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COMPUTING LONG-TERM DAYLIGHTING SIMULATIONS FROM HIGH DYNAMIC RANGE IMAGERY USING DEEP NEURAL NETWORKS

Yue Liu[1], Alex Colburn[2], and Mehlika Inanici[1]
[1]University of Washington, Seattle, WA
[2]Zillow Group, Seattle, WA

ABSTRACT

Compared with illuminance-based metrics, luminance-based metrics and evaluations provide better understandings of occupant visual experience. However, it is computationally expensive and time consuming to incorporate luminance-based metrics into architectural design practice because annual simulations require generating a luminance map at each time step of the entire year. This paper describes the development of a novel prediction model to generate annual luminance maps of indoor space from a subset of images by using deep neural networks (DNNs). The results show that by only rendering 5% of annual luminance maps, the proposed DNNs model can predict the rest with comparable accuracy that closely matches those high-quality point-in-time renderings generated by Radiance (*RPICT*) software. This model can be applied to accelerate annual luminance-based simulations and lays the groundwork for generating annual luminance maps utilizing High Dynamic Range (HDR) captures of existing environments.

INTRODUCTION

Architectural daylighting design is not only driven by energy concerns but also motivated by a desire to improve human comfort. The presence of daylight can improve occupants' health, awareness, and feelings of well-being (Boyce 2014). However, uncontrollably maximizing daylight penetration into buildings can lead to undesirable luminous environments that can impair vision or create visual discomfort.

Attitudes and research practices in architectural lighting field are shifting towards luminance-based metrics and evaluations. Compared with illuminance-based metrics, luminance-based metrics provide more meaningful information about occupant visual experience. For example, the primary source of indoor visual discomfort is discomfort glare caused by excessive light or contrast in an occupant's field of view. Therefore, glare can be better understood through luminance distribution-based metrics (Wienold and Christoffersen 2006, Jakubiec and Reinhart 2012, Suk et al. 2013, Konis 2014, Van Den Wymelenberg and Inanici 2015).

Practitioners and researchers need long-term daylighting simulations to predict the effectiveness of their design strategies and decisions. Although illuminance-based annual simulations are accessible to lighting professionals through a number of programs and metrics, luminance-based annual simulations remain computationally expensive. Long-term simulations are computed through multi-phased daylight coefficient methodologies, which have steep learning curves and long simulation times. There is a need to improve the availability and accessibility of long-term luminance-based lighting simulations.

Accelerating annual daylight simulations is an active area of research. *Daylight coefficient* (DC) approach has been developed as a numerical methodology to perform annual daylight predictions in a more efficient manner (Tregenza and Waters 1983). The classic DC concept is to divide the celestial hemisphere into discrete sky segments and calculate the contribution of each segment to the illuminance level at various sensor points. Further developments of *dynamic daylight simulation methods* (DDS) divide the light flux transfer process into multiple phases to better model complex fenestration systems (Laouadi et al. 2008, Ward et al. 2011).

More recent developments in lighting simulation acceleration depend on advances in modern computing technology. Two recent trends in lighting acceleration research are: 1) Increasing rendering efficiency by tracing multiple primary rays in parallel on a *graphics processing units* (GPU) (Jones and Reinhart 2014). Modern GPUs with highly parallel structure make them more efficient than general purpose central processing units (CPUs) for parallel computing of large data blocks. 2) Predicting lighting performance using machine learning based-algorithms.

This research follows the second trend and develops a workflow to generate annual luminance maps of indoor space from a subset of data using *artificial neural networks* (ANNs). Machine learning, and specifically ANN, has been used recently within the architectural lighting field. ANNs have been investigated for predicting indoor daylight illuminances (Kazanasmaz et al. 2009, Zhou and Liu 2015, Navada et al. 2016, Ahmad et al. 2017) and for developing and classifying sky models (Li et al. 2010, Satilmis et al. 2016).

Although statistical methods have been utilized to predict long-term luminance maps from limited number of imagery (Inanici 2013), the predictions were more successful for overcast skies and need further improvements to better model the sun patches under sunny skies. To the best of the authors' knowledge, there are no previous studies that utilize machine learning to predict long-term luminance distributions. This study provides a novel solution for long-term luminance simulation accelerations. It is inspired by Ren et al. (2015), which is based on learning the non-linear mapping of pixel-scale luminance values from local and contextual attributes of surface points. The accuracy, applicability, and usefulness of the proposed DNNs model is demonstrated, and its effectiveness is exemplified through computer-generated images.

