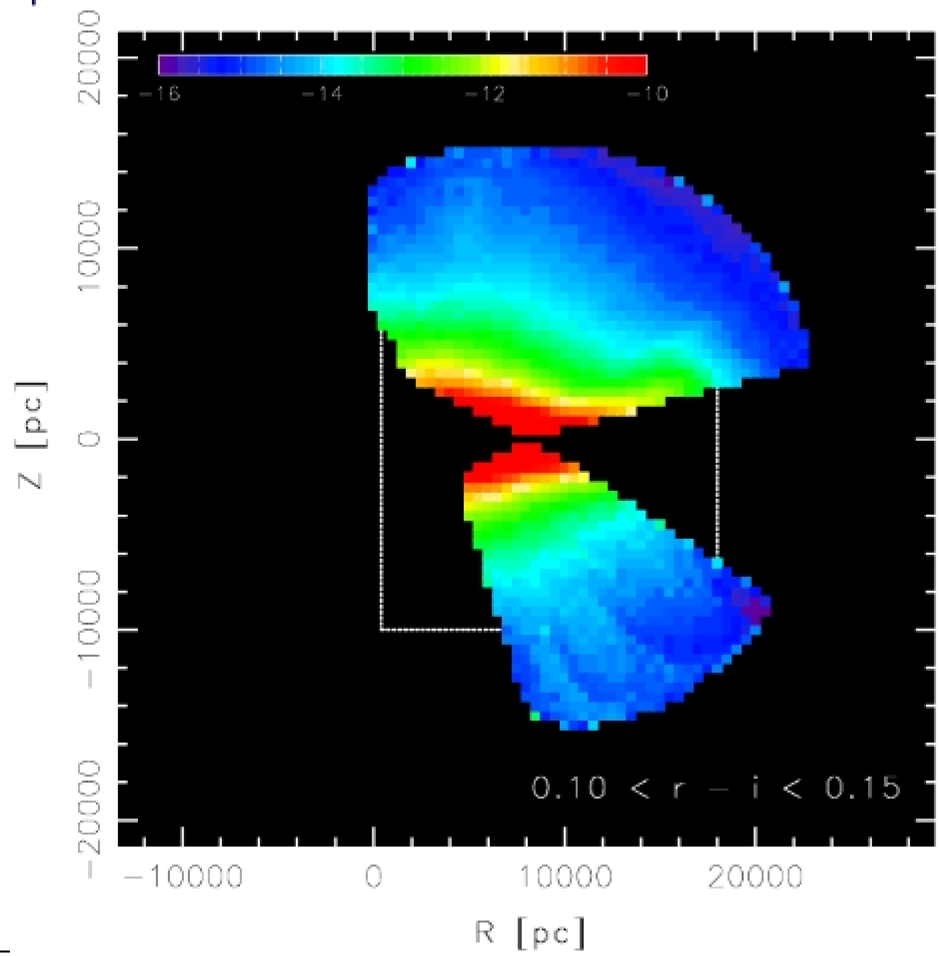


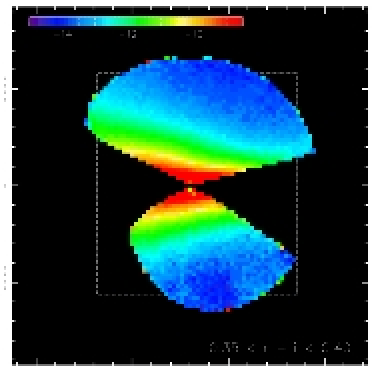


The r-i color bins sample a variety of scales

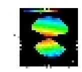
19 r−i color bins − 19 density maps

r - i	0.1	0.2	0.4	0.6	0.8	1	1.2	1.4
D [kpc]	15	13	7	4	3	2	1.5	1
SpT	~F9	G6	K4	K7	M0	M1	M2	M3





