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The Value of Reputation in an Online Freelance Marketplace

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Online freelance marketplaces are websites that match buyers of electronically deliverable services with freelancers. Although freelancing has grown in recent years, it faces the classic "information asymmetry" problem—buyers face uncertainty over seller quality. Typically, these markets use reputation systems to alleviate this issue, but the effectiveness of these systems is open to debate. We present a dynamic structural framework to estimate the returns to seller reputations in freelance sites. In our model, a buyer decides in each period whether to choose a bid from her current set of bids, cancel the auction, or wait for more bids. In the process, she trades off sellers' price, reputation, and other attributes, as well as the costs of waiting and canceling. Our framework addresses *dynamic selection*, which can lead to underestimation of reputation, through two types of persistent unobserved heterogeneities: bid arrival rates and buyers' unobserved preference for bids. We apply our framework to data from a leading freelance firm. We find that buyers are forward looking, that they place significant weight on seller reputation, and that not controlling for dynamics and selection can bias reputation estimates. Using counterfactual simulations, we infer the dollar value of seller reputations and provide guidelines to managers of freelance firms.

Key words: structural models; dynamic programming; empirical IO methods; auctions; online reputation systems; mixture models; dynamic selection; freelance marketplaces
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1. Introduction

Online freelance marketplaces are websites that match buyers of services that can be delivered electronically with sellers or *freelancers*—self-employed individuals or teams who offer their services on a per-job basis or for a fixed hourly rate. The most popular freelance marketplaces are Elance, Guru, vWorker, ODesk, and Freelancer, and the most popular categories of jobs are Web development, programming, writing, translation, design, and multimedia (Kozierok 2011). These websites typically use auction mechanisms to match buyers and sellers, although their mechanism differs from traditional auctions in three important ways. First, they follow a reverse auction format, where buyers post jobs and sellers bid for jobs. Second, buyers do not wait for the auction to end to make decisions; rather, in every period, they decide whether to terminate the auction (by choosing a submitted bid or canceling the auction) or continue waiting. Third, the lowest-priced bidder is not the default winner; rather, the buyer chooses the winner based on her discretion, and in doing so, she may trade off sellers' reputations, bid prices, and other bid attributes, as well as the costs of waiting and canceling. Yoganarasimhan (2013) refers to such auctions as beauty contest auctions.

Online freelancing has grown tremendously in the last few years. Industry revenues in 2010 were over $360 million, with a 61% increase from Q1 to Q4 (Morgan 2011). This surge can be attributed to two factors. First, technological innovations such as electronic deliverability of jobs and fast Internet connections have increased the supply of jobs that can be performed by freelancers. Second, freelance markets offer a low-cost way for geographically distant players to trade, especially since there is an abundance of unemployed skilled workers in emerging economies (e.g., Indian subcontinent, eastern Europe) that have low costs of living and a healthy demand for skilled workers in developed countries, where local labor is expensive.

Despite this recent growth, online freelancing faces many challenges, the primary one being the classic *information asymmetry* or "lemon" problem (Akerlof 1970). Note that buyers face considerable risks in these marketplaces—sellers may deliver low-quality services, abscond with advance payments, hold up the job without completing it and/or delay it, and steal intellectual property (IP) given to them during the job and sell it to a competitor or use it themselves. Although it is theoretically possible to contract on quality, service contracts are notoriously difficult to spell out and enforce (Brousseau and Glachant 2002).

Even if contracts could be made, because most sellers are geographically distant from buyers and belong to developing countries (where IP rights are comparatively lax and legal systems corrupt; see Park 2008), it is exceedingly difficult for injured buyers to obtain legal restitution. These frictions can preclude most transactions.

Online freelance marketplaces seek to mitigate these risks through reputation mechanisms designed to decrease the information asymmetry between players.[1] Typically, reputation systems follow a two-way feedback mechanism—after each transaction, both the seller and buyer are allowed to numerically rate each other, and these ratings are made available to their prospective clients. Intuitively, these feedback systems are designed to incentivize players to behave well in the current period using the threat of future punishment.

However, there is no clear consensus on the effectiveness of these reputation mechanisms. They can fail for many reasons—imperfect monitoring (Holmstrom 1999, Cripps et al. 2004), cheap identities (Friedman and Resnick 2001, Dellarocas 2003), reliance on unverifiable and voluntary feedback (Resnick and Zeckhauser 2002), vulnerability to Sybil attacks (Douceur 2002), and retaliation concerns (Cabral and Hortaçsu 2004), to mention a few. Some solutions have been proposed to these problems (Jøsang et al. 2007), few of which have also been adopted by freelance sites (see §3.1), but it is not clear how far they go in building robust reputations. Given that the entire freelancing industry is sustained by feedback-based reputation systems, a good understanding of their effectiveness has implications for the users of the freelance sites, the freelance sites themselves, and policy makers interested in regulating this industry. However, to our knowledge, there exists no systematic analysis of returns to seller reputation in freelance marketplaces.

This gap in the literature largely stems from the two key challenges involved in estimating returns to seller reputations in freelance marketplaces. First, in this setting, we need to account for the *option value of waiting* because buyers face dynamic considerations. Figure 1 shows the relationship between the exit period of the buyer and her probability of choosing

Figure 1 **Probability of Choosing a Bid (as Opposed to Canceling) by Buyers' Exit Period**

a bid in our data. Note the unmistakable downward trend—buyers who wait longer are more likely to cancel the auction rather than pick a bid. One reason for this could be that buyers are forward looking; i.e., only those buyers who receive a mediocre set of bids remain in the system (hoping to receive better bids), many of whom eventually cancel as good bids fail to materialize. Hence, ignoring these dynamics can bias the estimates of reputation.

Second, we need to account for *dynamic selection*. The downward trend in Figure 1 could also be due to selection bias—buyers who repeatedly wait are a self-selected group, and their persistent tendency to not choose a bid could be due to unobservable (to the researcher) factors that affect their utility from bids. For instance, some buyers may have good unobserved outside options (other freelance sites, own coding abilities), whereas others may specify their auctions poorly, and the bids they attract may not satisfy their requirements. Yet others may have private information on the number of bids they expect to receive in future or have an inherently low taste (value) for bids. There may also be significant differences in the average unobserved quality of the bids received by buyers. Such persistent unobservables lead to dynamic selection—the surviving buyers in any period are not random but are a self-selected group that has low unobserved taste for bids, and thus members of this group are more likely to cancel the auction than early exiters are. Not controlling for dynamic selection can bias the estimates of reputation; it can lead to systematic underestimation of reputation if a significant fraction of buyers have low unobserved preference for bids.

These challenges cannot be addressed by hedonic regressions and static discrete choice models because they ignore both intertemporal trade-offs and dynamic selection. Dynamic models without persistent unobserved heterogeneity can handle intertemporal trade-off, but not selection. In our setting, we

[1] Some online freelance marketplaces also offer escrow and arbitration. Escrow solves the problem of what comes first—work or payment. Arbitration can partially solve the problem of quality control if the buyer and seller can agree beforehand to abide by the site's decision in case of future differences. However, neither escrow nor arbitration solves the fundamental information asymmetry problem; they simply allow for better distribution of risk among the buyer, seller, and freelance site. So any site using these two mechanisms exclusively would have to charge high commissions to cover its costs, thereby precluding low-value transactions.

find that a static model considerably overpredicts cancellation for early deciders (by 49.92%) and underpredicts cancellation for late deciders (by 44.9%). Similarly, we find that a dynamic model without persistent unobservables underpredicts bid choice in the earlier periods (by 34.71% in period 1) and overpredicts it in the later periods (by 44.06% in the 13th period). Thus, these models not only furnish biased estimates of reputation but are also very poor in terms of fit.

In this paper, we present a structural framework to estimate the returns to seller reputations in reverse auction settings that can accommodate both dynamics and self-selection. In our partial equilibrium framework, we model buyers' decisions while taking sellers' behavior as given. Buyers face uncertainty over the number of bids they expect to receive in the future and the attributes of those bids (price, seller reputation, etc.). Each period, they solve a dynamic programming problem to decide whether to terminate the auction (by choosing one of the submitted bids or canceling) or continue to wait for another period. Our model allows for two types of persistent unobserved heterogeneities—in bid arrival rates across auctions and in buyers' unobserved preference for bids.

Estimation of the model is complicated by the size of the state space and the presence of persistent unobservables. In general, high-dimensional state spaces are intractable with nested fixed-point algorithms (Rust 1987) and are estimated using computationally light two-step methods (Hotz and Miller 1993). However, our state space is intractable even with standard two-step methods. Moreover, two-step methods have traditionally suffered from their inability to account for persistent unobservables (Aguirregabiria and Mira 2010). To address these issues, we adapt the two-step estimation framework recently developed by Arcidiacono and Miller (2011) as follows. First, to ensure computational tractability, we reformulate our value function by exploiting the finite-dependence properties of our data. Second, we employ an augmented expectation-maximization (EM) loop that nests a two-step estimator; i.e., we recursively calculate and update the conditional choice probabilities (CCPs) and structural parameters till convergence. A key issue with the use of two-step CCP-based methods in models with persistent unobserved heterogeneity is the nonparametric identification of CCPs. Whereas general proofs that allow unrestricted state transitions are available (Kasahara and Shimotsu 2009), they are not applicable to our setting. So we derive the conditions for nonparametric identification of CCPs and state transitions in reverse auction settings. Finally, we discuss the identification of the unobserved types and the exclusion restrictions that allow us to empirically estimate the discount factor.

We apply our empirical framework to data from a leading online freelance firm and present six key findings. First, we find that buyers place significant weight on seller reputations. Buyers not only value sellers with high average ratings but also value those with a large number of ratings. The returns to reputation manifests itself in higher probabilities of being chosen as well as the ability to charge higher prices. Second, we find that buyers prefer sellers with low bid prices, sellers with whom they have interacted in the past, and sellers from developed countries. Third, we find that about 30% of the buyers have high unobserved value for the jobs posted, whereas 70% have low unobserved value. Fourth, we find that not controlling for dynamics and persistent unobserved differences between buyers can lead to serious biases in the estimates of reputation. Fifth, we find that, on average, 87% of buyers considering entry actually choose to enter the market and that this number varies significantly with their unobserved type. Finally, we estimate the daily discount factor to be 0.88; i.e., buyers are forward looking but impatient, especially when compared with the discount factor implied by yearly interest rates. Our finding highlights the importance of estimating (as opposed to assuming) the discount factor, especially in settings unrelated to banking.

Next, we present results from a series of counterfactual experiments that quantify the impact of regime changes on auction cancellation rates and site revenues. We note that because we have a partial equilibrium model, these results are contingent on the assumption that sellers' side behavior remains the same. First, we switch off the site's reputation system and find that site revenues fall by 11.1%; that revenue loss is the highest from high-value auctions. This suggests that the site's reputation system is a significant source of revenue. Second, and surprisingly, we find that increasing the supply of sellers lowers cancellation rates but has no significant impact on revenues. This is because a higher supply of sellers decreases average transaction prices and hence the site's commissions. On the other hand, increasing the fraction of high-reputation sellers, while keeping supply constant, has the opposite impact—cancellation rates remain the same, but revenues increase. This is because buyers now replace low-reputation, low-price sellers with high-reputation, high-price sellers, thereby driving up the transaction prices and the site's commissions. Together, these two results provide valuable guidelines to managers of freelance sites. First, increasing the supply of sellers uniformly can negatively affect revenues. So, decreasing commission rates across the board to either attract sellers from other sites or incentivize all existing sellers to bid more is not a profitable strategy. Second, it is important to incentivize high-reputation sellers alone to bid more and win auctions at higher prices. Incentive mechanisms that selectively lower commission rates for high-reputation sellers and/or provide better services to them are recommended.

Finally, we examine whether the site can benefit from charging buyers a fee to post an auction. We consider two types of fees—fixed fees and a percentage of maximum bid. Auction fees have two opposing effects on revenue. On the one hand, the site has a new revenue stream. On the other hand, some buyers who might have previously procured from the site now do not even enter the auction. This leads to lower revenues from commissions. These two opposing forces give rise to an inverted U-shaped curve. Specifically, we find that (a) fixed auction fees dominate auction fees based on the percentage of the maximum bid, and (b) revenues are maximized at an auction fee of approximately $2.75.

In sum, our paper makes three key contributions to the literature. First, from a methodological perspective, we provide a dynamic structural framework to model and estimate the value of bidder attributes in reverse auctions. Our framework not only allows for large state spaces but also controls for the option value of waiting and dynamic selection. The framework is fairly general and can be adapted to a large class of optimal stopping problems (e.g., generalized search models, procurement auctions with unspecified end dates, dating or marriage decisions). Second, from a substantive perspective, we quantify the returns to reputation and other seller attributes in freelance markets. We also estimate the extent of buyer impatience and the distribution of persistent unobservable types in these markets. As far as we know, this is the first paper in marketing to study freelance markets. Because these markets are becoming increasingly popular, we believe our substantive findings will be of value to researchers, managers, and policy makers interested in this area.

Third, from a normative perspective, our work offers guidelines to sellers and managers of freelance sites and policy makers. From a seller's perspective, our estimates of buyer utility clarifies how buyers' trade off reputation and price and can help them optimize their efforts toward improving their own reputation and bidding strategies. From the freelance site's perspective, our framework can be used to gauge the effectiveness of their current reputation systems and evaluate the value of implementing a more robust one. Moreover, our counterfactual results can help sites design better incentive mechanisms for their members. Finally, from a policy maker's perspective, online freelancing is a large and growing industry that contributes significantly to offshore outsourcing of jobs, thereby putting it at the center of the raging debate on the impact of offshore outsourcing on local economies (Mankiw and Swagel 2006, Lacity and Rottman 2008). Although a complete analysis of the costs and benefits of offshore outsourcing is outside the scope of this paper, our estimates can serve as inputs in the larger cost–benefit analysis that policy makers must undertake to settle this debate.

2. Related Literature

Our paper relates to four broad streams of literature. First, our paper relates to the growing literature on online auctions in marketing. Park and Bradlow (2005) and Bradlow and Park (2007) examine bidder behavior in online auctions, and Zeithammer (2006, 2007) examines the dynamics of optimal bidding strategies and buyer behavior with forward-looking sellers. Zeithammer and Adams (2010) investigate the validity of the ubiquitous assumption that online sealed-bid auctions are strategically equivalent to second-price auctions. Finally, Yao and Mela (2008) consider a model of both seller and buyer behavior and estimate the impact of varying commission rates and the value of sellers to the marketplace.

Second, it relates to the literature on eBay auctions that uses hedonic regressions to evaluate the value of reputations on eBay (Kalyanam and McIntyre 2001, Eaton 2002, Jin and Kato 2002, Melnik and Alm 2002, Cabral and Hortaçsu 2004, Bajari and Hortaçsu 2004, Lucking-Reiley et al. 2007). Our paper differs from this research both substantively and methodologically. Substantively, we study seller reputations in a marketplace for services rather than goods, and our setting involves reverse auctions as opposed to traditional auctions. Buyers of services face higher risks, because unlike physical goods, services often have no external brand value, no physical attributes that can be shown in photos, or any third-party valuations. Seller reputations may therefore play a more central role in transactions of services. The reverse auction mechanism not only is a substantive difference but also presents methodological challenges that cannot be addressed by hedonic regressions—buyers face dynamic considerations in this setting and can exit the auction at each period, giving rise to dynamic selection. It is also difficult to interpret the results from hedonic regressions as buyer valuations or some other primitive construct unless we make strong (and unrealistic) assumptions regarding the auction setting (Bajari and Hortaçsu 2004, Rezende 2008). In contrast, our structural framework based on the utility-maximization framework is capable of handling both dynamics and selection. Moreover, our results can be directly interpreted as primitives that determine buyer utilities and therefore can be used to conduct counterfactual experiments to evaluate the impact of policy changes.

Third, our paper relates to the sequential search literature, where an agent who has imperfect information on a set of alternatives sequentially searches for the best option among them by paying a fixed cost for each search. Empirical models of sequential search endogenize choice sets in demand estimation and allow researchers to estimate search costs, furnish better estimates of price elasticity, and explain price dispersion for homogeneous goods (Hong and Shum 2006, Koulayev 2009). Our model differs from search models in two important respects. First, the sequence of bid arrivals in our model is not determined by the buyer, so unlike search models, sequence provides no information on the reservation prices of bids. Second, in search models, agents have perfect information on the observed attributes of the products and are assumed to be searching only for the realization of the error term (Kim et al. 2010). Together, these two assumptions ensure that search models have an analytical solution (Weitzman 1979), and hence estimation does not involve numerical solutions to the Bellman equation. This greatly simplifies the modeling challenges. However, in our case, it is unreasonable to assume that buyers have perfect information on the attributes of future bids; in fact, buyers do not even know the number of bids they will receive in future periods. We thus have to specify and estimate a full-blown dynamic discrete choice model.

Fourth, our paper relates to the small but growing literature that controls for dynamic selection in dynamic discrete choice contexts, e.g., Arcidiacono (2005) in college admissions, Carro and Mira (2006) in couples' contraception and sterilization decisions, and Arcidiacono et al. (2012) in teenage sex and contraception choices. It also relates to the broader literature on finite mixture models. Starting with Dempster et al. (1977), researchers in a variety of fields have employed finite mixtures to accommodate latent unobserved heterogeneity. In the marketing literature, finite mixtures were pioneered by Kamakura and Russell (1989) and Chintagunta et al. (1991) and have since been used extensively. See McLachlan and Peel (2004) and Allenby and Rossi (1998) for detailed discussions of finite mixture models.