OBJECTIVES

The objective of this research is to demonstrate the utilization of *deep neural networks* (DNNs) (a.k.a deep ANNs) techniques to predict long-term luminance maps from a small subset of data. The ultimate goal is to generate annual luminance maps from *High Dynamic Range* (HDR) photographs of existing environments, which will enable quantitative analysis of daylit environments without the time-consuming modeling process. To reach this final goal, in this paper, simulated images (instead of captures of existing buildings) are utilized to facilitate the development of the algorithm with adequate number of imageries under controlled settings.

Specifically, the contributions of this paper are as follows:

- A presentation of the first deep learning framework to conduct architectural luminance predictions: The proposed DNNs model accelerates annual luminance-based simulations by generating annual luminance maps extracted from a small set of rendered inputs. The performance of the model is quantitatively investigated to meet the scientific accuracy requirement for applications in architectural lighting.
- An analysis of the sensitivity of prediction accuracy to sample data size, which inform the optimum sample size for data collection and generation: The study further evaluates the impact of lower-precision predictions on the design decision making processes by utilizing false color images and visual discomfort indices.
- An exploration and analysis of the various processing and design decisions of the proposed DNNs system.
- A demonstration of a workflow that accurately predicts annual luminance maps from a small number of rendered images.

METHODOLOGY

ANNs

Machine learning is an approach to analyze and model complex systems. One popular and powerful Machine learning technique is ANNs. ANNs were proposed in 1943 (McCulloch and Pitts 1943) and inspired by biological neurons of animal brains and consist of neuron-like connected computing units called nodes. These nodes are organized into layers: input, output, and the hidden layers (i.e. layers between input and output). The connection between two nodes has a weight which defines how much the previous node influences the next node. The network gathers information and finds relationships between inputs and outputs through a learning (training) process. During the learning process, information is transferred from layer to layer and learned knowledge is stored in node weights. This enables neural networks to solve complex problems, where numerical solutions are difficult to obtain. A DNNs model is an ANN model with multiple hidden layers between the input and output layers. Like shallow ANNs, DNNs can solve complicated nonlinear problems such as predicting long-term luminance. In this study, DNNs are used to approximate the non-linear relationship between the image pixel luminance and daylighting conditions with minimal inputs.

Fig. 1. gives an overview of the proposed methodology. The workflow starts with the generation of sample data (HDR renderings) using the Radiance simulation engine. Radiance (Ward, 1994) is a physically-based simulation engine that has been validated against illuminance measurements in full-scale spaces (Mardaljevic 2000). Each sample consists of the input parameters and a pair of images: a high-quality rendering and a sun patch image. The high-quality renderings are used as ground truth for training the neural networks, while the low-quality sun patch images and simulation parameters are