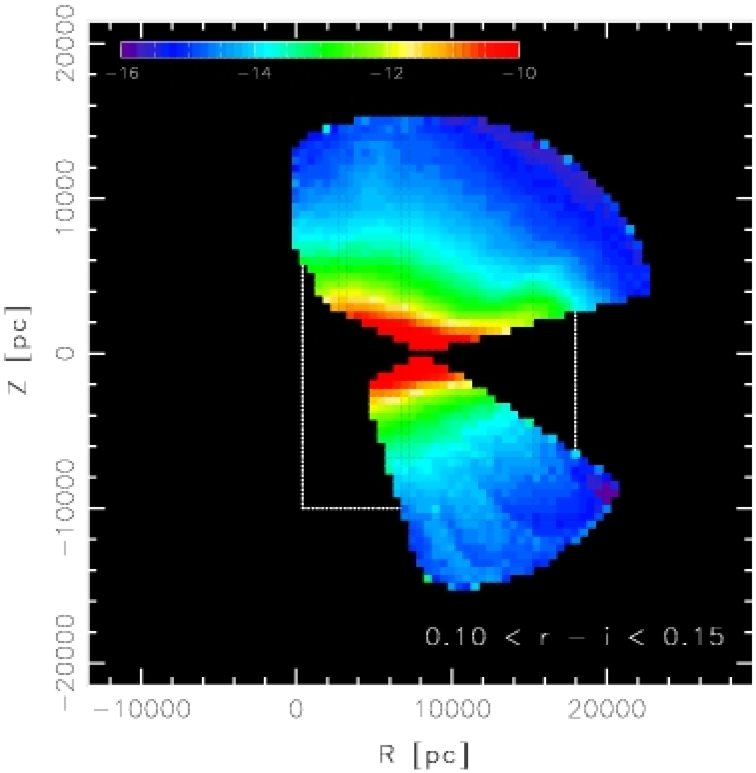
0.35 < r-i < 0.40

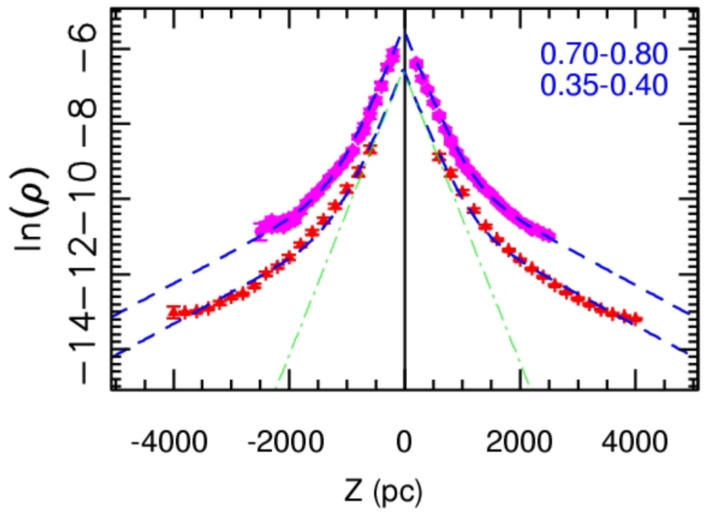


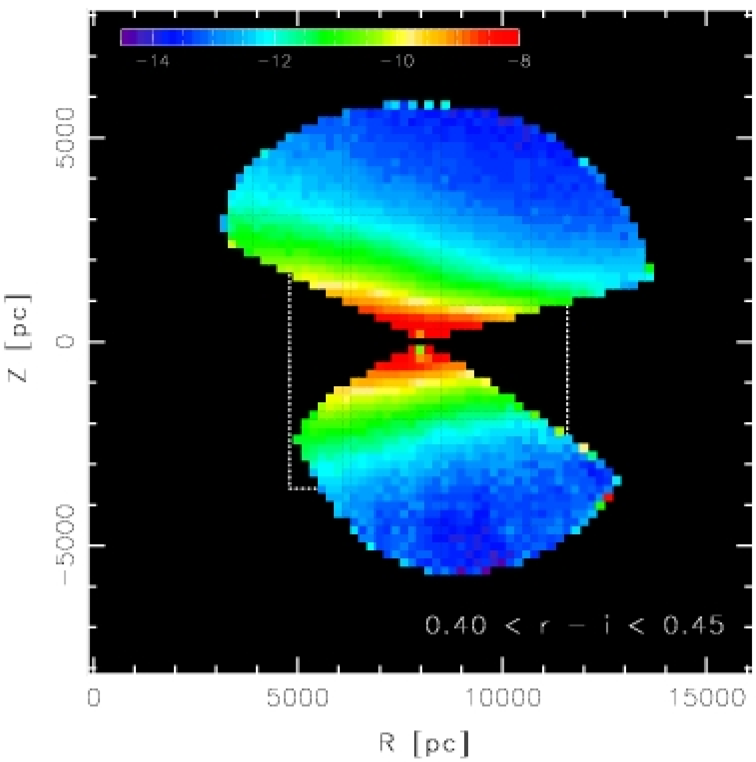
1.3 < r-i < 1.4

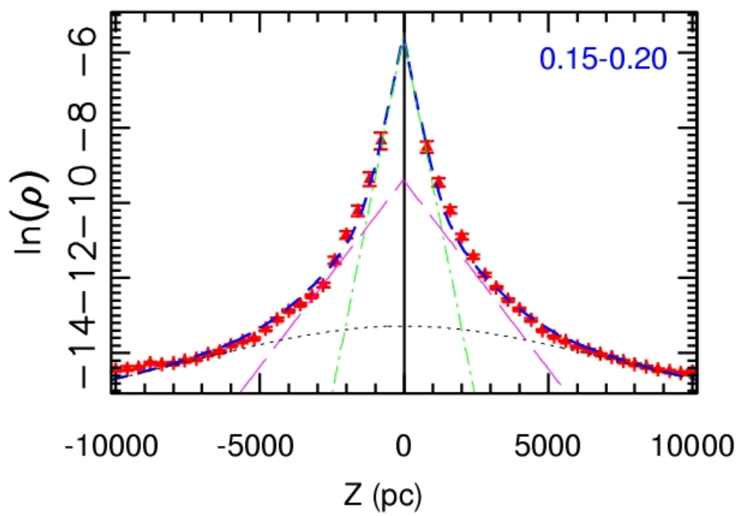
Thin to thick disk transition

- Yellow(ish) stars have intermediate luminosities: sample the transition from thin to thick disk at a few kpc





