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An Analysis of Price vs. Revenue Protection: Government Subsidies in the Agriculture Industry

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The agriculture industry plays a critical role in the U.S. economy and various industry sectors depend on the output of farms. To protect and raise farmers' income, the U.S. government offers two subsidy programs to farmers: the Price Loss Coverage (PLC) program which pays farmers a subsidy when the market price falls below a reference price, and the Agriculture Risk Coverage (ARC) program which is triggered when farmers' revenue is below a threshold. Given the unique features of PLC and ARC, we develop models to analyze their impacts on consumers, farmers, and the government. Our analysis generates several insights. First, while PLC always motivates farmers to plant more acres compared to the no-subsidy case, farmers may plant less acres under ARC, leading to a lower crop supply. Second, despite the prevailing intuition that ARC generally dominates PLC, we show that both farmers and consumers may be better off under PLC for a large range of parameter values, even when the reference price represents the historical average market price. Third, the subsidy that increases consumer surplus results in higher government expenditure. Finally, we calibrate our model with USDA data and provide insights about the effects of crop and market characteristics on the relative performance of PLC and ARC. We provide guidelines to farmers for enrolling crops in the subsidy programs, and show that our guidelines are supported by farmers' enrollment statistics. We also show that if the economic and political frictions caused by running the subsidy programs is significant, the subsidy that benefits both consumers and farmers may actually result in lower social welfare.

Key words: farming, agriculture, random yield, subsidy, PLC, ARC, social welfare

1. Introduction

Agriculture is an important sector of the U.S. economy. According to the U.S. Department of Agriculture (USDA), agriculture and agriculture-related industries contributed $789 billion to the U.S. GDP in 2013, a 4.7% share. The output of America's farms contributed $166.9 billion of this sum. Many industry sectors such as forestry, fishing, food, beverages, tobacco products, textiles, apparel, leather products, and food services and drinking places, rely on agricultural inputs in order to create added value to the economy.

The agriculture industry is characterized by uncertainty in the farm yield that arises from unfavorable weather conditions, natural disasters, and infestation of pests and diseases throughout the growing season. For instance, in an investigation of the revenue variations for corn and soybeans farmers from 1975 to 2012, Sherrick (2012) finds that a significant portion of variability in farmers' revenues is attributable to yield variability. Poor harvests not only hit farmers' revenues severely, but also lead to higher retail food prices and input costs for industries that depend on agriculture outputs. The 2012 drought impacted 80 percent of agricultural land in the U.S. and destroyed or damaged the quality of the major field crops in the Midwest, particularly field corn and soybeans. In 2014, California's drought cost the state's agriculture industry $2.2 billion in losses and added expenses, while cutting 3.8% of the state's farm jobs (Carlton, 2014).

To protect and raise farmers' income and limit revenue variability that arises from poor crop performance, the U.S. government offers subsidy programs for agricultural commodities. Every five years or so, Congress passes comprehensive legislation for agricultural programs (Bjerga, 2015(a)). In 2014, Congress approved the 2014 Farm Bill that changed the structure of programs that support farmers in the U.S., and the bill was signed into law by President Obama. The enacted farm bill provides payments for 13 crops: corn, soybeans, wheat, barley, oats, grain sorghum, rice, dry peas, lentils, small chickpeas, large chickpeas, other oilseeds, and peanuts (Shields, 2014). In particular, the bill introduced two major subsidy programs:[1]

1. **Price Loss Coverage (PLC)**: Under this program, farmers are paid a subsidy when the market price for a covered crop in a year falls below a reference price.
2. **Agriculture Risk Coverage (ARC)**: Under this program, farmers receive subsidy when their crop revenue in a given year drops below a reference revenue which is determined based on a multi-year moving average of historical crop revenue. ARC has two variations: the reference revenue can be calculated at the county level (County ARC or ARC-CO), or at the individual farm level (Individual ARC or ARC-IC).

Farmers of the covered crops were required to make a one-time irrevocable decision to choose between PLC and County ARC on a commodity-by-commodity basis for each farm. Alternatively, farmers could enroll *all* covered crops in Individual ARC. According to the USDA, among all farmers who have signed up for the new subsidy programs, 76% of U.S. farm acres (aggregated across all eligible crops) are enrolled in County ARC, 23% enrolled in PLC, and only 1% enrolled in Individual ARC (Bjerga, 2015(b)).

[1] The bill also offers interim financing for the commodities through the Marketing Assistance Loans program. Farm loans are beyond the scope of this paper and we leave their analysis for future research.

The subsidy programs protect farmers in two different ways. The advantage of PLC is that it sets a floor for commodity prices and protects farmers against having to sell at a loss when prices are too low. The disadvantage is that in a poor harvest year, when lower supply drives up commodity prices but farmers have less to sell, PLC offers no support. ARC cushions farmers against unfavorable weather conditions that destroy or severely degrade the harvest.

Prior to 2013, farmers could receive both fixed direct payments, paid irrespective of the harvest amount, and variable payments, which were contingent on farmers' earnings. The 2014 Farm Bill, however, eliminated the fixed payments and revised the variable payments. Both PLC and ARC subsidies depend on the realization of the farm yield during the growing season. As a result, farmers have to make their one-time enrollment decision under uncertainty in future yields. As J. Gordon Bidner, a farmer from Illinois, puts it: a farmer would need "two crystal balls" to decide because "farming is risky" (Bjerga, 2015(a)).

The structures of the subsidy programs also have implications for the government. Lack of appropriate support from the government compounded with uncertainty in weather conditions may prompt farmers to plant less crops, leading to scarce supply and high commodity prices that hurt consumers. Presumably, through PLC and ARC programs, the government is spending taxpayer money to enhance farmers' income and consumer welfare. However, given the uncertainty in farm yields, the cost of these programs to the government and their impact on farmers' decisions is not immediately clear. For example, the overly optimistic predictions by The Congressional Budget Office expected the new subsidies to cost $4.02 Billion in 2015, while the actual government expenditure reached $5 Billion (Bjerga, 2015(b)). This points to the pressing need to better understand these support mechanisms and the consequences they inflict on different stakeholders that are important from a policy standpoint.

Despite the important role government subsidies play in the agriculture industry, both in emerging and developed economies, this topic has received very limited attention in the literature. In a recent work, Tang et al. (2015) examine whether competing farmers in a developing economy should utilize market information or adopt agricultural advice. Although the authors do not model government subsidies, they allude to the role of subsidies in emerging economies and recognize the need for developing new models to study agricultural subsidy programs in developed countries: *"Because the contexts* [of emerging countries and developed countries] *are very different, there is a need to develop a different model to investigate the value of farm subsidies in developed economies, and we leave this question for future research."* In this paper, we address the need for analyzing the impacts of agricultural subsidies. We develop models to study PLC and ARC and compare these subsidies based on various performance measures that are important to policymakers.

The structures of PLC and ARC subsidies are interesting and unique. In particular, we are not aware of any other paper in the operations management literature that examines a subsidy program (in agriculture or other contexts) where the subsidy amount is contingent on the revenue (not price) realization. Although a number of papers in agricultural economics have looked at the interactions between different insurance coverages and futures and options under previous farm bills (e.g. Coble et al. 2000), such models do not represent the structures of PLC and ARC. To the best of our knowledge, we are the first to develop models that study current subsidy programs in the U.S. agriculture industry. Our research addresses the following questions:

1. What is the impact of PLC and ARC subsidies on the planting acreage of farmers who operate under yield uncertainty?
2. Under what conditions should farmers enroll their crops in PLC or ARC?
3. How does farmers' enrollment in one of the two subsidy programs impact consumer surplus and government expenditure?
4. What are the impacts of variations in crop and market characteristics on the relative performance of PLC and ARC? How do the subsidies compare in terms of social welfare?

In this paper, we construct models that capture the essence and most salient features of these subsidy mechanisms and yet allow us to derive analytical results and provide insights. We consider multiple farmers who compete in a Cournot fashion, and have to decide how many acres of a crop they want to plant in the beginning of the growing season under yield uncertainty. The farm yields are realized at the end of the season and market price is decreasing in the total amount of harvest. We characterize the farmers' equilibrium planting decisions under PLC and ARC, and compare the subsidies in several dimensions by linking the objectives of the subsidy stakeholders. Our analysis generates the following results and insights about the implications of the subsidy programs that can be used to offer practical guidelines to farmers and policymakers:

(i) ARC offers two-sided coverage; it protects farmers when crop revenue is very low, either because of a bad harvest or because the harvest is good but the market price is low (e.g. Zulauf 2014). The price protection in PLC, however, offers one-sided coverage. As a result, the prevailing intuition is that ARC generally dominates PLC, and that PLC can be better only in limited situations when price is very low and continues to stay low for consecutive years (e.g. Schnitkey et al. 2014). That is, if the market price is systematically lower than the reference price, then PLC may have an advantage over ARC. Contrary to this intuition, our results show that PLC can dominate ARC *even when* the reference price represents the historical average market price.

(ii) While the PLC program always motivates the farmers to plant more acres compared to when no subsidy is offered, the farmers may plant less acres under ARC. Therefore, the ARC subsidy does not necessarily lead to higher crop availability in the market. Furthermore, even if crop supply is higher under ARC compared to the no-subsidy scenario, it is still possible for PLC to result in a higher supply than ARC, thereby lowering market price and benefiting consumers. This, together with (i) discussed above, implies that PLC can create a win-win situation for the farming industry and consumers by increasing farmers' profit and reducing market price simultaneously.

(iii) The subsidy that increases consumer surplus results in higher government expenditure; therefore, while a win-win outcome for farmers and consumers can emerge in equilibrium, this comes at a higher cost to the government.

(iv) ARC's two-sided coverage induces the farmers to utilize the subsidy in two different ways: (1) the farmers may prefer to plant relatively smaller quantities. In this case, the farmers anticipate the trigger of the ARC subsidy mostly when yield realization is low; (2) the farmers may find it optimal to plant relatively larger quantities. In this case, it is mostly the high realizations of yield that trigger the subsidy by lowering the market price and farmers' revenue. We present the condition that determines when farmers adopt either of these two strategies under ARC. The condition is characterized by two types of parameters: (1) crop characteristics such as the distribution of the farm yield and the cost of planting, (2) market characteristics such as market size and market price sensitivity to crop supply.

(v) We use USDA historical data for crop yields, market price, number of farmers, and aggregate supply to calibrate our model, and conduct extensive numerical experiments to further explore the combined effects of variations in crop and market characteristics on the relative performance of the subsidies. Our experiments provide the following observations:

  (a) We observe that the win-win equilibrium for the farming industry and consumers under PLC does emerge for a significant range of parameter values.

  (b) We find that PLC (ARC) is the better program for farmers when variability in the crop yield is low (high) relative to the average yield and/or when the ratio of the planting cost to price sensitivity takes moderate (low or high) values. Based on this finding, we provide guidelines to farmers for enrolling their crops in PLC or ARC. Specifically, our model recommends that corn and soybean farmers enroll in ARC, due to the high ratio of planting cost to price sensitivity and high relative yield variability of these two crops. Our model also recommends that long-grain rice farmers enroll in PLC, due to the moderate ratio of planting cost to price sensitivity and low relative yield variability of long-grain rice. Our enrollment guidelines for these crops are indeed corroborated by the USDA subsidy enrollment statistics.

(c) We also compare social welfare in equilibrium, taking into account consumer surplus, farmers' profit, and the possible economic and political frictions caused by running the subsidy programs. Such frictions may be due to factors such as administrative costs of implementation, innovation and technology adoption implications, public resistance, international trade ramifications, and environmental externalities among others. We observe that when the friction created by subsidies is comparable to the subsidy amount, the program that incurs a lower expenditure for the government is better for the society despite entailing a lower profit for farmers and lower surplus for the consumers.

The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 describes our modeling framework and formulation of the subsidy programs. In Section 4, we characterize the equilibrium decision of the farmers for each subsidy program and examine the effects of model parameters on the outcomes. Section 5 includes our calibration exercise, and provides insights into the effects of the subsidy programs on consumers, government, and farming industry as crop and market characteristics vary. Section 6 concludes the paper with a summary of results.

2. Literature

Our paper relates and contributes to the literature that studies farmers' decision making under various sources of uncertainty, such as yield uncertainty, and protection schemes that are designed to support farmers. In agriculture economics, Coble et al. (2000), Coble et al. (2004), Mahul (2003), Mahul and Wright (2003), and Tiwari et al. (2017) study the interactions between crop protection schemes and futures and options markets. Sherrick et al. (2004) examine factors that influence the decision of corn and soybeans farmers in the Midwest to choose among different protection plans. Singerman et al. (2012) calibrate a structural model to examine organic crop insurance under the 2008 Farm Bill. The structures of the subsidies analyzed in these papers are different from PLC and ARC. Besides, these papers study subsidies, such as yield or hail insurance, that are no longer offered to farmers. In addition, these papers do not investigate the implications of subsidy programs on consumer welfare or government expenditures. Although recent publications such as Glauber and Westhoff (2015), Orden and Zulauf (2015), and Classen et al. (2016) discuss political economy, WTO considerations, and environmental quality implications of the 2014 Farm Bill, we are not aware of any analytical model in agriculture economics that examines the impacts of PLC and ARC subsidies on the farming industry, consumers, and government expenditures under different crop and market characteristics.

This work also relates to three streams of research in the operations management literature. First, our paper relates to papers in the agricultural operations management literature that investigate the impacts of uncertain farm yield on various decisions. Kazaz (2004) studies production planning with random yield and demand in the olive oil industry, assuming the sale price and cost of purchasing olives are exogenous and decreasing in yield. Kazaz and Webster (2011) study the impact of yield-dependent trading cost on selling price and production quantity. Boyabatli and Wee (2013) consider a firm that reserves the farm space under yield and open market price uncertainties and assume the production rate is non-decreasing in the yield. Boyabatli et al. (2014) study the processing and storage capacity investment and periodic inventory decisions in the presence of spot price and yield uncertainties. These papers do not address subsidy mechanisms.

Second, our paper relates to the growing body of work on the management of agricultural operations in developing economies. Huh and Lall (2013) study land allocation and applying irrigated water when the amount of rainfall and market prices are uncertain. Dawande et al. (2013) propose mechanisms to achieve a socially optimal distribution of water between farmers in India. Murali et al. (2015) determine optimal allocation and control policies for municipal groundwater management. An et al. (2015) investigate different effects of aggregating farmers through cooperatives. Chen et al. (2015) examine the effectiveness of peer-to-peer interactions among farmers in India. Chen and Tang (2015) study the value of public and private signals offered to farmers. Tang et al. (2015) investigate whether two farmers should use market information to improve production plans or adopt agricultural advice to improve operations. They show that agricultural advice improves welfare only when the upfront investment is sufficiently low, and the government should consider offering subsidies to reduce the investment cost. However, none of these papers model subsidies.

Third, this paper also relates to the growing stream of research in the operations literature that studies government subsidies in various contexts. In this stream, only a few papers have looked at agricultural subsidies. Kazaz et al. (2016) study various interventions including price support for improving supply and reducing price volatility of artemisinin-based malaria medicine. Guda et al. (2016) study the guaranteed support price scheme in developing countries where the government purchases crops from farmers at a certain price to support the underprivileged population. Akkaya et al. (2016a) study the effectiveness of government tax, subsidy, and hybrid policies in the adoption of organic farming. Akkaya et al. (2016b) analyze government interventions in developing countries in the form of price support, cost support, or yield enhancement efforts. They show that price and cost support are equivalent if the total budget is public information and that interventions cannot always generate positive return from the governments perspective. Our work is different from these papers in that we analyze the current price-protection and revenue-protection subsidies in the U.S. and examine the impacts of these subsidies on different stakeholders. Moreover, the PLC subsidy we study has a different structure than the price-based interventions studied in the aforementioned papers. We are not aware of any analytical work related to PLC and ARC subsidy programs in the literature. Using a model that incorporates the most important features of PLC and ARC, we analyze the implications of these subsidy payments on consumers, the government, and the farming industry in the U.S. Our work also expands the literature on the intersection of Cournot competition and yield uncertainty, which has received limited attention (Deo and Corbett, 2009).

Government support mechanisms have been studied in other contexts. For example, see Adida et al. (2013), Mamani et al. (2012) and Taylor and Xiao (2014) for subsidies in vaccines supply chain, Alizamir et al. (2015) for renewable energies, and Krass et al. (2012) and Cohen et al. (2015) for green technology adoption. Our work differs from these papers in that we compare two specific and unique subsidy mechanisms in the context of agriculture in which payments to the farmers are endogenous and depend on the historical market outcomes for each crop.

3. Modeling Framework and Subsidy Structures

We now introduce our framework for modeling PLC and ARC subsidies, and establish measures to assess their performance. Consider m homogenous profit-maximizing farmers who compete in a Cournot fashion, and must decide on their planting quantity at the beginning of a growing season while facing yield uncertainty. Our oligopolist setting allows us to examine the performance of different subsidy programs in the presence of competition among farmers and yield uncertainty. Cournot-based models have been commonly used in the agricultural economics (e.g. Shi et al. 2010, Agbo et al. 2015, Deodhar and Sheldon 1996, Dong et al. 2006) and operations (e.g. An et al. 2015, Chen and Tang, Tang et al. 2015) literature to study agricultural markets. Further, Cournot competition is particularly suitable for situations where there is a lag between the time decisions are made and the time uncertainty is resolved (Carter and MacLaren 1994).[2]

We denote the planting acreage of farmer j by q_j. We assume the cost of planting q_j acres is cq_j^2, which represents the total cost of securing all the resources and exerting the efforts needed to plant q_j acres. The quadratic cost function captures the increasing marginal cost of acquiring land and acts as a soft capacity constraint. Quadratic planting cost functions have been used in agricultural models (e.g. Wickens and Greenfield 1973, Parikh 1979, Holmes and Lee 2012, Agbo et al. 2015, Guda 2016, Akkaya et al. 2016b). For instance, estimates of total cost curves in the U.S. corn belt have provided evidence of diseconomies of scale (Peterson 1997). A recent article in BusinessWeek (Bjerga and Wilson 2016) reports that a strong U.S. dollar and higher borrowing costs, among other factors, have made it more difficult for farmers to finance operations or purchase land and

[2] A *farmer* in our model does not necessarily correspond to an individual with a small piece of land. Instead, it represents any influential decision-making entity (e.g., corporate farm, large producer, etc.) whose decision can meaningfully impact market equilibrium. There is increasing evidence that a large portion of agricultural farms in the United States are controlled and managed by a small number of farmers (Koba 2014).

equipment.[3] We point out that our results qualitatively hold for any increasing and strictly convex planting cost function in the form of cq^{β}. We focus on quadratic cost to obtain closed form solutions.

The amount of crop harvested at the end of the growing season depends on the farm yield, which is influenced by weather conditions and other unpredictable factors throughout the season. We represent the per-acre yield by random variable X with probability density function $f(X)$, defined over interval $[L, U]$. We denote the expected value and standard deviation of the yield distribution by μ and σ, respectively, and assume that the farm yields are perfectly correlated for all farmers. The assumption of perfect correlation is reasonable when the farms are located in counties that have similar weather conditions, and hence are exposed to the same sources of uncertainty. Allowing the farm yields to be partially correlated requires adding a multivariate yield distribution into the profit functions, which extremely complicates the analysis. Nevertheless, we extend our base model in Appendix A and revisit our results under independent or partially-correlated yields. Our numerical experiments illustrate that the qualitative nature of our results continue to hold under a more general structure of correlation.

The amount of crop harvested by farmer j at the end of the season is $q_j X$. This multiplicative form for random yield captures the proportional yield model that is commonly used in the literature (e.g. Yano and Lee 1995, Kazaz 2004, Kazaz and Webster 2011). It follows that the aggregate amount of crop available at the end of the harvesting season equals $\sum_{j=1}^{m} q_j X$. The market price for the crop depends on the aggregate supply through the following linear inverse demand curve

$$p\Big(\sum_{j=1}^{m} q_j, X\Big) = N - b\Big(\sum_{j=1}^{m} q_j\Big)X, \tag{1}$$

where N denotes the maximum possible price for the crop and b represents the sensitivity of market price to changes in crop supply. Using a linear (inverse) demand curve is a common approach in the literature of agriculture operations management (e.g. Kazaz 2004, An et al. 2015). The downward sloping relationship between supply and price is also supported by observations in practice. For example, the USDA periodically announces its forecast of weather conditions, farm yields, and production volumes for different crops. When new forecasts hint at a higher availability of crops, market prices decline (e.g. Newman 2015(a)-(c)). Table 1 summarizes the basic notations used throughout the paper; some of these notations will be introduced in the ensuing sections. We denote equilibrium values by a hat accent.

[3] In addition to the agriculture literature, quadratic production cost functions have been used in other contexts such as electricity generation cost in power-plants (Wood and Wollenberg 2012).