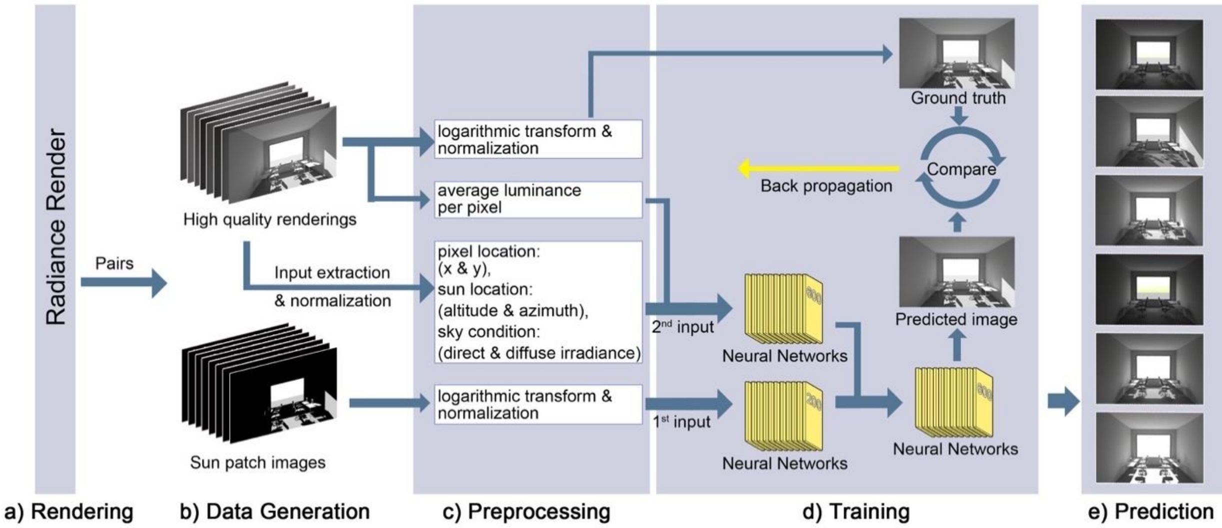

Figure 1. Overview of the framework: a) To train the DNN model, a small number of HDR images generated with the Radiance simulation engine are utilized. b) Pairs of high-quality renderings and quick sun patch renderings are generated. The high-quality renderings are used as ground truths for training the DNN, while the sun patch images are used as input to the DNN. c) The sun patch images and simulation parameters are combined to create input for the DNNs. d) The network is trained so that it accurately predicts the high-quality renderings from the input. e) Once trained, the network can be used to generate high-quality renderings from novel sun patch images and simulation parameters.

used as input to the network. The network is trained so that it accurately predicts the high-quality renderings from the simulation parameters and sun patch images. Once trained, the network can be used to generate high-quality renderings from other sun patch images and simulation parameters.

Light Transport Modeling

A light transport model (Eq. 1.) defines how light travels through space. When conducting daylight simulations in physically-based simulation engine, the light transport model is computed using complete scene information (scene geometries, light sources, and material properties). In contrast, the light transport model in this study is approximated from a limited number of rendered HDR images that exhibit scene appearance under various lighting conditions. Such a model can quickly and accurately predict a luminance map of the scene under different illumination conditions. The light transport model **M** of the scene is approximated with a machine learning driven neural networks model ($\mathbf{M_{NN}}$).

$$\boldsymbol{L} = \boldsymbol{M} \times \boldsymbol{I} \quad \text{(Equation 1)}$$

Where $\boldsymbol{M}$ is the light transport model, and $\boldsymbol{L}$ represents the luminance of image pixels. $\boldsymbol{I}$ is a feature vector which describes the illumination condition.

Input and Output of the DNNs Model

The first task in designing a DNNs model $\boldsymbol{M_{NN}}$ is to determine the input and output. $\boldsymbol{M_{NN}}$ encodes the non-linear relationship between the feature vector I and the image luminance L. The model M_{NN} is constructed the same way to approximate the relationship between L and I. Image luminance Lp of a pixel is the output, and the input is the feature vector Ip that consists of several parameters described the illumination conditions. These input parameters include: 1) the location of the sun, defined by sun altitude θ_{sun} and azimuth Ω_{sun}; 2) the luminance distribution of the sky, defined by direct irradiance Ir_{dir} and diffuse irradiance Ir_{dif}. Direct and diffuse irradiances can be easily retrieved from the weather files, and they are the essential parameters required by Perez all-weather sky models (Perez et al. 1993); 3) the location of the pixel defined by x and y coordinates (P_x, P_y). These six parameters describe the luminous environment within an indoor scene at a given date and time. Augmenting the input parameters with the average luminance of a pixel (Lum_{ave}) improves the accuracy of the model. This is consistent with a previous study which demonstrates that average color of a pixel indicates its similarity to other pixels in material and geometric properties (Ren et al. 2015). Thus, the proposed input feature vector includes 7 inputs $I_p = \{P_x, P_y, \Omega_{sun}, \theta_{sun}, Ir_{dir}, Ir_{dif}, Lum_{ave}\}$.

Data Generation

To develop the method under controlled settings, Radiance generated images are used. The test room model (Fig. 2) is located in Seattle (47.6°N, 122.3°W) and consists of a south-facing window and basic furniture. The reflectance values of walls, ceiling and floor are 50%, 80%, and 20%, respectively.