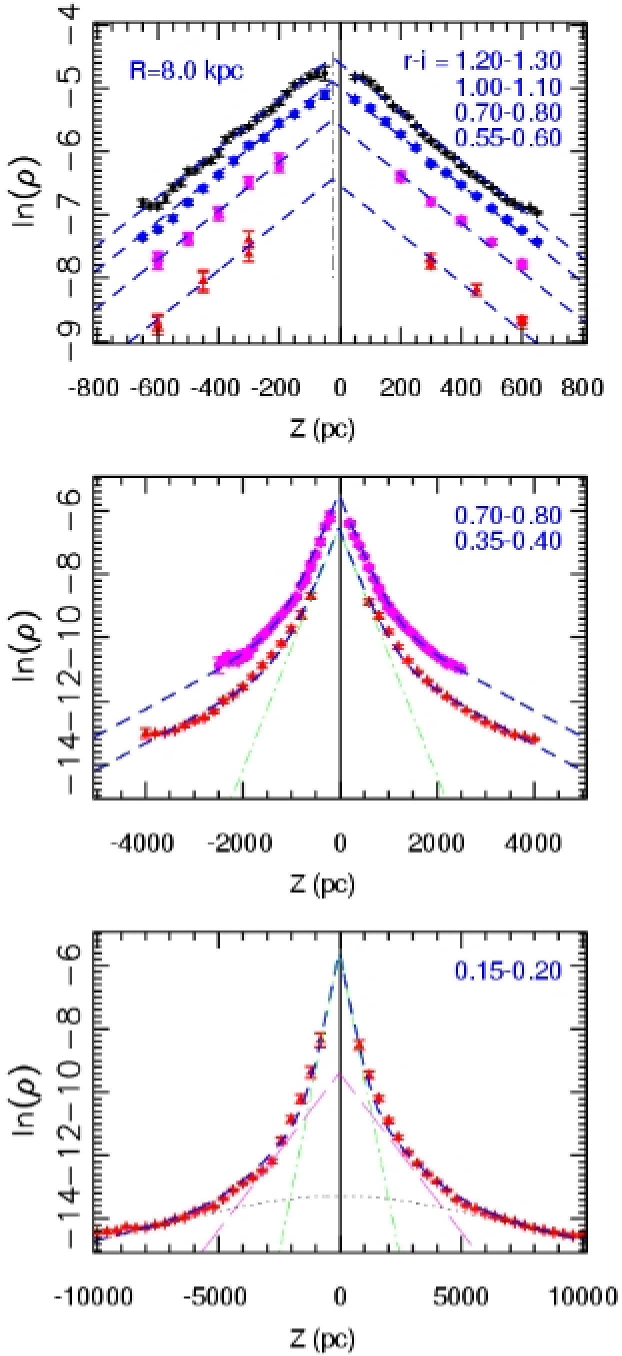




- Removal of obvious clumps
- Fit to least "contaminated" bins
- Exponential disks + halo models

$$\rho(R,Z) = \rho_{thin} e^{-\frac{R-R_e}{l_{thin}} - \frac{|Z+Z_0|}{h_{thin}}} + \rho_{thick} e^{-\frac{R-R_e}{l_{thick}} - \frac{|Z+Z_0|}{h_{thick}}} + \rho_{halo} \left(\frac{R_{GC}}{\sqrt{R^2 + (z+z_0)^2 / q^2}} \right)^n$$