3. Setting and Data

3.1. Setting

Our data come from a leading online freelance firm that had more than 320,000 registered freelancers and 150,000 registered buyers as of March 2011. Membership is free, and there is no fee for either posting an auction or bidding. The site receives about 400 new auctions every day, a vast majority (over 80%) of which are technology-oriented. It follows a sealed-bid reverse auction format; i.e., sellers have no information on other bids received by the buyer. Our data comprise a random sample[2] of all unabandoned[3] public auctions that have two-week expiry periods and maximum bids in the range of \$10 to \$100 (in increments of \$10) initiated from January 1, 2008 to December 31, 2010.[4]

We now describe the auction process in greater detail.

Step 1. A buyer with a procurement need initiates an auction with two key pieces of information.

- *Project title and description*: Most project descriptions are very short and generic. There are two reasons for this. First, it is costly (in time and effort) to write out all the project details, and most buyers find it easier to talk to the winning bidder by phone or email after the auction. Second, auction postings are visible to all site members. Therefore, revealing project details in the posting carries privacy risks. See Table 1 in the Web appendix for examples of project titles and descriptions (available as supplemental material at http://dx.doi.org/10.1287/mksc.2013.0809).
- *MaxBid*: This is the maximum amount that the buyer is willing to pay for the project. Although providing a MaxBid is optional, most serious buyers choose to do so because it conveys information to the seller about the size and difficulty of the project. Buyers can submit any MaxBid they want, but most of them choose multiples of 10. In our analysis, we exclude all auctions where MaxBids are not multiples of 10 because we need to estimate nonparametric joint distributions of bid attributes for each MaxBid included in the analysis to model buyers' expectation of future bids' attributes (see §5.4.2). Numbers that are not multiples of 10 have very few auctions that specify them as the MaxBid, making it difficult for us to generate bid distributions for them.

Step 2. After confirming that the project does not involve illegal activities, the site posts the auction on its public forum, which can be browsed by all

[2] We do not reveal the size of the full data set to preserve the privacy of the firm.

[3] Some buyers start auctions but do not monitor them or take any actions subsequently. We exclude such auctions from our analysis because we do not know whether the buyer was actually making any decisions or not and, if so, when exactly the buyer chooses to ignore the auction. Estimates from a model including these auctions are qualitatively similar to those presented here.

[4] The site also allows auctions where sellers bid for the hourly rate to be paid to them while they work on the project. This is in contrast to the pay-per-project auctions that we study, where sellers bid on the payment for completing the project. Usually, projects that can be well defined ahead of time are sold on a pay-per-project basis, whereas projects with undefined scope are sold on a pay-per-hour basis. However, the pay-per-hour format was only recently introduced by the site and forms a very small portion of their business. So we exclude them from our analysis.

its members. Sellers can also obtain up-to-date information on new project postings by subscribing to newsletters from the site. The posting contains information provided by the buyer (e.g., project description, auction start and expiry date, MaxBid) as well as information on the buyer herself (e.g., her past ratings, geographic location) through a link to her home page.

Step 3. Sellers can start bidding for the job as soon as the auction goes live. The bid, along with the seller's past average rating, becomes available to buyers through the site. The bid also contains a link to the seller's home page, which contains additional information about the seller.

Step 4. The buyer can stop the auction at any point in time by either picking one of the bids she has already received (if any) or canceling the auction. If she has not picked a bid after two weeks, the auction is automatically canceled.[5]

The site employs a mandatory escrow and offers free arbitration services for these auctions. It charges a fees for its services in the form of a percentage commission on the transaction amount, which is paid by the winning bidder. For example, if a seller with a bid of $50 wins a project, the buyer escrows $50 with the freelance site, and after the project is completed, the site releases $50 minus its commission to the bidder.

We now describe the reputation system used by the marketplace in detail. The site uses a symmetric numeric rating scale of 1–10 (and optional text comments) for both buyers and sellers. A rating of 1 stands for very bad and 10 for excellent. The site has implemented the following measures to make the reputation system robust. First, after a trade, both the buyers and the seller are given a fixed time period to rate each other, after which they lose the right to rate. The ratings are revealed publicly only after both parties have rated each other. If one of the parties fails to turn in its feedback, then the other party's rating is revealed only after the fixed time period, at which point the delinquent party cannot retaliate. Second, if there is a dispute following the trade and the case goes into arbitration, both parties lose the right to rate each other, although the neutral arbiter may rate either or both players. Third, members' home pages have information on all their past trades; i.e., members cannot selectively hide ratings. Overall, the site has a relatively high feedback rate compared with sites such as eBay, where the feedback rate is barely 50% (Resnick and Zeckhauser 2002). In our data, 92.4% of the buyers have rated their sellers, and 71.8% of the sellers have rated their buyers.

Table 1 Code for Buyer and Seller Geographic Regions

Region code	Countries
1	Indian subcontinent (India, Pakistan, Bangladesh, etc.)
2	Developed countries (United States, Canada, Australia, western Europe, etc.)
3	Eastern Europe (Romania, Russia, Estonia, etc.)
4	Everything else (Philippines, China, Korea, etc.)

3.2. Data

For each auction in our sample, we have the following information:

- The MaxBid of the auction
- Start and end dates of the auction
- Number of bids received for each day the auction is active
- The following buyer attributes:
  —Geographic region of the buyer (region codes are shown in Table 1).
  —Total number of past auctions initiated by the buyer.
  —Cancel ratio, which is the fraction of past auctions that the buyer canceled. A buyer who has initiated 10 auctions and picked winners in 7 auctions has a cancel ratio of 0.3; by default, it is zero for buyers with zero past auctions. Cancel ratio is indicative of the buyer's inherent choosiness and/or the quality of her outside options.
  —Number of past ratings and the sum of all past ratings.
  —Mean rating, defined as the "sum of all past ratings/total number of past ratings" if the buyer has at least one rating and is zero otherwise.
  —Tenure on the site, or the number of days since the buyer signed up.
- The following attributes for all bids received: the bid price, seller's geographic region (see Table 2), number of her past ratings, sum of her past ratings, mean rating, and an indicator for whether she has worked for the buyer in the past on this site[6]

Table 2 provides an overview of the number of auctions in our data, by MaxBid, and their outcomes. Note that only 78.23% of the auctions in the data end with the buyer picking a bid, whereas the rest are canceled. On average, auctions are active for about 2.67 periods and the average life span of an auction increases with MaxBid (see Table 3). That

[5] A winning bidder has 24 hours to reject the job without penalty. In such cases, the buyer may pick another bidder. In the data, we observe very few occurrences where winning sellers reject a won job. In those cases, we treat the first choice of the buyer as the winning bid. The qualitative results remain unchanged if we instead drop these auctions from our analysis.

[6] Both buyer and seller reputation metrics may change during the course of the auction if they receive new ratings. However, such changes are very small and infrequent. So for each auction, we obtain all buyer attributes at the beginning of the auction, and for each bid, we obtain seller's attributes at the date of bid submission. We then treat them as constant for the duration of the auction.

Table 2 **Summary Statistics of Auction Outcomes**

MaxBid	Total auctions	Total bids	No. of canceled auctions	No. of auctions where buyer picked a bid	% of auctions where buyer picked a bid	No. of auctions where buyer picked lowest bid	% of uncanceled auctions where buyer picked lowest bid
10	1,495	10,782	348	1,147	76.72	602	52.48
20	2,172	18,042	398	1,774	81.67	794	44.76
30	1,501	14,030	292	1,209	80.54	468	38.71
40	963	8,314	232	731	75.90	290	39.67
50	2,925	29,169	588	2,337	79.89	899	38.47
60	574	5,607	144	430	74.91	157	36.51
70	245	2,373	57	188	76.73	75	39.89
80	336	3,632	70	266	72.67	95	35.71
90	122	1,307	29	93	76.22	39	41.93
100	2,999	33,671	744	2,255	75.19	853	37.83
Total	13,332	126,927	2,902	10,430	78.23	4,272	40.96

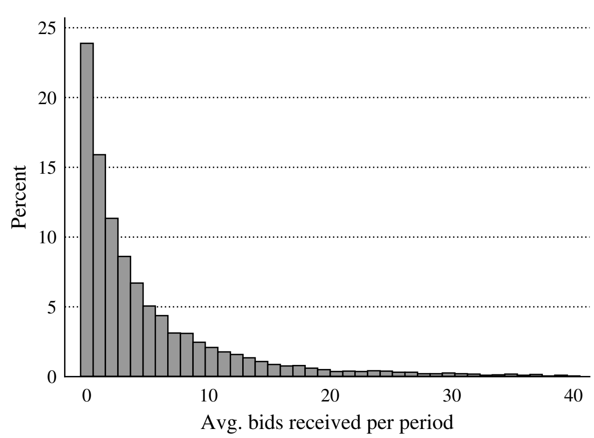
Table 3 **Summary Statistics of Periods Active and Bids Received**

	No. of periods active before choosing bid or canceling			No. of bids received		
MaxBid	Mean, Std. dev.	25th, 50th, 75th percentiles	Min, Max	Mean, Std. dev.	25th, 50th, 75th percentiles	Min, Max
10	2.03, 2.33	1, 1, 2	1, 14	7.30, 8.32	2, 5, 9	1, 88
20	2.12, 2.28	1, 1, 2	1, 14	8.34, 10.04	2, 5, 10	1, 140
30	2.46, 2.63	1, 1, 3	1, 14	9.32, 10.53	2, 5, 12	1, 80
40	2.52, 2.64	1, 1, 3	1, 14	8.76, 9.64	2, 6, 12	1, 74
50	2.64, 2.74	1, 1, 3	1, 14	10.02, 11.35	3, 6, 12	1, 86
60	2.89, 2.82	1, 2, 4	1, 14	9.76, 10.90	3, 6, 12	1, 88
70	3.17, 3.32	1, 2, 4	1, 14	9.59, 11.02	3, 6, 12	1, 71
80	3.40, 3.39	1, 2, 4	1, 14	10.89, 12.36	3, 6.5, 14	1, 94
90	3.54, 3.60	1, 2, 5	1, 14	10.98, 11.73	2.5, 6.5, 15	1, 59
100	3.35, 3.21	1, 2, 5	1, 14	11.19, 12.90	3, 7, 14	1, 131
Total	2.67, 2.81	1, 1, 3	1, 14	9.54, 11.09	3, 6, 12	1, 140

is, buyers who value the job more or those with larger projects seem to wait longer. Nevertheless, even within the same MaxBid, there is significant variation in when buyers close their auctions. An average auction receives about 9.54 bids, with the median being 6 (see Table 3). Because this number does not take into account the number of periods the auction was active, we present Figure 2, which shows the distribution of the average number of bids received per period by the auctions in our data. There is considerable heterogeneity across auctions in the average number of bids received per period, and the distribution in Figure 2 exhibits a long tail—a majority of the auctions (50.76%) receives no more than an average of 2 bids per period, whereas a few of them (18.06%) receive 10 or more.

Table 4 shows the summary statistics of buyer attributes.[7] The median buyer has about 10 ratings, with an average rating of 9.96. However, a big chunk of them (15.1%) have no past ratings, whereas another significant chunk (17.49%) has a mean rating of 10, with 10 or more ratings. Most buyers in the data have previous experience on the site; the median buyer has posted 11 successful auctions (in which she picked a bid) and 7 canceled auctions. Finally, as shown in Table 5, the majority (81.88%) of buyers belong to developed English-speaking countries, i.e., Region 2.

Figure 2 **Histogram of Average Number of Bids Received per Period by All Auctions**



Note. Bin width = 1.

[7] To preserve the privacy of buyers and the freelance site, summary statistics for buyer tenure are not shown.

Table 4 Summary Statistics of Buyer Attributes

	Summary statistics					
Buyer attributes	Mean	Std. dev.	25th	50th	75th	Min, Max
No. of ratings	40.87	125.33	2	10	34	0, 1,335
Avg. ratings	8.3	3.57	9.54	9.96	10	0, 10
Avg. ratings (if rated)	9.79	0.58	9.85	10	10	1, 10
No. of uncanceled auctions	39.92	149.56	2	11	36	0, 2,121
No. of canceled auctions	25.58	55.00	2	7	23	0, 555

Table 5 Distributions of Buyer and Seller Regions

	Seller region (%)			
Buyer region (%)	Region 1	Region 2	Region 3	Region 4
1 (6.44)	65.37	9.11	11.53	13.98
2 (81.88)	54.79	14.68	15.21	15.32
3 (2.47)	54.42	12.94	17.67	14.97
4 (9.21)	57.81	11.95	14.85	15.38

Table 6 shows the summary statistics of seller attributes. A large percentage (24.87%) of bidders have no ratings, and 20.97% of them have a very good reputation (with a mean rating of 10 with 10 or more ratings). About 1.98% of the bidders have interacted with the buyer in the past. Unlike buyers, the majority of the bidders (56.69%) belong to the Indian subcontinent (see Table 5). The other three regions are about equally represented (approximately 14%–15% each). Moreover, the distributions of the sellers' geographic region varies with the buyer's region. Finally, although a large number of bidders quote the MaxBid as their price (40.34%), many of them also quote much lower prices, with 4 being the lowest observed quote (see Table 7).

Table 6 also shows the summary statistics of accepted bids. There are systematic differences in the attributes of winning bidders compared with the full distribution of bidders. On average, they quote lower prices (see Table 7), have significantly better reputations, are more likely to belong to developed countries, and have a higher likelihood (10.46%) of past interaction with the buyer.

4. Model-Free Evidence

We now present some model-free evidence in support of the effectiveness of the reputation system. First, note that only 40.96% of the buyers who eventually pick a bid (i.e., do not cancel the auction), choose the lowest-priced bid (see Table 2). This tendency is also more pronounced for larger projects (higher MaxBids). For example, whereas 52.48% of the buyers in the category MaxBid = 10 pick the lowest-priced bid, only 37.83% in the category MaxBid = 100 do the same. These patterns in the data suggest that considerations other than price—potentially sellers' reputations—play an important role in buyers' decisions.

Second, we examine whether there is sufficient variation in the distributions of equilibrium prices. When reputation systems fail, high-quality or truthful sellers desert the market, leaving only low-quality sellers, who in turn receive low prices in equilibrium (Akerlof 1970). In contrast, a healthy marketplace with an informative reputation system can support both high- and low-quality sellers, with the former receiving a premium for reputation. So the absence of price variation in a market usually indicates the failure of its reputation system. Table 7 gives the summary statistics of the trading price for all the MaxBids, and Figure 3 shows the distribution of the prices received by sellers in equilibrium for MaxBid = 100. Together, they confirm that equilibrium prices are reasonably diffuse.

Third, we check whether buyers are more willing to pick a high-reputation seller compared with a

Table 6 Summary Statistics of Seller Attributes of All Bids and Accepted Bids

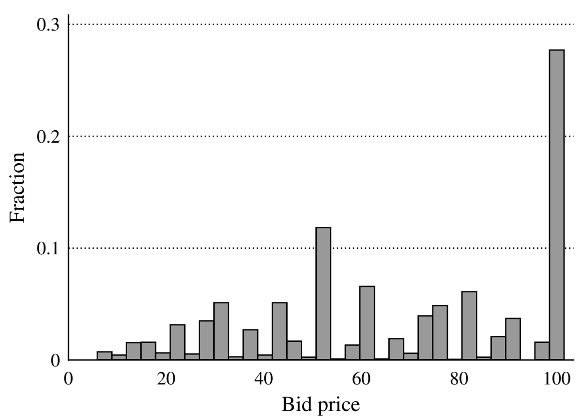
	Summary statistics					
Bidder attributes	Mean	Std. dev.	25th percentile	50th percentile	75th percentile	Min, Max
			All bidders			
No. of ratings	32.32	73.65	1	6	31	0, 1,679
Avg. ratings	6.94	4.14	1	9.33	9.9	0, 10
Avg. ratings (if rated)	9.23	1.3	9.03	9.66	10	1, 10
			Accepted bidders			
No. of ratings	51.7	106.29	3	16	54	0, 1,618
Avg. ratings	8.45	3.15	9.13	9.75	10	0, 10
Avg. ratings (if rated)	9.59	0.68	9.45	9.82	10	1, 10

Note. For all bidders, 1.98% have interacted with the buyer in the past; for accepted bidders, 10.46% have interacted with buyer in the past.

Table 7 Summary Statistics of Bid Prices

	MaxBid									
Summary statistics	10	20	30	40	50	60	70	80	90	100
					All bids received					
Mean	8.77	16.44	24.24	32.07	38.71	47.25	54.45	59.26	68.46	72.52
Std. dev.	1.84	4.4	6.67	9	12.12	13.59	15.44	19.73	21.62	27.1
25th percentile	8	13	20	25	30	40	45	50	50	50
50th percentile	10	19.99	25	35	40	50	60	60	75	80
75th percentile	10	20	30	40	50	60	70	80	90	100
Min, Max	4, 10	4, 20	6, 30	4, 40	4, 50	4, 60	4, 70	4, 80	4, 90	4, 100
					Accepted bids					
Mean	8.87	16.34	24.35	31.98	37.1	47.3	56.27	60.05	67.24	65.82
Std. dev.	1.87	4.61	7.05	9.41	13.05	14.74	15.78	20.69	25.34	28.87
25th percentile	8	13	20	25	25	40	45	49	50	40
50th percentile	10	20	29	35	40	50	60	65	75	70
75th percentile	10	20	30	40	50	60	70	80	90	100
Min, Max	4, 10	4, 20	6, 30	4, 40	4, 50	4, 60	5, 70	5, 80	8, 90	4, 100

Figure 3 Histogram of Prices of Winning Bids for MaxBid = 100



Note. Bin width = 3.