Table 1 Notations

m	number of farmers
N	maximum possible market price (intercept of the inverse demand curve)
b	market price sensitivity to change in crop supply (slope of the inverse demand curve)
c	farmers' planting cost coefficient
$f(x)$	probability density function of the random yield distribution
L, U	lower and upper bounds for the per-acre yield
μ, σ	expected value and standard deviation of the yield distribution
$\phi(x), \Phi(x)$	p.d.f and c.d.f. of the Normal distribution with mean μ and standard deviation σ
α	subsidy payment coefficient
q_j	acres planted by farmer j, $j = 1, \cdots, m$
$p\Big(\sum_{j=1}^{m} q_j, x\Big)$	crop price given planted acres and farm yield realization
$\lambda\Big(\sum_{j=1}^{m} q_j\Big)$	reference price in PLC given farmers' aggregate planted acres
$r\Big(\sum_{j=1}^{m} q_j\Big)$	reference revenue in ARC given farmers' aggregate planted acres
$\Gamma_{PLC}, \Gamma_{ARC}$	government's total subsidy payment under PLC and ARC, respectively
$\pi^i_{ns}, \pi^i_{PLC}, \pi^i_{ARC}$	farmer i's profit under no-subsidy, PLC and ARC, respectively, for $i = 1, \cdots, m$
$\Delta_{PLC}, \Delta_{ARC}$	total consumer welfare under PLC and ARC, respectively
Π_{sc}	social welfare

3.1. No Subsidies

As a benchmark scenario, we first formulate the farmers' problem when no subsidies is offered by the government. In this case, farmer i finds the planting decision that maximizes the following expected profit function

$$\pi^i_{ns}(q_i, \mathbf{q}_{-i}) = \int_L^U \left[N - b\Big(\sum_{j=1}^{m} q_j\Big) x \right] q_i x f(x) dx - c q_i^2 = N q_i \mu - b q_i \Big(\sum_{j=1}^{m} q_j\Big)(\mu^2 + \sigma^2) - c q_i^2 \,, \qquad (2)$$

where $\mathbf{q_{-i}} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_m)$ represents the decisions of other farmers. The integral multiplies the market price by farmer i's harvest and takes the expectation over all possible yield realizations to determine the farmer's expected revenue.

Lemma 1. *When no subsidy is offered, the symmetric equilibrium quantity is unique. Each farmer's planting decision and profit in equilibrium are given, respectively, by*

$$\begin{aligned} \widehat{q}_i = \widehat{q}_{ns} &= \frac{N\mu}{(m+1)b(\mu^2+\sigma^2)+2c} \,, \\ \widehat{\pi}^i_{ns} = \widehat{\pi}_{ns} &= (\widehat{q}_{ns})^2 \Big(b(\mu^2+\sigma^2) + c \Big) \,. \end{aligned} \qquad (3)$$

Lemma 1 shows that in the absence of any subsidies, higher uncertainty in the yield leads to a lower planting quantity. This is not surprising because the farmers' equilibrium quantity makes the marginal revenue equal to the marginal cost. Hence, as yield becomes more variable, marginal revenue declines and farmers react by planting less acres.

3.2. Subsidy Program Structures

To establish our models for analyzing PLC and ARC subsidies, we start this section by describing their detailed structures and our approach for capturing their most essential features. In the following sections, we formulate each subsidy and derive the corresponding performance measures.

The PLC subsidy program shields farmers against low market prices. In particular, if the market price of a crop in a selling season falls below a *reference price*, then the government pays each farmer who is enrolled in PLC a subsidy amount equal to the product of four terms: (1) the difference between the reference price and the realized market price; (2) the farmer's base acres for the crop which is the historical average planted acreage; (3) the average farm yield; and (4) a subsidy payment coefficient set by the government which we denote by $\alpha \leq 1$.[4]

The ARC subsidy program, on the other hand, protects farmers when per-acre revenue drops below a *reference revenue*. Under ARC County (also referred to as ARC-CO), the reference revenue per acre is defined as the five-year Olympic average market price multiplied by the five-year Olympic average yield, where the Olympic average excludes the lowest and highest values and calculates the simple average of the remaining three values. If a farmer becomes eligible for ARC subsidy, then the government pays the farmer a subsidy that is obtained by multiplying: (1) the difference between a percentage of the reference revenue and the actual revenue subject to a cap;[5] (2) the base acres; and (3) the subsidy payment coefficient α. In current practice, the subsidy payment coefficient α is set to 85% by the government for both PLC and ARC. We do not restrict the value of α in our analysis, and treat it as a model parameter in order to derive more general results.

The ARC program has another category, referred to as Individual ARC or ARC-IC, which calculates the reference revenue at the individual farm level. Farmers who choose ARC-IC are required to enroll *all* crops in this program. ARC-IC has been criticized by farming experts for its low payment rate and all-or-nothing restriction that limits farmers' choices (Kiser 2015). The fact that only 1% of farmers enrolled in ARC-IC clearly indicates that farmers also share these concerns and are not interested in ARC-IC. Therefore, we focus our analysis on PLC and County ARC (hereafter, ARC).

Our modeling approach for formulating the two subsidy mechanisms focuses on a stationary setting in which farmers' decisions are time-independent. It should be noted that the primary objective of our work is to compare the two subsidy mechanisms from a policy perspective by analyzing their implications on consumers, the government, and the farming industry. Subsequently, we use the insights that we gain from our analysis to derive normative policy recommendations. With this objective in mind, a stationary framework that studies the farmers' decision making process in the long-run would best serve our purpose. More precisely, given the history-dependent nature of the subsidies, our model assumes the subsidy programs have been in place for sufficiently long period of time so that possible impacts of the system's initial history are phased out. In

[4] Farmers who choose PLC can also purchase the Supplemental Coverage Option (SCO) insurance. SCO is an add-on insurance and its analysis is beyond the scope of this paper.

[5] The percentage is currently 86% and the cap is 10% of the reference revenue.

Appendix A1, we provide further justification for this modeling choice by providing the detailed formulation of the more general dynamic game and explaining its complexity. We then argue why our stationary approach is a reasonable approximation of the complex dynamic game.

The advantage of adopting such a modeling approach is twofold. First, it allows us to abstract away from the inherent complexities of dynamic games and dealing with multiple equilibria, thereby facilitating analytical tractability. Second, and more importantly, it isolates the main tradeoffs between the two subsidy programs while capturing the most important features of their design. That is, the only reason the farmers (and/or consumers, the government) may prefer PLC over ARC (or vice versa) in our model is the fundamental differences in the structure of the subsidies. Given our stationary approach, we can replace farmer i's base acre, which is the historical average of his planted acres, by his planting decision q_i. We are now ready to present our formulations for PLC and ARC subsidies.

3.3. PLC Subsidy Formulation

The reference price in PLC is chosen to achieve a number of objectives, the most important of which is to protect farmers against yield variability that may drive market price for crops below their expected price. In conjunction with our stationary framework, we set the reference price to be equal to the long-run average price, i.e., $\lambda(\sum_{j=1}^m q_j) = N - b\left(\sum_{j=1}^m q_j\right)\mu$, to isolate the impact of yield variability and eradicate other incentives that may distort farmers' planting decisions. In fact, the reference prices for most crops in the 2014 Farm Bill are also set close to their 5-year Olympic average prices prior to 2014. It is noteworthy that some reference prices may be set at a higher level to achieve other goals such as providing more support to crops that have not directly benefited from U.S. biofuels policy or those that lose the most from eliminating previous direct payments (Zulauf 2013); however, such objectives are beyond the scope of this paper. Furthermore, the highest ratio of reference price to average price in the 2014 Farm Bill was for peanut where the reference price was only 4% higher than the average price. Finally, we note that if the reference price for a crop is higher than its average price, one can alternatively achieve a similar tradeoff in our model by selecting a higher value for the payment coefficient α. In Appendix A2, we allow the reference price to be exogenous, and investigate its impact on the equilibrium outcome.

As mentioned earlier, in this paper we aim to inform policy discussions by analyzing the outcome of each subsidy program through the lens of consumers, the government, and the farming industry. Therefore, we next derive the performance measure for each of these stakeholders.

3.3.1. Consumers. Consumers' utility is mainly driven by the aggregate supply harvested at the end of the growing season, which also determines the market price of the crop (through Equation (1)). More precisely, given aggregate supply $\sum_{j=1}^m q_j X$, the total consumer surplus can

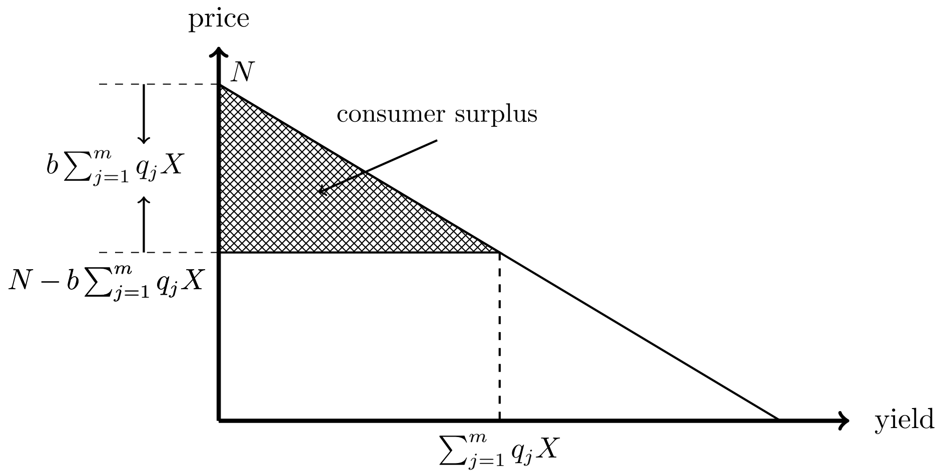


Figure 1 Consumer welfare

be obtained by integrating the utility in excess of price $p(\sum_{j=1}^m q_j, X)$ for those consumers who purchase the crop. This is shown as the shaded area in Figure 1. Taking the expectation over all yield realizations, we obtain the expected consumer surplus, denoted by Δ:

$$\Delta(\mathbf{q}) = \frac{1}{2}\mathbb{E}\left[bX^2\Big(\sum_{j=1}^m q_j\Big)^2\right] = \frac{b}{2}(\mu^2+\sigma^2)\Big(\sum_{j=1}^m q_j\Big)^2, \tag{4}$$

where $\mathbf{q} = (q_1, \ldots, q_m)$. Not surprisingly, Equation (4) shows that the expected consumer surplus is quadratically increasing in farmers' planting decision.

3.3.2. Government. We denote the total government payment under the PLC subsidy by $\Gamma_{PLC}(\mathbf{q})$. The government only provides subsidy to farmers if the realized market price $p\left(\sum_{j=1}^m q_j, x\right)$ is below the reference price $\lambda\left(\sum_{j=1}^m q_j\right)$. The amount of the subsidy paid to each farmer is proportional to the gap between the reference and the actual price, the farmer's planted acres, and the average yield. That is,

$$\Gamma_{PLC}(\mathbf{q}) = \alpha\sum_{i=1}^m \int_L^U \max\left\{0, \lambda\Big(\sum_{j=1}^m q_j\Big) - p\Big(\sum_{j=1}^m q_j, x\Big)\right\} q_i \mu f(x)dx, \tag{5}$$

where $p\left(\sum_{j=1}^m q_j, x\right) = N - b\sum_{j=1}^m q_j x$ and $\lambda\left(\sum_{j=1}^m q_j\right) = N - b\sum_{j=1}^m q_j\mu$. Therefore, the government payment can be simplified to

$$\Gamma_{PLC}(\mathbf{q}) = \alpha b\mu\Big(\sum_{j=1}^m q_j\Big)^2 \int_\mu^U (x-\mu)f(x)dx\,. \tag{6}$$

3.3.3. Farming Industry. The total expected profit that the farming industry enjoys is equal to the summation of individual farmer profits. To correctly represent the oligopoly market, we first derive an individual farmer's expected profit when he enrolls in the PLC subsidy program. We then use this to find the farming industry's total expected profit evaluated at the equilibrium solution for the oligopoly market. Farmer i's expected profit can be formulated as

$$\pi^i_{PLC}(q_i, \mathbf{q}_{-i}) = \int_L^U \left[N - b\Big(\sum_{j=1}^m q_j\Big)x \right] q_i x f(x)dx - cq_i^2$$
$$+ \alpha \int_L^U \max\left\{0, \lambda\Big(\sum_{j=1}^m q_j\Big)\mu - p\Big(\sum_{j=1}^m q_j, x\Big)\mu \right\} q_i f(x)dx\,.$$

The first part of the profit function is identical to the farmer's profit in (2) when no subsidy is offered. The second integral represents the amount of subsidy paid by the government - the summand in (5). Using the same simplifications as for the government payment, the farmer's profit can be expressed as follows

$$\pi^i_{PLC}(q_i, \mathbf{q}_{-i}) = Nq_i\mu - bq_i\Big(\sum_{j=1}^m q_j\Big)(\mu^2+\sigma^2) - cq_i^2 + \alpha b\mu q_i\Big(\sum_{j=1}^m q_j\Big)\int_\mu^U (x-\mu)f(x)dx\,. \tag{7}$$

The farming industry's total expected profit in equilibrium is then given by $\sum_{i=1}^m \pi^i_{PLC}(\widehat{q}_i, \widehat{\mathbf{q}}_{-i})$.

3.4. ARC Subsidy Formulation

The ARC subsidy protects the farmers in both extremes of the yield realization spectrum: when yield realization is very high or very low. When yield is very low, scarcity of the crop supply drives up the market price. Even though the price is high, the poor harvest reduces the farmers' revenue. On the other hand, when yield is very high, the farmers have a good harvest. However, a large supply results in a low market price, thereby reducing the farmers' revenue below the reference revenue. In either of these cases, the farmers qualify for subsidy from the government. For ease of exposition and analytical tractability, we drop the minor adjustments that are used in practice to calculate the final amount of subsidy (e.g., the 86% coefficient for the reference revenue), and simply assume the subsidy amount is proportional to the difference between reference and actual per-acre revenues. Our objective is to set up our model in a way to capture all essential elements of PLC and ARC programs in an analytically tractable model, while abstracting away from minor adjustments that can ultimately be amended to both programs in practice.

3.4.1. Consumers. Similar to PLC, consumers' utility is contingent on the supply of crop at the end of the growing season, as represented in Equation (4). Note that while the expression for the expected consumer surplus is the same for PLC and ARC ($\Delta_{PLC}(\mathbf{q}) = \Delta_{ARC}(\mathbf{q})$ for a given $\mathbf{q}$), their values will be different in equilibrium since PLC and ARC induce different planting quantities by the farmers.

3.4.2. Government. We denote the total government payment under the ARC subsidy by $\Gamma_{ARC}(\mathbf{q})$. If the farmers' realized per-acre revenue, $p\Big(\sum_{j=1}^m q_j, x\Big)x$, is below the reference revenue $r\Big(\sum_{j=1}^m q_j\Big) = \Big[N - b\left(\sum_{j=1}^m q_j\right)\mu\Big]\mu$, then the farmers receive a subsidy proportional to the gap between the reference and actual revenues and the planted acres. Therefore,

$$\Gamma_{ARC}(\mathbf{q}) = \alpha \sum_{i=1}^{m} \int_L^U \max\left\{0, r\Big(\sum_{j=1}^m q_j\Big) - p\Big(\sum_{j=1}^m q_j, x\Big)x\right\} q_i f(x)dx. \tag{8}$$

Plugging in the expressions for price and reference revenue, the government's total subsidy payment simplifies to:

$$\Gamma_{ARC}(\mathbf{q}) = \alpha \sum_{i=1}^{m} \int_L^U \max\left\{0, b\Big(\sum_{j=1}^m q_j\Big)(x^2-\mu^2) + N(\mu - x)\right\} q_i f(x)dx.$$

The expected amount of subsidy becomes positive when the farm yield is either low or high. To simplify $\Gamma_{ARC}(\mathbf{q})$ further, we note that the amount of subsidy is quadratic in x and becomes zero when $x = \mu$ or $x = \frac{N}{b\sum_{j=1}^m q_j} - \mu$. The order of the two roots depends on the farmers' planting quantity and cannot be determined a priori. Nevertheless, we can expand the subsidy term to

$$\begin{aligned}\Gamma_{ARC}(\mathbf{q}) = \alpha &\int_L^{\min\left\{\frac{N}{b\sum_{j=1}^m q_j} - \mu, \mu\right\}} \left[b\Big(\sum_{j=1}^m q_j\Big)^2 (x^2-\mu^2) + N\Big(\sum_{j=1}^m q_j\Big)(\mu - x)\right] f(x)dx \\ &+ \alpha \int_{\max\left\{\frac{N}{b\sum_{j=1}^m q_j} - \mu, \mu\right\}}^U \left[b\Big(\sum_{j=1}^m q_j\Big)^2 (x^2-\mu^2) + N\Big(\sum_{j=1}^m q_j\Big)(\mu - x)\right] f(x)dx. \end{aligned} \tag{9}$$

Furthermore, using $\int_L^U \left[b\Big(\sum_{j=1}^m q_j\Big)^2 (x^2-\mu^2) + N\Big(\sum_{j=1}^m q_j\Big)(\mu - x)\right] f(x)dx = b\Big(\sum_{j=1}^m q_j\Big)^2 \sigma^2$, we can write the expected subsidy payment as

$$\Gamma_{ARC}(\mathbf{q}) = \alpha b\Big(\sum_{j=1}^m q_j\Big)^2 \sigma^2 - \alpha \int_{\min\left\{\frac{N}{b\sum_{j=1}^m q_j} - \mu, \mu\right\}}^{\max\left\{\frac{N}{b\sum_{j=1}^m q_j} - \mu, \mu\right\}} \left[b\Big(\sum_{j=1}^m q_j\Big)^2 (x^2-\mu^2) + N\Big(\sum_{j=1}^m q_j\Big)(\mu - x)\right] f(x)dx.$$

3.4.3. Farming Industry. In order to find the farming industry's total expected profit, we first derive an individual farmer's expected profit when he enrolls in the ARC subsidy program. This is then used to find the farming industry's total expected profit evaluated at the equilibrium solution for the oligopoly market. The expected profit of farmer i under ARC is given by

$$\pi^i_{ARC}(q_i, \mathbf{q}_{-i}) = \int_L^U \left[N - b\Big(\sum_{j=1}^m q_j\Big)x\right] q_i x f(x)dx - cq_i^2 + \alpha \int_L^U \max\left\{0, r\Big(\sum_{j=1}^m q_j\Big) - p\Big(\sum_{j=1}^m q_j, x\Big)x\right\} q_i f(x)dx.$$

The first part of the profit function is identical to the farmer's profit in (2) when no subsidy is offered. The second integral represents the amount of subsidy paid by the government —the summand in (8). Using the same simplifications as for the government payment, the farmer's profit can be expressed as

$$\begin{aligned}
\pi^i_{ARC}(q_i,\mathbf{q}_{-i}) &= Nq_i\mu - bq_i\Big(\sum_{j=1}^m q_j\Big)(\mu^2+\sigma^2) - cq_i^2 \\
&\quad + \alpha\int_L^{\min\left\{\frac{N}{b\sum_{j=1}^m q_j}-\mu,\mu\right\}}\left[b\Big(\sum_{j=1}^m q_j\Big)(x^2-\mu^2)+N(\mu-x)\right]q_i f(x)dx \qquad (10)\\
&\quad + \alpha\int_{\max\left\{\frac{N}{b\sum_{j=1}^m q_j}-\mu,\mu\right\}}^{U}\left[b\Big(\sum_{j=1}^m q_j\Big)(x^2-\mu^2)+N(\mu-x)\right]q_i f(x)dx \\
&= Nq_i\mu - bq_i\Big(\sum_{j=1}^m q_j\Big)(\mu^2+\sigma^2) - cq_i^2 + \alpha b\Big(\sum_{j=1}^m q_j\Big)q_i\sigma^2 \\
&\quad - \alpha\int_{\min\left\{\frac{N}{b\sum_{j=1}^m q_j}-\mu,\mu\right\}}^{\max\left\{\frac{N}{b\sum_{j=1}^m q_j}-\mu,\mu\right\}}\left[b\Big(\sum_{j=1}^m q_j\Big)(x^2-\mu^2)+N(\mu-x)\right]q_i f(x)dx. \qquad (11)
\end{aligned}$$

The farming industry's total expected profit in equilibrium is then given by $\sum_{i=1}^m \pi^i_{ARC}(\widehat{q}_i,\widehat{\mathbf{q}}_{-i})$.

4. Analysis

In this section, we characterize the symmetric equilibrium outcome under both PLC and ARC regimes, and explore their implications on different stakeholders to guide policy decisions. As it is evident from (7) and (11), the farmers' expected profit function depends on integrals of the yield distribution, and cannot be further simplified without full knowledge of the distribution. Moreover, the profit function for ARC is not necessarily well-behaved in the planting quantity. In order to proceed with our analysis and obtain analytical results, make the following assumption about the yield distribution:

Assumption 1. *The random yield has a Normal distribution with p.d.f $\phi(x)$ and c.d.f $\Phi(x)$. Also, $\mu \geq 3\sigma$ so that the probability of negative yield values is negligible.*[6]

While using a Normal distribution makes the analysis tractable, it is also strongly supported by data from USDA National Agricultural Statistics Service (NASS). More specifically, we found data on NASS website[7] for the historical yield of seven eligible crops. We conducted the Lilliefors test (Lilliefors, 1967) on the yield values for the past 10 years to see if the yield values for each crop follow a Normal distribution. Table 2 summarizes the value of the test statistic for each crop and the critical value at the 5% level of significance. For all the crops, the test statistic is smaller than the critical value. Therefore, the Lilliefors test clearly shows that the Normal distribution is a good fit for random yield.