R=8 kpc cross-sections and model predictions



Best fit parameters

$$Z_0 = 24\,pc \pm 5\,pc$$

$$h_{thin} = 270 \pm 10\,pc$$

$$l_{thin} = 2400 \pm 200\,pc$$

$$h_{thick} = 1400 \pm 50\,pc$$

$$l_{thick} = 3500 \pm 500\,pc$$

$$\rho_{thick} / \rho_{thin} = 0.06 \pm 0.01$$

$$\rho_{halo} / \rho_{thin} \sim 0.0001$$

$$q \cong 2$$

$$n \sim 1-3$$

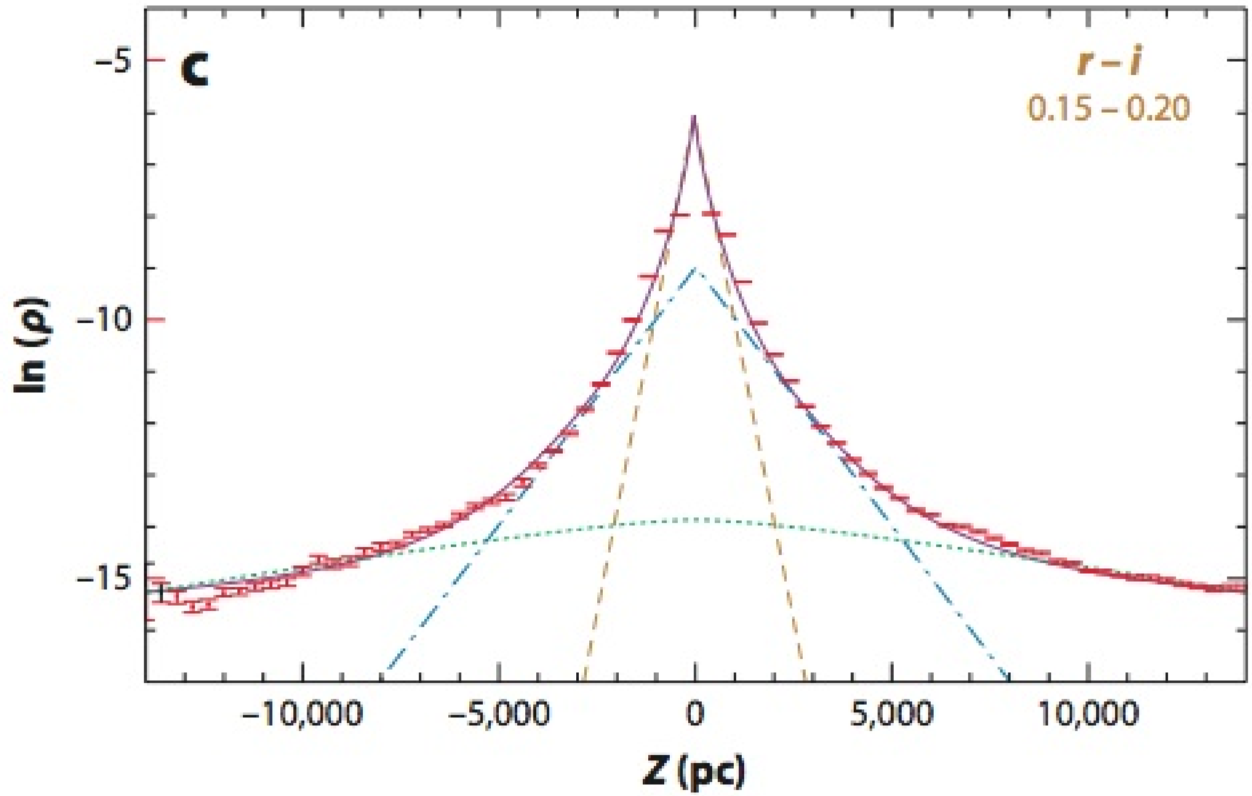


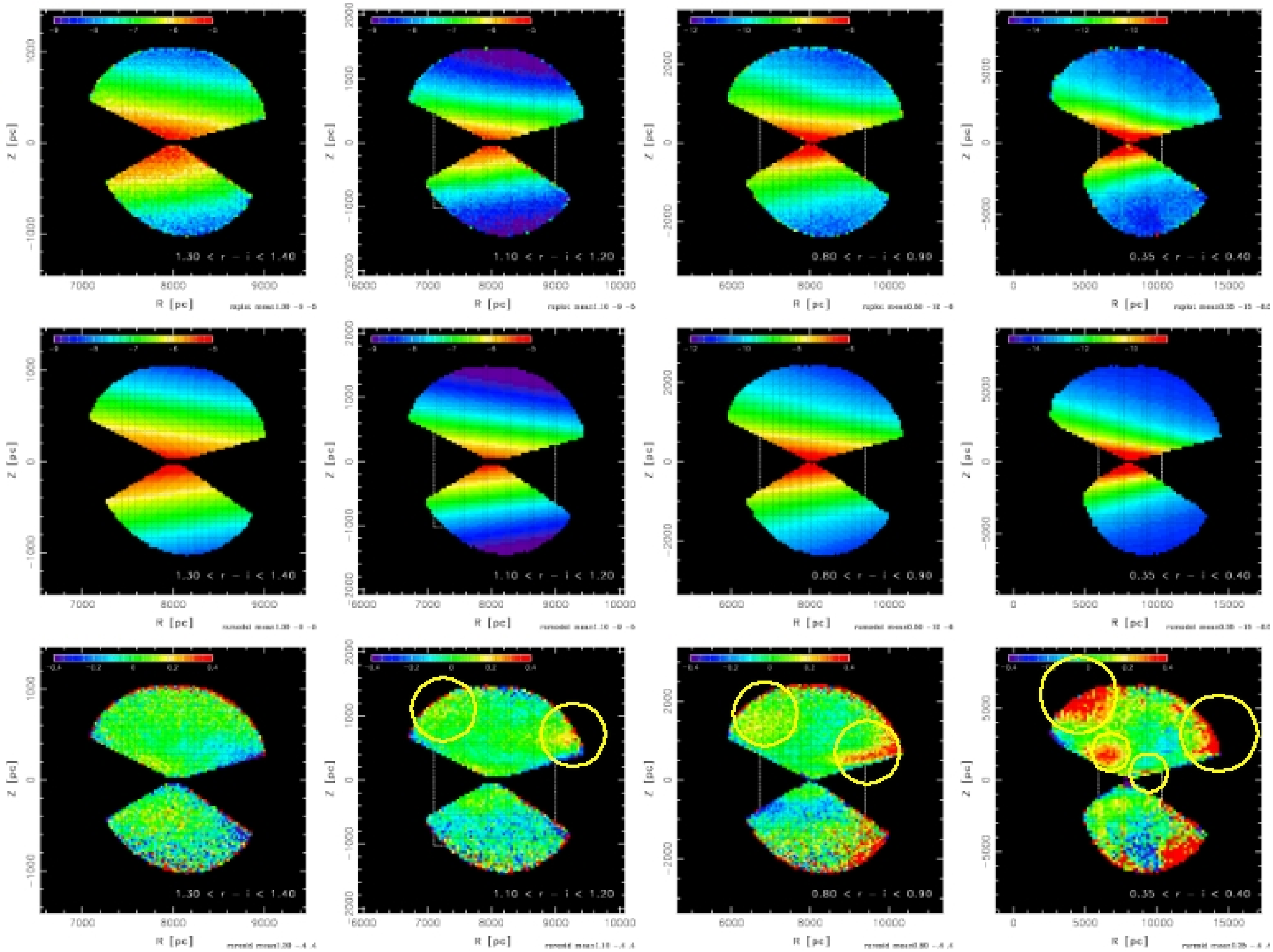
Figure 4

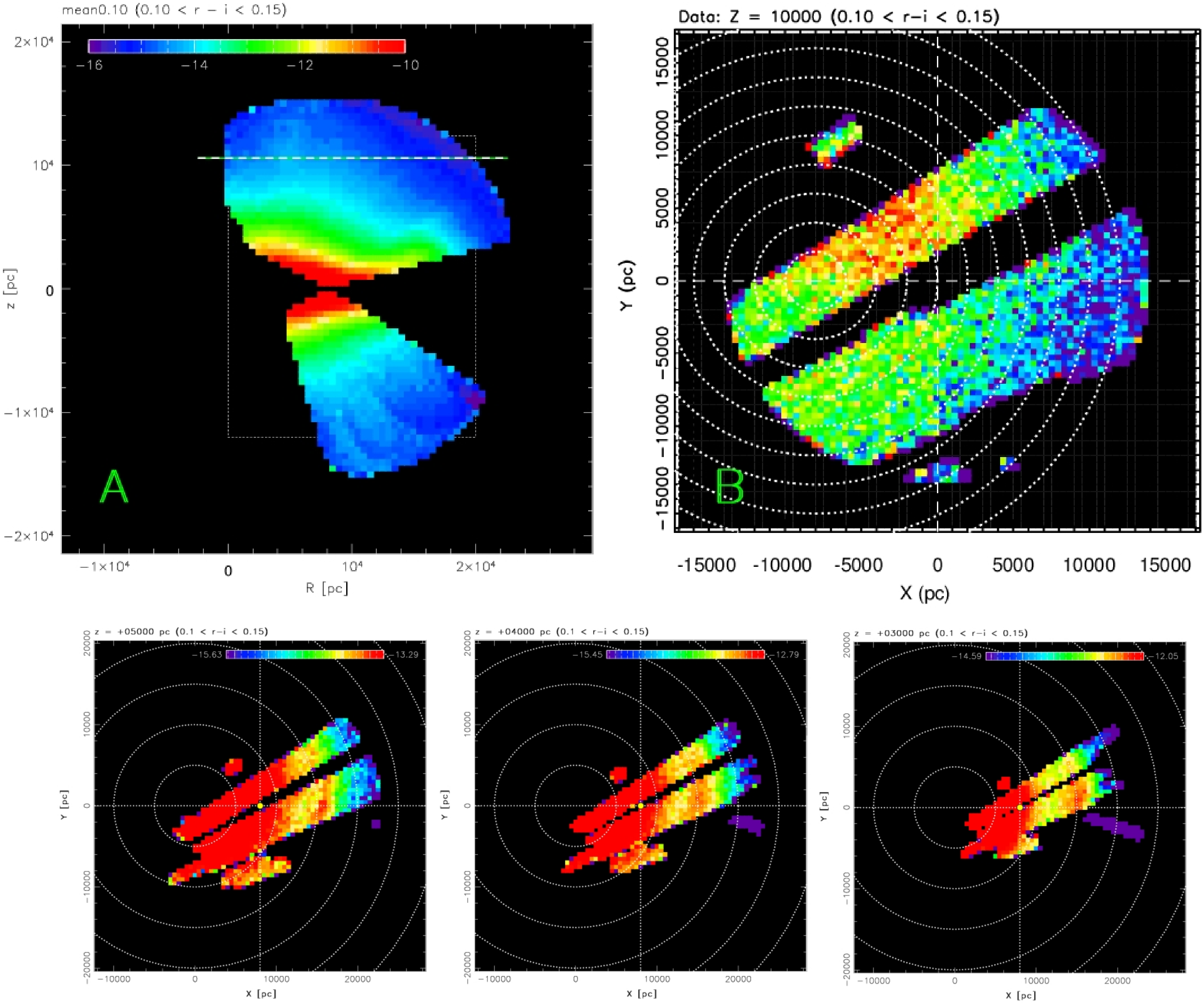
Cross sections through maps, similar to those shown in **Figure 3**, showing the vertical ($|Z|$) distribution at $R = 8$ kpc and for different $r - i$ color bins. The orange dashed lines are exponential models fitted to the red points (the sech^2 function is not a good fit; see footnote 28 of Jurić et al. 2008). The orange dashed lines in panel *a* correspond to a fit with a single, exponential disk. The vertical dashed line shows the best-fit position of the maximum density (not at 0 due to the Sun's offset from the disk midplane). The blue dot-dashed line in panel *b* corresponds to an additional disk component, and the data are fit with a sum of two disks with scale heights of 270 pc and 1,200 pc, respectively, and a relative normalization of 0.04 (the "thin" and the "thick" disks). The purple line in the bottom panel (closely following the data points) corresponds to a sum of two disks and a power-law spherical halo. The orange dashed line and the blue dot-dashed line are the disk contributions, and the halo contribution is shown by the green dotted line. For the best-fit parameters, see **Table 1**. Reprinted from Jurić et al. (2008).