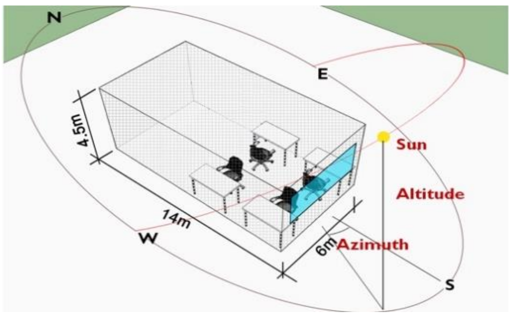

Figure 2 Room model used in Radiance simulations

Perez all-weather sky models are generated using the direct and diffuse irradiance values extracted from Seattle EnergyPlus weather file (EPW). Two sets of images are then rendered using Radiance RPICT method: high-quality renderings with 4 ambient bounces (-ab 4) and quick sun patch renderings with 0 ambient bounce (-ab 0). These two sets of images share the other rendering parameters (-ps 2 -pt .05 -pj .9 -dj .7 -ds .15 -dt .05 -dc .75 -dr 3 -st .15 -aa .1 -ar 512 -ad 2048 -as 1024 -lr 8 -lw .005). The HDR renderings of the scene are generated in 1-hour intervals for the entire year. The total number of these images is 4379 for both high-quality renderings and sun patch renderings. This database is prepared for model development purposes only. When applying the workflow, the user only needs to generate a small subset of these images.

The generated images are divided into three groups: training, validation and test groups. The images in the training group are the input to feed into the model. During the training process, only the model with the improved performance on validation sets are saved. The test group images are used to evaluate the performance of the method. There is no overlap between the test group and training/validation group. K-means method (divide the training samples into evenly distributed clusters and uniformly select samples from each cluster) is used to reduce the bias in the training, validation and test sets. This makes selected training samples well-distributed over the light domain.

Data Preprocessing

After the data generation step, the input parameters and the output luminance are preprocessed and normalized to the range of [0,1]. This is done to reduce the variations in the input values so that they contribute more proportionately to the final results. However, due to the nature of high dynamic range renderings, the luminance values in one image can span several orders of magnitude (e.g., the brightest part of the solar corona compared to the shadows in the room). Therefore, after the normalization, most of the indoor luminance values are very low. A gamma-correction of 2.2 is applied to increase the contrast and spread the range of luminance more evenly over the range of [0,1]. After the training process, the inverse of the preprocessing transform is applied to reconstruct the final luminance maps.

ANNs Architecture

It is known that a neural network can fit any function with arbitrary accuracy, given adequate network size and training data. An ANNs model with more trainable parameters may provide more computational power during the training process, but meanwhile, require more input data to avoid overfitting (Turmon and Fine 1994). Initially, the study started by designing a model with the same number of learnable parameters as described in Ren et al. (2015). But the proposed DNN model differs from Ren et al. in that it utilizes a single DNN rather than an ensemble model created via a hierarchical clustering mechanism. The model achieves the same level of accuracy and can be trained in hours on a single machine rather than CPU cluster nodes. The DNNs model is created with five hidden layers, each of 600 nodes. In each layer, the network applies a linear convolution to the output of the previous layer and then applies a nonlinear activation function Rectified linear unit (ReLU). ReLUs are simple and fast to evaluate and have been shown to achieve state-of-the-art performance in many optimization tasks.

Initial experiments show that a model based on the feature vector composed of 7 inputs can capture the smoothly varying illumination, but it fails to adequately capture the sharp shadows and sun penetrations caused by the direct sunlight (Fig. 3 (b)(d)). Although this model can be improved by using more high-quality rendering samples that cover more sun positions, this defeats the purpose of training with a limited number of high-quality renderings. To improve the result, a second feature vector (pixel luminance of sun patch renderings $Lum_{sun\ patch}$) is added. Compared with high-quality renderings, quick sun patch renderings have two advantages: 1) they can be quickly simulated, and 2) they can be generated using a crude model constructed from either HDR