[6] This inequality is supported by our estimates for μ and σ in Section 5.

[7] The website address is: http://www.nass.usda.gov.

Crop	Corn	Soybeans	Barely	Oats	Rice	Peanuts	Sorghum
Test Statistic	0.2038	0.2104	0.1788	0.1463	0.1518	0.2146	0.1349
Critical Value (5%)	0.2616						

Table 2 Results of the Lilliefors test for normality of the crop yield distribution

4.1. Equilibrium Characterization

In this section, we address our first research question about the impact of PLC and ARC on the planting quantity of farmers. We first present the equilibrium planting quantity and profit for the farmers under PLC.

Proposition 1. *Under the PLC program, the symmetric equilibrium quantity is unique. Each farmer's planting quantity in equilibrium is characterized by*

$$\widehat{q}_i = \widehat{q}_{PLC} = \frac{N\mu}{(m+1)b\left(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}\right)+2c} \qquad \text{for } i=1,\dots,m\,. \tag{12}$$

Each farmer's equilibrium expected profit is given by

$$\widehat{\pi}^i_{PLC} = \widehat{\pi}_{PLC} = N\widehat{q}_{PLC}\mu - bm(\widehat{q}_{PLC})^2(\mu^2+\sigma^2) - c(\widehat{q}_{PLC})^2 + \frac{\alpha b m\mu\sigma(\widehat{q}_{PLC})^2}{\sqrt{2\pi}}\,. \tag{13}$$

Furthermore, $\widehat{q}_{PLC} > \widehat{q}_{ns}$ *and* $\widehat{\pi}_{PLC} > \widehat{\pi}_{ns}$.

Proposition 1 shows that offering price protection motivates the farmers to plant more acres and increases their profit compared to when no subsidy is offered. The subsidy achieves its intended objective by inducing a higher supply of the crop, which in turn, reduces the market price and benefits the consumers. Therefore, in comparison with the no-subsidy case, both the farming industry and consumers would be better off in the presence of PLC.

As mentioned earlier, yield uncertainty plays a crucial role in the farmers' decision making process since they have to decide on their planting quantity before yield is realized. We next investigate the effect of yield uncertainty on the farmers' equilibrium quantity and profit under PLC.

Proposition 2. *Suppose expected yield* μ *remains constant while yield variability* σ *increases. Then, there exists threshold* $\bar{\sigma} = \frac{\alpha\mu}{2\sqrt{2\pi}}$ *so that* $\widehat{q}_{PLC}$ *and* $\widehat{\pi}_{PLC}$ *both increase with* σ *if* $\sigma \le \bar{\sigma}$*, and decrease with* σ *otherwise.*

Proposition 2 highlights a sharp contrast between PLC and the no-subsidy scenario. We showed in Lemma 1 that when the farmers are not insured by the government, higher uncertainty in yield always leads to a decline in the farmers' quantity. When the farmers enroll in PLC, however, higher uncertainty in yield does not immediately reduce the planting quantity. In fact, as the yield distribution gets more dispersed, the farmers may be encouraged to utilize the PLC subsidy by planting more acres. This implies that unlike the no-subsidy scenario, both the farmers as well as the consumers would actually benefit from higher variability in yield as long as the increase in variability is not excessive.

The intuition behind this reaction by the farmers can be explained as follows. When yield variability goes up, the likelihoods of both low and high yield realizations increase. This has two effects on the farmers' revenue. On the one hand, higher variability reduces the portion of the farmers' marginal revenue that they would automatically earn irrespective of the subsidy program (see (2)). On the other hand, because the likelihood of high yield values increases, it becomes more likely that the market price will fall below the reference price, making the farmers eligible for the PLC subsidy. Thus, higher variability increases the farmers' marginal revenue from subsidy (see the last expression in (7)). For low values of σ, the positive effect of higher variability outweighs the negative effect and the farmers benefit from planting more acres. This also benefits the consumers because the market price goes down. However, for larger values of yield uncertainty, the negative effect dominates and the farmers' planting acres and profit decline.

The next proposition characterizes the farmers' equilibrium planting decision under ARC.

Proposition 3. *The symmetric equilibrium under the ARC program, denoted by $\widehat{q}_i = \widehat{q}_{ARC}$ for $i = 1, \ldots, m$, is unique and satisfies the following equation*

$$\begin{aligned} &N\mu - (m+1)b\widehat{q}_{ARC}(\mu^2+\sigma^2) - 2c\widehat{q}_{ARC} + \alpha(m+1)b\widehat{q}_{ARC}\sigma^2 \\ &- \alpha\int_{\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}}^{\max\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}} \left[(m+1)b\widehat{q}_{ARC}(x^2-\mu^2) + N(\mu-x)\right]\phi(x)dx = 0\,. \end{aligned} \tag{14}$$

Each farmer's equilibrium expected profit is given by

$$\begin{aligned} \widehat{\pi}^i_{ARC} = \widehat{\pi}_{ARC} = &N\widehat{q}_{ARC}\mu - bm(\widehat{q}_{ARC})^2(\mu^2+\sigma^2) - c(\widehat{q}_{ARC})^2 + \alpha bm(\widehat{q}_{ARC})^2\sigma^2 \\ &- \alpha\int_{\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}}^{\max\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}} \left[bm(\widehat{q}_{ARC})^2(x^2-\mu^2) + N\widehat{q}_{ARC}(\mu-x)\right]\phi(x)dx\,. \end{aligned} \tag{15}$$

As Proposition 3 suggests, the equilibrium under ARC is more involved, and can be presented only implicitly through (14). The equilibrium condition is driven by the revenue-based piece-wise structure of ARC; the subsidy is triggered when yield realization is either high or low, but remains inactive for moderate values of the yield.

The intricate structure of the ARC subsidy may also lead to some unintended consequences. As formally stated in the following corollary, unlike the PLC program, the farmers may be prompted to plant less acres under ARC compared to when no subsidy is offered.

Corollary 1. *Define*

$$G(q) = (m+1)bq\sigma^2 - \int_{\min\left\{\frac{N}{bmq}-\mu,\mu\right\}}^{\max\left\{\frac{N}{bmq}-\mu,\mu\right\}} \left[(m+1)bq(x^2-\mu^2) + N(\mu-x)\right]\phi(x)dx\,.$$

Then, $\widehat{q}_{ARC} < \widehat{q}_{ns}$ if and only if $G(\widehat{q}_{ns}) < 0$, where $\widehat{q}_{ns}$ is defined in (3).

The farmers receive the ARC subsidy when their revenues fall, either because of a low price or because of a low harvest. The farmers may then find it optimal to strategically plant less acres in order to reduce their revenue and take advantage of the subsidy payment. Therefore, while PLC subsidy is always favored by consumers (compared to no subisdy), ARC may result in a lower supply and a higher market price for consumers. Subsequently, the introduction of the ARC subsidy may hinder total consumer surplus.

4.2. Subsidy Comparison and Policy Implications

Now that we have established the equilibrium outcomes under both subsidy mechanisms, we are ready to investigate their implications on different stakeholders that are relevant from a policy perspective. In particular, we are interested in exploring how consumers, the farming industry, and the government are impacted under each subsidy regime. Throughout this section, it is natural to assume that the subsidy coefficient α is the same for both PLC and ARC, which is consistent with the current implementation under 2014 Farm Bill. Hence, any difference in the performance of the two subsidies is merely driven by their inherent structural design, independently of coefficient α.

To proceed, it is critical to note that the two-sided structure of the ARC payments may persuade the farmers to exploit the subsidy in two distinct ways. In fact, it can be shown that the farmers' strategy in response to ARC subsidy is specifically driven by whether or not model parameters belong to set $\mathcal{S}$, where

$$\mathcal{S} = \left\{ (c, b, \mu, \sigma, m, \alpha) \middle| \left(\frac{c}{b}\right) \geq \frac{1}{2} \left[(m-1)\mu^2 - (1-\alpha)(m+1)\sigma^2 \right] \right\}.$$

When $(c, b, \mu, \sigma, m, \alpha) \in \mathcal{S}$ (e.g., when $\frac{c}{b}$ is large while other parameters are fixed), it is most profitable for the farmers to plant less in equilibrium, and induce a high market price. The low revenues that trigger the subsidy in this case are mainly caused by small harvest quantities (despite a high market price). In particular, $\widehat{q}_{ARC} \leq \frac{N}{2bm\mu}$ and any yield realization below the mean leads to a subsidy payment. On the other hand, when model parameters belong to $\bar{\mathcal{S}}$ (the complement of $\mathcal{S}$), the farmers' strategy flips, and they prefer to plant more acres thereby driving down the market price. In this case, the farmers anticipate the trigger of subsidy when revenue drops due to low market price (despite high level of production). More precisely, we have $\widehat{q}_{ARC} \geq \frac{N}{2bm\mu}$ and any yield realization above the mean entails a subsidy payment.

The next theorem, which presents one of our main analytical results, formalizes the link between different stakeholders' payoffs under both subsidies.

Theorem 1. *Suppose model parameters belong to set $\mathcal{S}$. The following statements hold in equilibrium:*

(a) If $\widehat{\pi}_{PLC} \geq \widehat{\pi}_{ARC}$, then $\widehat{\Delta}_{PLC} \geq \widehat{\Delta}_{ARC}$. Equivalently, if $\widehat{\Delta}_{ARC} \geq \widehat{\Delta}_{PLC}$, then $\widehat{\pi}_{ARC} \geq \widehat{\pi}_{PLC}$.

Conversely, suppose model parameters belong to set $\bar{\mathcal{S}}$. The following statements hold in equilibrium:

(b) If $\widehat{\Delta}_{PLC} \geq \widehat{\Delta}_{ARC}$, then $\widehat{\pi}_{PLC} \geq \widehat{\pi}_{ARC}$. Equivalently, if $\widehat{\pi}_{ARC} \geq \widehat{\pi}_{PLC}$, then $\widehat{\Delta}_{ARC} \geq \widehat{\Delta}_{PLC}$.

The findings of Theorem 1 provide valuable insights about the policy implications of the subsidy programs. More specifically, the theorem relates the desired subsidy scheme for the two main stakeholders, depending on crop and market characteristics. Recall that set $\mathcal{S}$ characterizes parameter values for which the farmers find it beneficial to plant relatively smaller quantities under ARC. Naturally, one would expect that the farmers would be better off under ARC in more scenarios than consumers due to higher crop prices. Theorem 1 formalizes this intuition. When model parameters belong to $\mathcal{S}$, statement (a) in the theorem indicates that consumers are in a favored position under PLC. This is because in this case, if the farmers are better off under PLC, the same must be true for consumers as well. On the other hand, the farmers are in a favored position under ARC; when consumers are better off under ARC, so are the farmers. Conversely, when model parameters belong to $\bar{\mathcal{S}}$, the farmers find it beneficial to plant relatively larger quantities in equilibrium under ARC. Therefore, the direction of the deductions reverse when model parameters fall in $\bar{\mathcal{S}}$, so that the farmers (consumers) are in a favored position under PLC (ARC).

Putting all together, Theorem 1 helps to better understand the two subsidy mechanisms through the lens of different stakeholders, and enables the policymakers to compare them based on their specific preferences and objectives (e.g., consumer surplus vs. farmers' payoffs). Furthermore, the theorem highlights the possibility of having a win-win situation that can simultaneously benefit consumers and farmers. In the next three subsections, we focus on different stakeholders separately, and establish regions of model parameters that enable the comparison of the subsidy programs.

4.2.1. Consumers. Although the primary objective behind agricultural subsidies is to protect farmers against unpredictable market conditions, their corresponding impact on consumer surplus is of policy interest and should not be overlooked. As discussed in Section 3, consumer surplus in our model is driven by the aggregate supply, which in turn, determines the market price of the crop through the inverse demand curve. It follows from Equation (4) that the larger the total quantity produced, the higher the consumer surplus.

PROPOSITION 4. *There exist thresholds $\gamma_1 < \gamma_2 < \gamma_3 < \gamma_4$, which only depend on μ, σ, m, and α such that*

(i) $\widehat{\Delta}_{PLC} \geq \widehat{\Delta}_{ARC}$ if $\gamma_2 \leq \frac{c}{b} \leq \gamma_3$;

(ii) $\widehat{\Delta}_{ARC} \geq \widehat{\Delta}_{PLC}$ if $\frac{c}{b} \leq \gamma_1$ or $\frac{c}{b} \geq \gamma_4$.

Furthermore, γ_1 is increasing in σ, whereas $\gamma_3 - \gamma_2$ and γ_4 are decreasing in σ.

Note that one unit increase in production quantity by farmer i, in addition to the extra revenue it generates, negatively impacts profits in two ways: (i) it increases the planting cost, which is an individual effect incurred only by farmer i, and (ii) it lowers the market price, which is a collective effect incurred by all the farmers. In essence, the ratio $\frac{c}{b}$ captures the relative importance of these two forces. As it turns out, this ratio plays an important role in the comparison between the two subsidy mechanisms. Proposition 4 implies that consumer surplus is higher under ARC if the ratio $\frac{c}{b}$ is high or low. Moderate values of this ratio, on the other hand, entail a higher consumer surplus under PLC.[8] Moreover, keeping everything else constant, the region over which PLC dominates ARC in terms of consumer surplus shrinks as yield variability increases.

To understand the role that the ratio $\frac{c}{b}$ plays in Proposition 4, one should focus on varying these two parameters in isolation. First consider small values of price sensitivity, b. Comparing the government subsidies paid under PLC in (6) and ARC in (9), we notice that the farmers receive a small amount of subsidy under PLC, whereas the ARC payment could be significant even for low values of b. Therefore, the farmers have an incentive to plant more under ARC. Conversely, for higher values of b, the second subsidy term in (9) becomes the dominant term as its lower integral limit approaches μ. Proposition 4 states that the gap between the reference revenue and actual revenue grows faster than the gap between the reference price and actual price when b is large; this can be attributed to the fact that the (positive) quadratic coefficient of b in the second integral of (9) is small compared to the linear coefficient in (6). Similarly, when planting cost c is low, the farmers plant more acres under both subsidy mechanisms and the second subsidy term in (9) becomes the dominant term as its lower integral limit approaches μ. Therefore, the gap between the reference revenue and actual revenue grows faster than the gap between the reference price and actual price for higher values of aggregate supply. Finally, when the planting cost c is large, the farmers' margins fall under both subsidies. This means that even for small to moderate yield variability, ARC has an advantage over PLC due to a large difference between the reference revenue and actual revenue.

4.2.2. Government. A critical factor in assessing a subsidy is the total expenditure it imposes on the government during implementation. Given the non-trivial interaction between the subsidy structures and the farmers' incentives, combined with the competition among farmers, the comparison between government expenditures under PLC and ARC is challenging. However, as the next result shows, the total cost of these policies can be partially tied to the aggregate production.

PROPOSITION 5. *Define ratios* $\underline{\beta} = \frac{m}{m+1}$ *and* $\overline{\beta} = \frac{m+1}{m}$. *Then,* $\widehat{\Gamma}_{ARC} \geq \widehat{\Gamma}_{PLC}$ *if* $\widehat{q}_{ARC} \geq \overline{\beta}\widehat{q}_{PLC}$, *and* $\widehat{\Gamma}_{ARC} \leq \widehat{\Gamma}_{PLC}$ *if* $\widehat{q}_{ARC} \leq \underline{\beta}\widehat{q}_{PLC}$.

[8] Please see Appendix B for closed form expressions of thresholds $\gamma_1, \ldots, \gamma_4$.

Proposition 5 relates to our research question about the effects of the subsidies on consumer surplus and government expendituers. It delivers a useful and general insight, indicating that for sufficiently large m (which holds for the practical values obtained in our model calibration in Section 5), the subsidy that entails a larger quantity (hence, higher consumer surplus) would be more expensive to the government. Note that this result holds irrespective of the profit that is generated for the farmers. This result also complements Theorem 1 by ruling out, for a large combination of model parameters, the possibility of a situation where both consumers and the government are better off under the same policy.

4.2.3. Farming Industry. We now turn our attention to the primary objective of the agricultural subsidies (i.e., to protect farmers' profit), and investigate how the farmers' profit is impacted under PLC and ARC.

PROPOSITION 6. *Suppose $m \geq 10$.*[9] *There exist thresholds $\rho_1 < \rho_2 < \rho_3$, which only depend on μ, σ, m, and α such that*

(i) $\widehat{\pi}_{PLC} \geq \widehat{\pi}_{ARC}$ if $\rho_1 \leq \frac{c}{b} \leq \rho_2$;

(ii) $\widehat{\pi}_{ARC} \geq \widehat{\pi}_{PLC}$ if $\frac{c}{b} \geq \rho_3$.

Furthermore, $\rho_2 - \rho_1$ is decreasing in σ. Similarly, ρ_3 decreases with σ as long as $\alpha \leq \frac{2\sigma\sqrt{2\pi}}{\mu}$.

The above result is presented for the farmers' profit in parallel to Proposition 4 for the consumers' surplus, and further highlights the importance of ratio $\frac{c}{b}$ in comparing the two subsidy mechanisms. In particular, the proposition characterizes ranges of $\frac{c}{b}$ over which the farmers would enroll in PLC or ARC. Crops with high values of this ratio are better candidates for ARC, whereas the farmers would prefer to enroll in PLC for crops with moderate $\frac{c}{b}$.[10] Moreover, for a wide range of realistic parameters, the ARC-dominant interval (PLC-dominant interval) expands (shrinks) as yield variability increases. The intuition for Proposition 6 is similar to the intuition for Proposition 4, because when the farmers are better off under a subsidy program, they plant more under that program, making the consumers better off as well.

5. Model Calibration and Managerial Insights

In this section, we report the results of the numerical experiments that further explore the impacts of model parameters on the subsidy programs. To use practical values in the experiments, we calibrated our model using available data on USDA website. We used annual yield data for the past 10 years to estimate the mean and standard deviation (in bushels per acre) of the yield distribution for crops. The estimates are summarized in Table 3.

[9] This is not restrictive at all since in all realistic settings there are multiple thousand farmers planting the same crop.

[10] See Appendix B for closed form expressions of thresholds $\rho_1, \ldots, \rho_3$.

Crop	Corn	Soybeans	Barely	Oats	Long-grain Rice	Peanuts	Sorghum
Mean	153.84	43.36	67.92	63.68	160.00	169.23	64.42
Standard Deviation	13.71	2.83	4.93	4.14	7.31	20.89	9.22

Table 3 Parameter estimates for crop yield distributions (bushels per acre)

To estimate the price function, we searched for data on market price and aggregate supply of crops. We were able to find both data for corn, barley, oats, sorghum, soybeans and long-grain rice. For these crops, we estimated N and b by fitting a linear regression equation with inflation-adjusted price as the dependent variable and supply as the independent variable. The estimates are provided in Table 4 and more details about the data are provided in Appendix C. For m, we used the number of influential farmers (as defined in Koba 2014) reported in the last Census of Agriculture (USDA, 2014). For the cost coefficient, we used USDA per-acre cost and average planting acreage data for each crop and determined the value of c such that the quadratic cost value became equal to USDA's reported per-acre cost for the average planting acreage. The estimates in Tables 3 and 4 formed a basis for the range of parameter values in our experiments. We performed sensitivity analysis to evaluate the effects of varying different parameters on the model outcomes.

Crop	Corn	Barely	Oats	Sorghum	Soybeans	Long-grain Rice
N ($ per bushel)	11.67	7.18	5.96	6.55	21.54	61.909
b ($ per bushels (in millions) squared)	0.00054	0.00967	0.01432	0.00625	0.00336	0.0867
m	41192	2009	1805	2512	34571	2264
c ($ per square acres)	4.26	57.644	47.493	2.014	2.973	2.764

Table 4 Estimates of price function parameters, number of farmers, and cost coefficient

The analytical results in Section 4 described the effect of variations in individual model parameters on the subsidies when other parameters are held constant. In the first experiment, we investigated the effects of simultaneous changes in model parameters on the equilibrium profits of PLC and ARC, which determine farmers' subsidy enrollment decision. More specifically, we fixed the values of N, b, and c. Then, we calculated the equilibrium profits for PLC and ARC and determined the optimal subsidy over a two-dimensional space of μ and σ values that include the estimates in Table 3 and satisfy $\mu \geq 3\sigma$. For the farmers (consumers), the optimal subsidy refers to the subsidy with the higher equilibrium profit (consumer welfare). For the government, the subsidy with the lower government expenditure at the farmers' equilibrium planting quantity is considered the optimal policy. We then repeated the experiment for different values of b when N and c were held constant, and for different values of c when N and b were held constant. In addition, we held N constant and simultaneously changed c and b such that the ratio $\frac{c}{b}$ had the same values as when only b or c varied. In all experiments, we used $\alpha = 0.85$. Across our experiments, the equilibrium planting quantity and farmers' profit under the optimal subsidy were on average 5.72% and 11.74% higher than their corresponding values under no subsidy.