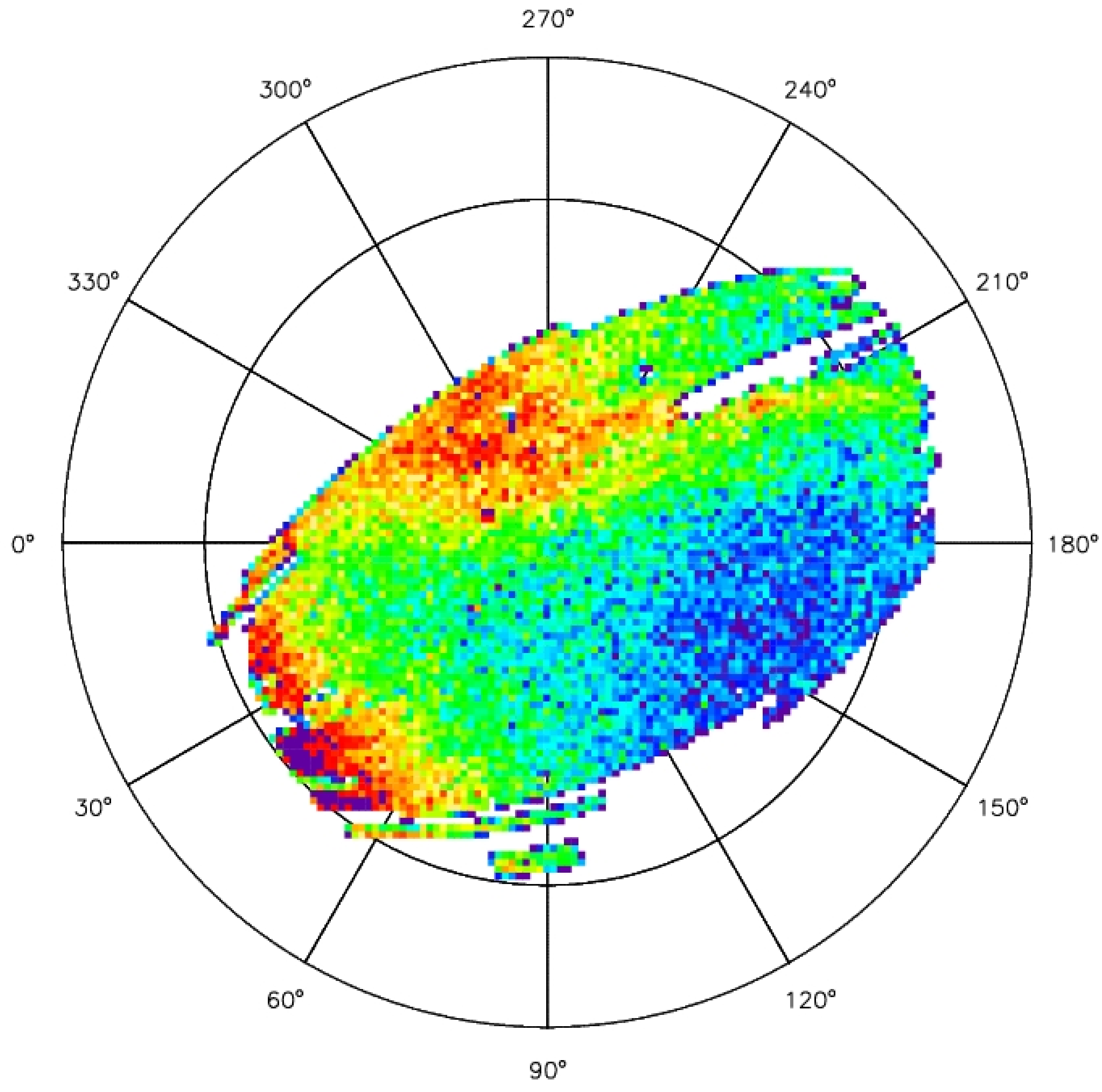
Model fit residuals

Scale increases this way

	1.3 < r-i < 1.4	1.1 < r-i < 1.2	0.8 < r-i < 0.9	0.35 < r-i < 0.40
Data				
Model				
D/M-1				




mean0.10 (0.10 < r − i < 0.15)
A
z [pc]
R [pc]
Data: Z = 10000 (0.10 < r−i < 0.15)
B
Y (pc)
X (pc)
z = +05000 pc (0.1 < r−i < 0.15)
z = +04000 pc (0.1 < r−i < 0.15)
z = +03000 pc (0.1 < r−i < 0.15)
Y [pc]
X [pc]



SDSS DR5: 0.3<g-r<0.4 & 21<r<21.5

Summary of the spatial distribution of stars:

- 3D stellar number density maps of the Milky Way from SDSS photometric observations of $\sim$50 million stars

- A two-component exponential disk model is in fair agreement with the data

- Halo properties poorly constrained due to rich substructure and limited sky coverage; however, an oblate halo is always preferred (no strong evidence for triaxial halo)

- A remarkable localized overdensity in the direction of Virgo over $\sim$1000 deg^2 of the sky

- Clumps/overdensities/streams are an integral part of Milky Way structure, both of halo and the disk(s)

Velocity measurements

- Velocity can be expressed as a (vector) sum of the component along the line of sight, or radial velocity (v_{rad}), and the component perpendicular to the line of sight, or tangential velocity (v_{tan}).