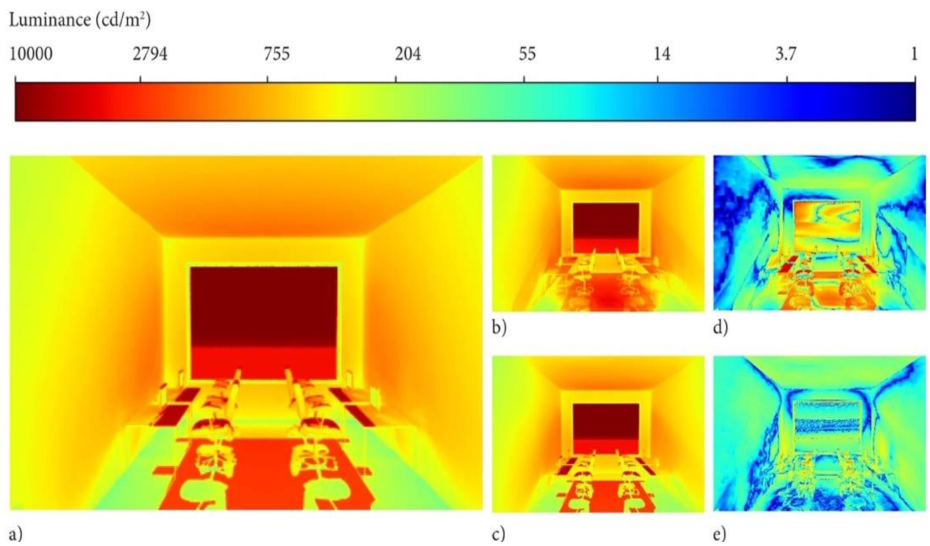

Figure 3. An example illustrates the comparison of the predictions made by the 1st DNNs model and the final proposed DNNs model. The ground truth luminance map (a) is poorly predict as in (b) with blurry shadows and sun penetrations by the 1st DNNs model. With the help of quick sun patch renderings, the final proposed model generates more satisfying results as in (c). (d) and (e) are errors maps for (b) and (c).

photographs or basic CAD models. This information can aid neural networks in modeling the light transport of scenes with sharp shadows and sun penetrations, as shown in Fig. 3 (c)(e). This second feature vector also provides the network with more data to avoid the overfitting problem.

The proposed DNNs model (Fig. 4) contains two input vectors: a vector with 7 parameters and 2nd vector consisting of the luminance of the sun patch image ($Lum_{sun\ patch}$). The 1st and 2nd input first go through 4 convolutional layers of 600 filters and 1 convolutional layer of 200 filters; respectively. All convolutional layers using the same kernel size of 1 and stride of 1. A concatenation layer then merges these input layers. The combined features are followed by two additional

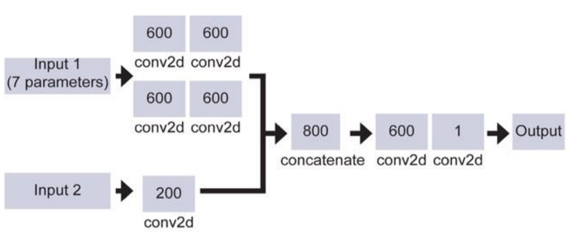

Figure 4 The proposed DNNs architecture. After a series of 2-dimensional convolutional layers, a concatenate layer merges two heads of input: one for regressing the sun patch images (Input2) and the other for the location, sun and sky parameters, etc. (Input1). The number indicates the amount of filters in each layer (e.g., 600 conv2d).

convolutional layers of 600 filters and 1 filter; respectively. The ReLU activation functions are applied to all layers. Increasing the kernel size lowered the model accuracy during model design studies. Because this model yields satisfactory results, adding more complicated types of layers is not necessary.

Training

The main idea of training the neural network is to minimize the difference between the predicted image and the ground truth image (high-quality rendering). The backpropagation algorithm is used to train the network via examples. The idea is to repeatedly update the neural networks by comparing the predicted image and the ground truth images, until a stopping condition is achieved. The metric to evaluate this difference during the training process is called the loss function. Choices of loss functions greatly impact DNNs and their learning dynamics (Janocha and Czarnecki 2017). After evaluating several loss functions, a combination of mean square error (MSE) (Eq. 2.) and relative error rate (RER) (Eq. 3.) produced the best results. The final loss function is defined as f = MSE+ RER * 0.01. The training process takes 2 hours on NVIDIA Tesla K80 GPU. After the training is finished, prediction of a new lighting condition can be made in seconds.

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(I_i - \tilde{I}_i)^2 \qquad \text{(Equation 2)}$$

$$\text{RER} = \sqrt{\frac{\sum_i (I_i - \tilde{I}_i)^2}{\sum_i {I_i}^2}} \qquad \text{(Equation 3)}$$

where n is the number of elements, and I_i and $\tilde{I}_i$ are the ground truth and predicted luminance of pixel i.