Figure 4 CDFs of ln(Number of Ratings + 1) for Accepted and Rejected Bidders

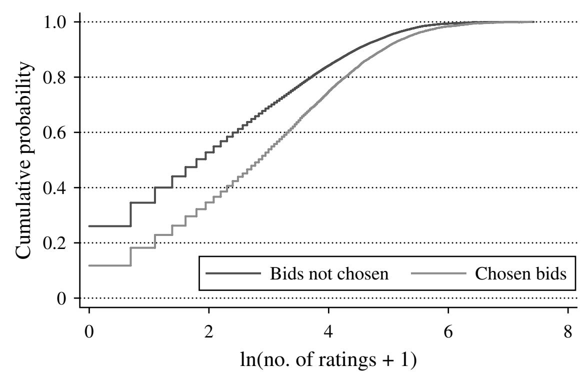
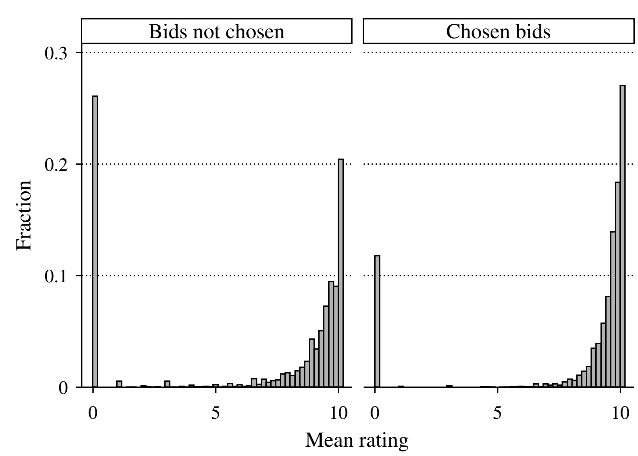


Figure 5 Histogram of Mean Ratings for Accepted and Rejected Bidders

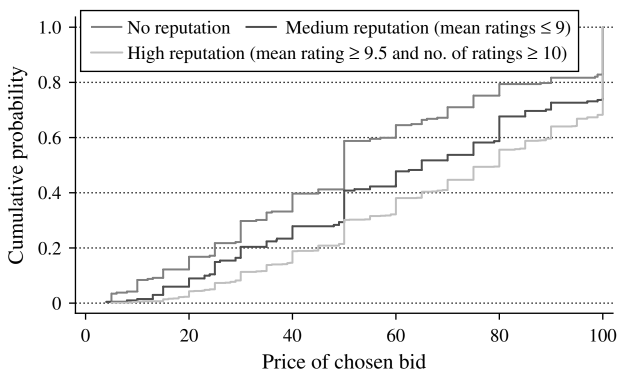


low-reputation seller. Figure 4 presents the cumulative distribution functions (CDFs) of the number of past ratings for two sets of sellers: (a) sellers whose bids were chosen by the buyer and (b) sellers whose bids were not chosen. Note that there is a clear difference in the CDFs—on average, winning bidders have more ratings than the losing bidders. Figure 5 presents the distributions of the mean ratings for the same two sets of sellers. Here, we find that, on average, chosen bidders have a better mean rating than those who were not chosen. Taken together, these findings suggest that buyers value and prefer sellers with good reputations.

Finally, we examine whether buyers are willing to pay higher prices to high-reputation sellers; i.e., are reputations and equilibrium prices positively correlated? In Figure 6, we present the distributions of prices, for chosen bids, for three types of sellers: (a) sellers who have no past ratings, (b) sellers who have some reputation, and (c) sellers who have an excellent reputation. On average, sellers with no past ratings receive the lowest prices, whereas those with medium reputations receive somewhat higher prices, and those with the highest reputations receive the best prices.

These preliminary findings are suggestive rather than conclusive, but they nevertheless support the

Figure 6 CDFs of Prices Received by Sellers Based on Reputation



Note. MaxBid = 100.

existence of positive returns to reputation for sellers in this market.

5. Model

We now specify and estimate a formal model of buyer behavior that rationalizes the choices observed in the data. We briefly summarize our empirical framework. First, in §5.1, we present the model setup and timeline. Second, in §5.2, we specify the set of variables that affect buyers' decisions. We also specify which of these vary with time, which remain constant, and which of these are observable to the researcher. Next, in §5.3, we present buyers' flow utilities or per-period utilities as functions of observable and unobservable state variables. Then, we dive into the dynamics of the problem. In §5.4, we explain how we capture buyers' future expectations. We model buyers' beliefs on the number of bids they expect to receive using fixed effects Poisson in §5.4.1. To model buyers' beliefs on the attributes of the bids they expect to receive in the future, we use a combination of nonparametric joint distributions (see §5.4.2) and multinomial logit models (see §§5.4.3 and 5.4.4), depending on whether the bid attribute is continuous or discrete. Finally, in §5.5, we combine all these pieces to formulate buyers' dynamic optimization problem.

5.1. Model Setup

Time is discrete and indexed by $t = \{1, 2, \ldots, T\}$. There are N buyers, who are indexed by i and have defined preferences over a sequence of states of the world from $t = 1$ to T. Each buyer initiates one auction and with some abuse of notation, we often use buyer i and auction i synonymously. Auctions go live at the beginning of period 1, and buyer i makes her decision d_{it} at the end of every period till she makes a terminal decision or the auction expires at T. In our data and analysis, one period is equivalent to a day, and the expiry period is $T = 14$.[8] Each period, the buyer's options are as follows: cancel the auction, pick one of the bids she has already received (if any), or wait for another period. The former two decisions are terminal; i.e., once she cancels or picks a bid, she makes no more decisions. The option of waiting is unavailable at T, whence the buyer has to either choose one of her bids or cancel. Buyer i's alternatives are indexed by j. For periods $1 \le t \le T-1$, $j \in \mathcal{J}_{it} = \{1, 2, \ldots, J_{it}\}$, where alternative $j = 1$ represents waiting, $j = 2$ canceling, and $j = \{3, \ldots, J_{it}\}$ choosing the jth bid that has arrived so far. For period T, the alternatives are $\mathcal{J}_{iT} = \{2, \ldots, J_{iT}\}$. Finally, note that the buyer faces a nonstationary dynamic optimization problem because the horizon is finite; there is a clear terminal period T.

5.2. State Variables

Let $\{x_{it}, \zeta_{it}\}$ denote the state of the world for buyer i at period t, where x_{it} is the set of observable state variables and ζ_{it} is the set of unobservable (to the researcher) state variables.

5.2.1. Observed State Variables. We consider two sets of observable state variables that influence the buyer's decision: time-invariant and time-varying variables. The time-invariant state variables are (a) m_i, the MaxBid specified by the buyer; (b) n_i, the number of ratings received by the buyer; (c) r_i, the buyer's mean rating; (d) a_{si}, the number of past auctions in which i picked a bidder (i.e., did not cancel); (e) a_{ci}, the number of past auctions canceled by i; (f) g_i, buyer i's geographic region; and (g) l_i, the buyer's tenure length on the site. These time-invariant state variables remain constant through the auction; i.e., they transition into the same values every period.

The time-varying state variables are time t, the total number of bids received so far, B_{it}, and a $B_{it} \times 5$ matrix of bid attributes. The jth row of this matrix captures five key attributes of the jth bid received by i: (a) bp_{ij}, the bid price; (b) bn_{ij}, the number of ratings received by bidder j; (c) br_{ij}, the mean rating of j; (d) bg_{ij}, bidder j's geographic region; and (e) bt_{ij}, an indicator variable that is 1 if i and j have traded in the past.[9]

[8] In practice, time is continuous, and a buyer may make decisions at any point in time, as many times as she wishes, till she makes a terminal decision or the auction ends. However, we do not observe buyers making decisions. Although we have data on when a terminal action was taken, we have no information on the intermediate decision points at which the buyer chose to wait. We therefore aggregate the data over 24-hour intervals and assume that all buyers make their decisions on a daily basis.

[9] For past interactions, it is possible to include the rating given to j by i as another state variable as well. However, in general, we find that sellers do not bid on auctions by buyers who gave them low ratings previously. So the ratings from past interactions do not provide much information than the indicator of past interactions.

5.2.2. Unobserved State Variables. Again, we consider time-invariant and time-varying variables. Time-invariant unobserved state variables capture the persistent unobserved heterogeneity between buyers. We consider two types of persistent unobserved state variables: $\{\eta_i, s_i\}$. The former, η_i, is a time-invariant buyer (or auction) fixed effect that affects the number of bids received by buyer i, although it does not affect i's flow utility. The latter, $s_i = \{1, \ldots, S\}$, denotes i's unobserved type, drawn from a set of finite types. It captures the unobserved aspects of the buyer, auction, and bidders that influence a buyer's tendency to pick a bid. For example, some buyers may have better outside options through other freelancing sites or their own coding abilities. Such buyers would exhibit a persistent tendency to cancel or wait, and thus not choose bids. Or a buyer may specify her auction poorly, and the bids she attracts may not satisfy her requirements, in which case she would persistently avoid choosing a bid. There might be significant differences in the unobserved quality of the bids across auctions (e.g., some buyers/auctions may receive bids that, on average, have lower unobserved quality) and therefore exhibit a persistent tendency to not pick bids. All of these factors are captured by s_i because it links buyers' choices over time and thereby addresses the dynamic selection problem discussed in §1. In our application, we allow for two unobserved types: a low type, which has low unobserved preference for bids ($s_i = 1$), and a high type, which has high unobserved preference for bids ($s_i = 2$).

With sufficiently long panels for buyers, it is possible to avoid persistent unobservables and instead estimate buyer-specific CCPs à la Misra and Nair (2011). However, we do not have sufficiently long panels for a large fraction of buyers to adopt this approach. Moreover, this method would not allow us to control for auction-specific persistent unobservables.

The time-varying unobserved state variable is ϵ_{it}, which is a $J_{it} \times 1$ vector with support $R^{J_{it}}$, whose jth component forms the mean-zero additive error to buyers' per-period utility from alternative j. We assume that the errors ϵ_{it} are independent and identically distributed (i.i.d.) over time and drawn from a generalized extreme value (GEV) distribution, which yields nested logit probabilities in a static setting. All the bid options are in one nest, and the cancel and wait options are in two separate singleton nests. Let $\sigma \in [0, 1]$ be the correlation of errors in the nest with the bid options, where $\sigma = 0$ implies perfect correlation and $\sigma = 1$ indicates no correlation. Errors across nests are not correlated. Assuming a GEV distribution considerably eases the computational burden for two reasons. First, it gives us closed-form expressions for choice probabilities (McFadden 1978, Rust 1987). Second, it allows us to derive an analytical relationship between choice probabilities and the future value function (Hotz and Miller 1993, Arcidiacono and Miller 2011).

5.3. Flow Utility

There are two possible ways to model buyers' flow utility: either derive it from a micromodel of buyer and seller behavior based on assumptions on economic primitives (e.g., distribution of seller types and buyer's rating policies and learning behavior) or choose a convenient parameterization that is flexible enough to capture the patterns in data. The former is more theoretically appealing, but it has three drawbacks (Berry and Reiss 2007, Ellickson and Misra 2011). First, it requires much more structure and additional data on the primitives of the market and information on repeated interactions between buyers and sellers, which are not available to us. Second, it is likely to become unwieldy in most real-world settings such as ours, rendering estimation intractable. Third, without information on true seller types, bidding strategies, and buyer's rating behavior, identification of primitives will still be driven by functional form assumptions. Hence, we adopt the latter approach in our paper.

Let buyer i's present discounted value of her lifetime utilities at period t from making decision $d_{it} \in \mathcal{J}_{it}$ be $\sum_{k=t}^{T} \delta^{k-t} U(d_{ik}, x_{ik}, s_i, \epsilon_{ik})$, where $\delta \in (0, 1)$ is the discount rate and $U(d_{it}, x_{it}, s_i, \epsilon_{it})$ is buyer i's flow utility in period t. Then $U(d_{it}, x_{it}, s_i, \epsilon_{it})$ is additively separable as follows:

$$U(d_{it}, x_{it}, s_i, \epsilon_{it}) = u(d_{it}, x_{it}, s_i) + \epsilon_{it}(d_{it}), \qquad (1)$$

where $\epsilon_{it}(d_{it})$ is the d_{it}th component of the $J_{it} \times 1$ i.i.d. GEV error vector ϵ_{it}.

There are two points of note here. First, flow utilities are not functions of the unobserved time-invariant state variable η_i, because buyers do not receive any instantaneous utility in anticipation of future bids. Of course, optimal decisions will be functions of η_i and time through the dynamics of the problem. Second, time does not enter the flow utility as a state variable; i.e., we do not assume duration dependence. There are two reasons for this. (a) Duration dependence in consumption usually comes into play when consumers accumulate stocks of utilities over time through experience goods such as vacations or recreational golf (Hartmann 2006). In our context, there is a one-time exit decision, and there is no rational underpinning for a utility stock from waiting. (b) More importantly, we cannot separately identify persistent unobserved heterogeneity in buyers' preference for bids (s_i) and duration dependence in utilities because we do not have data on multiple auctions for a large fraction of buyers.

We now specify the deterministic components of buyers' per-period utility from waiting ($j = 1$), canceling ($j = 2$), and picking a bid ($j \geq 3$). In all discrete choice models, only differences in utilities matter. We therefore need to normalize the utility from one alternative to zero, and the utilities from all other alternatives are specified in relation to this option. This is done as follows:

$$u(2, x_{it}, s_i) = 0. \tag{2}$$

Although we can choose any alternative as this base choice, we chose cancel as that option for a key reason—it gives us finite dependence; i.e., it gives us a decision that resets the state space to a known configuration and has a well-defined utility. As Arcidiacono and Ellickson (2011) point out, a clever choice of a base decision that exploits finite dependence can greatly simplify estimation. See §6.1 for details on how this normalization simplifies estimation.

Next, we specify the flow utility from waiting (in comparison to that from canceling). Canceling allows buyers to close the auction and finish the work through other means immediately, e.g., do it themselves, hire a local programmer, or visit another freelance site. A buyer who chooses to wait may also explore outside options without any monetary costs (because the site does not charge any fees for keeping an auction active), but there are other hassles associated with waiting and maintaining an active auction. For example, every time the buyer signs into the site (even for activities unrelated to this particular auction), she is urged to review bids and close the auction. The site also sends emails and messages with information on new bids, urging buyers to pick a bid, which some buyers may want to avoid. In our data, we find that most buyers cancel the auction well before the two-week expiry period, which suggests that there are indeed costs to waiting. Therefore, we do not restrict the flow utilities from waiting and canceling to be the same. Rather, we allow the utility from waiting (in comparison to canceling) to vary with buyer- and auction-specific variables, and we let the estimates from the data inform us of buyers' relative preference for both:

$$\begin{aligned} u(1, x_{it}, s_i) &= W_{iw}(x_{it}, s_i) = W_{iw}(x_{it}) \\ &= \alpha_{w1} + \alpha_{w2} m_i + \alpha_{w3} \ln(n_i \cdot r_i + 1) \\ &\quad + \alpha_{w4} I(a_{si} + a_{ci} = 0) + \alpha_{w5} a_{si} \\ &\quad + \alpha_{w6} c_i + \alpha_{w7} l_i + \alpha_{w8} I(g_i = 1) \\ &\quad + \alpha_{w9} I(g_i = 2) + \alpha_{w10} I(g_i = 3). \end{aligned} \tag{3}$$

The flow utility from waiting is allowed to depend on MaxBid (m_i); buyer reputation ($\ln(n_i \cdot r_i + 1)$; ln(sum of buyer ratings); the buyer's previous experience on the site, an indicator for a new buyer ($I(a_{si} + a_{ci} = 0)$); the number of uncanceled past auctions (a_{si}); cancel ratio (c_i); the length of the buyer's tenure on the site (l_i); and her geographic location ($I(g_i = 1), I(g_i = 2), I(g_i = 3)$). By modeling the wait utility as a function of buyer- and project-specific variables, we are allowing the buyers' cost of waiting to vary with these factors. For example, buyers with larger projects (higher MaxBids) may have a lower cost of waiting, whereas buyers from developing countries may have a higher cost of waiting, as they can easily obtain cheap local labor. Note that W_{iw} is independent of s_i because the latter is defined as i's unobserved taste for picking a bid.