- Radial velocity is measured from spectra; large modern stellar spectroscopic surveys, such as SDSS and RAVE, obtain errors of a few km/s (a revolution: close to 10^6 spectra)

- Tangential velocity is measured from proper motion: angular displacement of stars on the sky (typically a tiny fraction of an arcsecond per year, but the record holder, Barnard's star, moves at 10 arcsec/yr); the two best large proper motion catalogs are based on the Hipparcos survey (an astrometric satellite, accuracy of $\sim$milliarcsec/yr for $V < 10$), and the SDSS-POSS catalog (5×10^7 stars, 3-5 mas/yr to $V < 20$)

Velocity measurements

- To get tangential velocity, v, from proper motion, μ, distance D must be known:

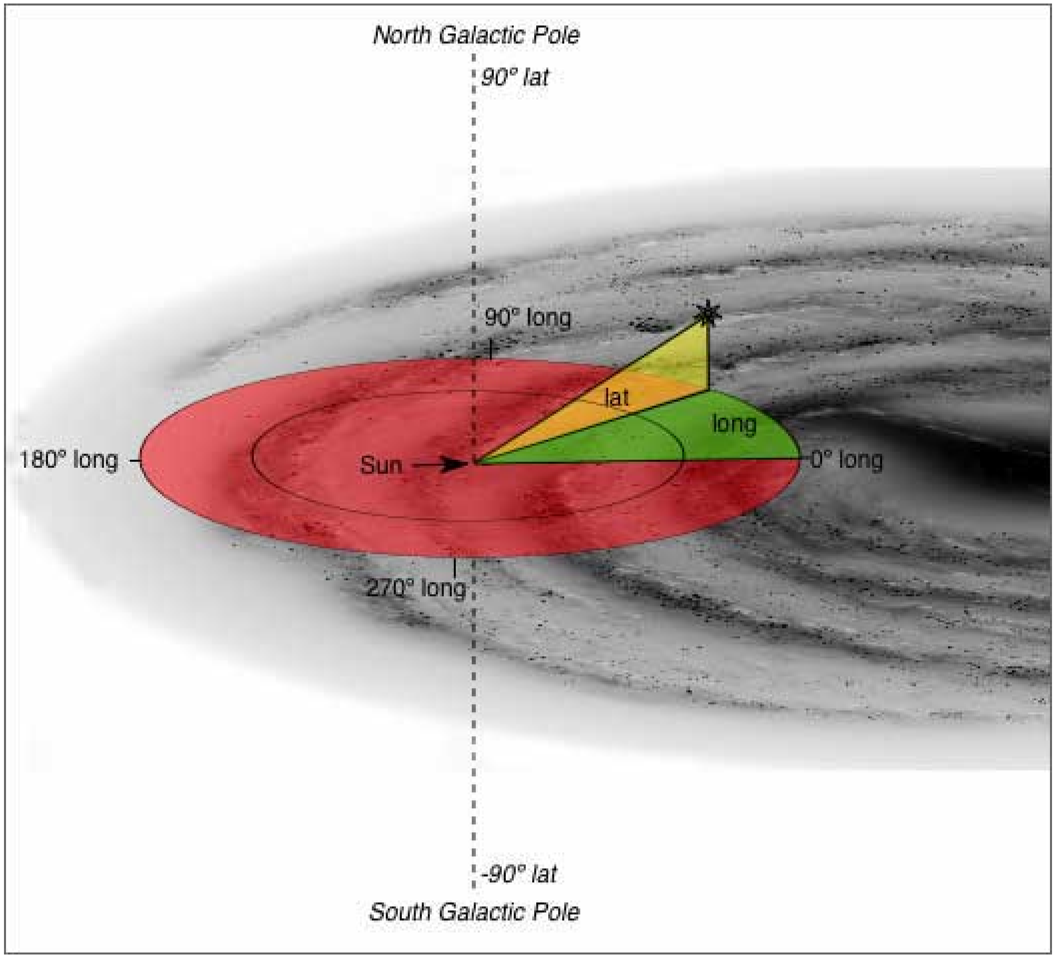
$$v = 4.74 \, \frac{\mu}{\text{mas/yr}} \, \frac{D}{\text{kpc}} \quad \text{km/s} \tag{1}$$

- At a distance of 1 kpc, and for proper motions good to $\sim$mas/yr, the tangential velocity errors are similar to radial velocity errors from SDSS and RAVE

- The advantage of radial velocity is that its measurement does not require distance, while the advantage of proper motion measurements is that they are much "cheaper"

Velocity measurements

- Assume that v_{rad} and the two components of tangential velocity, v_l (in the direction of the galactic longitude) and v_b (in the direction of the galactic latitude), are known.



Velocity measurements

- Assume that v_{rad} and the two components of tangential velocity, v_l (in the direction of the galactic longitude) and v_b (in the direction of the galactic latitude), are known.