RESULTS AND DISCUSSION

The neural network trained using a limited number of samples from the training group results in an approximation of the light transport model M_{NN}. The model is then tested on images under new illumination conditions that the networks have not seen before. The test sets contain 500 samples randomly selected from the test group. The model's performance is measured by MSE (Eq. 2.) and RER (Eq. 3.) between the predicted images and the ground truth images. The results are evaluated using false color images and image subtraction operations.

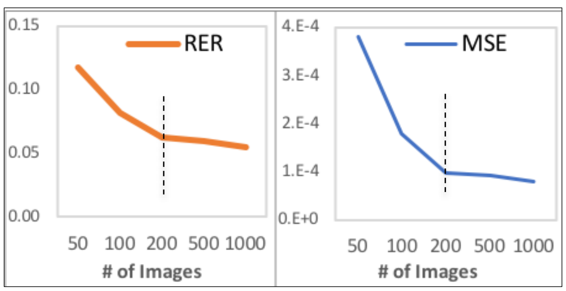

Figure 5. Sensitivity tests show the relationship between the number of input images and the accuracy of test sets evaluated by RER (left) and MSE (right). The dashed lines indicate an optimal sample size of 200.

A sensitivity analysis is performed using the training sets of 50, 100, 200, 500, and 1000 images (out of 4379 images) and the test set of 500 images. The purpose of the analysis is to understand how the number of training samples influences the prediction accuracy, and to find the optimum sample size. Fig. 5 shows the result error curves with respect to the number of training samples. The predicted image accuracy, measured by MSE and RER, increases with the number of training samples. However, the error curves decrease slowly after the number of images reach to 200. Therefore, 200 images are selected as the optimum sample size, with predictions MSE of 1e-05 and RER of 0.063 using a three-fold cross validation. This demonstrates that using 5% (200 out of 4379) of high-quality renderings, and the model can predict a luminance map of any time of the year with an average 93% accuracy.

Additionally, to evaluate how the results can influence lighting design decisions, the daylight glare probability (DGP) (Wienold and Christoffersen, 2006) is added as a visual comfort indicator, In DGP analysis, luminance maps are processed to determine the potential glare sources, and subsequently, to determine the percentage of the population who would find the scene as glary. DGP is applied to both the ground truth images and the predicted images. Note that both images have the same field of view. The results show that DGPs evaluated using predicted images closely match those using ground truth images with an average absolute error of 2.6e-08.

To better illustrate the results, Fig. 6 shows three selected samples of different sky conditions (clear sky, intermediate sky, and cloudy sky) from the test set of this study. Predicted images are compared to ground truth images, with error maps illustrating the absolute differences. All images are shown in false color with logarithmic scale and each image is labeled with MSE, RER, and DGP values. The results show that: 1) It is hard to differentiate predicted images from ground truth images in false color without the help of error maps. False color images are commonly used by architects or lighting professionals to make design decisions. The proposed model is able to make predictions that will lead to the same design decisions of those real high-quality renderings, even with a small number of input samples (50 images out of 4379 images). 2) Similarly, DGPs of predicted images closely match those of ground truth images. This leads to a similar conclusion that same design decision can be made with predictions based on small training samples. 3) As more clearly shown in error maps, the differences decrease when the number of images increases. This matches the conclusion of the previous sensitivity analysis. 4) Among the three sky conditions, predicted images of a sunny sky with high direct irradiance have the highest errors. 5) While the result generated by this method looks almost indistinguishable from the ground truth, the hardest part to predict is the outdoor ground plane seen through the window. This planar geometry is not the Radiance generated ground hemisphere, which can be more easily predicted.

CONCLUSION

This study presents the development of a novel machine learning based method to generate annual luminance maps. Annual luminance maps are essential for qualitative and quantitative lighting evaluations, but they are computationally expensive to generate. The results demonstrate that by using a DNNs model, it is possible to shorten the annual simulation time by using 5% high-quality renderings evenly sampled over the year as input and reach a comparable accuracy in the test set. This method can be used as an alternative to accelerate annual luminance-based simulations.