Next, we assume that i's utility from picking a bid j (in comparison to canceling) is

$$u(\text{bid}_j, x_{it}, s_i) = W_{ib}(x_{it}, s_i) + Y_{ij}(x_{it}) \quad \text{if } j \geq 3. \tag{4}$$

Here, $W_{ib}(x_{it}, s_i)$ refers to the aspects of the utility function that are constant across all bids; i.e., they are the same within the nest of bid options. $W_{ib}(x_{it}, s_i)$ is specified as

$$\begin{aligned} W_{ib}(x_{it}, s_i) = {} & \alpha_{b1} + \alpha_{b2} m_i + \alpha_{b3} \ln(n_i \cdot r_i + 1) \\ & + \alpha_{b4} I(a_{si} + a_{ci} = 0) + \alpha_{b5} a_{si} + \alpha_{b6} c_i \\ & + \alpha_{b7} l_i + \alpha_{b8} I(g_i = 1) + \alpha_{b9} I(g_i = 2) \\ & + \alpha_{b10} I(g_i = 3) + \alpha_{b11} I(s_i = 2). \end{aligned} \tag{5}$$

It is analogous to W_{iw}, with the unobserved type s_i thrown in. Since we allow for two unobserved types, α_{b11} is the relative preference of the high type ($s_i = 2$) for the bid nest compared with the low type. While s_i enters the flow utility only as an intercept, it should be noted that the tendency to pick a bid is not merely shifted by the intercept, because s_i also enters the utility from waiting (through the value function) in a highly nonlinear way. Thus the overall impact of the unobserved type is more involved than it appears at first glance.[10]

The second part of bid j's utility, $Y_{ij}(x_{it})$, consists of terms that vary across alternatives within the bid nest. Since s_i does not vary within the bid nest, $Y_{ij}(x_{it})$ does not depend on it:

$$\begin{aligned} Y_{ij}(x_{it}) = {} & \beta_1 \ln(bp_{ij}+1) + \beta_2 I(bn_{ij}=0) + \beta_3 \ln(bn_{ij}+1) \\ & + \beta_4 (br_{ij} - \overline{br}) + \beta_5 (br_{ij} - \overline{br})^2 \\ & + \beta_6 (br_{ij} - \overline{br}) \ln(bn_{ij}+1) + \beta_7 m_i \ln(bn_{ij}+1) \end{aligned}$$

[10] In general, because of the inherent nonlinearity of dynamic discrete choice models, unobserved heterogeneity is usually included as an intercept; what may appear to be a restrictive formulation in a static model is far from it in a dynamic model. The reasoning for this is very similar to the reason static logit models suffer from independence of irrelevant alternatives (IIA) but dynamic logit models with the same formulation are free from IIA. Please see Carro and Mira (2006), Arcidiacono (2005), and Arcidiacono et al. (2007, 2012) for examples of this kind of formulation.

$$
\begin{aligned}
&+\beta_8 r_i(br_{ij}-\overline{br})+\beta_9 b_{ti}+\beta_{10} I(bg_{ij}=1)\\
&+\beta_{11} I(bg_{ij}=2)+\beta_{12} I(bg_{ij}=3)\\
&+\beta_{13} I(g_i=2, bg_{ij}=1)+\beta_{14} I(g_i=bg_{ij}=2)\\
&+\beta_{15} I(g_i=2, bg_{ij}=3)+\beta_{16} I_3(g_i=bg_{ij}\neq 2). \quad (6)
\end{aligned}
$$

$Y_{ij}(x_{it})$ is a concave function of bid j's price, $\ln(bp_{ij}+1)$, to allow for nonlinearities in price sensitivity and the following reputational attributes of the seller: (a) $I(bn_{ij}=0)$, an indicator for no past ratings; (b) $\ln(bn_{ij}+1)$, a concave function of her total past ratings; (c) $(br_{ij}-\overline{br})$, her mean-centered average past rating; (d) its square $(br_{ij}-\overline{br})^2$; and (e) the interaction term $(br_{ij}-\overline{br})\ln(bn_{ij}+1)$. (Mean centering aids in the interpretation of parameter values after estimation.) We also allow for interaction between seller's reputation and project size through $m_i \ln(bn_{ij}+1)$, because buyers may value seller reputations more for larger projects. Similarly, we allow for interaction between seller's and buyer's reputations through $r_i(br_{ij}-\overline{br})$, because high-reputation buyers may place a higher value on seller's reputations. Utility from a bid is also allowed to depend on the past history of the buyer and the bidder because buyers may prefer sellers with whom they have interacted in the past. Finally, we also include geographic region dummies and interaction effects between buyer and seller regions. Although the utility function can be tweaked further, we found the above specification to be superior to others with which we experimented.

Note that Y_{ij} depends only on the attributes of seller j, not her identity. Our specification is similar in spirit to Arcidiacono (2005), who allows students' utilities from attending a specific college to depend on the attributes of the college, but not its identity. We cannot allow for seller identifiers, either through seller dummies or through unobserved seller types, because (a) we do not see enough repeat bids from sellers to identify seller dummies or seller-specific unobservable types, and (b) attaching an unobserved type to each seller is infeasible given the number of bids in the data. For example, even if we were to allow two unobserved types of sellers, for a typical auction with 10 bids, the number of possible unobserved states would equal 2^{10}. We would then have to integrate out 2^{10} possible states in our likelihood, which is infeasible.

Given that we have included all the key seller-specific variables that a buyer observes when making her decision, and given the nature of our setting, where most sellers are small players who have no brand equity beyond the site, this assumption seems reasonable. Moreover, as discussed earlier, even though we cannot control for the identity or unobserved quality of each bid, we are able to control for the average unobserved quality of a buyer's bids through s_i.

5.4. State Transitions

Buyers' decisions not only influence future state transitions but are also influenced by their beliefs on the evolution of future states. We thus need to model state transitions and specify buyers' beliefs over them. We assume that buyers have rational expectations; i.e., their expectations are consistent with the true state transition probabilities inferred from data.

First, note that buyer- and auction-specific state variables (observed and unobserved) remain constant for the duration of the auction. Next, consider the time-varying observed state variables. Time transitions deterministically, increasing by one every period. However, bids arrive stochastically, and buyers face uncertainty over the attributes of future bids. Bid arrivals are modeled using a fixed effects Poisson. Buyers' expectations on the attributes of future bids are modeled as follows: (a) Price, mean rating, and the number of ratings are modeled using three-dimensional nonparametric joint distributions. (b) Indicators for the sellers' geographic region and the past interaction are modeled using two separate logit distributions. (c) Finally, as discussed earlier, the unobserved state variables ϵ_{it} are assumed to be i.i.d. over time.

We discuss these state-transition models in detail below.

5.4.1. Poisson Bid Arrival Process. We use a Poisson process to model bid arrival because (a) bids arrive independently in sealed-bid auctions, and (b) we do not see evidence for overdispersion in data (see Table 3). The Poisson model, however, needs to capture two important patterns in the data. First, it should allow for auction-level heterogeneity since some auctions receive many more bids than others (Figure 2). Furthermore, because only some variation in the number of bids received can be explained by observed buyer and auction characteristics, we also need to account for unobserved heterogeneity in auctions. Second, the model should capture the fact that bid arrival rates vary over different time periods. For example, we observe that bid arrivals generally slow down with time. To accommodate these considerations, we employ a fixed effects Poisson model.

The conditional probability function of b_{it}, the number of bids received by buyer i in period t, is

$$h_p(b_{it}\mid z_{it}, \eta_i, \theta_p)=\frac{\exp(-\eta_i\lambda_{it})(\eta_i\lambda_{it})^{b_{it}}}{b_{it}!}, \quad (7)$$

where $\lambda_{it}=\exp(z_{it}'\theta_p)$, η_i is an unobserved buyer/auction-specific fixed effect, z_{it} is a set of time-varying buyer attributes, and θ_p is the parameter vector to be estimated. Because bids arrive independently and are not visible to other bidders, the number and characteristics of previous bids cannot affect the number of bids arriving in the current time period. Therefore, the only time-varying buyer/auction attribute is time t. Hence, we omit the subscript i in z_{it} (and λ_{it}) and define z_t as the set of $T-1$ time dummies. In the Poisson model, all time-invariant buyer and auction attributes (including buyer's unobserved type, s_i) are subsumed by the fixed effect η_i. Overall, $\{\theta_p, \eta_1, \ldots, \eta_N\}$ is the set of parameters to be estimated in this context.

5.4.2. Nonparametric Joint Distributions of Price, Number of Ratings, and Mean Rating. Our preliminary analysis revealed that the distributions of these three attributes do not follow any specific parametric forms. Therefore, we model their joint distributions using multivariate kernel density functions.

First, we classify all the bids according to the MaxBid (10–100) of the corresponding auction. Table 2 shows the total number of auctions and bids in each MaxBid over the observation period. We consider each MaxBid to be a separate class because the range and distribution of bid prices is very different across MaxBids (see Table 7). Moreover, the correlations between the three bid attributes vary by project size or MaxBid. Next, we subclassify the bids based on the buyer's reputation to capture the differences in buyers' expectations about the bids they might receive in future as follows: (a) in subclass 1, the number of ratings $= 0$; (b) in subclass 2, the number of ratings > 0 and the average rating < 9.5; (c) in subclass 3, the number of ratings < 15 and the average rating ≥ 9.5; and (d) in subclass 4, the number of ratings ≥ 15 and the average rating ≥ 9.5. Our classification allows buyers with a large number of high ratings (subclass 4) to receive better bids (low bid price, high bidder reputation, etc.), on average, compared with those with no ratings (subclass 1). In fact, Kolmogorov–Smirnov tests comparing the distributions for the four subclasses of bids (for a given MaxBid) reject the null hypothesis of equality of the distributions. Ideally, of course, we would like to have a finer-grained subclassification. However, further classification is not feasible given the size of our data.[11]

Let $\mathscr{C} = \{1, \ldots, C\}$ be the set of categories, where $C = 40$, since we have 10 MaxBids, each with four subclasses; $c = 1$ denotes MaxBid $= 10$ and subclass 1, $c = 2$ denotes MaxBid $= 10$ and subclass 2, and so on. The observed bids in each category are indexed by $q \in \{1, 2, \ldots, M_c\}$, where M_c is the total number of bids in category c. The qth bid in any category is denoted by the vector $A_q = (bp_q, bn_q, br_q)'$, where the three elements denote bid price, the number of ratings, and mean rating of the seller. We model the probability density function at a point A in the three-dimensional space, in category c, using the multivariate kernel density estimator:

$$\psi_c(A, h_c) = \frac{1}{M_c \cdot h_c^3 \cdot r(k_c, A)^3} \sum_{q=1}^{M_c} K\left\{\frac{A - A_k}{h_c \cdot r(k_c, A)}\right\}, \tag{8}$$

where h_c is the optimal bandwidth window for category c and $K(\cdot)$ is the three-dimensional kernel function satisfying the property $\int_{R^3} K(A)\, d(A) = 1$. (We choose the standard trivariate normal in our estimation.) The scaling parameter $r(k_c, A)$ represents the Euclidean distance from A to the kth-nearest point in the data. In the absence of $r(k_c, A)$, the same bandwidth is used for all parts of the distribution. This is problematic in finite samples because it is difficult to pick one optimal bandwidth for the entire range of the distribution; low bandwidths lead to spurious noise in the tails of the distribution, whereas high bandwidths cause oversmoothing in the main parts of the distribution (Silverman 1986). Scaling the bandwidth locally using $r(k_c, A)$ provides a simple but effective solution to this problem. Furthermore, as is common in the literature, we set $k_c = \sqrt{M_c}$.

The choice of the bandwidth is crucial to the quality of the kernel density estimator. In §6.3.2, we discuss the estimation of the optimal bandwidth h_c.

5.4.3. Multinomial Logit Model of Bidder's Geographic Region. Sellers can belong to one of four discrete geographic regions (see Table 1). Conditional on buyer-specific state variables and a given draw of bid price, bidder mean rating, and number of bidder ratings, we can model the distribution of the seller's geographic region using a multinomial logit model. Let $h_g(bg_{ij} \mid gx_{ij}, \theta_g)$ be the conditional probability of bg_{ij}, where gx_{ij} is the set of state variables that influences the draw of the bidders' geographic region and $\theta_g = \{\theta_{g1}, \theta_{g2}, \theta_{g3}\}$ are the parameter vectors associated with the regions 1, 2, and 3, respectively. Then, the probability that bidder j in auction i belongs to geographic region q is

$$h_g(bg_{ij} = q \mid gx_{ij}, \theta_g) = \frac{e^{gx_{ij}\theta_{gq}}}{1 + e^{gx_{ij}\theta_{g1}} + e^{gx_{ij}\theta_{g2}} + e^{gx_{ij}\theta_{g3}}} \quad \forall\, q \in \{1, 2, 3\},$$

$$h_g(bg_{ij} = 4 \mid gx_{ij}, \theta_g) = \frac{1}{1 + e^{gx_{ij}\theta_{g1}} + e^{gx_{ij}\theta_{g2}} + e^{gx_{ij}\theta_{g3}}}. \tag{9}$$

[11] Results are robust to modifications in cutoffs used to subclassify the data, such as using $n_i > 20$ as the cutoff point between subclasses 3 and 4. Moreover, preliminary analysis of the data did not reveal any systematic differences between the distributions of early and later bids. So we do not further classify bids based on their time of arrival.

Note that $\theta_{g4} = 0$ because $q = 4$ is the base region. In our estimation, we include buyer-specific variables and the three bid attributes drawn from the nonparametric joint distribution in gx_{ij}. The set of parameters to be estimated in this context is $\theta_g = \{\theta_{g1}, \theta_{g2}, \theta_{g3}\}$.

5.4.4. Logit Model of Buyer-Bidder Past Interaction Indicator. The buyer–seller past interaction is characterized using the indicator variable bt_{ij} which is 1 only if the buyer and seller have interacted in the past. Let $h_b(bt_{ij} \mid tx_{ij}, \theta_t)$ be the conditional probability of bt_{ij} given state variables tx_{ij} and parameter vector θ_t; tx_{ij} consists of buyer-specific state variables and bid price, bidder mean rating, number of bidder ratings, and bidder country. Then, the logit probability of $bt_{ij} = 1$ is

$$h_b(bt_{ij} = 1 \mid tx_{ij}, \theta_t) = \frac{e^{tx_{ij}\theta_t}}{1 + e^{tx_{ij}\theta_t}}. \tag{10}$$

Here, $bt_{ij} = 0$ is the base option, with probability $1/(1 + e^{tx_{ij}\theta_t})$.

5.4.5. State Transitions and Unobserved Buyer Type s_i. Note that bid arrivals are allowed to be correlated to a buyer's unobserved type (s_i) because the Poisson fixed effect (η_i) subsumes all time-invariant buyer attributes, including s_i. We thus allow the number of bids received by a buyer to be correlated to her unobserved preference for bids. For example, a buyer who puts up an ill-specified auction may not receive many bids and may also exhibit a tendency to not pick bids.

However, we do not allow the attributes of bids received by a buyer to be correlated to her unobserved type s_i. Recall that we model three of the bid attributes using nonparametric multivariate joint distributions. Allowing for correlations between these bid attributes and unobserved type s_i would require us to specify and estimate nonparametric mixture models at each step of the EM algorithm, which would nontrivially increase the computational tractability of the model.

5.5. The Buyer's Problem

In each period, buyer i picks a decision that maximizes the present discounted value of her lifetime utilities. The set of these optimal decision is d^*, whose elements $d_t^*(x_{it}, \eta_i, s_i, \epsilon_{it})$ are

$$d_t^*(x_{it}, \eta_i, s_i, \epsilon_{it}) = \arg\max_d E_d\left(\sum_{k=t}^{T} \delta^{k-t}\left\{\left[u(d_{ik}, x_{ik}, s_i) + \epsilon_{ik}(d_{ik})\right]\middle| x_{it}, \eta_i, s_i, \epsilon_{it}\right\}\right), \tag{11}$$

where the expectation is taken over the future states induced by d_i^*. The value function at time t, $V(x_{it}, \eta_i, s_i, \epsilon_{it})$, is the expected present discounted value of lifetime utility from following d_i^* and is given by

$$V(x_{it}, \eta_i, s_i, \epsilon_{it}) = \max_d E_d\left(\sum_{k=t}^{T} \delta^{k-t}\left\{\left[u(d_{ik}, x_{ik}, s_i) + \epsilon_{ik}(d_{ik})\right]\middle| x_{it}, \eta_i, s_i, \epsilon_{it}\right\}\right). \tag{12}$$

By Bellman's principle of optimality, the value function can also be obtained using the recursion:

$$\begin{aligned} &V(x_{it}, \eta_i, s_i, \epsilon_{it}) \\ &= \max_{d_{it}} \Big\{ u(d_{it}, x_{it}, s_i) + \epsilon_{it}(d_{it}) \\ &\quad + \delta E_d[V(x_{it+1}, \eta_i, s_i, \epsilon_{it+1}) \mid d_{it}, x_{it}, \eta_i, s_i] \Big\}. \end{aligned} \tag{13}$$

Buyers face this dynamic optimization problem—waiting is costly but may fetch better bids in the future. For example, in the future the buyer may receive a low-priced bid from a high-reputation worker. However, she faces uncertainty over the number of bids she will receive in the future and the attributes of those bids.

6. Estimation

We face two key challenges in estimating the model—the size of the state-space and the presence of persistent unobserved heterogeneity. In general, high-dimensional state spaces are intractable with nested fixed-point algorithms and are estimated using computationally light two-step methods pioneered by Hotz and Miller (1993). However, our state space is intractable even with standard two-step methods. Moreover, two-step methods have traditionally suffered from their inability to account for persistent unobservables (Aguirregabiria and Mira 2010). Although Aguirregabiria and Mira (2007) have proposed a recursive two-step nested pseudo-likelihood estimator, it is not applicable here because it requires stationarity and involves large matrix inversions. We therefore adapt the two-step estimation framework recently developed by Arcidiacono and Miller (2011). Their methodology offers two key innovations that we implement:

- First, they generalize Altug and Miller (1998) and provide a framework to exploit finite dependence for a large class of problems, including those with correlated GEV errors that lead to nested logit probabilities (such as ours).
- Second, they present an EM-like algorithm that allows for finite unobserved types within a two-step estimator.