Figure 2 illustrates the patterns we observed in the experiments for the relative performance of the subsidies. First, Figures 2(a) and (c) reflect the result in Proposition 5, that for sufficiently large m, the subsidy with higher consumer surplus corresponds to higher government expenditures. Second, Figures 2(a) and (b) provide more intuition into the effects of changing various model parameters on subsidy performances and their comparison as stated in Propositions 4 and 6, respectively. In our extensive experiments, we observed that the overwhelming majority of parameter value combinations resulted in a win-win outcome for farmers and consumers, either under PLC or under ARC. In the remainder of this section, we address our fourth research question by discussing the effect of various parameters on the subsidy of choice from the viewpoint of the farming industry and consumers.

We first discuss the effects of the yield distribution on the optimal subsidy regions. For all practical values, holding everything else constant, the farmers and consumers are better off with ARC when variability in farm yield is high and they are better off with PLC when yield variability is low. This result is driven by two effects: (1) The PLC subsidy payment is triggered only when the farm yield is high (above the average) and the market price falls below the reference price, whereas the ARC subsidy payment is triggered when the farm yield is either significantly lower or higher than the average; (2) Because the PLC subsidy is based on price, change in the yield has a unidirectional effect on the amount of PLC subsidy; as yield increases from the average, the market price decreases from the reference price. In contrast, because the ARC subsidy is calculated based on revenue as opposed to price, an increase in yield creates two opposing forces; it increases the amount of harvest but reduces the market price. For small changes in yield, the gap between the reference price and actual price in PLC grows faster than the gap between the reference revenue and actual revenue in ARC. For large changes in yield, however, the gap between the reference revenue and actual revenue grows faster. When σ is low, yield values near the average yield are more likely to occur. Due to the second effect described above, the rate of increase in PLC subsidy is faster when the yield increases from the average compared to the rate of increase in the ARC subsidy when the yield decreases from the average. As a result, PLC dominates ARC when yield variability is low. On the other hand, when variability is high, extreme values of yield (both low and high) become more likely than values near the average. In this situation, the first and second effects are both in favor of ARC, making it a better choice for the farmers.

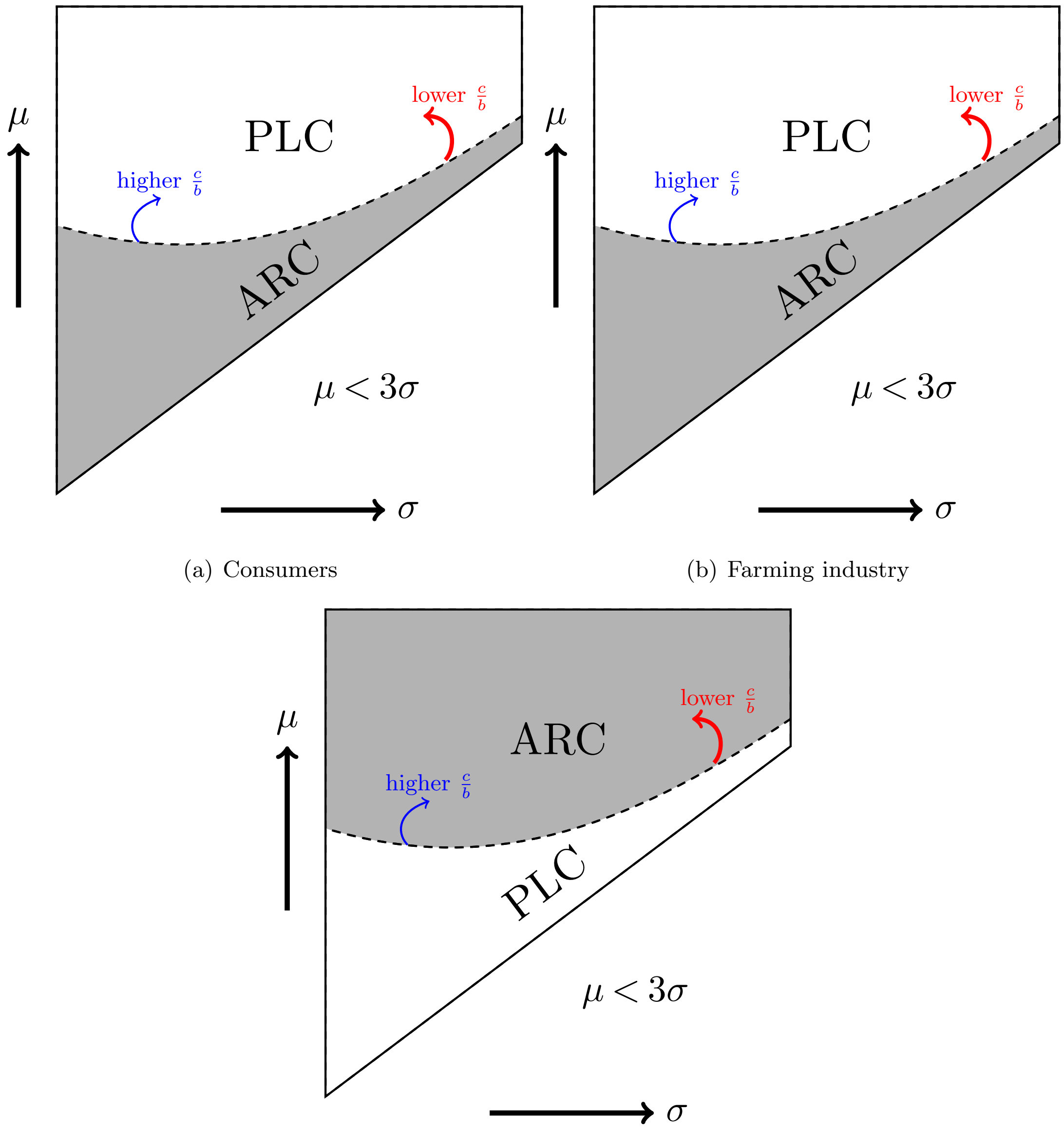
Next we observe that as yield variability remains constant and yield average increases, PLC becomes more beneficial to the farmers. When average yield goes up, the reference price in PLC decreases so the yield needs to be higher to trigger the subsidy payment. Nevertheless, because the PLC subsidy is proportional to the average yield, the net effect of an increase in average yield is a larger amount of PLC subsidy. In contrast, when average yield increases, the reference revenue in ARC goes down and the amount of ARC subsidy decreases. Therefore, for sufficiently large values of average yield, PLC dominates ARC.

Our observations about the effects of yield average and variability are supported by practitioners. As Bjerga (2015a) states, when yields plummet and harvest is destroyed (which is more likely to occur when yield variability is high), ARC is more beneficial to farmers than PLC. However, because ARC payouts are based in part on average yield, ARC becomes less beneficial to farmers when average yield is low. The combined effects of yield parameters in Figure 2(b) suggest that, holding other parameters constant, farmers should enroll crops that have a low relative variability in yield (coefficient of variation) in the PLC program and enroll crops that have a high relative variability in the ARC program.

The boundary between the optimal subsidy regions depends on the ratio of the planting cost coefficient to market price sensitivity to crop supply. As the cost of planting decreases and/or market price becomes more responsive to the crop supply, the region for PLC expands and PLC dominates ARC for high values of yield average and variability; the reverse is true for low yield average and variability where ARC dominates PLC. Stated differently, as illustrated in Figure 2, the boundary that determines the subsidy dominance moves clockwise as the value of $\frac{c}{b}$ increases.

An important question that farmers face is whether they should enroll their crops in ARC or PLC. Based on the combination of observations about the optimal subsidy for farmers in Figure 2(b), we provide rules of thumb to farmers in Figure 3 for choosing between PLC and ARC. Farmers should enroll crops with moderate $\frac{c}{b}$ and/or low relative yield variability in PLC. In contrast, farmers should enroll crops with low or high $\frac{c}{b}$ and/or high relative variability in yield in ARC.

To further highlight the value of our guidelines for subsidy enrollment, we compare the results of our model to the USDA statistics for farmers' enrollment in the subsidy programs (USDA 2015). The statistics highlight four crops for which the vast majority of farmers prefer one subsidy over the other: soybeans, corn, long-grain rice, and peanuts. For corn, soybeans, and long-grain rice, we checked our analytical results in Section 4. Table 5 shows that all three crops belong to $\mathcal{S}$ when farmers enroll in ARC, meaning that if farmers choose ARC, then they utilize ARC by planting relatively smaller quantities. For corn and soybeans, our model recommends that farmers enroll



(a) Consumers

(b) Farming industry

(c) Government

Figure 2 Optimal subsidy regions

in ARC. The USDA enrollment statistics indeed support our recommendations: 96% of soybeans farmers and 91% of corn farmers have enrolled in ARC. In addition, our model determines that ARC results in higher consumer surplus and government expenditure is lower under PLC. The ARC equilibrium quantities for corn and soybeans are both higher than their no-subsidy quantities. For long-grain rice, although the sufficient conditions for farmers' profit are not satisfied, we numerically determine that PLC is the optimal subsidy for farmers and that the increase in farmers' profit from the no-subsidy scenario is around 6.31 times higher under PLC. We have also provided the farmers' optimal subsidy region for long-grain rice in Figure 4. According to USDA, 99% of long-

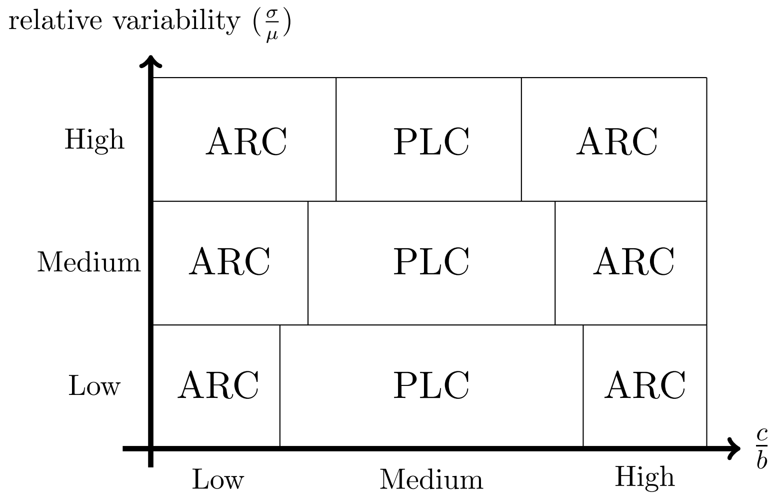


Figure 3 Subsidy enrollment guideline for farmers

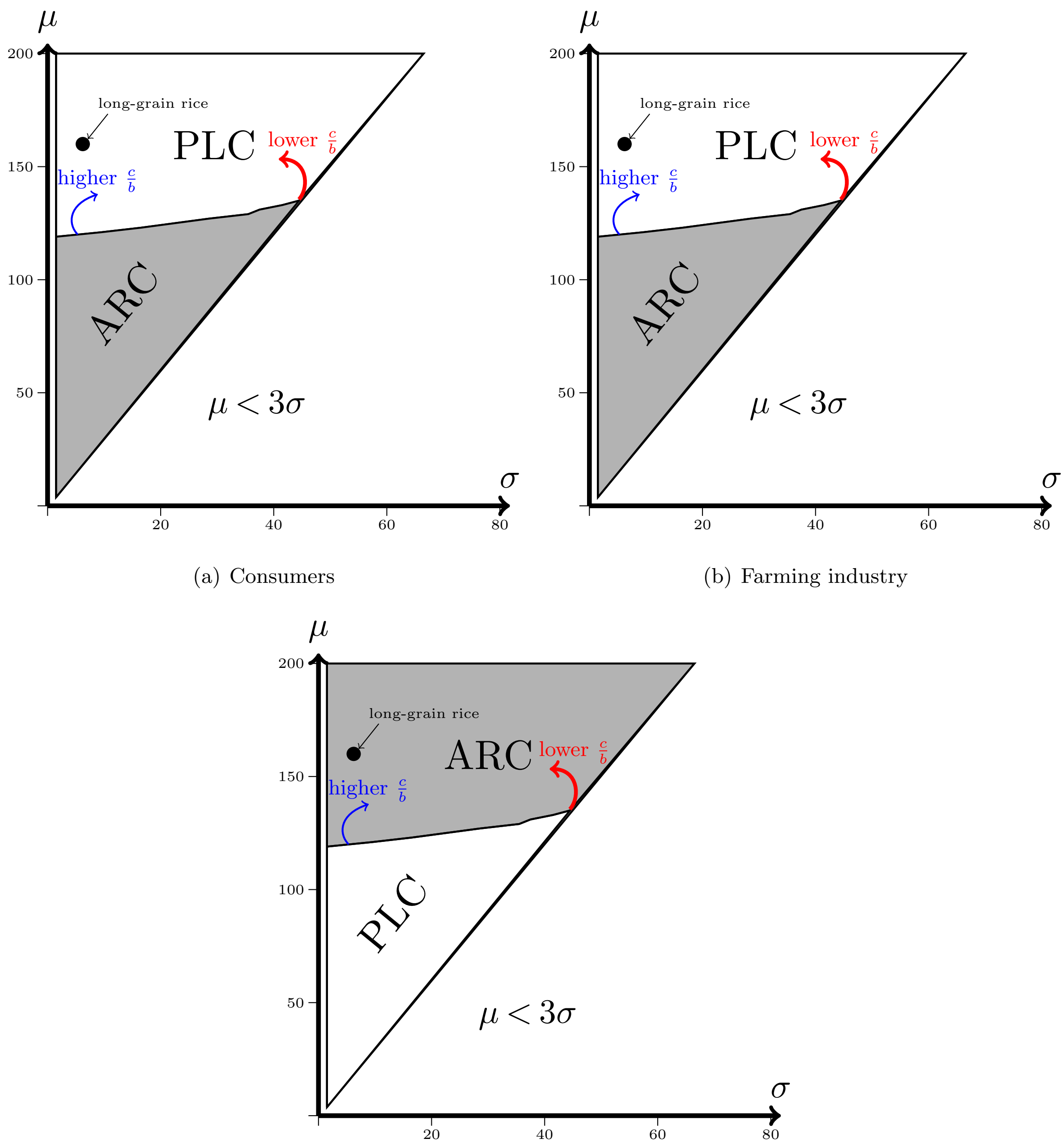
grain rice farmers have enrolled in PLC, which supports our recommendation. PLC also results in higher consumer surplus for long-grain rice and, as a result, government expenditure is lower under ARC. For peanuts, although we do not have the price function parameters, we conjecture that PLC should be better than ARC for farmers, given the high value of the estimated average yield for peanuts. Our conjecture is supported by the enrollment of 99% of peanuts farmers in PLC.

Crop	Belongs to $\mathcal{S}$	Farmers	Consumers	Government	$\widehat{q}_{ARC}$ vs. $\widehat{q}_{ns}$
Corn	Yes	ARC	ARC	PLC	$\widehat{q}_{ARC} > \widehat{q}_{ns}$
Soybeans	Yes	ARC	ARC	PLC	$\widehat{q}_{ARC} > \widehat{q}_{ns}$
Long-grain rice	Yes	PLC	PLC	ARC	$\widehat{q}_{ARC} > \widehat{q}_{ns}$

Table 5 Calibration results for corn, soybean, and long-grain rice

In response to our last research question, we conclude this section by discussing the social welfare implications of PLC and ARC policies. We define social welfare as the sum of consumer surplus and farmers' profit minus the economic and political frictions caused by running the subsidy programs. Such frictions represent the economic inefficiencies that are inflicted by subsidies and can arise from various factors that are not explicitly captured in our model, including the administrative costs of implementing the programs (such as documenting farmers' enrollment, determining farmers' eligibility, issuing payments to farmers, etc.), innovation and technology adoption implications, international trade ramifications, public resistance, and environmental externalities among others (see, for example, Glauber and Westhoff 2015, Peterson 2009, and Sunding and Zilberman 2001).

We introduce $0 \leq \zeta \leq 1$ to denote the friction caused by each dollar spent on the subsidies. The total friction due to offering PLC and ARC is thus given by $\zeta\,\widehat{\Gamma}_{PLC}$ and $\zeta\,\widehat{\Gamma}_{ARC}$, respectively. This



(a) Consumers

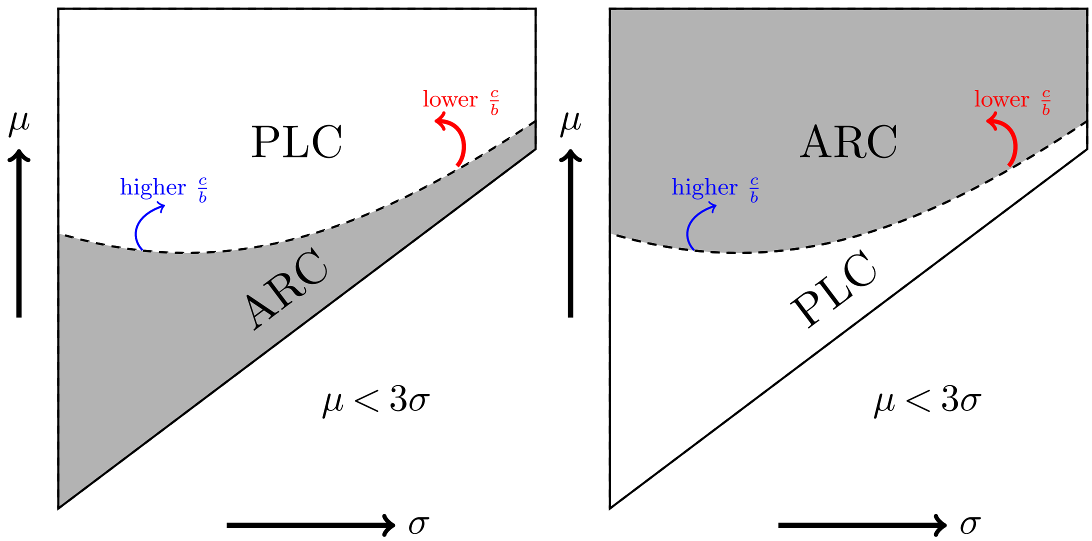
(b) Farming industry

(c) Government

Figure 4 Optimal subsidy regions calibrated for long-grain rice

approach has been used in the literature to study social welfare implications of subsidy programs in other contexts, such as renewable energy technology adoption (Alizamir et al 2016), and includes $\zeta = 0$ (e.g., van Benthem et al. 2008, Wand and Leuthold 2011) and $\zeta = 1$ (e.g., Lobel and Perakis 2011) as special cases. Thus social welfare equals $m\widehat{\pi}_{PLC} + \widehat{\Delta}_{PLC} - \zeta\,\widehat{\Gamma}_{PLC}$ under PLC and equals $m\widehat{\pi}_{ARC} + \widehat{\Delta}_{ARC} - \zeta\,\widehat{\Gamma}_{ARC}$ under ARC.

In our experiments, we used several low, medium, and high values for ζ. Figure 5 summarizes our observations about social welfare. When ζ is low (i.e. low friction), the subsidy program that benefits both farmers and consumers also results in higher social welfare. On the other hand, when ζ is high (i.e., when the friction is comparable to the amount spent on subsidies), the program that incurs a lower expenditure for the government is the preferred one for the society despite entailing lower consumer surplus and lower farmers' profit.



(a) Low friction ($\zeta = 0$) (b) High friction ($\zeta = 1$)

Figure 5 Optimal subsidy regions for social welfare

6. Conclusion

This work develops the first analytical model to study the current crop subsidies in the US agriculture industry, namely, PLC and ARC. These subsidies are offered by the government to protect farmers' income against unfavorable weather conditions that adversely impact harvest. These subsidies, in particular ARC, have unique features and have received limited attention in the literature; PLC protects farmers against a low market price, whereas ARC protects farmers against low revenues. Using a Cournot model, we characterize the equilibrium planting decisions of competing farmers that face uncertainty in the yield under each subsidy program. We then examine the policy implications of the subsidies by comparing their effects on consumer welfare, farming industry profit, and government expenditure.

Contrary to the prevailing intuition that ARC is likely to be superior because it offers a two-sided risk coverage as opposed to PLC, our model suggests that PLC could be a better subsidy in many scenarios. From farmers' point of view, we show that the PLC program can lead to a higher expected profit even when the reference price represents the historical average market price. This is especially true for crops that exhibit low relative yield variability and/or when the ratio of the planting cost to price sensitivity to supply takes moderate values. Consumers can also be better off under PLC in many cases as PLC leads to a higher planting quantity and therefore lower market price. In addition, the win-win outcome under PLC also benefits the society if the economic and political frictions caused by running the subsidy programs is sufficiently low.

There are a number of potential research directions for extending our work. For example, we assume that a population of homogenous farmers compete in a Cournot fashion. In reality, there are also smaller farmers who act as price takers, and their individual quantity decisions have no influence on the market. Further, farmers' heterogeneity can be captured in multiple dimensions including size, planting cost, and yield variability. It would be interesting to study how the two subsidy programs segment a heterogeneous pool of farmers, and which farmers benefit the most. While weather conditions are a primary cause for variability in farm yield, farmers can exert effort and adopt farming practices that can potentially improve the farm yield. It would also be a fruitful direction to model the interaction between the subsidy programs and such efforts by farmers.

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Appendix A: Extensions

In this appendix, we present extensions of our main model and discuss the effects of relaxing some of the assumptions.