- The Cartesian velocity components can be computed from

$$\begin{aligned} v_X^{obs} &= -v_{rad}\cos(l)\cos(b) + v_b\cos(l)\sin(b) + v_l\sin(l) \\ v_Y^{obs} &= -v_{rad}\sin(l)\cos(b) + v_b\sin(l)\sin(b) - v_l\cos(l) \\ v_Z^{obs} &= -v_{rad}\sin(b) + v_b\cos(b) \end{aligned}$$

- For completeness (right-handed coordinate system!):

$$\begin{aligned} X &= R_\odot - D\cos(l)\cos(b) \\ Y &= -D\sin(l)\cos(b) \\ Z &= D\sin(b) \end{aligned}$$

Velocity measurements

- Assume that v_{rad} and the two components of tangential velocity, v_l (in the direction of the galactic longitude) and v_b (in the direction of the galactic latitude), are known.

- The Cartesian velocity components can be computed from

$$
\begin{aligned}
v_X^{obs} &= -v_{rad}\,\mathsf{cos}(l)\,\mathsf{cos}(b) + v_b\,\mathsf{cos}(l)\,\mathsf{sin}(b) + v_l\,\mathsf{sin}(l) \\
v_Y^{obs} &= -v_{rad}\,\mathsf{sin}(l)\,\mathsf{cos}(b) + v_b\,\mathsf{sin}(l)\,\mathsf{sin}(b) - v_l\,\mathsf{cos}(l) \\
v_Z^{obs} &= -v_{rad}\,\mathsf{sin}(b) + v_b\,\mathsf{cos}(b)
\end{aligned}
$$

- Locally, these components are related to more traditional nomenclature as $v_X = -U$, $v_Y = -V$, and $v_Z = W$.

Velocity measurements

- How do we go from the measured v_X, v_Y, and v_Z for a star, to **its own galactocentric** v_R, v_ϕ, and v_Z?

- First, we need to account for our motion. When reporting radial velocity, the projection of Earth's orbital motion (up to 30 km/s!) is typically corrected. Hence, we *only* need to correct for the solar motion around the center of the Milky Way ($v^\odot$):

$$v_X^{obs} = v_X^* + v_X^\odot$$
$$v_Y^{obs} = v_Y^* + v_Y^\odot$$
$$v_Z^{obs} = v_Z^* + v_Z^\odot$$

  where v^* corresponds to a **star's own motion** around the center of the Milky Way (this is what we want to get)

Velocity measurements

- The solar motion is traditionally decomposed into the **rotational** motion of the Local Standard of Rest and the solar **peculiar** motion:

$$
\begin{aligned}
v_X^{\odot} &= v_X^{\odot,pec} \\
v_Y^{\odot} &= -v_{LSR} + v_Y^{\odot,pec} \\
v_Z^{\odot} &= v_Z^{\odot,pec}
\end{aligned}
$$

- **Note the minus sign in front of v_{LSR}!** Usually it is assumed that $v_{LSR} = 220$ km/s (based on HI measurements by Gunn, Knapp & Tremaine 1979), but some recent papers claim that it could be off by as much as 20-30 km/s (some methods are sensitive to uncertain $R_* = 8.0$ kpc!)

Velocity measurements

- The solar peculiar motion is obtained by averaging the motions of a large number of stars from the (local) solar neighboorhood (so that their peculiar velocities cancel out)

- Currently the best measurement of the solar peculiar motion is based on Hipparcos data (Dehnen & Binney 1998): $v_X^{\odot,pec} = -10.0$ km/s, $v_Y^{\odot,pec} = -5.3$ km/s, $v_Z^{\odot,pec} = 7.2$ km/s.

- But recently they revisited this problem (Schönrich, Binney & Dehnen 2010): $v_X^{\odot,pec} = -11.1$ km/s, $v_Y^{\odot,pec} = -12.2$ km/s, $v_Z^{\odot,pec} = 7.3$ km/s

- The measured mean Y velocity component depends greatly on the selected type of stars (the so-called *asymmetric drift*).

Velocity measurements

- How do we go from the measured v_X, v_Y, and v_Z for a star, to **its own galactocentric** v_R, v_ϕ, and v_Z?

- First, we need to account for our motion:

$$v_X^* = v_X^{obs} - v_X^{\odot}$$
$$v_Y^* = v_Y^{obs} - v_Y^{\odot}$$
$$v_Z^* = v_Z^{obs} - v_Z^{\odot}$$

- After (v_X^*, v_Y^*, v_X^*) are known, and assuming that the position of the star, (X^*, Y^*, Z^*), is known too, this is simply a coordinate system transformation $(R = \sqrt{X^2 + Y^2})$

$$v_R^* = v_X^* \frac{X^*}{R^*} + v_Y^* \frac{Y^*}{R^*}$$
$$v_\phi^* = -v_X^* \frac{Y^*}{R^*} + v_Y^* \frac{X^*}{R^*}$$

Oort's constants:

$$A \equiv \frac{1}{2}\left(\frac{v_c}{R} - \frac{dv_c}{dR}\right)_{R_\odot} \tag{2}$$

$$B \equiv -\frac{1}{2}\left(\frac{v_c}{R} + \frac{dv_c}{dR}\right)_{R_\odot}, \tag{3}$$

where $v_c(R)$ is the rotational velocity.

In the solar neighborhood,

$$A = 14.5 \pm 1.5 \text{ km/s/kpc}, \quad B = -12 \pm 3 \text{ km/s/kpc}. \tag{4}$$

Since $A - B$ is not vanishing, locally the radial velocity curve is not flat.