Applications of this framework have been limited. Murphy (2013), Beresteanu et al. (2010), and Ellickson et al. (2012) exploit finite dependence to perform value function reformulations to simplify computation in large state-space problems. Finger (2008) and Chung et al. (2013) employ the EM-like algorithm to accommodate persistent unobservables in two-step methods. The latter is a notable early application of this method in the marketing context. In our estimation, we exploit both aspects of this framework.

This estimation strategy has three important benefits in our context. First, given our high-dimensional state space, it makes estimation feasible without having to resort to artificial discretization of the state space or other state-space reduction methods (such as compressing all bid-related variables into one inclusive value as in Gowrisankaran and Rysman 2012). Second, even in our extremely large state space, it allows us to incorporate persistent unobserved heterogeneity.[12] Third, it is robust to misspecifications in buyers' future expectations far out in the future. For example, in our model, the number of bids that first-period dropouts expect to receive in future periods is based on estimates derived from the number of bids received by those who did not drop out. However, the expectations of these first-period dropouts could be very different. Hence, forecasts of future states are always susceptible to errors because they involve predictions of off-equilibrium paths that are never observed in the data. Because this estimation procedure only projects finite periods into the future (one in our case), it is robust to such misspecifications. Moreover, because it only simulates one period ahead, it is subject to less simulation bias, even in cases where the state space is sparsely populated. See Arcidiacono and Ellickson (2011) for details.

6.1. CCP Representation of the Problem Using Finite Dependence

We define the ex ante value function, $V'(x_{it}, \eta_i, s_i)$, as follows:

$$V'(x_{it}, \eta_i, s_i) = \int V(x_{it}, \eta_i, s_i, \epsilon_{it}) g(\epsilon_{it})\, d\epsilon_{it}. \quad (14)$$

$V'(x_{it}, \eta_i, s_i)$ is i's continuation value of being in state $\{x_{it}, \eta_i, s_i\}$, after integrating out ϵ_{it}. The continuation value gives us the choice-specific value function, $v(d_{it}, x_{it}, \eta_i, s_i)$, as

$$\begin{aligned} v(d_{it}, x_{it}, \eta_i, s_i) &= u(d_{it}, x_{it}, s_i) + \delta \int V'(x_{it+1}, \eta_i, s_i) \\ &\quad \cdot f(x_{it+1} \mid d_{it}, x_{it}, \eta_i, s_i)\, dx_{it+1} \\ &= u(d_{it}, x_{it}, s_i) + \delta \int V'(x_{it+1}, \eta_i, s_i) \\ &\quad \cdot f(x_{it+1} \mid d_{it}, x_{it}, \eta_i)\, dx_{it+1}. \end{aligned} \quad (15)$$

Here, $v(d_{it}, x_{it}, \eta_i)$ is i's present discounted value from choosing action d_{it} in period t (net of $\epsilon_{it}(d_{it})$) and following the set of optimal actions d_i^* from period $t+1$. The second line of Equation (15) follows from the fact that state transitions, $f(\cdot)$, are independent of s_i. Since choosing a bid and canceling are terminal options, there is no continuation value once either of these options are chosen. Thus,

$$\begin{aligned} v(\text{bid}_j, x_{it}, \eta_i, s_i) &= u(\text{bid}_j, x_{it}, s_i) \\ &= W_{ib}(x_{it}, s_i) + Y_{ij}(x_{it}), \end{aligned} \quad (16)$$

$$v(2, x_{it}, \eta_i, s_i) = u(2, x_{it}, s_i) = 0. \quad (17)$$

However, waiting does have a continuation value. Thus,

$$\begin{aligned} v(1, x_{it}, \eta_i, s_i) &= u(1, x_{it}, s_i) + \delta \int V'(x_{it+1}, \eta_i, s_i) \\ &\quad \cdot f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1} \\ &= W_{iw}(x_{it}) + \delta \int V'(x_{it+1}, \eta_i, s_i) \\ &\quad \cdot f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1}. \end{aligned} \quad (18)$$

We use these choice-specific value functions to derive the choice probabilities. Rust (1987) shows that, for GEV errors, there exist analytical relationships between choice probabilities and choice-specific value functions and that these choice probabilities are analogous to the relationships in static discrete choice models. Let $P(1 \mid x_{it}, \eta_i, s_i)$, $P(2 \mid x_{it}, \eta_i, s_i)$, and $P(\text{bid}_j \mid x_{it}, \eta_i, s_i)$ be the respective probabilities of waiting, canceling, and picking bid j, given state variables $\{x_{it}, \eta_i, s_i\}$. Then the GEV error structure gives the following nested logit probabilities:

$$P(1 \mid x_{it}, \eta_i, s_i) = \frac{e^{[v(1, x_{it}, \eta_i, s_i)]}}{1 + e^{[v(1, x_{it}, \eta_i, s_i)]} + e^{[W_{ib}(x_{it}, s_i) + \sigma I(x_{it})]}}, \quad (19)$$

$$P(2 \mid x_{it}, \eta_i, s_i) = \frac{1}{1 + e^{[v(1, x_{it}, \eta_i, s_i)]} + e^{[W_{ib}(x_{it}, s_i) + \sigma I(x_{it})]}}, \quad (20)$$

$$\begin{aligned} P(\text{bid}_j \mid x_{it}, \eta_i, s_i) &= \frac{e^{[W_{ib}(x_{it}, s_i) + \sigma I(x_{it})]}}{1 + e^{[v(1, x_{it}, \eta_i, s_i)]} + e^{[W_{ib}(x_{it}, s_i) + \sigma I(x_{it})]}} \\ &\quad \cdot \frac{e^{Y_{ij}(x_{it})/\sigma}}{\sum_{q=3}^{J_{it}} e^{Y_{iq}(x_{it})/\sigma}}, \end{aligned} \quad (21)$$

where $I(x_{it}) = \ln[\sum_{q=3}^{J_{it}} e^{Y_{iq}(x_{it})/\sigma}]$ is the inclusive value of the bid nest. Using these analytical expressions, we can estimate the model primitives, $\{\alpha, \beta, \delta\}$, if we can compute $\int V'(x_{it+1}, \eta_i, s_i) f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1}$ and

[12] Note that even if we could reduce the state space somewhat, full solution models are still infeasible in our setting because they would require us to embed a standard nested fixed-point algorithm inside the EM loop, which is highly computationally expensive. Indeed, EM algorithms are known to be slow even in nondynamic settings.

integrate out the unobserved state variable s_i. Arcidiacono and Miller (2011) show that, for GEV errors, there exists a simple analytical relationship between the continuation value $V'(x_{it}, \eta_i, s_i)$ and the conditional choice probabilities. In our setting, we can express $V'(x_{it}, \eta_i, s_i)$ as

$$\begin{aligned} V'(x_{it}, \eta_i, s_i) &= \gamma + v(2, x_{it}, s_i) - \ln(p(2 \mid x_{it}, \eta_i, s_i)) \\ &= \gamma - \ln(p(2 \mid x_{it}, \eta_i, s_i)), \end{aligned} \tag{22}$$

where γ is the Euler's constant and $p(2 \mid x_{it}, \eta_i, s_i)$ is the conditional probability of choosing the terminal action (cancel) given observed state variables x_{it} and the unobserved state variables $\{\eta_i, s_i\}$. Because of the finite dependence in the model, the choice-specific value function for cancel has no continuation value, and $\int V'(x_{it+1}, \eta_i, s_i) f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1}$ simplifies to

$$\begin{aligned} &\int V'(x_{it+1}, \eta_i, s_i) f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1} \\ &= \int [\gamma - \ln(p(2 \mid x_{it+1}, \eta_i, s_i))] f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1}. \end{aligned} \tag{23}$$

To evaluate the right-hand side of Equation (23), we only need one-period-ahead CCPs, $p(2 \mid x_{it+1}, \eta_i, s_i)$, and state transition probabilities, $f(x_{it+1} \mid 1, x_{it}, \eta_i)$.

The choice of "cancel" as the base option helps in computation because $v(2, x_{it}, s_i) = u(2, x_{it}, s_i) = 0$. So we do not need estimates of the structural parameters associated with $u(2, x_{it}, s_i)$ to obtain numerical estimates of $\int V'(x_{it+1}, \eta_i, s_i) f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1}$. On the other hand, if we had chosen "wait" as the base option, we would need to use updated estimates of $u(2, x_{it}, s_i)$ at each step of the EM to derive $\int V'(x_{it+1}, \eta_i, s_i) f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1}$, which is computationally more intensive.[13]

6.2. Log-Likelihood and Estimation Outline

Let π_k be the population probability of a buyer being of unobserved type k, where $k \in \{1, \dots, S\}$. Because we have the full history for each auction, we do not condition the initial distribution of types on observed state variables.[14] The joint log-likelihood of i's decision d_{it} and observed state variables x_{it}, conditional on x_{it-1} and unobserved state variables $\{\eta_i, s_i\}$ is given by

$$\begin{aligned} &L_i(d_i, x_i \mid \alpha, \beta, \delta, \theta) \\ &= \ln\Bigg[\sum_{k=1}^{S} \pi_k \Bigg[\prod_{t=1}^{T_i} [\Pr(d_{it} \mid x_{it}, \eta_i, s_i = k)^{I(d_{it})} \\ &\qquad \cdot f(x_{it} \mid d_{it-1}, x_{it-1}, \eta_i)]\Bigg]\Bigg], \end{aligned} \tag{24}$$

where θ is the set of parameters and the nonparametric distributions associated with the state transitions; i.e., $\theta = \{\theta_p, \theta_g, \theta_t, \psi_c\}$, $\Pr(d_{it} \mid x_{it}, \eta_i, s_i)$ are the choice probabilities for alternatives available to i at period t, and $I(d_{it})$ is an indicator variable that is 1 if d_{it} is the observed choice in the data. Note that we have integrated out the unobserved state variable s_i from the log-likelihood but not η_i because η_i can be consistently estimated from the data. (See §6.3.1 for details.) Since state transitions are independent of s_i after accounting for η_i, we can write Equation (24) as

$$\begin{aligned} &L_i(d_i, x_i \mid \alpha, \beta, \delta, \theta) \\ &= \ln\Bigg[\sum_{k=1}^{S} \pi_k \prod_{t=1}^{T_i} \Pr(d_{it} \mid x_{it}, \eta_i, s_i = k)^{I(d_{it})}\Bigg] \\ &\quad + \ln\Bigg[\prod_{t=1}^{T_i} f(x_{it} \mid d_{it-1}, x_{it-1}, \eta_i)\Bigg]. \end{aligned} \tag{25}$$

Because the log-likelihoods of state transitions and observed choices are additively separable, they can be maximized separately as follows.

- *Estimate state transitions*: First, we estimate the Poisson bid arrival process and also obtain consistent estimates, $\hat{\eta}_i$, of η_i. Next, we estimate the nonparametric distributions of bids, the multinomial logit model of seller's geographic region, and the logit model of buyer–seller past interaction. Because these models do not depend on s_i, they can be consistently estimated at the first stage.
- *Augmented two-step EM estimator*: Recursively compute and update the CCPs, π_k, structural parameters $\{\alpha, \beta\}$, and discount factor δ. At this stage, estimates from the state transition models are used to calculate the future continuation values in conjunction with CCPs.

[13] While we have represented $V'(x_{it}, \eta_i, s_i)$ as a function of the terminal action cancel, it can also be represented as a function of the terminal action of choosing a bid; i.e., we can write it as $V'(x_{it}, \eta_i, s_i) = \gamma + v(\text{bid}_j, x_{it}, s_i) - \sigma \ln(p(\text{bid}_j \mid x_{it}, \eta_i, s_i)) - (1-\sigma)\ln(\sum_{q=3}^{J_{it}} p(\text{bid}_q \mid x_{it}, \eta_i, s_i))$. However, since $v(\text{bid}_j, x_{it}, s_i)$ and σ are unknown prior to the second-stage estimation, this would require us to substitute the updated values for these expressions at each iteration of the EM algorithm, which increases computational time and complexity. In contrast, specifying $V'(x_{it}, \eta_i, s_i)$ as a function of a terminal action that is part of a singleton nest, and whose flow utility has been normalized to zero, simplifies the computation considerably. We are now only required to substitute the updated values of CCPs at each step of the EM algorithm.

[14] All buyers start with the same set of observed time-varying state variables; i.e., everyone starts at time $t = 1$ and zero bids. However, we could still condition the initial distribution of types on buyer-specific state variables. The results from such an expanded model (available from the author upon request) are not very different from those presented here.

Without persistent unobserved heterogeneity s_i, we could have used a standard two-step estimator by estimating CCPs along with the state transition models as functions of x_{it} and $\hat{\eta}_i$. (Because η_i does not affect flow utilities, a consistent estimate, $\hat{\eta}_i$, of η_i is available after the estimation of the Poisson bid arrival process.) However, the inclusion of s_i necessitates the use of the EM algorithm, since we can no longer obtain consistent estimates of CCPs from the data (s_i and π_k are unknown).

Estimating the model in stages does not affect the consistency of the results (Rust and Phelan 1997, Rothwell and Rust 1997), although it does lead to lower standard errors for structural parameters because estimates of state transition probabilities and CCPs are treated as data in the estimation. Hence, we bootstrap these standard errors.[15] We now describe each estimation step in detail.

6.3. Estimation of State Transitions: Models of Bid Arrival and Bid Attributes

6.3.1. Estimation of Fixed Effects Poisson. The set of parameters to be estimated is $\theta_1 = \{\theta_p, \eta_1, \ldots, \eta_N\}$, which includes the fixed effects (η_i). We use the maximum likelihood approach to estimate θ_1. Below, we present an overview of the estimation and refer interested readers to Winkelmann (2003) for details.

Let b_i be the vector of the number of bids received by i, where $b_i = (b_{i1}, b_{i2}, \ldots, b_{iT_i})$, and T_i is the last period in which the auction is still active. For example, if i cancels her auction two days after posting it, then $T_i = 2$. Equation (26) gives us the conditional log-likelihood contribution of i as

$$L_{ip}(b_{i1}, \ldots, b_{iT_i} \mid \theta_p, \eta_i) = -\eta_i \sum_{t=1}^{T_i} \lambda_t + \ln \eta_i \sum_{t=1}^{T_i} b_{it} + \sum_{t=1}^{T_i} b_{it} \ln \lambda_t - \sum_{t=1}^{T_i} (\ln b_{it}). \quad (26)$$

Setting the first derivative of $L_{ip}(b_{i1}, \ldots, b_{iT_i} \mid \theta_p, \eta_i)$ to zero, we have $\hat{\eta}_i = \sum_{t=1}^{T_i} b_{it} / \sum_{t=1}^{T_i} \lambda_t$. Substituting this back into Equation (26) gives us a conditional log-likelihood, $L_{ip}(b_{i1}, \ldots, b_{iT_i} \mid \theta_p)$, which is independent of η_i. ($\sum_{t=1}^{T_i} b_{it}$ is a sufficient statistic for η_i.) Then, the conditional log-likelihood of all the bid arrivals observed in the data is given by

$$L_p(\theta_p) = \sum_{i=1}^{N} L_{ip}(b_{i1}, \ldots, b_{iT_i} \mid \theta_p, \hat{\eta}_i). \quad (27)$$

We then maximize $L_p(\theta_p)$ to estimate θ_p. Once we have a consistent estimate, $\hat{\theta}_p$, of θ_p, we can use it to consistently estimate η_i using $\hat{\eta}_i = \sum_{t=1}^{T_i} b_{it} / \sum_{t=1}^{T_i} \hat{\lambda}_t$, where $\hat{\lambda}_t = \exp(z_t' \hat{\theta}_p)$.

6.3.2. Estimation of Nonparametric Joint Distributions. To obtain the nonparametric joint distributions, we first estimate the optimal bandwidth h_c $\forall c \in \mathscr{C}$ using likelihood cross-validation (Duin 1976, Silverman 1986). Let $\hat{\psi}_c(h, A)$ and $\hat{\psi}_{c,-q}(h, A)$ be the probability density function estimate of point A from the cth category using bandwidth h and data sets $\{A_1, \ldots, A_{M_c}\}$ and $\{A_1, \ldots, A_{q-1}, A_{q+1}, \ldots, A_{M_c}\}$, respectively. Then the cross-validation score of h for category c is given by averaging the log-likelihood $\hat{\psi}_{c,-q}(h, A)$ over all q:

$$CV_c(h) = M_c^{-1} \sum_{q=1}^{M_c} \ln[\hat{\psi}_{c,-q}(h, A)]. \quad (28)$$

The likelihood cross-validation choice of the optimal bandwidth is the value that maximizes $CV_c(h)$. Intuitively, the cross-validation score $CV_c(h)$ is the log-likelihood of observing the data set. The probability of drawing the data point A (assuming it is not part of the data set) is $\hat{\psi}_{c,-q}(h, A)$. So $M_c^{-1} \Pi_{q=1}^{M_c} \hat{\psi}_{c,-q}(h, A)$ is the total probability of observing the data set.