A1. Dynamic Model

Throughout the paper, our focus has been on a stationary setting in which farmers' planting decisions are time-independent. More precisely, we have assumed that the subsidy programs have been in place for a sufficiently long period of time, which makes it reasonable to restrict attention to stationary equilibria. This assumption allows us to reduce a complex dynamic game to a one-shot game that captures the key features of the two subsidies, and is analytically tractable.

In this appendix, we provide further justification for our modeling choice by: (*i*) deriving the detailed formulation of the dynamic game model and explaining its complexity (both analytical as well as numerical), and (*ii*) describing the link between our stationary framework and the more general dynamic model, which highlights why our assumption is a reasonable approximation. The dynamic model that we consider allows the system to start from an arbitrary state when the subsidies are introduced, and we are interested in finding the Markov Perfect Equilibria (MPEs). We only present the details of the game formulation for the ARC subsidy since it is more general and requires further derivations. The approach for formulating PLC is simpler and can be derived by following similar steps.

Assume the reference revenue under the ARC program is calculated as the product of average market price and average yield realization over the last τ periods.[11] Then, the payoff-relevant history of the game at each point in time that summarizes the uncertainty realizations as well as actions taken by farmers in all previous periods can be captured by two τ-dimensional vectors $(\mathbf{p}, \mathbf{x})$ where $\mathbf{p} = (p_1, p_2, \ldots, p_\tau)$ and $\mathbf{x} = (x_1, x_2, \ldots, x_\tau)$ represent market price history and (per-acre) yield history, respectively. Hence, one can construct an infinite-horizon discrete-time stochastic dynamic game in which the state is $\mathbf{z} = (\mathbf{p}, \mathbf{x})$. The reference revenue is then calculated as $r = \left(\sum_{i=1}^{\tau} p_i/\tau\right)\left(\sum_{i=1}^{\tau} x_i/\tau\right)$.

Suppose the subsidy is introduced at time zero when the system state is $\mathbf{z}_0 = (\mathbf{p}_0, \mathbf{x}_0)$. When the system is in state $\mathbf{z}$ and the farmers collectively take actions $\mathbf{q} = (q_1, \ldots, q_m)$, farmer j's stage payoff becomes $\pi^j_{ARC}(q_j, \mathbf{q}_{-j}; r)$, which is farmer j's expected single period payoff as defined in (10) as a function of the reference revenue r. Further, the transition probability density function $h(\mathbf{z}'|\mathbf{z}, \mathbf{q})$ can be derived as

$$h(\mathbf{z}'|\mathbf{z}, \mathbf{q}) = \begin{cases} \phi(x') & \text{for } \mathbf{z}' = \left(\left(p_2, p_3, \ldots, p_\tau, N - b\left[\sum_{j=1}^{m} q_j\right] x'\right), \left(x_2, x_3, \ldots, x_\tau, x'\right)\right) \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathbf{z}^t$ and $\mathbf{q}^t$ represent the state and action vector at time t, respectively, and define s^j as farmer j's strategy, which is a mapping from histories to actions. Then, the expected discounted payoff for farmer j is

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t \pi^j_{ARC}\left(s^1(\mathbf{z^t}), s^2(\mathbf{z^t}), \ldots, s^m(\mathbf{z^t}); r\right)\right],$$

where δ is the discount factor.

[11] Under the current Farm Bill, τ is set equal to five years.

Finally, the value function for farmer j in equilibrium, which is the highest payoff she can achieve starting from a given state, can be obtained by solving the following Bellman equation:

$$V_j(\mathbf{z};\mathbf{s}_{-j}) = \max_{q_j} \mathbb{E}\left[\pi^j_{ARC}(q_j, \mathbf{s}_{-j}(\mathbf{z}); r) + \delta \int_{\mathbf{z}'} h\big(\mathbf{z}'|q_j, \mathbf{s}_{-j}(\mathbf{z}), \mathbf{z}\big) V_j(\mathbf{z}'; \mathbf{s}_{-j})\right]. \tag{16}$$

We denote by $\widehat{s}^j(\mathbf{z})$ the optimal strategy of farmer j in equilibrium, which entails action $\widehat{q}_j$. In particular, we are interested in symmetric equilibria in which $\widehat{q}_j(\mathbf{z}) = \widehat{q}(\mathbf{z})$ for all j.

Next, we argue that solving the dynamic game described above is extremely complex, both analytically and numerically. First, note that we are facing a stochastic dynamic game (of complete information), which is known to be very difficult in most cases (Fudenberg and Tirole 2000, Basar and Olsder 1999). In this case, in particular, the state is 2τ dimensional and continuous. The uniqueness of the equilibrium is not guaranteed for such games since the state space and the action space are continuous and not finite. For the specific game described above, due to the inherent stochasticity caused by yield uncertainty, any history may emerge in equilibrium. Therefore, identifying the equilibria requires full characterization of functions $\widehat{q}(.)$ and $\widehat{V}(.)$ for all possible histories, which are multi-dimensional and continuous. Furthermore, these functions will not be even differentiable because of non-differentiability of the single-period payoff function.

Tackling these problems numerically also introduces several challenges since it requires discretizing the state space as well as the action space, and iteratively solving for either $\widehat{q}(.)$ or $\widehat{V}(.)$ while taking the other one as given. Aside from the computational challenge of each iteration (since the single period payoff function is highly nonlinear and non-differentiable) and loss of accuracy due to discretization, the convergence of the numerical procedure to an equilibrium is not guaranteed, and depends on the starting point of the algorithm.

To overcome these challenges while maintaining the essential features of the underlying problem, we have incorporated the following simplifications that reduce the above complex dynamic game to a static one. First, as mentioned earlier, we restrict attention to MPEs, which implies that the subsidy policies have been in place for sufficiently long period of time and the effect of history prior to the introduction of subsidies has phased out. More precisely, the focus is on stationary settings where farmers' planting decisions become time-independent. Focusing on stationary equilibria in dynamic games is a common approach in the literature (see, for example, Huang et al. 2001, Plambeck and Wang 2009, Agrawal et al. 2012, 2016).

Second, we approximate the reference revenue $r = \left(N - bQ\overline{X}\right)\overline{X}$ by $\left(N - bQ\mu\right)\mu$, where $\overline{X} = \left(\sum_{i=1}^{\tau} X_i\right)/\tau$. To understand why this is reasonable in our setting, first note that X_i's are normally distributed and τ is set equal to five under the current bill, which implies $\overline{X} \sim \mathcal{N}(\mu, \sigma/\sqrt{5})$. More importantly, even for unlikely sequences of yield realizations for which $\overline{X}$ happens to be very high (resp. very low) compared to μ, the average price will, in turn, be very low (resp. very high). Hence, even if $\overline{X}$ is distant from μ, the product of average yield and average price will still remain very close to $\big(N - bQ\mu\big)\mu$. In particular, our simple simulation of 1000 normally distributed random values indicates that the coefficient of variation for the observed distribution of $\Big(N - bQ\overline{X}\Big)\overline{X}$ is around 0.002, which justifies our approximation. Finally, the fact that the policy is using olympic averages for price and yield instead of regular averages also suggests that the policymakers' intention is to smooth out possible fluctuations caused by very low or very high realizations of the annual yield, and maintain average price and average yield around their means.

These two simplifications essentially make the problem that farmers face in each period identical and reduce our complex dynamic game to a repeated game. As a result, the equilibrium of the stage game (i.e., the equilibrium that we study in the paper) constitutes an equilibrium of this repeated game.

A2. Exogenous Reference Price in PLC

We now consider the case of an exogenous reference price for the PLC subsidy. Let λ denote the exogenous reference price that is independent of the average market price. Then, the expected profit of farmer i becomes

$$\pi^i_{PLC}(q_i, \mathbf{q}_{-i}) = \int_L^U \left[N - b\Big(\sum_{j=1}^m q_j\Big)x\right] q_i x f(x)dx - cq_i^2 + \alpha \int_L^U \max\left\{0, \lambda - p\Big(\sum_{j=1}^m q_j, x\Big)\mu\right\} q_i \phi(x)dx\,.$$

Using the first derivative of the profit function, we show that the equilibrium quantity under PLC exogenous, denoted by $\widehat{q}_{exg}$, satisfies the following equation

$$\begin{aligned} &N\mu - (m+1)b(\mu^2+\sigma^2)\widehat{q}_{exg} - 2c\widehat{q}_{exg} + \alpha\mu\left(\lambda - N + (m+1)b\mu\widehat{q}_{exg}\right)\left(1 - \Phi\left(\frac{N-\lambda}{bm\widehat{q}_{exg}}\right)\right) \\ &+ \alpha\mu(m+1)b\sigma^2\widehat{q}_{exg}\phi\left(\frac{N-\lambda}{bm\widehat{q}_{exg}}\right) = 0. \end{aligned} \tag{17}$$

Based on the equilibrium condition for $\widehat{q}_{exg}$, the equilibrium farmer profit under PLC exogenous, denoted by $\widehat{\pi}_{exg}$, equals

$$\begin{aligned} \widehat{\pi}_{exg} = &N\widehat{q}_{exg}\mu - mb(\mu^2+\sigma^2)(\widehat{q}_{exg})^2 - c(\widehat{q}_{exg})^2 + \alpha\mu\widehat{q}_{exg}\left(\lambda - N + mb\mu\widehat{q}_{exg}\right)\left(1 - \Phi\left(\frac{N-\lambda}{bm\widehat{q}_{exg}}\right)\right) \\ &+ \alpha\mu mb\sigma^2(\widehat{q}_{exg})^2\phi\left(\frac{N-\lambda}{bm\widehat{q}_{exg}}\right). \end{aligned} \tag{18}$$

We now compare the exogenous PLC model with our base PLC model.

PROPOSITION 7. *The equilibrium quantity and profit of exogenous PLC compare to those under endogenous PLC as follows:*

(a) If $\lambda \geq N - bm\mu\widehat{q}_{PLC}$*, then* $\widehat{q}_{exg} > \widehat{q}_{PLC}$ *and* $\widehat{\pi}_{exg} \geq \widehat{\pi}_{PLC}$*,*

(b) If $N - b(m+1)\mu\widehat{q}_{PLC} < \lambda < N - bm\mu\widehat{q}_{PLC}$*, then* $\widehat{q}_{exg} > \widehat{q}_{PLC}$ *and* $\widehat{\pi}_{exg} < \widehat{\pi}_{PLC}$*,*

(c) If $\lambda \leq N - b(m+1)\mu\widehat{q}_{PLC}$*, then* $\widehat{q}_{exg} \leq \widehat{q}_{PLC}$ *and* $\widehat{\pi}_{exg} < \widehat{\pi}_{PLC}$*.*

This result shows that if the exogenous reference price is equal to or larger than the equilibrium average market price in our model, then the farmers plant more and their profit is equal to or higher than their profit when the reference price is set to the average market price. When the exogenous reference price is below the endogenous equilibrium average market price, farmers' planting acreage and profit is lower than those under the endogenous reference price model. Based on this result, if our model determines that PLC is better than ARC, then a higher-than-average exogenous reference price makes PLC even more superior than ARC. It is conceivable that if the exogenous reference price significantly decreases from the endogenous average price, then ARC can become the optimal policy. In the Farm Bill, we observe the largest gap between the exogenous reference price and historical average market price for two crops: corn and soybean, whose values of λ are approximately 30% below the average market price. Interestingly, for these crops we already showed in Section 5 that even under the endogenous reference price, ARC is the better subsidy for corn and soybean farmers. In fact, we numerically solved the PLC model with the exogenous reference prices from the Farm Bill for corn ($\lambda = 3.7$ \$/bushel), soybean ($\lambda = 8.4$ \$/bushel), and long-grain rice ($\lambda = 31.11$ \$/bushel), three crops for which the overwhelming majority of farmers enrolled in one of the subsidy programs. The optimal subsidy program for farmers does not change under the exogenous model, meaning that ARC remains the better program for corn and soybean farmers, and PLC remains the better program for long-grain rice farmers.

A3. Correlated Yield

Throughout the paper, we assumed that farm yields were perfectly correlated between m farmers. In this section, we examine the situation where the farm yields follow a general multivariate normal distribution. In this case, the expected amount of ARC subsidy for farmer i is given by

$$\alpha \int \cdots \int_{x_1,\cdots,x_m \geq 0} \max\left[0, \Big(N - b\mu \sum_{j=1}^{m} q_j\Big)\mu - \Big(N - b\sum_{j=1}^{m} q_j x_j\Big) x_i\right] q_i\, \phi(x_1,\cdots,x_m)\, dx_1 \cdots dx_m, \tag{19}$$

in which $\phi(x_1,\cdots,x_m)$ is a multivariate normal distribution with $\mu_i = \mu, \sigma_i = \sigma$ $(\forall i = 1,\cdots,m)$, and $\rho_{ij} = \rho$ $(\forall i \neq j)$ where ρ denotes the correlation between the farm yields. We consider values $0 < \rho < 1$ because crops require certain climates to grow successfully (temperature, precipitation, etc.); therefore, a crop can only be planted in areas that have similar climates, hence the positive correlation. It is easy to see that the ARC subsidy expression in (19) is extremely complex because it involves the sum of products of correlated normal random variables; even when the yield variables are independent the ARC subsidy payment remains intractable. Consequently, to investigate the effect of correlated yields we resort to Monte Carlo simulation in Matlab. In each iteration of simulation, we generated 100,000 random values of $(x_1,\cdots,x_m)$ from the multivariate normal distribution and found the equilibrium quantities that solved the first order conditions for ARC and

PLC and calculated their corresponding expected profits. We repeated this procedure for different values of ρ. Our experiments indicate that the qualitative nature of our observations in Section 5 remains valid when correlation between farm yields is smaller than 1 (see Figure 6(a)). In addition, we observe that, holding everything else constant, the region where the farming industry is better off with PLC expands as correlation approaches 1. When yields are strongly positively correlated, the probability of a large total harvest realization (above average) increases significantly. A large supply drops the market price significantly below the reference price and results in a substantial subsidy payment under PLC. For ARC, however, a large total harvest entails two opposing effects; the positive effect of a low market price on the revenue subsidy payment is dampened by the negative effect of a large above-average yield realization. Consequently, PLC outperforms ARC.

A4. Demand Uncertainty

In our model, we captured the uncertainty in market price that arises from uncertainty in the farm yields. Market price can also be influenced by other factors such as the overall domestic and global economic conditions and other public policies. In this section, we examine the effect of incorporating demand uncertainty into the main model by assuming that the market price equals $N + \epsilon - b\sum_{j=1}^{m} q_j X$ where ϵ is a random shock that is independent of X and takes on values $-\theta N$, 0, and θN, each with probability 1/3. For PLC, the equilibrium quantity satisfies the following equation

$$\begin{aligned}\sum_{i=1}^{3}\beta_i\Bigg[&(N+\epsilon_i)\mu-(m+1)b\widehat{q}_{PLC}(\mu^2+\sigma^2)-2c\widehat{q}_{PLC}\\&+\alpha(m+1)b\mu\sigma^2\widehat{q}_{PLC}\phi\left(\mu+\frac{\epsilon_i}{bm\widehat{q}_{PLC}}\right)-\alpha\mu\epsilon_i\left(1-\Phi\left(\mu+\frac{\epsilon_i}{bm\widehat{q}_{PLC}}\right)\right)\Bigg]=0,\end{aligned}\tag{20}$$

and for ARC, the equilibrium condition is

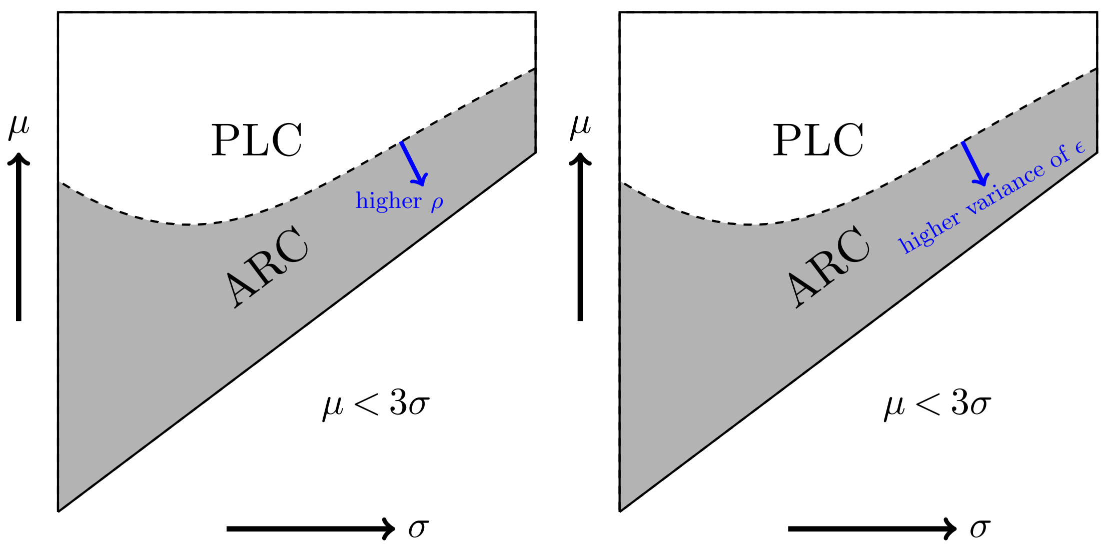
$$\begin{aligned}\sum_{i=1}^{3}\beta_i\Bigg[&(N+\epsilon_i)\mu-(m+1)b\widehat{q}_{ARC}(\mu^2+\sigma^2)-2c\widehat{q}_{ARC}+\alpha(m+1)b\widehat{q}_{ARC}\sigma^2-\alpha\epsilon_i\mu\\&-\alpha\int_{\min\{\chi_1(\widehat{q}_{ARC},\epsilon_i),\chi_2(\widehat{q}_{ARC},\epsilon_i)\}}^{\max\{\chi_1(\widehat{q}_{ARC},\epsilon_i),\chi_2(\widehat{q}_{ARC},\epsilon_i)\}}\left[(m+1)b\widehat{q}_{ARC}(x^2-\mu^2)+N(\mu-x)-\epsilon_i x\right]\phi(x)dx\Bigg]=0,\end{aligned}\tag{21}$$

where $\epsilon_1=-\theta N$, $\epsilon_2=0$, $\epsilon_3=\theta N$, $\beta_i=\frac{1}{3}$ for $i=1,2,3$, and

$$\chi_1(\widehat{q}_{ARC},\epsilon_i)=\frac{N+\epsilon_i-\sqrt{\epsilon_i^2+2N\epsilon_i+(N-2bm\mu\widehat{q}_{ARC})^2}}{2bm\widehat{q}_{ARC}},$$
$$\chi_2(\widehat{q}_{ARC},\epsilon_i)=\frac{N+\epsilon_i+\sqrt{\epsilon_i^2+2N\epsilon_i+(N-2bm\mu\widehat{q}_{ARC})^2}}{2bm\widehat{q}_{ARC}}.$$

Because adding a second source of variability made the analysis complicated, we used numerical experiments. We selected low, moderate, and high values for θ from interval $(0,1)$ to capture different degrees of variability in demand and at the same time keep the numerical experiments manageable. After repeating our numerical experiments, we observed that our main results in Section 5 continue to hold qualitatively. In addition, we observed that, everything else held constant, as the variance of ϵ increases (i.e. higher θ), the regions in which the farming industry and consumers are better off with PLC expand and, as a result, the region in which the government is better off with ARC also expands (see Figure 6(b)). The reason is that as θ increases, the magnitude of uncertainty increases. While the negative realization of ϵ may work in favor of ARC if it is accompanied by a low farm yield, the positive realization of ϵ works against ARC, but it does not create a negative impact on PLC since PLC subsidy is always triggered when market price is low.

In summary, our extensive experiments indicate that our results are quite robust and the insights and observations generated from our model continue to hold under more general settings.