This study is a first successful step towards using DNNs for generating annual luminance maps from a limited

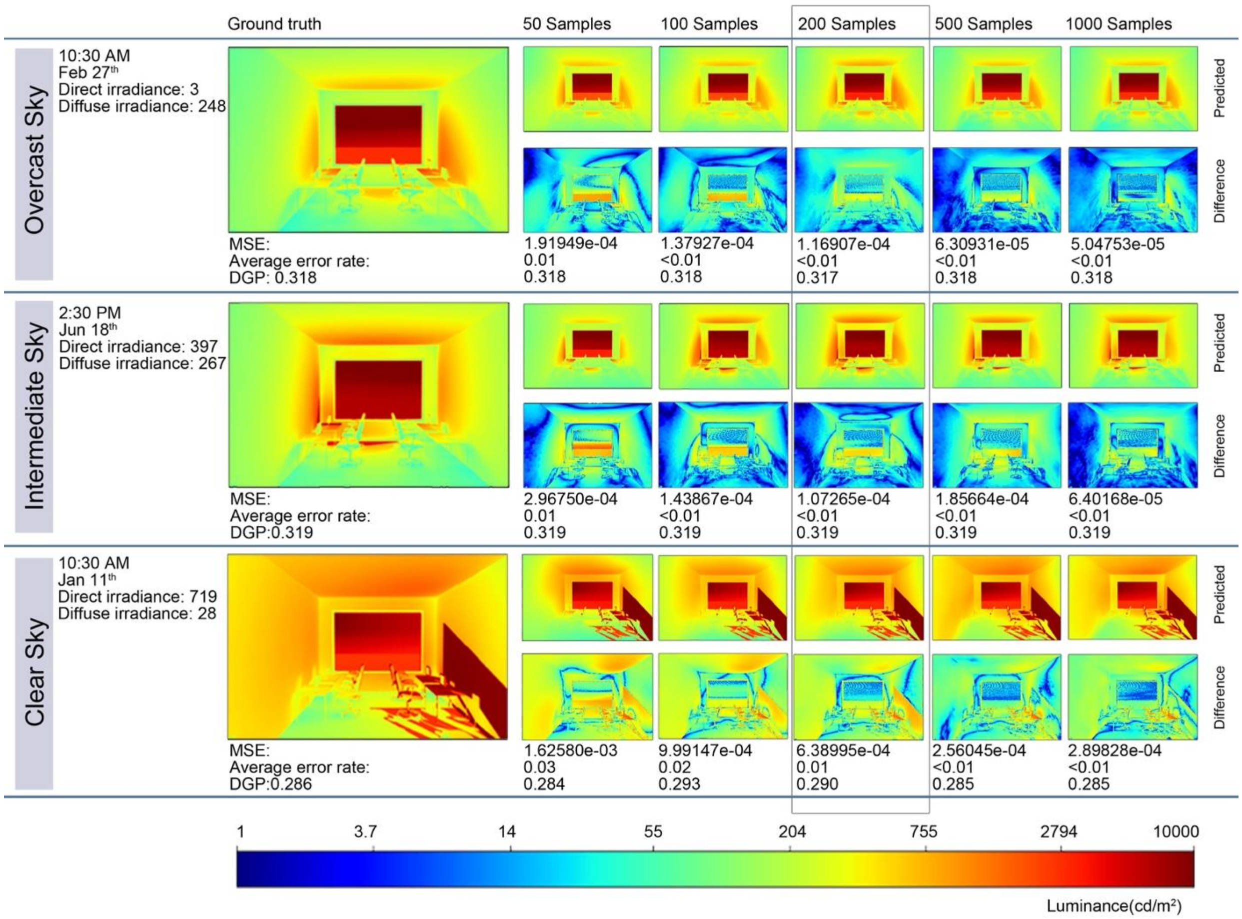

Figure 6. Sensitivity experiment results. Three exemplary test cases with different sky types are displayed in false color with logarithmic scale. The bounding box highlights an optimal sample size of 200.

number of HDR captures. It demonstrates that DNNs can be applied to solve complicated rendering problems. Specifically, the study shows a DNN approach is able to recognize and approximate the fundamental, underlying non-linear relationship between the image pixel luminance and daylighting conditions without overfitting, and generate high-quality renderings.

The next step is to apply the method to real world data capture with HDR photographs instead of rendering images. It is clear that this will be more difficult than developing and testing the model in a controlled setting. One major challenge is the duration of the collection period. In the current training data, though sparse, the input is distributed over the entire year in order to cover wide ranging sun positions. However, it is not practical to capture the scene multiple times throughout the year. Further studies will be conducted to minimize the data collection period without compromising the accuracy. The successful development of a practical workflow will enable quantitative analysis of daylit environments without requiring a time-consuming modeling process.

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