Although the maximization is conceptually simple, it is computationally intensive. To evaluate the likelihood at a given bandwidth, we need to evaluate the density at each data point at that bandwidth and then sum over the density contributions of all data points. Moreover, at each data point, we need to find the kth-nearest point in the Euclidean space to calculate its density contribution. This becomes prohibitively expensive as the size of the data set and the number of dimensions increase. Moreover, this has to be done multiple times to reach the optimal h_c. Many algorithms have been proposed to address these computational issues, but we follow the recent method proposed by Gray and Moore (2003), which is based on *k-dimensional trees* and has been shown to be much faster than previous methods. We use the MATLAB-based kernel density estimation (KDE) toolbox to perform the estimation (Ihler 2003).

6.3.3. Estimation of Logit Models of Seller Region and Past Buyer–Seller Interaction. The estimation of the multinomial logit model of seller region (bg_{ij}) and the binary logit model of past buyer–seller interaction indicator (bt_{ij}) are straightforward. The log-likelihood of drawing the sellers' geographic regions observed in the data is

$$L_g(\theta_g) = \sum_{i=1}^{N} \sum_{j=3}^{J_{iT_i}} \sum_{q=1}^{4} \ln[h_g(bg_{ij} = q \mid gx_{ij}, \theta_g)^{I(bg_{ij}=q)}], \quad (29)$$

[15] We sample on auctions and use 250 replications in our bootstrap procedure.

where $J_{iT_i} - 2$ is the total number of bids that i has accumulated in the last period that she is active (T_i). Similarly, the log-likelihood of the buyer–seller interactions observed in the data is given by

$$L_b(\theta_t) = \sum_{i=1}^{N} \sum_{j=3}^{J_{iT_i}} \sum_{q=1}^{2} \ln[h_b(bt_{ij} = q \mid tx_{ij}, \theta_t)^{I(bt_{ij}=q)}]. \quad (30)$$

Maximizing the above log-likelihoods gives us consistent estimates of θ_g and θ_t.

6.4. Two-Step EM Estimator

In the second stage, we estimate the CCPs, the population probabilities of unobserved types (π_k), and the structural parameters and discount factor $\{\alpha, \beta, \delta\}$. The first part of the log-likelihood from Equation (25) suggests the maximization:

$$(\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\pi}) = \arg\max_{\alpha,\beta,\delta,\pi} \sum_{n=1}^{N} \ln\left[\sum_{k=1}^{S} \pi_k \prod_{t=1}^{T_i} \Pr(d_{it} \mid x_{it}, \eta_i, s_i = k)^{I(d_{it})}\right]. \quad (31)$$

Dempster et al. (1977) note that the first-order condition for the above maximization problem is the same as that of the following maximization if $\rho(k \mid d_i, x_i; \hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\pi})$ were treated as known:

$$(\hat{\alpha}, \hat{\beta}, \hat{\delta}) = \arg\max_{\alpha,\beta,\delta} \sum_{n=1}^{N} \prod_{t=1}^{T_i} \sum_{k=1}^{S} \rho(k \mid d_i, x_i; \hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\pi}) \cdot \ln[\Pr(d_{it} \mid x_{it}, \eta_i, s_i = k)^{I(d_{it})}]. \quad (32)$$

Here, $\rho(k \mid d_i, x_i; \hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\pi})$ is the posterior probability of i belonging to unobserved type k given data (d_i, x_i), population-type estimates $\hat{\pi}$, and structural parameter estimates $\{\hat{\alpha}, \hat{\beta}, \hat{\delta}\}$. Since ρ is unknown, Dempster et al. (1977) propose a recursive EM algorithm that starts with a set of assumed structural parameters, based on which ρ's are updated, which in turn are substituted back into the maximization problem in Equation (32) to obtain a new set of parameters. This process is repeated till the parameters and π's converge.

In a standard finite-mixture setting, this is relatively straightforward to implement. However, in a dynamic setting, the choice probabilities, $\Pr(d_{it} \mid x_{it}, \eta_i, s_i)$, are functions of the unknown continuation values through $\int V'(x_{it+1}, \eta_i, s_i) f(x_{it+1} \mid 1, x_{it}, \eta_i)\, dx_{it+1}$, which have to be calculated using the analytical expression shown in Equation (23). To do so, we first need estimates of the CCP of cancellation, $p(2 \mid x_{it+1}, \eta_i, s_i)$, which cannot be directly obtained from the data since s_i are unknown. Arcidiacono and Miller (2011) propose an expanded version of the EM algorithm where the CCPs are also updated at each step of the EM algorithm. We follow their approach. In §1 of the Web appendix, we provide the step-by-step details of the estimation process.

6.5. Identification

6.5.1. Identification of CCPs, State Transitions, and Population Distribution of Types. Nonparametric identification of CCPs, state transitions, and the population distribution of types is an important prerequisite for using the two-step methods. Whereas persistent unobserved heterogeneity can lead to nonidentification, Kasahara and Shimotsu (2009) and Arcidiacono and Miller (2011) show that in many cases identification can be restored. Specifically, they prove that if all states are reachable after all decisions (i.e., unconstrained state-space evolution), then the number of decision-state sequences available for identification expands exponentially with time and state-space size. In this context, they show that data covering three or more time periods are sufficient for nonparametrically identifying CCPs and population probabilities.

However, in our setting, all states are not reachable after all decisions. Waiting is the only continuation decision in our model and the only decision after which the state space can change. Moreover, once a buyer has received a certain number of bids, she cannot go back to fewer bids or change the attributes of the bids she has already received. This constrains state-space evolution and restricts the number of decision-state sequences that are observable in the data. So the general proofs by Kasahara and Shimotsu (2009) and Arcidiacono and Miller (2011) are not directly applicable to our setting.[16] However, we do have a large number of time periods, the number of bids that a buyer can receive is high, and for a given number of bids, the possible combinations of bid attributes is very large. Therefore, we still have sufficient observed decision-state sequences to identify CCPs and state transitions.

In §2 of the Web appendix, we present a formal proof. Here, we provide some intuition on how our CCPs are identified. Since state space advances only through wait decisions, by default, CCPs of waiting

[16] We present a small example to demonstrate the differences in the identification of problems with constrained state-space evolution and those without constraints. Consider a general case with 10 possible states and 5 possible decisions that the agent can make at any point in time. Let each decision be a continuation decision such that the agent can transition into any of the states in the next period following any decision. Then, with T periods, the number of observable decision-state sequences is $(10 \times 5)^T$. If the number of CCPs and state transitions that we need to identify is less than this number, then we have ensured that the first-stage estimates are nonparametrically identified. Now consider a modification of the previous case so that only one decision is a continuation decision and the rest are terminal. Then the number of decision-state sequences is $\Sigma_{t=1}^{T} 10^t$. This is considerably lower than $(10 \times 5)^T$. Hence, constraints on the state-space evolution can interfere with the nonparametric identification of first-stage estimates and should be verified for each application setting.

appear in many decision-state sequences and are easily identified; i.e., a buyer in a given state could have jumped into it in many ways, and with the wait decision, can also jump out of it in many ways. However, termination CCPs (i.e., the CCPs of canceling and choosing a bid) appear only in sequences that end in the current state and action. Hence, one might suspect that they are not identified. However, that is not the case because the same terminal decision-state combination can be reached in many ways. For example, the CCP of canceling for a buyer of a specific unobserved type with three bids in period 2 will appear in all of the following sequences: (a) buyer receives zero bids in $t = 1$, waits, and receives three in $t = 2$; (b) buyer receives one in $t = 1$, waits, and receives two in $t = 2$; (c) buyer receives two in $t = 1$, waits, and receives one in $t = 2$; and (d) buyer receives three in $t = 1$, waits, and receives zero in $t = 2$. Note that although all these sequences end with the same state and terminal decision, there are still enough sequences for identification. Of course, with bid attributes, the number of such sequences increases even more. This allows terminal CCPs to be identified.

Nevertheless, we do not have sufficient degrees of freedom to nonparametrically identify more than two types in this context. We formally show why this is the case in §2 in the Web appendix. Thus, in our application, we only allow for two unobserved types.

6.5.2. Identification of Discount Rate and Utility Parameters. The discount factor is identified through exclusion restrictions (Rust 1994, Magnac and Thesmar 2002). The most important restriction we employ is the exclusion of time from flow utility; i.e., time affects state transitions through bid arrival rates but not flow utilities. Apart from time, buyer/auction fixed effects (η_i) also help in identifying the discount factor since η_i affect bid arrival rates but are not included in flow utilities. (The first- stage estimate, $\hat{\eta}_i$, of η_i is treated as known in the second stage.) Note that part of the variation in η_i is explained by buyer- and auction-specific variables that are also included in the flow utility (e.g., buyer reputation, MaxBid, unobserved auction type s_i), and hence this variation cannot contribute to the identification of the discount factor. However, we find that even after accounting for observed time-invariant buyers, auction-specific state variables, and s_i, a significant amount of variation in η_i still remains unexplained. It is this remnant variation that acts as an exclusion restriction (i.e., it affects bid arrivals but not flow utility) and helps in identifying the discount factor.

Utility parameters associated with observed buyer and bid attributes are identified as usual—from the variation in data and buyers' choices—so we do not go into their details. Instead, we focus on unobserved types (s_i), which are identified through the dynamics of the model, realizations of bid arrivals, bid attributes, and buyers' decisions. For example, a buyer with high unobserved preference for bids may get only one bid in period 1 and pick it right away, whereas a buyer with low unobserved preference for bids may get many bids over time and repeatedly wait. Consider two buyers who are identical on $(x_{it}, \hat{\eta}_i)$ and receive similar bids in the first period. Suppose one picks a bid at $t = 1$ and the other continues for two weeks without picking bids. Without unobserved heterogeneity, both of these buyers would have the same predicted bid choice probabilities. However, with unobserved heterogeneity, we learn more about both buyers—the first buyer has a clear preference for choosing bids, either because the bids she receives are of high unobserved quality or because she has no good outside options, and the second buyer clearly has low inherent preference for choosing bids, either because her bids are of low unobserved quality or because she has good outside options. These kinds of variations in the data help identify types.[17]

7. Results

7.1. State Transition Estimates: Bid Arrival and Bid Attributes

We now discuss a few highlights from the first-step results. We refer interested readers to Tables 2–4 in the Web appendix for details on the parameter estimates.

First, in the context of the Poisson model, our estimates suggest that bid arrivals slow down considerably with time. We also find that there is significant variation in the estimated fixed effects, $\hat{\eta}_i$. This affirms the importance of including auction-specific unobserved heterogeneity (η_i) in our bid arrival model. Overall, we find that the Poisson model captures the patterns very well. Second, from the multinomial logit model of seller region, we find that bid price and many of the bidder attributes are correlated with the bidder's geographic region. A seller's geographic region is also influenced by the geographic region of the buyer and her reputation. For example, everything else being constant, buyers are more likely to attract sellers from their own region. This effect likely stems from lower communication costs and similarities in IP restrictions within a region. Third, the estimates from

[17] Note that allowing for unobserved heterogeneity in flow utilities across buyers through s_i and holding discount rates constant across buyers is analogous to allowing for unobserved heterogeneity in discount rates and keeping flow utilities independent of persistent unobservables. So buyers who exhibit an aversion to picking bids (type $s_i = 1$; see Table 8) can be interpreted as patient buyers, and vice versa. In other words, unobserved heterogeneity in buyers' patience and preference for bids cannot be separately identified in our setting.

Table 8 Estimates of Structural Parameters

		Model 1		Model 2		Model 3	
Coefficients varying within the bids' nest		Coefficient	Std. error	Coefficient	Std. error	Coefficient	Std. error
β_1	ln(*Bidprice* + 1)	−0.5852	0.0255	−0.2744	0.0063	−0.6513	0.0166
β_2	*No. seller ratings* = 0 (indicator)	0.2660	0.1824	0.0738	0.0779	0.1751	0.188
β_3	ln(*No. of seller ratings* + 1)	0.1131	0.0094	0.0508	0.0040	0.1205	0.0095
β_4	*Seller mean rating* (centered)	0.0994	0.0244	0.0448	0.0107	0.1063	0.0249
β_5	*Squared seller mean rating* (centered)	0.0065	0.0037	0.0035	0.0017	0.0083	0.0038
β_6	*Seller mean rating* (centered) × ln(*No. of seller ratings* + 1)	0.0709	0.0090	0.0321	0.0039	0.0762	0.0099
β_7	*MaxBid* × ln(*No. of seller ratings* + 1)	−0.0001	0.0001	−0.0000	0.0000	−0.0001	0.0001
β_8	*Buyer mean rating* × *Seller mean rating* (centered)	0.0012	0.0004	0.0006	0.0002	0.0013	0.0004
β_9	*Indicator for i and j past interaction*	0.7650	0.0400	0.3337	0.0155	0.792	0.0272
β_{10}	*Seller region* = 1	−0.0541	0.0413	−0.0202	0.0189	−0.0480	0.0434
β_{11}	*Seller region* = 2	0.2792	0.0557	0.1268	0.0249	0.3010	0.0548
β_{12}	*Seller region* = 3	0.0926	0.0163	0.0415	0.0085	0.0986	0.0176
β_{13}	*Seller region* = 1 and *Buyer region* = 2	0.0098	0.0580	−0.0053	0.0283	−0.0126	0.0578
β_{14}	*Seller region* = 2 and *Buyer region* = 2	−0.0344	0.0445	−0.0202	0.0212	−0.0480	0.0455
β_{15}	*Seller region* = 3 and *Buyer region* = 2	−0.1980	0.0522	−0.0883	0.0273	−0.2095	0.0591
β_{16}	*Seller region* = *Buyer region* ≠ 2	0.1390	0.0352	0.0609	0.0135	0.1446	0.0279
	Coefficients common across nests						
Wait nest							
α_{w1}	*Constant*			2.3629	0.1784	−0.5042	0.1811
α_{w2}	*MaxBid*			0.0031	0.0006	0.0024	0.0007
α_{w3}	ln(*Sum of buyer ratings* + 1)			−0.1671	0.0154	−0.0360	0.0152
α_{w4}	*No. of previous auctions* = 0 (indicator)			−1.0086	0.1147	−0.1018	0.1140
α_{w5}	*No. of uncanceled past auctions*			−0.0003	0.0004	−0.0006	0.0003
α_{w6}	*Cancel ratio*			−1.0721	0.1004	−0.0818	0.1000
α_{w7}	*Buyer tenure on site* (in years)			0.0323	0.0132	0.0058	0.0132
α_{w8}	*Buyer region* = 1			−0.2659	0.0800	−0.0722	0.0812
α_{w9}	*Buyer region* = 2			0.0038	0.0620	−0.0202	0.0629
α_{w10}	*Buyer region* = 3			−0.0488	0.0441	−0.0415	0.0448
Bids nest							
α_{b1}	*Constant*	5.2908	0.3329	3.9790	0.1232	3.0548	0.1310
α_{b2}	*MaxBid*	0.0025	0.0009	−0.0028	0.0007	−0.0028	0.0008
α_{b3}	ln(*Sum of buyer ratings* + 1)	−0.3275	0.0186	−0.2449	0.0153	−0.2629	0.0161
α_{b4}	*No. of previous auctions* = 0 (indicator)	−2.7496	0.1326	−2.1635	0.1140	−3.0430	0.1212
α_{b5}	*No. of of uncanceled past auctions*	0.0023	0.0006	0.0013	0.0004	0.0012	0.0003
α_{b6}	*Cancel ratio*	−3.1816	0.1121	−2.7108	0.0958	−3.6312	0.1042
α_{b7}	*Buyer tenure on site* (in years)	0.0715	0.0152	0.0717	0.0014	0.1164	0.0152
α_{b8}	*Buyer region* = 1	−0.6953	0.1046	−0.6520	0.0954	−0.9982	0.1043
α_{b9}	*Buyer region* = 2	0.3869	0.0840	0.3817	0.0700	0.6210	0.0764
α_{b10}	*Buyer region* = 3	−0.0600	0.0523	−0.0444	0.0489	−0.0080	0.0531
α_{b11}	$s_i = 2$ (inidcator)					3.5232	0.0436
π_1	*Prob. of high type*					0.2926	0.0121
δ	*Discount factor*			0.2960	0.0292	0.8823	0.0314
σ	*Nest correlation*	0.4177	0.0169	0.1918	0.0094	0.4553	0.0126

Note. $N = 13,550$.

the logit model for the indicator of past buyer–seller interaction suggest that only a small percentage of sellers are likely to have interacted with the buyer. However, the probability of drawing such sellers is higher for buyers who specify higher MaxBids, have initiated a large number of uncanceled auctions in the past, and have a good reputation. Similarly, sellers who belong to regions 1 and 2, quote slightly higher prices than average, and have a good reputation on the site are more likely to have interacted with the buyer in the past.

Finally, we discuss our estimates of the nonparametric joint distributions of bid price, number of bidder ratings, and bidder average rating. There are no parametric results in this context except the bandwidths (h_c) for the 40 subclasses. Since these bandwidths are not very informative in and of themselves, we do not present them here. However, we note that the kernel density estimates are, in general, very good at approximating the joint distributions of these bid attributes; Kolmogorov–Smirnov tests comparing the equality of the original and estimated distributions for all of the subclasses confirm them to be indistinguishable.

7.2. CCP Estimates

Because of the size of the state space and the presence of a large number of continuous state variables, CCPs are estimated using flexible logits. We include all of the state variables, their higher-order terms, and interactions in our flexible logit. Use of flexible logits to model CCPs has precedence in the literature (Arcidiacono and Miller 2011, Fang and Wang 2013, Murphy 2013). Because of the large number of CCP parameters and the difficulty in interpreting them, we do not present the CCP estimates here. Instead, we highlight the main effects. A buyer's probability of canceling an auction increases concavely with the prices of the bids received. However, it decreases with the mean ratings of those bidders, the MaxBid of the auction, the number of bids received so far, and the number of past ratings received by the buyer. The probability of cancellation also varies with unobserved buyer type and time.