(a) Effect of correlation, ρ (b) Effect of variance of ϵ

Figure 6 Optimal subsidy regions for farmers under correlated yield and uncertain demand

Appendix B: Proofs

Proof of Lemma 1: $\pi^i_{ns}(.)$ is concave in q_i. We have

$$\frac{\mathrm{d}}{\mathrm{d}q_i}\pi^i_{ns}(q_i, \mathbf{q}_{-i}) = Nq_i\mu - b\Big(\sum_{j\neq i}^{m} q_j + 2q_i\Big)(\mu^2+\sigma^2) - 2cq_i\,.$$

Solving $\frac{\mathrm{d}}{\mathrm{d}q_i}\pi^i_{ns}(q_i, \mathbf{q}_{-i}) = 0$ subject to $q_i = q$ for all $i = 1, \ldots, m$ gives us the equilibrium quantity and profit in (3). $\square$

Proof of Proposition 1: For PLC, the expected profit of farmer i is concave in the planting quantity q_i. The derivative of $\pi^i_{PLC}(q_i,\mathbf{q}_{-i})$ with respect to q_i equals

$$\frac{\mathrm{d}}{\mathrm{d}q_i}\pi^i_{PLC}(q_i,\mathbf{q}_{-i}) = Nq_i\mu - b\Big(\sum_{j\neq i}^m q_j + 2q_i\Big)(\mu^2+\sigma^2) - 2cq_i + \alpha b\Big(\sum_{j\neq i}^m q_j + q_i\Big)\mu\int_\mu^\infty (x-\mu)\phi(x)dx$$

where $\int_\mu^\infty (x-\mu)\phi(x)dx = \sigma^2\phi(\mu) = \frac{\sigma}{\sqrt{2\pi}}$. We set $q_i = q$ for all $i = 1,\dots,m$ in $\frac{\mathrm{d}}{\mathrm{d}q_i}\pi^i_{PLC}(q_i,\mathbf{q}_{-i})$ and solve for q which gives us (12) as the equilibrium planting acreage. Substituting the equilibrium quantity in the profit function, we get the equilibrium profit for PLC in (13). It is straightforward to show $\widehat{q}_{PLC} > \widehat{q}_{ns}$ and $\widehat{\pi}_{PLC} > \widehat{\pi}_{ns}$. □

Proof of Proposition 2: Taking the derivative of $\widehat{q}_{PLC}$ with respect to σ, we see that the sign of the derivative depends on $2\sigma - \frac{\alpha\mu}{\sqrt{2\pi}}$; therefore, $\widehat{q}_{PLC}$ increases with σ for $\sigma \le \frac{\alpha\mu}{2\sqrt{2\pi}}$ and decreases with σ afterwards. We then take the derivative of (13) with respect to σ and find that it has three roots, two of which are negative, hence unacceptable, and the third one is $\frac{\alpha\mu}{2\sqrt{2\pi}}$. We verify that the second derivative of PLC equilibrium profit with respect to σ is negative at $\frac{\alpha\mu}{2\sqrt{2\pi}}$. Therefore, the behavior of the equilibrium PLC profit when σ varies is identical to the behavior of $\widehat{q}_{PLC}$. □

Proof of Proposition 3: Going back to (11), we have to analyze the following two cases:
Case 1: $0 \le q \le \frac{N}{2bm\mu}$, i.e., $\min\left\{\frac{N}{bmq}-\mu,\mu\right\} = \mu$ and

$$\begin{aligned}\frac{d}{dq_i}\pi^i_{ARC}(q_i,\mathbf{q}_{-i}) =& N\mu - b\Big(\sum_{j\neq i}^m q_j + 2q_i\Big)(\mu^2+\sigma^2) - 2cq_i + \alpha b\Big(\sum_{j\neq i}^m q_j + 2q_i\Big)\sigma^2 \\ & - \alpha\int_\mu^{\frac{N}{b\sum_{j=1}^m q_j}-\mu}\left[b\Big(\sum_{j\neq i}^m q_j + 2q_i\Big)(x^2-\mu^2) + N(\mu-x)\right]\phi(x)dx. \qquad (22)\end{aligned}$$

Setting $q_i = q$ for all $i = 1,\dots,m$ and using $\frac{\mathrm{d}}{\mathrm{d}x}\phi(x) = -\frac{x-\mu}{\sigma^2}\phi(x)$, we get the following derivatives of the profit function

$$\begin{aligned}\frac{d}{dq_i}\pi^i_{ARC}(q) =& N\mu - (m+1)bq(\mu^2+\sigma^2) - 2cq + (m+1)\alpha bq\sigma^2 \\ & - \alpha\int_\mu^{\frac{N}{bmq}-\mu}\big[(m+1)bq(x^2-\mu^2) + N(\mu-x)\big]\phi(x)dx\end{aligned}$$

$$\begin{aligned}\frac{d}{dq}\Big(\frac{d}{dq_i}\pi^i_{ARC}(q)\Big) =& -(m+1)b(\mu^2+\sigma^2) - 2c + (m+1)\alpha b\sigma^2 - \alpha(m+1)b\int_\mu^{\frac{N}{bmq}-\mu}(x^2-\mu^2)\phi(x)dx \\ & + \frac{\alpha N^2}{b^2m^3q^3}(N - 2bmq\mu)\phi\left(\frac{N}{bmq}-\mu\right)\end{aligned}$$

$$\frac{d^2}{dq^2}\Big(\frac{d}{dq_i}\pi^i_{ARC}(q)\Big) = \frac{\alpha N^2\left(-2\,m^3q^3\mu\,\sigma^2\,(m-1)\,b^3 + Nm^2q^2\,(m\sigma^2 + 4\,\mu^2 - 2\,\sigma^2)\,b^2 - 4\,N^2bmq\mu + N^3\right)}{b^4m^5q^6\sigma^2}\phi\left(\frac{N}{bmq}-\mu\right)$$

First, we note that $\lim_{q\to 0}\frac{d}{dq_i}\pi^i_{ARC}(q) > 0$, $\frac{d}{dq_i}\pi^i_{ARC}(q)|_{q=\frac{N}{2bm\mu}} = \frac{\big(\big((\alpha-1)(m+1)\sigma^2+\mu^2(m-1)\big)b-2c\big)N}{2bm\mu}$, and $\lim_{q\to 0}\frac{d}{dq}\big(\frac{d}{dq_i}\pi^i_{ARC}(q)\big) < 0$. The equation $\frac{d^2}{dq^2}\big(\frac{d}{dq_i}\pi^i_{ARC}(q)\big) = 0$ has only one real solution in the interval. Substituting this solution into $\frac{d}{dq}(\frac{d}{dq_i}\pi^i_{ARC}(q))$ and after some algebra, we get a function $\frac{N}{b}g(\alpha,\mu,\sigma,m)-2c$. Using Maple's optimization toolbox, we can show $g(\alpha,\mu,\sigma,m)$ is always negative. Therefore, $\frac{d}{dq_i}\pi^i_{ARC}(q)$ is a decreasing convex-concave function and has exactly one root in $(0,\frac{N}{2bm\mu})$ if and only if $\frac{d}{dq_i}\pi^i_{ARC}(q)|_{q=\frac{N}{2bm\mu}}<0$. This means that π_{ARC} is a unimodal function of q in interval $0\le q\le\frac{N}{2bm\mu}$.

Case 2: $q>\frac{N}{2bm\mu}$, i.e., $\min\left\{\frac{N}{bmq}-\mu,\mu\right\}=\frac{N}{bmq}-\mu$ and when we set $q_i=q$ for all $i=1,\ldots,m$ in the first derivative of the profit function, we get the following

$$\begin{aligned}\frac{d}{dq}(\frac{d}{dq_i}\pi^i_{ARC}(q))&=N\mu-(m+1)bq(\mu^2+\sigma^2)-2cq+(m+1)\alpha bq\sigma^2\\&\quad-\alpha\int_{\mu}^{\frac{N}{bmq}-\mu}\left[(m+1)bq(x^2-\mu^2)+N(\mu-x)\right]\phi(x)dx\end{aligned}\tag{23}$$

$$\begin{aligned}\frac{d^2}{dq^2}(\frac{d}{dq_i}\pi^i_{ARC}(q))&=-(m+1)b(\mu^2+\sigma^2)-2c+(m+1)\alpha b\sigma^2-\alpha(m+1)b\int_{\mu}^{\frac{N}{bmq}-\mu}(x^2-\mu^2)\phi(x)dx\\&\quad-\frac{\alpha N^2}{b^2m^3q^3}(N-2bmq\mu)\phi\left(\frac{N}{bmq}-\mu\right).\end{aligned}$$

Also, the third derivative of the profit function in this case equals the negative of the third derivative in case 1 so the third derivatives have the same real root. If we substitute the root in the second derivative above, we again obtain a function that is negative for any m, μ, and σ. Thus, the first derivative for case 2 is also a decreasing convex-concave function which means (23) has a unique solution in $q>\frac{N}{2bm\mu}$ if and only if the value of (23) at $q=\frac{N}{2bm\mu}$ is positive. Note that the first derivatives are case 1 and case 2 are equal to each other when $q=\frac{N}{2bm\mu}$. Therefore, putting everything together, the ARC expected profit function is unimodal in q and attains its maximum value in interval $(0,\frac{N}{2bm\mu})$ when $(((\alpha-1)(m+1)\sigma^2+\mu^2(m-1))b+2c)<0$ and in interval $[\frac{N}{2bm\mu},\infty)$ otherwise. □

Proof of Corollary 1: When $\alpha=0$, the farmers plant $\widehat{q}_{ns}$. Because $\widehat{q}_{ns}$ maximizes (2), $\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{ns})=\alpha G(\widehat{q}_{ns})$. Since the farmers' profit function under ARC is unimodal, if $G(\widehat{q}_{ns})>0$, then $\widehat{q}_{ARC}>\widehat{q}_{ns}$ and $\widehat{q}_{ARC}$ increases with α; otherwise, $\widehat{q}_{ns}$ decreases from $\widehat{q}_{ns}$ as α increases. □

Proof of Theorem 1: To proceed with the proof, we first use the results of Proposition 1 and Proposition 3 for PLC and ARC, respectively, to simplify an individual farmer's profit in equilibrium. For PLC, from (12) and (13) it follows that:

$$\widehat{\pi}_{PLC}=(\widehat{q}_{PLC})^2\Big(b(\mu^2+\sigma^2)+c-\alpha b\sigma^2\mu\phi(\mu)\Big).\tag{24}$$

For ARC, we plug in the FOC (14) into (15) to obtain:

$$\widehat{\pi}_{ARC}=(\widehat{q}_{ARC})^2\left(b(\mu^2+\sigma^2)+c-\alpha b\sigma^2+\alpha b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right).\tag{25}$$

Define

$$\kappa = \left(b(\mu^2+\sigma^2) + c - \alpha b\sigma^2 + \alpha b \int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}} (x^2-\mu^2)\phi(x)dx \right) - \left(b(\mu^2+\sigma^2) + c - \alpha b\sigma^2\mu\phi(\mu) \right)$$
$$= \alpha b \left(\sigma^2\mu\phi(\mu) - \sigma^2 + \int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}} (x^2-\mu^2)\phi(x)dx \right).$$

Further, note that

$$\int_s^t (x^2-\mu^2)\phi(x)dx = \sigma^2\left[(s+\mu)\phi(s) - (t+\mu)\phi(t)\right] + \sigma^2\Big(\Phi(t)-\Phi(s)\Big).$$

First consider $(c,b,\mu,\sigma,m,\alpha)\in\mathcal{S}$. This implies that

$$\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\} = \mu \qquad \text{and} \qquad \max\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\} = \frac{N}{bm\widehat{q}_{ARC}}-\mu\,.$$

After expanding the integral, we get

$$\kappa = \alpha b\sigma^2\left(\mu\phi(\mu) - 1 + 2\mu\phi(\mu) - \frac{N}{bm\widehat{q}_{ARC}}\phi\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big) + \Phi\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big) - \Phi(\mu)\right).$$

Define function $g(.)$ as

$$g(x) = (x+\mu)\phi(x) - \Phi(x).$$

Then,

$$\kappa = \alpha b\sigma^2\left(3\mu\phi(\mu) - 1 - g\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big) - \Phi(\mu)\right).$$

Using simple algebra, we can show that $g'(x)\le 0$ for $x\ge\mu$, and hence $g(.)$ is decreasing over $[\mu,\infty)$. Thus, it takes its maximum value within set $\mathcal{S}$ at $x=\mu$ (Note that $(c,b,\mu,\sigma,m,\alpha)\in\mathcal{S}$ implies $\frac{N}{bm\widehat{q}_{ARC}}-\mu\ge\mu$). It follows from Assumption 1 that κ is always positive. As a result, if $\widehat{q}_{ARC}\ge\widehat{q}_{PLC}$ (or, equivalently, $\widehat{\Delta}_{ARC}\ge\widehat{\Delta}_{PLC}$) then $\widehat{\pi}_{ARC}\ge\widehat{\pi}_{PLC}$. This completes the proof for part (a). The proof for the case where $(c,b,\mu,\sigma,m,\alpha)\notin\mathcal{S}$ is exactly similar except we need to show that $\kappa<0$. This completes the proof for part (b) of the theorem. □

Proof of Proposition 4: Recall that the equilibrium quantity for PLC satisfies the following equation:

$$N\mu - (m+1)b(\mu^2+\sigma^2)\widehat{q}_{PLC} - 2c\widehat{q}_{PLC} + \frac{\alpha(m+1)b\mu\sigma\widehat{q}_{PLC}}{\sqrt{2\pi}} = 0. \tag{26}$$

Since $\pi_{ARC}(q)$ is unimodal in q, we make use of the sign of the first derivative of $\pi_{ARC}(q)$ to derive inequalities for the relationship between $\widehat{q}_{PLC}$ and $\widehat{q}_{ARC}$. Note that consumer welfare for each subsidy is proportional to the planting acreage under that subsidy. For this purpose, we use the following integral simplifications which are obtained through integration by parts:

$$\begin{aligned}
&\int_{y_1}^{y_2} (\mu - x)\phi(x)dx = \sigma^2 \left(\phi\left(y_2\right) - \phi\left(y_1\right)\right), \\
&\int_{y_1}^{y_2} (x^2 - \mu^2)\phi(x)dx = \sigma^2 \left((y_1+\mu)\phi\left(y_1\right) - (y_2+\mu)\phi\left(y_2\right)\right) + \sigma^2 \left(\Phi\left(y_2\right) - \Phi\left(y_1\right)\right).
\end{aligned} \tag{27}$$

We first present the proof for part (i). Note that $\min\left\{\mu, \frac{N}{bm\widehat{q}_{PLC}} - \mu\right\} = \mu$ if and only if $\frac{c}{b} \geq \overline{\gamma}$ where $\overline{\gamma} = \frac{(m-1)\mu^2-(m+1)\sigma\left(\sigma-\frac{\alpha\mu}{\sqrt{2\pi}}\right)}{2}$. First, suppose $\frac{c}{b} \geq \overline{\gamma}$. Then, evaluating (14) at $\widehat{q}_{PLC}$ and using (27), we get

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) = {} & N\mu - (m+1)b(\mu^2+\sigma^2)\widehat{q}_{PLC} - 2c\widehat{q}_{PLC} + \alpha(m+1)b\widehat{q}_{PLC}\sigma^2 - \frac{2\alpha(m+1)b\mu\sigma\widehat{q}_{PLC}}{\sqrt{2\pi}} \\
& + \frac{\alpha N\sigma^2}{m}\phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right) + \frac{\alpha N\sigma}{\sqrt{2\pi}} - \alpha(m+1)b\widehat{q}_{PLC}\sigma^2\left(\Phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right) - 0.5\right)
\end{aligned}$$

Using (26), we can simplify the right-hand side above to

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) = {} & \alpha(m+1)b\widehat{q}_{PLC}\sigma^2\left(1.5 - \Phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right)\right) - \frac{3\alpha(m+1)b\mu\sigma\widehat{q}_{PLC}}{\sqrt{2\pi}} \\
& + \frac{\alpha N\sigma^2}{m}\phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right) + \frac{\alpha N\sigma}{\sqrt{2\pi}}
\end{aligned} \tag{28}$$

Since $\min\left\{\mu, \frac{N}{bm\widehat{q}_{PLC}} - \mu\right\} = \mu$, we have $\Phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right) > 0.5$. Also $\phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right) < \phi(\mu)$. Thus

$$\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) < \alpha(m+1)b\sigma\widehat{q}_{PLC}\left(\sigma - \frac{3\mu}{\sqrt{2\pi}}\right) + \frac{\alpha N\sigma(m+1)}{m\sqrt{2\pi}} \tag{29}$$

We now substitute the expression for $\widehat{q}_{PLC}$ in the right-hand side above and find that $\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) < 0$ if

$$\mu\left(\sigma\sqrt{2\pi} - 3\mu\right) + \frac{m+1}{m}\left(\mu^2 + \sigma^2 - \frac{\alpha\mu\sigma}{\sqrt{2\pi}} + \frac{2c}{(m+1)b}\right) < 0. \tag{30}$$

Letting $\alpha = 0$ in the left-hand side and solving the inequality, we see that if $\frac{c}{b} < \gamma_3$ where $\gamma_3 = \frac{(2m-1)\mu^2-(m+1)\sigma^2-\mu\sigma m\sqrt{2\pi}}{2}$, then $\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) < 0$. It is easy to see that $\gamma_3 > \overline{\gamma}$ when $m \geq 4$ which is a non-restrictive assumption. Therefore, if $\overline{\gamma} \leq \frac{c}{b} < \gamma_3$, then $\widehat{q}_{PLC} \geq \widehat{q}_{ARC}$ and $\widehat{\Delta}_{PLC} \geq \widehat{\Delta}_{ARC}$. Second, suppose $\frac{c}{b} < \overline{\gamma}$ and $\min\left\{\mu, \frac{N}{bm\widehat{q}_{PLC}} - \mu\right\} = \frac{N}{bm\widehat{q}_{PLC}} - \mu$. Then two cases can occur: (1) From Proposition 3, if $\frac{c}{b} \geq m\left(\mu^2 - (1-\alpha)\sigma^2\right) - \left(\mu^2 + (1-\alpha)\sigma^2\right)$, then $\widehat{q}_{ARC} < \frac{N}{2bm\mu}$ and $\widehat{q}_{ARC} < \widehat{q}_{PLC}$ holds; (2) If $\widehat{q}_{ARC} \geq \frac{N}{2bm\mu}$, then using (27), we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) = {} & N\mu - (m+1)b(\mu^2+\sigma^2)\widehat{q}_{PLC} - 2c\widehat{q}_{PLC} + \alpha(m+1)b\widehat{q}_{PLC}\sigma^2 + \frac{2\alpha(m+1)b\mu\sigma\widehat{q}_{PLC}}{\sqrt{2\pi}} \\
& - \frac{\alpha N\sigma^2}{m}\phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right) - \frac{\alpha N\sigma}{\sqrt{2\pi}} + \alpha(m+1)b\widehat{q}_{PLC}\sigma^2\left(\Phi\left(\frac{N}{bm\widehat{q}_{PLC}} - \mu\right) - 0.5\right)
\end{aligned}$$

Given (26),

$$\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) = \alpha(m+1)b\widehat{q}_{PLC}\sigma^2\left(0.5+\Phi\left(\frac{N}{bm\widehat{q}_{PLC}}-\mu\right)\right)+\frac{\alpha(m+1)b\mu\sigma\widehat{q}_{PLC}}{\sqrt{2\pi}} - \frac{\alpha N\sigma^2}{m}\phi\left(\frac{N}{bm\widehat{q}_{PLC}}-\mu\right)-\frac{\alpha N\sigma}{\sqrt{2\pi}} \tag{31}$$

Since $\min\left\{\mu, \frac{N}{bm\widehat{q}_{PLC}}-\mu\right\} = \frac{N}{bm\widehat{q}_{PLC}}-\mu$, $\Phi\left(\frac{N}{bm\widehat{q}_{PLC}}-\mu\right) < 0.5$. Thus

$$\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) < \alpha(m+1)b\sigma\widehat{q}_{PLC}\left(\sigma+\frac{\mu}{\sqrt{2\pi}}\right)-\frac{\alpha N\sigma}{\sqrt{2\pi}} \tag{32}$$

Substituting the expression for $\widehat{q}_{PLC}$ in the right-hand side above, we see that $\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) < 0$ if

$$\mu\left(\sigma\sqrt{2\pi}+\mu\right)-\left(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{(m+1)b}\right) < 0. \tag{33}$$

Letting $\alpha = 1$ in the left-hand side and solving the inequality, we see that if $\frac{c}{b} > \frac{(m+1)\sigma(2.906\mu-\sigma)}{2}$, then $\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) < 0$. Combining cases 1 and 2, if $\frac{c}{b} > \gamma_2$ where $\gamma_2 = \frac{1}{2}\min\left\{(m+1)\sigma(2.906\mu-\sigma), m(\mu^2-(1-\alpha)\sigma^2)-(\mu^2+(1-\alpha)\sigma^2)\right\}$, then $\widehat{q}_{PLC} \geq \widehat{q}_{ARC}$. It is straightforward to show that $\overline{\gamma} > \gamma_2$. Putting everything together, if $\gamma_2 \leq \frac{c}{b} \leq \gamma_3$, then $\widehat{\Delta}_{PLC} \geq \widehat{\Delta}_{ARC}$. Note that the interval $[\gamma_2, \gamma_3]$ is non-empty because

$$\begin{aligned}\frac{\gamma_3-\gamma_2}{2} &\geq (2m-1)\mu^2-(m+1)\sigma^2-\mu\sigma m\sqrt{2\pi}-m\left(\mu^2-(1-\alpha)\sigma^2\right)+\left(\mu^2+(1-\alpha)\sigma^2\right)\\ &= m\mu^2-m\mu\sigma\sqrt{2\pi}-\alpha(m+1)\sigma^2,\end{aligned}$$

which is positive as long as $\mu \geq 3\sigma$ and $m \geq 2$.