7.3. Structural Parameter Estimates

The estimates of the structural parameters associated with buyers' utility are presented in Table 8. The first column (Model 1) presents a static model that ignores both dynamics and persistent unobservables. The second column (Model 2) presents a model with dynamics but without the unobserved state variable s_i. Here, we do not control for persistent unobserved heterogeneity in buyers' taste for bids or the average quality of bids received by a buyer. In the third column, we consider a model (Model 3) with two unobserved types $s_i \in \{1, 2\}$. Unless specified otherwise, we discuss the results from Model 3 throughout.

7.3.1. Bottom-Level Utility Parameters. The top set of rows in Table 8 shows the estimates for coefficients that vary within the bid nest. First, we find that buyers' utility from bids is decreasing in price, although concavely; i.e., the same price increase is less painful at higher prices. Next, we find that sellers with more ratings are more attractive; i.e., the coefficient of $\ln(bn_{ij} + 1)$ is positive. This suggests that buyers value the site's reputation system—they prefer sellers who have been in the system for some time and on whom information, in the form of past ratings, is available. Note that this positive effect is consistent with the cheap-identity problem prevalent in Internet marketplaces. Sellers with few past ratings are not only untested (and therefore less trustworthy) but are also more likely to be low-quality sellers who have simply reappeared as new after milking their old reputations. However, the marginal value of each additional rating is decreasing, possibly because the new information in each new rating decreases as the number of ratings increases. This effect is consistent with "imperfect monitoring" because the number of reviews would be irrelevant if reviews were perfectly informative. (We also tried other functions of number of ratings and found the fit of the model with $\ln(bn_{ij} + 1)$ to be the best.)

Buyers also derive value from the mean rating of the seller (i.e., the coefficient of $(br_{ij} - \overline{br})$ is positive). If the seller's mean rating is higher than that of the average seller in the marketplace, then the buyer derives a positive value; otherwise, the buyer derives a negative value. Note that this effect suggests that buyers adjust for the "rating inflation" problem, commonly observed in online reputation systems. The squared term of the centered mean rating term also has a positive effect. That is, any increase in a seller's mean rating above the mean $(\overline{br})$ has a convex benefit, whereas the opposite is true for decreases below the mean rating. Moreover, the interaction of the natural log of the number of ratings and the centered mean rating, $(br_{ij} - \overline{br}) \ln(bn_{ij} + 1)$, is also positive. This implies that buyers' valuation of a marginal change in a seller's rating (over the population mean) is proportional to the natural log of the number of ratings she has received. That is, a seller with a mean rating of 9.5 and 20 ratings is valued less than a seller of mean rating 9.5 and 25 ratings, even after controlling for the main effects of the number of ratings. Again, this effect reflects the fact that ratings are noisy.

MaxBid has no impact on a buyer's valuation for seller reputation. If we interpret MaxBid as project size or a buyer's value for the project, then we can conclude that buyers do not necessarily place more value on seller reputations for larger or more important projects. A buyer's own reputation also does not influence her valuation of sellers' reputations. Buyers have a strong preference for sellers with whom they have worked in the past. In principle, the direction of this effect could go either way, depending on whether the previous interaction went well or not. However, in the data, we find that sellers usually avoid buyers who gave them low ratings previously. So the indicator for previous interaction almost always indicates a "good" previous interaction. The strong positive effect is then understandable since the information asymmetry problem is alleviated when the buyer has traded with the seller in the past and liked her.

Finally, we find that buyers prefer sellers from developed countries the most, followed by those from eastern European countries. Buyers in developing countries have a small preference for sellers from their own geographic regions, possibly because such sellers might be easier to communicate with and/or have a better understanding of the local intellectual property regulations.

7.3.2. Top-Level Utility Parameters. The two bottom sets of rows in Table 8 show the top-level coefficients for the wait and bid nests. These coefficients represent the relative attractiveness of the two nests when compared with canceling.

We find that buyers' flow utility from waiting is increasing in MaxBid and decreasing in the number of buyer ratings. Buyers who are new to the site, those who have few ratings, those who have few uncanceled auctions, and those with high cancel ratios receive lower utility from picking a bid. This is possibly because, after canceling, buyers with a good reputation and history in this specific freelance community have higher costs of finding alternative workers from outside sources. Furthermore, buyers from region 1 are less likely to choose a bid and those from region 2 are more likely. This might be due to local labor costs, which are high in region 2 and low in region 1.

We also find that the high types ($s_i = 2$) have high unobserved preference for bids compared with the low types ($s_i = 1$). The two types are distributed in the ratio of about 3:7, with the high type being in the minority.

7.3.3. Nest Correlation and Discount Factor. The last two rows in Table 8 show the estimates of the bid nest's correlation parameter σ and the discount factor δ. The nesting parameter σ in Model 3 is 0.4553, significantly less than 1, which suggests that the unobservable preferences for bids, ϵ_{ijt}, have a component that is correlated across the bid options. Moreover, note that the addition of persistent unobserved heterogeneity in buyers' taste for bids, in the form of s_i, reduces the contemporaneous correlation between the ϵ_{ijt}. This suggests that σ was picking up some of the persistence in buyers' preference for bids in Model 2.

The model without persistent unobserved heterogeneity in preferences (Model 2) attributes the early exit of a large fraction of buyers to impatience and significantly underestimates the discount factor. This underestimation is rectified in Model 3, where the daily discount factor is estimated to be 0.8823. Although this seems reasonable for this context, it is lower than the discount rate implied by the yearly interest rate. Given the size of these jobs (at most worth \$100), this level of impatience is understandable—it is relatively easy for a buyer to complete the job herself in a few hours.[18] Our findings highlight the importance of empirically estimating the discount factor, especially in less conventional nonmonetary settings, where traditional discount factors calculated from interest rates are unlikely to be applicable. Please see Frederick et al. (2002) for an exhaustive review of experimental studies on time discounting. Dubé et al. (2012), Chung et al. (2013), and Yao et al. (2012) also provide excellent discussions on the magnitude of estimated discount factors using field and survey data.

[18] As a robustness check, we also estimated a version of the model with the discount factor fixed at one and confirmed that reputation effects are qualitatively similar to those presented here.

8. Validation

We now compare the fit and performance of our model with two inferior models: a static model and a dynamic model without persistent unobserved heterogeneity.

In the static model, only the final outcomes are analyzed—cancel or choose a submitted bid (Model 1 in Table 8). We find that the static model considerably overpredicts cancellation for early deciders (by 49.92% for cohort E1) and underpredicts it for late deciders (by 44.9% for cohort E13). See Table 9 for details. As mentioned in §1, there are two issues here—dynamics and self-selection. First, buyers who choose to exit early are likely to have drawn a very good set of bids, making them less likely to cancel. In contrast, those who wait are likely to have drawn a poor set of bids and therefore are more likely to cancel. Second, buyers who wait longer are a self-selected group; their repeated decisions to not choose a bid indicate that they have good unobserved outside options and/or an inherently low taste for bids. Hence, when they do exit, they are more likely to cancel. Because the static model cannot account for these factors, it does a poor job of predicting the realized outcomes in the data.

Next, in Table 10, we present the fit of two dynamic models, with and without persistent unobserved heterogeneity (Models 2 and 3 from Table 8, respectively). Actual and model-predicted probabilities of canceling and choosing a bid for surviving buyers for each period are shown. To calculate the

Table 9 **Model Fit for the Static Model: Sample and Model-Predicted Probabilities of Canceling**

	Probability of canceling				
	Full data	Cohort E1	Cohort E5	Cohort E9	Cohort E13
Data	0.2177	0.1320	0.3459	0.4689	0.5321
Prediction	0.2177	0.1979	0.2363	0.2786	0.2932

Notes. Cohort Et refers to the set of buyers who *exited* the auction in period t by choosing a bid or canceling. Note that the probability of bidding = 1 − probability of canceling.

Table 10 Fit of Dynamic Models Without and With Unobserved Heterogeneity

Cohort (no. of obs.)	Probability of canceling			Probability of bidding		
	Data	One type	Two types	Data	One type	Two types
S1 ($N = 13{,}350$)	0.0721	0.0724 (0.0003)	0.0532 (0.0189)	0.4921	0.3213 (0.1708)	0.4461 (0.046)
S2 ($N = 6{,}076$)	0.0718	0.0744 (0.0026)	0.0647 (0.0071)	0.2572	0.2893 (0.0321)	0.2949 (0.0377)
S3 ($N = 4{,}114$)	0.0702	0.0743 (0.004)	0.0668 (0.0034)	0.1854	0.2802 (0.0948)	0.2170 (0.0317)
S4 ($N = 3{,}078$)	0.0663	0.0775 (0.0112)	0.0778 (0.0113)	0.1558	0.2822 (0.1264)	0.185 (0.0292)
S5 ($N = 2{,}407$)	0.0764	0.0779 (0.0015)	0.0815 (0.0051)	0.1528	0.2820 (0.1292)	0.1576 (0.0048)
S6 ($N = 1{,}857$)	0.0797	0.0811 (0.0014)	0.0929 (0.0132)	0.1444	0.2838 (0.1394)	0.146 (0.0016)
S7 ($N = 1{,}445$)	0.0920	0.0808 (0.0112)	0.7921 (0.0046)	0.1435	0.2757 (0.1322)	0.1229 (0.0206)
S8 ($N = 1{,}105$)	0.0869	0.0828 (0.0041)	0.0981 (0.0112)	0.1489	0.2813 (0.1324)	0.1242 (0.0247)
S9 ($N = 846$)	0.0981	0.0861 (0.012)	0.1083 (0.0102)	0.1160	0.2813 (0.1652)	0.1165 (0.0004)
S10 ($N = 664$)	0.113	0.0911 (0.0219)	0.1321 (0.0191)	0.144	0.2945 (0.1505)	0.1294 (0.0145)
S11 ($N = 489$)	0.1309	0.0947 (0.0362)	0.1525 (0.0216)	0.1474	0.2934 (0.146)	0.1303 (0.0172)
S12 ($N = 353$)	0.1813	0.1032 (0.0781)	0.1961 (0.0148)	0.2249	0.3125 (0.0877)	0.1505 (0.0743)
S13 ($N = 208$)	0.2788	0.1259 (0.153)	0.3621 (0.0833)	0.2576	0.3711 (0.1136)	0.2539 (0.0037)
S14 ($N = 99$)	0.9681	0.3094 (0.6587)	0.7026 (0.2655)	0.032	0.6906 (0.6587)	0.2974 (0.2655)

Notes. Difference between model-predicted probabilities and data are shown as absolute errors in parentheses. Cohort St refers to the set of *surviving* buyers at period t.

predicted probabilities, we first calculate the continuation values using our estimates of the CCPs and state transitions. Then we plug them and our utility estimates into the closed-form expressions of nested choice probabilities. For the model with unobserved heterogeneity, we weigh the choice probabilities of a buyer (or auction) i with her ex post probabilities of belonging to the two unobserved types conditional on the observed outcomes in data.

Both models perform better than the static model, but the model without persistent unobserved heterogeneity is still very poor. Because it does not recognize that early bid choosers likely have a high unobserved preference for bids and late exiters likely have low unobserved preference for bids, it underpredicts bid choice initially and overpredicts bid choice in later periods. The underprediction in the first period is especially severe, when the observed choice probability for the bid nest is 0.4921 and the model predicted probability is 0.3213. Although it performs better in predicting cancellations, the predictions worsen with time. For example, at $t = 12$, the observed probability of cancellation is 0.1813, whereas the model predicted probability is 0.1032. Overall, these fit values suggest that a dynamic model alone cannot explain the realized outcomes in the data.

Finally, note that the fit of the dynamic model with persistent unobserved heterogeneity is very good, especially when compared with previous models. It slightly underpredicts bid choice in the first period and overpredicts bid choice in the middle periods. However, it generally does not deviate from the observed choice probabilities by large amounts. The improvement in the fit of this model highlights the importance of controlling for dynamic selection in this context.

9. Simulation Results

Given that the model predicts the realized outcomes reasonably well, we now use our estimates to derive additional results by running simulations. In this section, we run simulations without making any regime changes; we consider regime changes in §10.

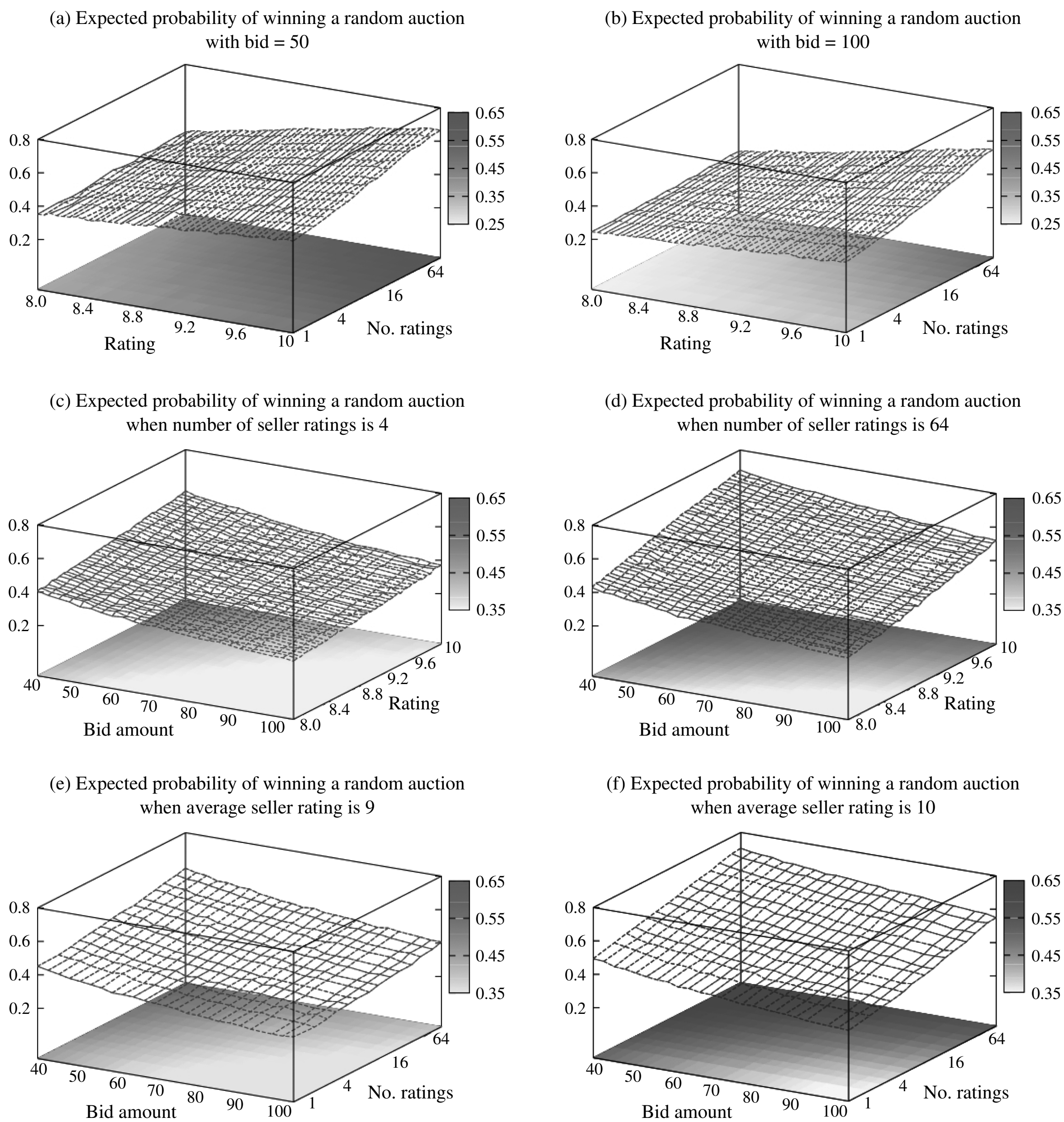
9.1. Sellers' Perspective: Returns to Reputation

Below, we present two types of results to highlight the returns to reputation for a seller.

9.1.1. Aggregate Choice Probability Maps. We simulate the expected probability of being successfully picked for a focal bidder, with a certain set of reputation attributes (number of ratings and average rating) and bid price, if she were to be the first bidder in a randomly chosen auction. The variation in the winning probabilities across different reputation values and bid prices is used to calculate the returns to reputation in this market. Simulation details are described in §3 of the Web appendix, and the results are presented in Figure 7. We have chosen to report the results for MaxBid $= 100$ because it is the dominant category, although results for other MaxBids are similar.

In panels (a) and (b) of Figure 7, we set the bid prices at 50 and 100, respectively, and vary the average rating and number of ratings. It is clear that increases in both the average rating and the number of ratings result in higher winning probabilities. More importantly, the positive interaction effect between number of ratings and the average rating leads to greater returns at higher values of these metrics. For example, at a bid price of \$50 and an average rating of 9, going from 1 to 128 ratings improves the

Figure 7 Probability of Being Successfully Picked for a Focal Bidder with a Certain Set of Reputation Attributes and Bid Price When MaxBid $= 100$



probability of success by 0.14, which translates to an increase of 35% in expected revenue. On the other hand, the same increase in the number of ratings at an average rating of 10 improves expected revenue by about 50%.