We now present the proof for part (ii). We start with the first case assuming $\frac{c}{b} \geq \overline{\gamma}$ so $\min\left\{\mu, \frac{N}{bm\widehat{q}_{PLC}}-\mu\right\} = \mu$. Going back to (28), we use $\Phi\left(\frac{N}{bm\widehat{q}_{PLC}}-\mu\right) < 1$ and $\phi\left(\frac{N}{bm\widehat{q}_{PLC}}-\mu\right) > 0$ to get

$$\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) > \alpha(m+1)b\sigma\widehat{q}_{PLC}\left(\frac{\sigma}{2}-\frac{3\mu}{\sqrt{2\pi}}\right)+\frac{\alpha N\sigma}{\sqrt{2\pi}} \tag{34}$$

Next we substitute the expression for $\widehat{q}_{PLC}$ and see that $\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) > 0$ if

$$\mu\left(\sigma\sqrt{2\pi}-6\mu\right)+2\left(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{(m+1)b}\right) > 0. \tag{35}$$

Letting $\alpha = 1$ and solving the inequality, we get $\frac{c}{b} \geq \gamma_4$ where $\gamma_4 = \frac{m+1}{2}\left(2\mu^2-\sigma^2-\frac{\mu\sigma(\pi-1)}{\sqrt{2\pi}}\right)$. It is easy to show that $\gamma_4 > \gamma_3$. Thus if $\frac{c}{b} \geq \gamma_4$, then $\widehat{q}_{ARC} > \widehat{q}_{PLC}$ and $\widehat{\Delta}_{ARC} > \widehat{\Delta}_{PLC}$. Now suppose $\frac{c}{b} < \overline{\gamma}$ so $\min\left\{\mu, \frac{N}{bm\widehat{q}_{PLC}}-\mu\right\} = \frac{N}{bm\widehat{q}_{PLC}}-\mu$. Going back to (31) and using $\Phi\left(\frac{N}{bm\widehat{q}_{PLC}}-\mu\right) > 0$ and $\phi\left(\frac{N}{bm\widehat{q}_{PLC}}-\mu\right) < \phi(\mu)$ we get

$$\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) > \alpha(m+1)b\sigma\widehat{q}_{PLC}\left(\frac{\sigma}{2}+\frac{\mu}{\sqrt{2\pi}}\right)-\frac{\alpha N\sigma(m+1)}{m\sqrt{2\pi}} \tag{36}$$

Next we substitute the expression for $\widehat{q}_{PLC}$ and find that $\frac{\mathrm{d}}{\mathrm{d}q}\pi_{ARC}(\widehat{q}_{PLC}) > 0$ if

$$\mu\left(\sigma\sqrt{2\pi}+2\mu\right) - 2\left(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{(m+1)b}\right)\left(\frac{m+1}{m}\right) > 0. \tag{37}$$

Letting $\alpha = 0$ and solving the inequality, we get $\frac{c}{b} \leq \gamma_1$ where $\gamma_1 = \frac{m\mu\sigma\sqrt{2\pi}}{4} - \frac{\mu^2+(m+1)\sigma^2}{2}$. It is easy to show that $\gamma_1 < \gamma_2$. Thus if $\frac{c}{b} \leq \gamma_1$, then $\widehat{q}_{ARC} > \widehat{q}_{PLC}$ and $\widehat{\Delta}_{ARC} > \widehat{\Delta}_{PLC}$. Putting the two cases together, $\widehat{\Delta}_{ARC} > \widehat{\Delta}_{PLC}$ if $\frac{c}{b} \leq \gamma_1$ or $\frac{c}{b} \geq \gamma_4$.

Finally, we have $\frac{\mathrm{d}}{\mathrm{d}\sigma}\gamma_1 = m\mu\frac{\sqrt{2\pi}}{2} - 2(m+1)\sigma > 0$ and $\frac{\mathrm{d}}{\mathrm{d}\sigma}\gamma_4 = (m+1)\left(-2\sigma - \frac{\mu(\pi-1)}{\sqrt{2\pi}}\right) < 0$. Also, with simple algebra, one can see that $\frac{\mathrm{d}}{\mathrm{d}\sigma}(\gamma_3-\gamma_2) = -\mu\left((2m+1)\sqrt{2\pi}+\frac{m+1}{\sqrt{2\pi}}\right)$ or $\frac{\mathrm{d}}{\mathrm{d}\sigma}(\gamma_3-\gamma_2) = -\sqrt{2\pi}m\mu - 2\alpha(m+1)\sigma$, depending on which term in the definition of γ_2 is smaller. As it can be seen, in both cases, $\frac{\mathrm{d}}{\mathrm{d}\sigma}(\gamma_3-\gamma_2) < 0$. □

Proof of Proposition 5: Similar to the proof of Theorem 1, we first simplify an individual farmer's profit in equilibrium to get:

$$\widehat{\pi}_{PLC} = (\widehat{q}_{PLC})^2\left(b(\mu^2+\sigma^2)+c-\alpha b\sigma^2\mu\phi(\mu)\right),$$

$$\widehat{\pi}_{ARC} = (\widehat{q}_{ARC})^2\left(b(\mu^2+\sigma^2)+c-\alpha b\sigma^2+\alpha b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right).$$

Note that the farmer's profit in equilibrium consists of two parts: (i) the profit earned by selling the crop on the market, and (ii) the subsidy received from the government. The subsidy that the government pays in equilibrium to each farmer under ARC can be obtained by subtracting the first part from the farmer's actual profit. That is,

$$\begin{aligned}\text{subsidy}_{ARC} &= \widehat{\pi}_{ARC} - \left(N\widehat{q}_{ARC}\mu - bm(\widehat{q}_{ARC})^2(\mu^2+\sigma^2) - c(\widehat{q}_{ARC})^2\right)\\ &= (\widehat{q}_{ARC})^2\left((m+1)b(\mu^2+\sigma^2)+2c-\alpha b\sigma^2+\alpha b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right) - N\widehat{q}_{ARC}\mu\,.\end{aligned}$$

It worth mentioning that an alternative way of deriving the above expression is to directly start with the subsidy payment term in the farmer's objective function, and then use the ARC's first-order-condition to simplify the integral term. That approach is more complicated, but leads to the same outcome.

Now, assume that $\widehat{q}_{ARC} = \beta\widehat{q}_{PLC}$, where β can be smaller or bigger than one. It follows that

$$\begin{aligned}\text{subsidy}_{ARC} &= \beta^2(\widehat{q}_{PLC})^2\left((m+1)b(\mu^2+\sigma^2)+2c-\alpha b\sigma^2+\alpha b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right) - N\beta\widehat{q}_{PLC}\mu\\ &= \beta^2(\widehat{q}_{PLC})^2\left((m+1)b(\mu^2+\sigma^2)+2c-\alpha b\sigma^2+\alpha b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right)\end{aligned}$$

$$
\begin{aligned}
&-\beta(\widehat{q}_{PLC})^2\big((m+1)b(\mu^2+\sigma^2-\alpha\sigma^2\mu\phi(\mu))+2c\big)\\
&=(\widehat{q}_{PLC})^2\left((\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\alpha\beta^2 b\sigma^2+\alpha\beta^2 b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right)\\
&\quad+(\widehat{q}_{PLC})^2\alpha(m+1)b\beta\mu\sigma^2\phi(\mu)\,.
\end{aligned}
$$

Similarly, for PLC we have

$$
\begin{aligned}
\text{subsidy}_{PLC}&=\widehat{\pi}_{PLC}-\Big(N\widehat{q}_{PLC}\mu-bm(\widehat{q}_{PLC})^2(\mu^2+\sigma^2)-c(\widehat{q}_{PLC})^2\Big)\\
&=(\widehat{q}_{PLC})^2\Big(\alpha mb\mu\sigma^2\phi(\mu)\Big)\,.
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\text{subsidy}_{ARC}-\text{subsidy}_{PLC}=&\;(\widehat{q}_{PLC})^2\left((\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\alpha\beta^2 b\sigma^2+\alpha\beta^2 b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right)\\
&+(\widehat{q}_{PLC})^2\Big(\left[(m+1)\beta-m\right]\alpha b\mu\sigma^2\phi(\mu)\Big)\,.
\end{aligned}
$$

We need to consider two separate cases depending on whether or not model parameters belong to set $\mathcal{S}$:

Case 1: $(c,b,\mu,\sigma,m,\alpha)\in\mathcal{S}$. Then

$$
\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}=\mu\quad\Leftrightarrow\quad\widehat{q}_{ARC}\le\frac{N}{2bm\mu}\quad\Leftrightarrow\quad(m-1)\mu^2-(m+1)\sigma^2(1-\alpha)\le\frac{2c}{b}\,.
$$

In this case,

$$
\begin{aligned}
\frac{\text{subsidy}_{ARC}-\text{subsidy}_{PLC}}{(\widehat{q}_{PLC})^2}=&\;(\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\frac{3}{2}\alpha\beta^2 b\sigma^2+\alpha\beta^2 b\sigma^2\Phi\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big)\\
&-\frac{\alpha N\beta^2\sigma^2}{m\widehat{q}_{ARC}}\phi\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big)+\left[2\beta^2+(m+1)\beta-m\right]\alpha b\mu\sigma^2\phi(\mu)\,.
\end{aligned}
$$

Define function $g(.)$ as

$$
g(x)=(x+\mu)\phi(x)-\Phi(x)\,,
$$

and note that $g'(x)\le 0$ for $x\ge\mu$, and hence $g(.)$ is decreasing over $[\mu,\infty)$. Thus, we have

$$
(c,b,\mu,\sigma,m,\alpha)\in\mathcal{S}\quad\Leftrightarrow\quad\mu\le\frac{N}{bm\widehat{q}_{ARC}}-\mu<\infty\quad\Rightarrow\quad-1\le g\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big)\le 2\mu\phi(\mu)-\frac{1}{2}\,.
$$

It follows from the above inequality that,

$$
\frac{\text{subsidy}_{ARC}-\text{subsidy}_{PLC}}{(\widehat{q}_{PLC})^2}\le(\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\frac{1}{2}\alpha\beta^2 b\sigma^2+\left[2\beta^2+(m+1)\beta-m\right]\alpha b\mu\sigma^2\phi(\mu)\,.
$$

and

$$
\frac{\text{subsidy}_{ARC}-\text{subsidy}_{PLC}}{(\widehat{q}_{PLC})^2}\ge(\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\alpha\beta^2 b\sigma^2+\left[(m+1)\beta-m\right]\alpha b\mu\sigma^2\phi(\mu)\,.
$$

Now, using simple algebra, it is straightforward to show that if $\beta\ge\overline{\beta}=\frac{m+1}{m}$, then $\text{subsidy}_{ARC}\ge\text{subsidy}_{PLC}$, and hence, $\widehat{\Gamma}_{ARC}\ge\widehat{\Gamma}_{PLC}$. Similarly, if $\beta\le\underline{\beta}=\frac{m}{m+1}$, then $\text{subsidy}_{ARC}\le\text{subsidy}_{PLC}$, and hence, $\widehat{\Gamma}_{ARC}\le\widehat{\Gamma}_{PLC}$.

Case 2: $(c,b,\mu,\sigma,m,\alpha) \notin \mathcal{S}$. The proof steps are very similar to that of Case 1. More precisely,

$$\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}=\frac{N}{bm\widehat{q}_{ARC}}-\mu \quad \Leftrightarrow \quad \widehat{q}_{ARC}\geq\frac{N}{2bm\mu} \quad \Leftrightarrow \quad (m-1)\mu^2-(m+1)\sigma^2(1-\alpha)\geq\frac{2c}{b}\,.$$

In this case:

$$\frac{\text{subsidy}_{ARC}-\text{subsidy}_{PLC}}{(\widehat{q}_{PLC})^2} = (\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\frac{1}{2}\alpha\beta^2 b\sigma^2-\alpha\beta^2 b\sigma^2\Phi\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big)$$
$$+\frac{\alpha N\beta^2\sigma^2}{m\widehat{q}_{ARC}}\phi\Big(\frac{N}{bm\widehat{q}_{ARC}}-\mu\Big)+\left[-2\beta^2+(m+1)\beta-m\right]\alpha b\mu\sigma^2\phi(\mu)\,.$$

It follows that,

$$\frac{\text{subsidy}_{ARC}-\text{subsidy}_{PLC}}{(\widehat{q}_{PLC})^2}\geq(\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\alpha\beta^2 b\sigma^2+\left[-2\beta^2+(m+1)\beta-m\right]\alpha b\mu\sigma^2\phi(\mu)\,.$$

and

$$\frac{\text{subsidy}_{ARC}-\text{subsidy}_{PLC}}{(\widehat{q}_{PLC})^2}\leq(\beta^2-\beta)\left[(m+1)b(\mu^2+\sigma^2)+2c\right]-\alpha\beta^2 b\sigma^2+\left[(m+1)\beta-m\right]\alpha b\mu\sigma^2\phi(\mu)\,.$$

Similar to Case 1, using simple algebra, it is straightforward to show that if $\beta\geq\overline{\beta}=\frac{m+1}{m}$, then $\text{subsidy}_{ARC}\geq\text{subsidy}_{PLC}$, and hence, $\widehat{\Gamma}_{ARC}\geq\widehat{\Gamma}_{PLC}$. Similarly, if $\beta\leq\underline{\beta}=\frac{m}{m+1}$, then $\text{subsidy}_{ARC}\leq\text{subsidy}_{PLC}$, and hence, $\widehat{\Gamma}_{ARC}\leq\widehat{\Gamma}_{PLC}$. □

Proof of Proposition 6: Recall from the proof of Proposition 5 that

$$\widehat{\pi}_{PLC}=(\widehat{q}_{PLC})^2\Big(b(\mu^2+\sigma^2)+c-\alpha b\sigma^2\mu\phi(\mu)\Big)\,,$$
$$\widehat{\pi}_{ARC}=(\widehat{q}_{ARC})^2\left(b(\mu^2+\sigma^2)+c-\alpha b\sigma^2+\alpha b\int_{\min\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}^{\max\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\}}(x^2-\mu^2)\phi(x)dx\right)\,.$$

First, we want to identify sufficient conditions under which $\widehat{\pi}_{PLC}\geq\widehat{\pi}_{ARC}$. We need to consider two separate cases depending on whether or not model parameters belong to set $\mathcal{S}$:

Case 1: $(c,b,\mu,\sigma,m,\alpha)\in\mathcal{S}$. Then

$$\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}=\mu \quad \Leftrightarrow \quad \widehat{q}_{ARC}\leq\frac{N}{2bm\mu} \quad \Leftrightarrow \quad (m-1)\mu^2-(m+1)\sigma^2(1-\alpha)\leq\frac{2c}{b}\,. \tag{38}$$

In this case, from (25) it follows that

$$\widehat{\pi}_{ARC}\leq(\widehat{q}_{ARC})^2\Big(b(\mu^2+\sigma^2)+c-\frac{\alpha b\sigma^2}{2}+2\alpha b\sigma^2\mu\phi(\mu)\Big)\,.$$

Define

$$\beta=\left(\frac{\mu^2+\sigma^2+\frac{c}{b}-\alpha\sigma^2\mu\phi(\mu)}{\mu^2+\sigma^2+\frac{c}{b}-\frac{\alpha\sigma^2}{2}+2\alpha\sigma^2\mu\phi(\mu)}\right)^{\frac{1}{2}}\,.$$

Then, we are looking for sufficient conditions that imply $\beta\widehat{q}_{PLC}\geq\widehat{q}_{ARC}$. It is not hard to show that β is increasing in $\frac{c}{b}$. Hence, letting $\frac{c}{b}=0$, and using Assumption 1, we can conclude that $0.86\leq\beta\leq1$.

Case 1-(a):

$$\min\left\{\frac{N}{bm\beta\widehat{q}_{PLC}}-\mu,\mu\right\}=\frac{N}{bm\beta\widehat{q}_{PLC}}-\mu \quad \Leftrightarrow \quad \beta\widehat{q}_{PLC}\geq\frac{N}{2bm\mu} \quad \Leftarrow \quad 0.86\widehat{q}_{PLC}\geq\frac{N}{2bm\mu}$$

$$\Leftrightarrow \quad (0.72m-1)\mu^2-(m+1)\sigma^2+\frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}}\geq\frac{2c}{b}\,. \tag{39}$$

In Case 1-(a), we have $\widehat{q}_{ARC}\leq\frac{N}{2bm\mu}$ and $\beta\widehat{q}_{PLC}\geq\frac{N}{2bm\mu}$. Thus, $\beta\widehat{q}_{PLC}\geq\widehat{q}_{ARC}$ is automatically satisfied. However, the intersection of (38) and (39) is empty. Therefore, in this case, we cannot identify any sufficient conditions for $\widehat{\pi}_{PLC}\geq\widehat{\pi}_{ARC}$.

Case 1-(b):

$$\min\left\{\frac{N}{bm\beta\widehat{q}_{PLC}}-\mu,\mu\right\}=\mu \quad \Leftrightarrow \quad \beta\widehat{q}_{PLC}\leq\frac{N}{2bm\mu} \quad \Leftarrow \quad \widehat{q}_{PLC}\leq\frac{N}{2bm\mu}$$

$$\Leftrightarrow \quad (m-1)\mu^2-(m+1)\sigma^2+\frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}}\leq\frac{2c}{b}\,. \tag{40}$$

In Case 1-(b), both $\widehat{q}_{ARC}$ and $\beta\widehat{q}_{PLC}$ are on the same side of $\frac{N}{2bm\mu}$. Thus, in order to compare them, we need to evaluate the sign of ARC's first-order-condition (Equation (14)) at $\beta\widehat{q}_{PLC}$. Denote $Z_{ARC}(q)$ to represent the ARC's first-order-condition evaluated at q. Then, after some algebra, we have

$$\begin{aligned}
Z_{ARC}(\beta\widehat{q}_{PLC})=\ldots=&N\mu(1-\beta)+\alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC}-\alpha(m+1)b\beta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}}\\
&-\alpha\int_{\mu}^{\frac{N}{bm\beta\widehat{q}_{PLC}}-\mu}\left[(m+1)b\beta\widehat{q}_{PLC}(x^2-\mu^2)+N(\mu-x)\right]\phi(x)dx\\
=&N\mu(1-\beta)+\alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC}-\alpha(m+1)b\beta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}}\\
&-\alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC}\left[2\mu\phi(\mu)-\frac{N}{mb\beta\widehat{q}_{PLC}}\phi\Big(\frac{N}{mb\beta\widehat{q}_{PLC}}-\mu\Big)+\Phi\Big(\frac{N}{mb\beta\widehat{q}_{PLC}}-\mu\Big)-\frac{1}{2}\right]\\
&-\alpha N\sigma^2\left[\phi\Big(\frac{N}{mb\beta\widehat{q}_{PLC}}-\mu\Big)-\phi(\mu)\right]\,.
\end{aligned}$$

Note that if the above term is negative, it implies that the FOC for ARC is negative at $\beta\widehat{q}_{PLC}$ and hence, $\beta\widehat{q}_{PLC}\geq\widehat{q}_{ARC}$. Using simple algebra, we can show that $Z_{ARC}(\beta\widehat{q}_{PLC})$ is negative if

$$(0.4m-1)\mu^2-(m+1)\sigma^2-1.74m\mu\sigma\geq\frac{2c}{b}\,. \tag{41}$$

Finally, since the intersection of (38) and (40) and (41) is empty, we cannot identify any sufficient condition in Case 1-(b) that ensures $\widehat{\pi}_{PLC}\geq\widehat{\pi}_{ARC}$.

Case 2: $(c,b,\mu,\sigma,m,\alpha)\notin\mathcal{S}$.

$$\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}=\frac{N}{bm\widehat{q}_{ARC}}-\mu \quad \Leftrightarrow \quad \widehat{q}_{ARC}\geq\frac{N}{2bm\mu} \quad \Leftrightarrow \quad (m-1)\mu^2-(m+1)\sigma^2(1-\alpha)\geq\frac{2c}{b}\,. \tag{42}$$

In this case, from (25) it follows that

$$\widehat{\pi}_{ARC} \leq (\widehat{q}_{ARC})^2 \Big(b(\mu^2+\sigma^2) + c - \frac{\alpha b \sigma^2}{2} \Big) .$$

Define

$$\beta = \left(\frac{\mu^2+\sigma^2+\frac{c}{b} - \alpha\sigma^2\mu\phi(\mu)}{\mu^2+\sigma^2+\frac{c}{b}-\frac{\alpha\sigma^2}{2}} \right)^{\frac{1}{2}} .$$

Then, we are looking for sufficient conditions that imply $\beta\widehat{q}_{PLC} \geq \widehat{q}_{ARC}$. It is not hard to show that β is increasing in $\frac{c}{b}$. Hence, letting $\frac{c}{b}=0$, and using Assumption 1, we can conclude that $0.96 \leq \beta \leq 1$.

Case 2-(a):

$$\min\left\{ \frac{N}{bm\beta\widehat{q}_{PLC}} - \mu, \mu \right\} = \mu \quad \Leftrightarrow \quad \beta\widehat{q}_{PLC} \leq \frac{N}{2bm\mu} \quad \Leftarrow \quad \widehat{q}_{PLC} \leq \frac{N}{2bm\mu}$$

$$\Leftrightarrow \quad (m-1)\mu^2 - (m+1)\sigma^2 + \frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}} \leq \frac{2c}{b} . \tag{43}$$

In Case 2-(a), we have $\widehat{q}_{ARC} \geq \frac{N}{2bm\mu}$ and $\beta\widehat{q}_{PLC} \leq \frac{N}{2bm\mu}$. Thus, $\beta\widehat{q}_{PLC} \geq \widehat{q}_{ARC}$ cannot hold.

Case 2-(b):

$$\min\left\{ \frac{N}{bm\beta\widehat{q}_{PLC}} - \mu, \mu \right\} = \frac{N}{bm\beta\widehat{q}_{PLC}} - \mu \quad \Leftrightarrow \quad \beta\widehat{q}_{PLC} \geq \frac{N}{2bm\mu} \quad \Leftarrow \quad 0.96\widehat{q}_{PLC} \leq \frac{N}{2bm\mu}$$

$$0.96\widehat{q}_{PLC} \leq \frac{N}{2bm\mu} \quad \Leftrightarrow \quad (0.92m-1)\mu^2 - (m+1)\sigma^2 + \frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}} \geq \frac{2c}{b} . \tag{44}$$

In this case, both $\widehat{q}_{ARC}$ and $\beta\widehat{q}_{PLC}$ are on the same side of $\frac{N}{2bm\mu}$. Thus, similar to Case 1-(b), their comparison requires evaluating the sign of the ARC's first-order-condition at $\beta\widehat{q}_{PLC}$.

$$\begin{aligned}
Z_{ARC}(\beta\widehat{q}_{PLC}) = \ldots =& N\mu(1-\beta) + \alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC} - \alpha(m+1)b\beta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}} \\
& - \alpha\int_{\frac{N}{bm\beta\widehat{q}_{PLC}}-\mu}^{\mu} \left[(m+1)b\beta\widehat{q}_{PLC}(x^2-\mu^2) + N(\mu-x) \right] \phi(x)dx \\
=& N\mu(1-\beta) + \alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC} - \alpha(m+1)b\beta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}} \\
& - \alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC}\left[-2\mu\phi(\mu) + \frac{N}{mb\beta\widehat{q}_{PLC}}\phi\Big(\frac{N}{mb\beta\widehat{q}_{PLC}}-\mu\Big) - \Phi\Big(\frac{N}{mb\beta\widehat{q}_{PLC}}-\mu\Big) + \frac{1}{2} \right] \\
& - \alpha N\sigma^2\left[\phi(\mu) - \phi\Big(\frac{N}{mb\beta\widehat{q}_{PLC}}-\mu\Big) \right] \\
\leq& N\mu(1-\beta) + \alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC} + \alpha(m+1)b\beta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}} - \frac{\alpha N\sigma^2}{m}\phi\Big(\frac{N}{mb\beta\widehat{q}_{PLC}}-\mu\Big) - \frac{\alpha N\sigma}{\sqrt{2\pi}} \\
\leq& N\mu(1-\beta) + \alpha(m+1)b\beta\sigma^2\widehat{q}_{PLC} + \alpha(m+1)b\beta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}} - \frac{\alpha N\sigma}{\sqrt{2\pi}} .
\end{aligned}$$

Dividing $Z_{ARC}(\beta\widehat{q}_{PLC})$ by $\alpha N\sigma$ and multiplying it by $(1+\beta)$, we have

$$\frac{(1-\beta)Z_{ARC}(\beta\widehat{q}_{PLC})}{\alpha N\sigma} = \frac{\mu^2 - \frac{\sqrt{2\pi}}{2}\mu\sigma}{\sqrt{2\pi}\Big(\mu^2+\sigma^2+\frac{c}{b}-\frac{\alpha\sigma^2}{2}\Big)} + \frac{(1+\beta)\Big[\sqrt{2\pi}\beta\mu\sigma+(\beta-1)\mu^2-\sigma^2+\frac{\alpha\mu\sigma}{\sqrt{2\pi}}-\frac{2c}{b(m+1)}\Big]}{\sqrt{2\pi}\Big(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{b(m+1)}\Big)}$$
$$\le \frac{\mu^2 - \frac{\sqrt{2\pi}}{2}\mu\sigma + (1+\beta)\Big[\sqrt{2\pi}\beta\mu\sigma+(\beta-1)\mu^2-\sigma^2+\frac{\alpha\mu\sigma}{\sqrt{2\pi}}-\frac{2c}{b(m+1)}\Big]}{\sqrt{2\pi}\Big(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{b(m+1)}\Big)}\,.$$

Since the denominator of the above term is positive, its sign is determined by the sign of the numerator. The numerator, in turn, can be bounded from above by

$$\mu^2 - \frac{\sqrt{2\pi}}{2}\mu\sigma + 2\sqrt{2\pi}\mu\sigma - 1.96\sigma^2 + \frac{2\mu\sigma}{\sqrt{2\pi}} - \frac{3.92c}{b(m+1)}\,.$$

The above term is negative if and only if

$$\mu^2 + 4.55\mu\sigma - 1.96\sigma^2 \le \frac{3.92c}{b(m+1)} \quad \Leftrightarrow \quad 0.51(m+1)\mu^2 + 2.32(m+1)\mu\sigma - (m+1)\sigma^2 \le \frac{2c}{b}\,. \quad (45)$$

Therefore, in order to have $\widehat{\pi}_{PLC} \ge \widehat{\pi}_{ARC}$, the three inequalities (42) and (44) and (45) must hold simultaneously. That is,

$$0.51(m+1)\mu^2+2.32(m+1)\mu\sigma-(m+1)\sigma^2 \le \frac{2c}{b} \le \min\left\{(m-1)\mu^2-(m+1)\sigma^2(1-\alpha), (0.92m-1)\mu^2-(m+1)\sigma^2+\frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}}\right\}.$$

We denote the left hand side of the above inequality by ρ_1, and its right hand side by ρ_2.

By simple algebra, we can show that $\rho_2-\rho_1$ is decreasing in σ (the details of the derivations are omitted here).

Next, we turn our attention to ARC, and identify sufficient conditions under which $\widehat{\pi}_{ARC} \ge \widehat{\pi}_{PLC}$. Similar to above, multiple cases have to considered.

Case 1: $(c,b,\mu,\sigma,m,\alpha)\in\mathcal{S}$. Then

$$\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}=\mu \quad \Leftrightarrow \quad \widehat{q}_{ARC} \le \frac{N}{2bm\mu} \quad \Leftrightarrow \quad (m-1)\mu^2-(m+1)\sigma^2(1-\alpha) \le \frac{2c}{b}\,.$$

In this case, from (25) it follows that

$$\widehat{\pi}_{ARC} \ge (\widehat{q}_{ARC})^2\Big(b(\mu^2+\sigma^2)+c-\alpha b\sigma^2\Big)\,.$$

Define

$$\theta = \left(\frac{\mu^2+\sigma^2+\frac{c}{b}-\alpha\sigma^2\mu\phi(\mu)}{\mu^2+\sigma^2+\frac{c}{b}-\alpha\sigma^2}\right)^{\frac{1}{2}}.$$

Then, we are looking for sufficient conditions that imply $\theta\widehat{q}_{PLC} \le \widehat{q}_{ARC}$. It is not hard to show that θ is increasing in $\frac{c}{b}$. Hence, letting $\frac{c}{b}=0$, and using Assumption 1, we can conclude that $0.98 \le \theta \le 1$.

Case 1-(a):

$$\min\left\{\frac{N}{bm\theta\widehat{q}_{PLC}}-\mu,\mu\right\}=\frac{N}{bm\theta\widehat{q}_{PLC}}-\mu \quad\Leftrightarrow\quad \theta\widehat{q}_{PLC}\geq\frac{N}{2bm\mu} \quad\Leftarrow\quad 0.98\widehat{q}_{PLC}\geq\frac{N}{2bm\mu}$$

$$\Leftrightarrow\quad (0.96m-1)\mu^2-(m+1)\sigma^2+\frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}}\geq\frac{2c}{b}\,. \tag{46}$$

In Case 1-(a), we have $\widehat{q}_{ARC}\leq\frac{N}{2bm\mu}$ and $\theta\widehat{q}_{PLC}\geq\frac{N}{2bm\mu}$. Thus, $\theta\widehat{q}_{PLC}\leq\widehat{q}_{ARC}$ cannot hold. Therefore, in this case, we cannot identify any sufficient conditions for $\widehat{\pi}_{ARC}\geq\widehat{\pi}_{PLC}$.

Case 1-(b):

$$\min\left\{\frac{N}{bm\theta\widehat{q}_{PLC}}-\mu,\mu\right\}=\mu \quad\Leftrightarrow\quad \theta\widehat{q}_{PLC}\leq\frac{N}{2bm\mu} \quad\Leftarrow\quad \widehat{q}_{PLC}\leq\frac{N}{2bm\mu}$$

$$\Leftrightarrow\quad (m-1)\mu^2-(m+1)\sigma^2+\frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}}\leq\frac{2c}{b}\,.$$

In Case 1-(b), both $\widehat{q}_{ARC}$ and $\theta\widehat{q}_{PLC}$ are on the same side of $\frac{N}{2bm\mu}$. Thus, in order to compare them, we need to evaluate the sign of ARC's first-order-condition (Equation (14)) at $\theta\widehat{q}_{PLC}$. Then, after some algebra, we have

$$\begin{aligned}
Z_{ARC}(\theta\widehat{q}_{PLC})=\ldots=\;& N\mu(1-\theta)+\alpha(m+1)b\theta\sigma^2\widehat{q}_{PLC}-\alpha(m+1)b\theta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}}\\
&-\alpha(m+1)b\theta\sigma^2\widehat{q}_{PLC}\left[2\mu\phi(\mu)-\frac{N}{mb\theta\widehat{q}_{PLC}}\phi\Big(\frac{N}{mb\theta\widehat{q}_{PLC}}-\mu\Big)+\Phi\Big(\frac{N}{mb\theta\widehat{q}_{PLC}}-\mu\Big)-\frac{1}{2}\right]\\
&-\alpha N\sigma^2\left[\phi\Big(\frac{N}{mb\theta\widehat{q}_{PLC}}-\mu\Big)-\phi(\mu)\right]\\
\geq\;& N\mu(1-\theta)+\frac{1}{2}\alpha(m+1)b\theta\sigma^2\widehat{q}_{PLC}-3\alpha(m+1)b\theta\widehat{q}_{PLC}\frac{\mu\sigma}{\sqrt{2\pi}}\\
&+\frac{\alpha N\sigma^2}{m}\phi\Big(\frac{N}{mb\theta\widehat{q}_{PLC}}-\mu\Big)+\alpha N\sigma^2\phi(\mu)\,.
\end{aligned}$$

Dividing $Z_{ARC}(\theta\widehat{q}_{PLC})$ by $\alpha N\sigma$ and multiplying it by $(1+\theta)$, we have

$$\begin{aligned}
\frac{(1+\theta)Z_{ARC}(\theta\widehat{q}_{PLC})}{\alpha N\sigma}=\;&\frac{-\sqrt{2\pi}\mu\sigma+\mu^2}{\sqrt{2\pi}\Big(\mu^2+\sigma^2+\frac{c}{b}-\alpha\sigma^2\Big)}+\frac{(1+\theta)\left[\frac{\sqrt{2\pi}}{2}\theta\mu\sigma-3\theta\mu^2+\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{b(m+1)}\right]}{\sqrt{2\pi}\Big(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{b(m+1)}\Big)}\\
\geq\;&\frac{\frac{2}{(m+1)}(\mu^2-\sqrt{2\pi}\mu\sigma)+(1+\theta)\left[\frac{\sqrt{2\pi}}{2}\theta\mu\sigma-3\theta\mu^2+\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{b(m+1)}\right]}{\sqrt{2\pi}\Big(\mu^2+\sigma^2-\frac{\alpha\mu\sigma}{\sqrt{2\pi}}+\frac{2c}{b(m+1)}\Big)}\,.
\end{aligned}$$

Note that if the above term is positive, it implies that the FOC for ARC is positive at $\theta\widehat{q}_{PLC}$ and hence, $\theta\widehat{q}_{PLC}\leq\widehat{q}_{ARC}$. Using simple algebra, we can show that the denominator of the above term is always positive. Moreover, we can multiply the numerator by $(m+1)$ to get

$$\begin{aligned}
&2(\mu^2-\sqrt{2\pi}\mu\sigma)+\frac{2(1+\theta)c}{b}+(m+1)\left[\frac{\sqrt{2\pi}}{2}\theta(1+\theta)\mu\sigma-3\theta(1+\theta)\mu^2+(1+\theta)(\mu^2+\sigma^2)-\frac{\alpha(1+\theta)\mu\sigma}{\sqrt{2\pi}}\right]\\
\geq\;&2(\mu^2-\sqrt{2\pi}\mu\sigma)+\frac{3.96c}{b}+(m+1)\left[2.42\mu\sigma-6\mu^2+1.98(\mu^2+\sigma^2)-\frac{2\mu\sigma}{\sqrt{2\pi}}\right]\\
=\;&(-4.02m-2.02)\mu^2+(1.62m-3.38)\mu\sigma+1.98(m+1)\sigma^2+\frac{3.96c}{b}\,.
\end{aligned}$$

The above term is positive if and only if

$$(2.03m+1.02)\mu^2-(0.82m-1.71)\mu\sigma-(m+1)\sigma^2\leq\frac{2c}{b}\,. \tag{47}$$

Therefore, the sufficient condition for $\widehat{\pi}_{ARC}\geq\widehat{\pi}_{PLC}$ is to have inequalities (38) and (40) and (47) satisfied simultaneously. That is, to have

$$\frac{2c}{b}\geq\max\left\{(m-1)\mu^2-(m+1)\sigma^2(1-\alpha),(m-1)\mu^2-(m+1)\sigma^2+\frac{\alpha(m+1)\mu\sigma}{\sqrt{2\pi}},(2.03m+1.02)\mu^2-(0.82m-1.71)\mu\sigma-(m+1)\sigma^2\right\}$$

We denote the right hand side of the above inequality by ρ_3. By simple algebra, it is straightforward to show that ρ_3 decreases with σ as long as $\alpha\leq\frac{2\sigma\sqrt{2\pi}}{\mu}$ and $m\geq 3$.

Case 2: $(c,b,\mu,\sigma,m,\alpha)\notin\mathcal{S}$.

$$\min\left\{\frac{N}{bm\widehat{q}_{ARC}}-\mu,\mu\right\}=\frac{N}{bm\widehat{q}_{ARC}}-\mu\quad\Leftrightarrow\quad\widehat{q}_{ARC}\geq\frac{N}{2bm\mu}\quad\Leftrightarrow\quad(m-1)\mu^2-(m+1)\sigma^2(1-\alpha)\geq\frac{2c}{b}\,.$$

In this case, from (25) it follows that

$$\widehat{\pi}_{ARC}\geq(\widehat{q}_{ARC})^2\Big(b(\mu^2+\sigma^2)+c-\frac{\alpha b\sigma^2}{2}-2\alpha b\sigma^2\mu\phi(\mu)\Big)\,.$$

Define

$$\theta=\left(\frac{\mu^2+\sigma^2+\frac{c}{b}-\alpha\sigma^2\mu\phi(\mu)}{\mu^2+\sigma^2+\frac{c}{b}-\frac{1}{2}\alpha\sigma^2-2\alpha\sigma^2\mu\phi(\mu)}\right)^{\frac{1}{2}}.$$

Then, we are looking for sufficient conditions that imply $\theta\widehat{q}_{PLC}\leq\widehat{q}_{ARC}$. It is obvious from the definition of θ that $\theta\geq 1$. Hence, in this case we are not able to identify conditions that implies $\theta\widehat{q}_{PLC}\leq\widehat{q}_{ARC}$. □

Proof of Proposition 7: First, note that $\lambda<N$ must hold because otherwise the farmers would receive PLC subsidy at any market price. Second, (17) reduces to the equilibrium condition for $\widehat{q}_{PLC}$ when $\lambda=N-b(m+1)\mu\widehat{q}_{PLC}$. Now we write

$$\begin{aligned}\frac{\mathrm{d}^2}{\mathrm{d}q\mathrm{d}\lambda}\pi^i_{PLC}(q)&=\alpha\mu\left[1-\Phi\left(\frac{N-\lambda}{bmq}\right)+\frac{\lambda-N+b(m+1)q\mu}{bmq}\phi\left(\frac{N-\lambda}{bmq}\right)-\frac{(m+1)\sigma^2}{m}\phi'\left(\frac{N-\lambda}{bmq}\right)\right]\\&=\alpha\mu\left[1-\Phi\left(\frac{N-\lambda}{bmq}\right)+\frac{N-\lambda}{bm^2q}\phi\left(\frac{N-\lambda}{bmq}\right)\right]>0.\end{aligned}$$

Thus, $\widehat{q}_{exg}$ increases in λ and $\widehat{q}_{exg}\geq\widehat{q}_{PLC}$ if and only if $\lambda\geq N-b(m+1)\mu\widehat{q}_{PLC}$.

Next, we compare the profits. When $\lambda=N-bm\mu\widehat{q}_{PLC}$, (18) reduces to the equilibrium profit under endogenous reference price. Because $\phi\left(\frac{N-\lambda}{bmq}\right)\leq\phi(\mu)$ and the expression $\alpha\mu q\left(\lambda-N+mb\mu q\right)\left(1-\Phi\left(\frac{N-\lambda}{bmq}\right)\right)$ increases in λ, we conclude that $\widehat{\pi}_{exg}\geq\widehat{\pi}_{PLC}$ if and only if $\lambda\geq N-bm\mu\widehat{q}_{PLC}$. □

Appendix C: Regression Models

Here, we provide scatter plots of data for price versus supply of corn, barely, oats, and sorghum from the past ten years that we used to estimate N and b. The unit of price is dollars per bushel and the unit of supply is million bushels. We found the data on the website of the USDA Economic Research Service at https://www.ers.usda.gov. We adjusted the price values for inflation using the Department of Labor CPI Inflation calculator and then fit a regression equation on price versus supply. In the equations that are displayed in the scatter plots, the variable S represents crop supply.

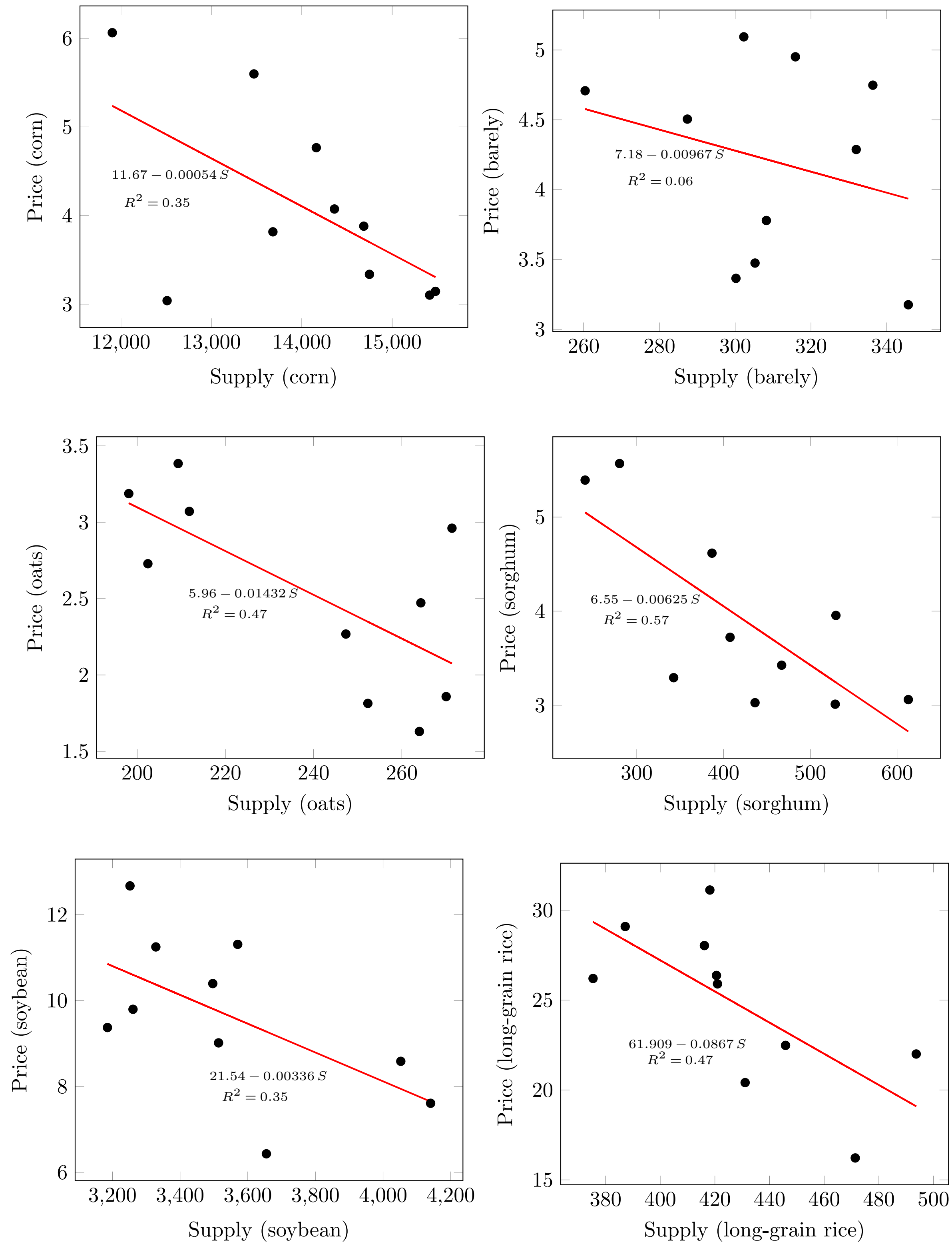


Figure 7 **Regression models of price versus supply**