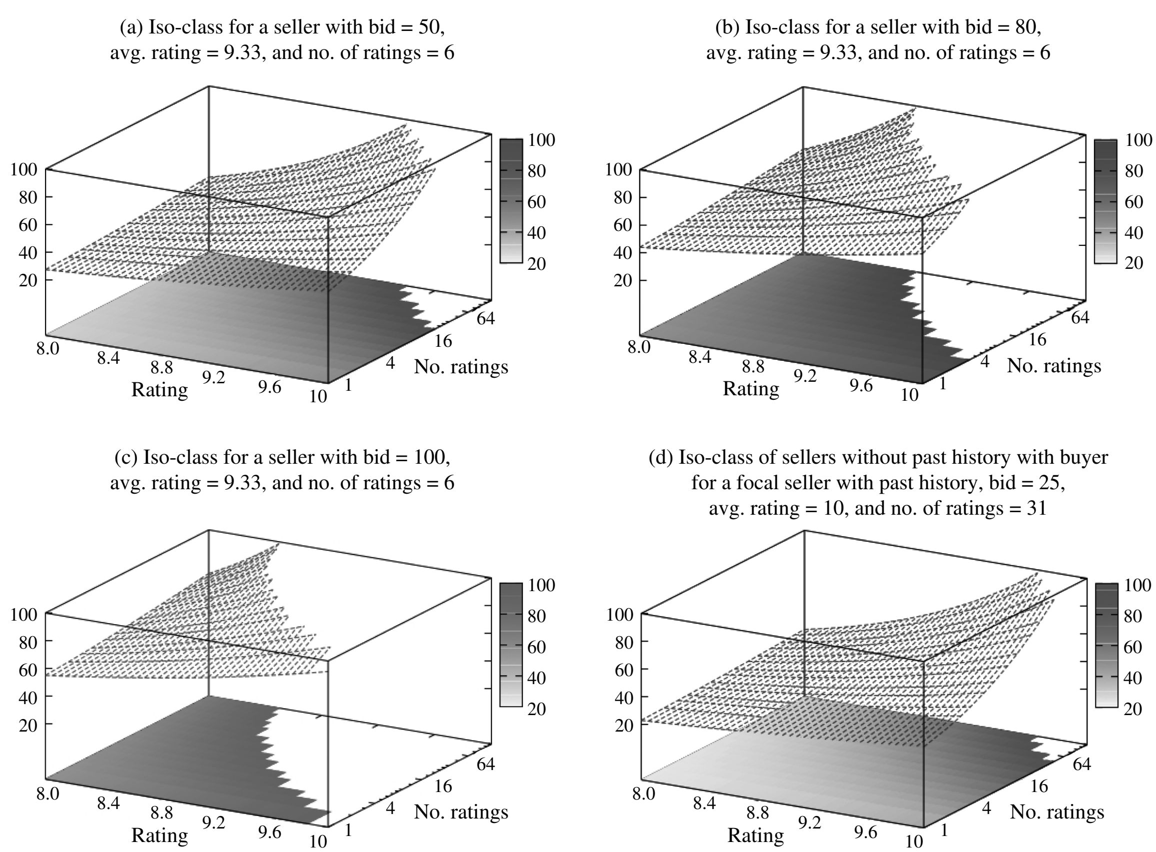
In Figure 7, panels (c) and (d), we set the number of ratings at 4 and 64, respectively, and we vary the bid price and average rating. With four ratings, a seller bidding $100 can go from an expected probability of success of 0.25 at an average rating of 8 to a probability of 0.37 at an average rating of 10, which translates to an increase of almost 50% in expected revenue. With 64 ratings, this increases to approximately 100%. The percentage increase in revenues at lower bid prices is also significant, though lower. For example, at a bid of $40 and 64 ratings, going from an average rating of 9 to 10 increases expected revenues by 20%.

In Figure 7, panels (e) and (f), we set the average ratings at 9 and 10, respectively, and vary the bid price and number of ratings. At an average rating of 9, going from 1 to 128 ratings increases expected probability of winning by approximately 0.1, which translates to an increase of between 20% and 40% in expected revenue depending on the bid price. At an average rating of 10, the expected increase in revenue is higher at 40%–70%. Overall, we find that there are significant returns to reputation in this marketplace.

9.1.2. Iso-Classes of Sellers. We now present *iso-classes* of sellers based on reputation and price. To derive these iso-classes, we pick a focal bidder and then vary the price and reputation metrics while holding the expected utility from the seller constant (using Equation (6)). This gives us a set of sellers who provide the same expected utility and have the same probability of being chosen as the initially chosen focal seller, within a given choice set.

Consider a focal bidder with median reputation who quotes $50. Figure 8, panel (a) shows the heat map of prices that bidders with different reputation metrics would have to quote to be in the same iso-class as the focal bidder. For example, a seller with a mean rating of 10 and 6 ratings who quotes $70 is in the same iso-class as the focal seller, which translates to a $20 premium for a 0.66 increase in mean rating. Similarly, panels (b) and (c) in Figure 8 show the iso-classes for median reputation sellers quoting $80 and $100, respectively. Figure 8, panel (d) shows the iso-class of sellers who have past history with the buyer, when the focal seller quotes $25, is at the 75th percentile of reputation, and has no past history with the

Figure 8 Iso-Classes for Different Focal Sellers



buyer. Note that even when high reputations and low prices are accounted for, it can be difficult to compete with sellers who have interacted with the buyer in the past. For instance, a seller with the same reputation as the focal seller but with a past interaction history is able to charge \$80 and be in the same iso-class.

9.2. Buyer Entry

We now present some results on buyers' entry decision. Even though we did not explicitly model buyer entry, we are able to examine entry decisions because of the structural nature of our model.

Buyer i chooses to post an auction or enter the market at time $t=0$ if his expected utility from doing so is greater than that from not entering. If we normalize the utility of not posting an auction to zero (similar to that from canceling) and assume that buyers' costs of making the actual post are negligible, we can write i's entry decision in period $t=0$ as

$$\delta\int V'(x_{i1},\eta_i,s_i)f(x_{i1}\mid \text{enter},x_{i0},\eta_i)+\epsilon_{i0}^{\text{enter}}>\epsilon_{i0}^{\text{no-enter}}, \tag{33}$$

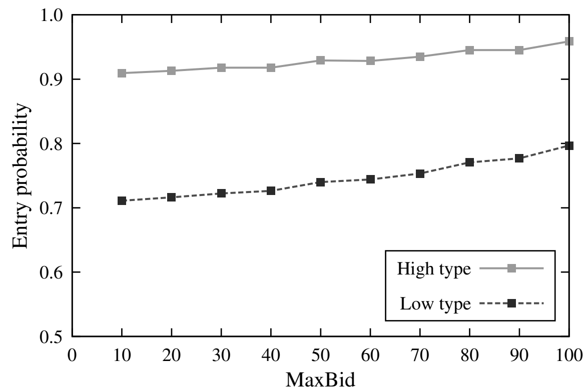
where the first term is the discounted future value of entering the auction (and making optimal decisions henceforth), and the right-hand side is the utility from not entering. The two error terms, $\epsilon_{i0}^{\text{enter}}$ and $\epsilon_{i0}^{\text{no-enter}}$, are assumed to be i.i.d. extreme value. This gives us the entry probability of buyer i:

$$P(\text{enter}\mid x_{i0},\eta_i,s_i)=\frac{e^{\delta\int V'(x_{i1},\eta_i,s_i)f(x_{i1}\mid\text{enter},x_{i0},\eta_i)}}{1+e^{\delta\int V'(x_{i1},\eta_i,s_i)f(x_{i1}\mid\text{enter},x_{i0},\eta_i)}}. \tag{34}$$

We can thus calculate the a priori expected probability of buyer i's entry using Equation (34).[19] In Figure 9, we present the average entry probabilities for the two types of buyers for all MaxBids. There are two points of note here. First, we find that buyers' likelihood of entry is increasing with MaxBid; i.e., buyers are more likely to enter higher-value auctions. Second, 92.9% of high-type buyers who consider entry in any given period actually enter, whereas only 74.6% of low-type buyers do. This discrepancy stems from the differences in the value that the two types place on bids. Recall that high-type buyers have a significantly higher value from choosing bids; consequently, they are more likely to enter the auction in anticipation of this future utility. This difference (almost 20%) further highlights the importance of accounting for persistent unobserved heterogeneity in buyers' differences.

[19] First-period CCPs are not identified, so we use the full solution method to obtain value functions at $t=1$. The size of the state space makes this computationally intensive, but it remains feasible because we need only to solve for the value functions once using a backward solution. See §4 in the Web appendix for details.

Figure 9 **Buyer Entry Probability by Unobserved Buyer Type**



10. Counterfactuals

We now consider a set of regime changes and document their impact on auction success rates and firm revenues. Specifically, we seek to answer the following questions:

- What is the value of the site's reputation system?
- Can the site improve its revenues by attracting more sellers?
- Can the site improve its revenues by ridding itself of low-reputation, low-price sellers and instead attracting high-reputation, high-price sellers?
- Can the site benefit from charging a fee to post auctions? If yes, should it charge a fixed fee for all auctions, or should it instead consider a fee based on the percent of the MaxBid?

Note that these questions do not have a priori obvious answers and hence require an empirical structural model to arrive at reasonable answers. For example, consider the issue of auction fees. On the one hand, auction fees are a direct source of revenue for the firm. On the other hand, they have a negative impact on buyer entry; i.e., some buyers who may have previously procured from the site may choose not to enter, thereby leading to a loss in revenue from commissions. Using our structural framework, we are able to empirically evaluate the impact of such opposing forces and make normative recommendations to the firm.

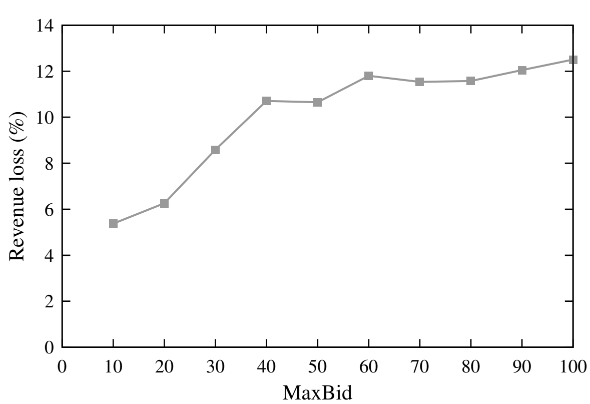
In each counterfactual, we simulate the auctions and re-solve the buyer's decisions under the new regime. CCPs from the original solution are not valid under regime changes, and we use the full solution method to obtain the value functions in our simulations. Because each counterfactual only requires us to solve for the value functions once (by specifying the continuation values as functions of inclusive values and employing some amount of discretization), we are able to make the problem computationally feasible. Furthermore, because this is a nonstationary dynamic programming problem, we use the backward solution method to solve for value functions, the details of which are given in §4 of the Web appendix. Finally, we also note that because we have a partial equilibrium model, the usual caveats apply when interpreting our results.

10.1. Value of Reputation System

Earlier, we saw that buyers value high-reputation sellers—they are not only more likely to pick them but are also willing to pay them higher prices. Since the freelance site generates revenues through percentage commissions on prices, a robust reputation system that sustains high-equilibrium prices can be a significant source of revenue and competitive advantage to freelance sites. Therefore, using our estimates on the primitives of buyer utility, we now quantify the value of the reputation system for the firm. We keep the distribution of sellers and buyers the same and set the average rating of all sellers to the population mean and the number of ratings to zero. Since we use mean-centered average ratings and ln(number of ratings + 1) in the buyers' utility model, this sets both the reputation effects to zero. We also assume that the bid arrival process remains the same. Then we re-solve for the buyers' decisions.

In the absence of a reputation system, buyers have a lower value from choosing bids, and more of them prefer to cancel the auction. This has a direct negative effect on the site's revenues. Furthermore, because the reputation attributes have now disappeared, buyers' relative weight on price increases. Thus successful auctions now clear at lower prices, which has an additional negative effect on the site's revenues though decreased commissions. Thus, the cumulative impact on revenues is negative and higher than that implied by the lower clearance rates. Overall, we find that revenues fall by 11.1%. Furthermore, we find that the reputation system is more valuable for high-value auctions (see Figure 10). Specifically, we find that the revenue loss for auctions with a \$10 MaxBid is about 5.37%, whereas it more than doubles to 12.51% for \$100 auctions. This suggests that the site may benefit from further investments in its reputation system, especially for high-value auctions.

Figure 10 Percentage Loss in Site Revenue When Reputation System Is Switched Off (by MaxBid)



Finally, a necessary caveat here is that our assumption that bid arrivals, distribution of sellers, and prices remain the same may not be reasonable. For instance, without the reputation system, high-quality sellers may leave the marketplace, leaving only low-quality ones, thereby increasing buyers' likelihood of canceling the auction even more and leading to even lower revenues. Thus, the current findings can be interpreted as a lower bound on the inferred value of the reputation system. Furthermore, if the manager has external information (from surveys or previous regime changes) on how sellers modify their behavior in response to changes in the reputation system, they may include it in counterfactual simulations, which can then be used to inform managerial initiatives. Thus, our model and estimation framework can be used as a managerial tool to test the impact of policy changes before implementing them.

10.2. Modifications to Sellers' Side

Next, we present two counterfactual experiments where we modify the supply (seller) side and examine the impact of the regime change on both cancellation rates and revenues.

10.2.1. Increasing the Supply of Sellers. In this simulation, we examine the impact of increasing the supply of sellers without changing the distribution of seller reputations and prices. The firm can achieve this by reducing the commission rates, which would attract new sellers as well as incentivize existing sellers to bid more. The key question that we seek to answer here is, if the firm could engineer each auction to have more bids (without changing the distribution of bids that it receives), would it increase the number of successful transactions and revenues?

We simulate all the auctions in the data numerous times with the following modification: for each auction, the bid arrival rate is increased by an inflation factor.[20] We track the success (bid choice or cancel, when the auction is terminated) and the transaction price (if a bid was chosen) for each simulated auction. The simulation results are presented in Figure 11. The number of successful auctions increases by approximately 2.5% when the bid arrival rate is inflated by 1.5 and by approximately 4% when it doubles. This is understandable because as the number of bids increase, a buyer is more likely to draw a high-value bid (high (H) reputation, low (L) price, etc.). Surprisingly, though, the increase in the transactions does not make much of a dent on the revenues, which

[20] Technically, in the simulation, we modify the Poisson parameters θ_p to Inf-Factor $\cdot\, \theta_p$.

Figure 11 Impact of Increasing the Supply of Sellers

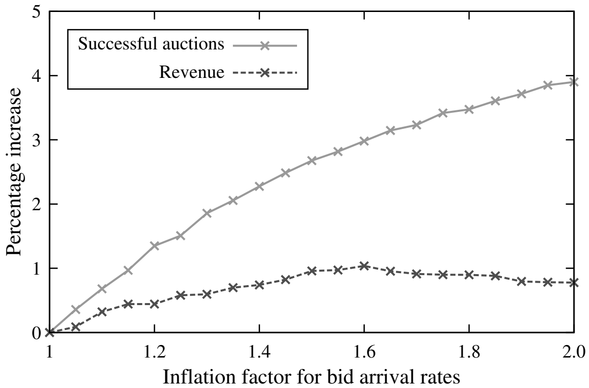
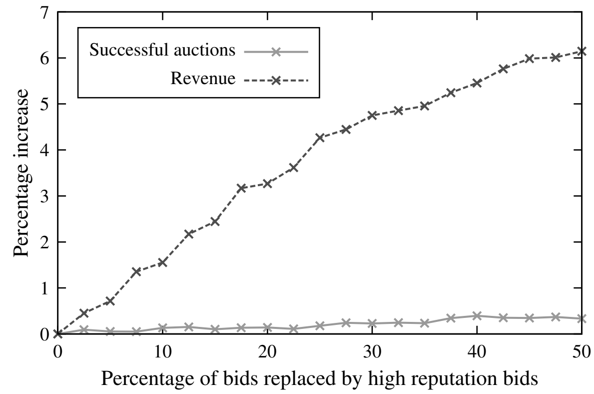


Figure 12 Impact of Increasing the Fraction of High-Reputation Sellers



increases by a small percentage initially (approximately 1% at an inflation factor of 1.6) and then starts decreasing. This is in spite of the monotonic increase in the number of successful transactions. The lukewarm response of revenues stems from the competition effect—when buyers have a larger number of bidders to choose from, they are more likely to find sellers quoting lower prices (and reasonable reputation values), which drives down the transaction price.[21] Since the firm's revenues come from percentage commissions on the transaction price, lower transaction prices lead to lower revenues.[22] Hence, our results suggest that increasing the supply of sellers on the site alone is not enough to increase revenues.

10.2.2. Increasing the Fraction of High-Reputation Sellers. We now examine the impact of keeping the supply of sellers the same and increasing the fraction of H-reputation sellers. Note that the firm can selectively incentivize H-reputation sellers to bid more through many mechanisms. It can provide monetary benefits such as lower commission rates to H-reputation sellers and nonmonetary incentives such as premium accounts and better services.

As before, we simulate all the auctions in the data a large number of times, but with the following modification: when generating the bid attributes (price, number of ratings, mean rating) for a fixed percentage of bids, instead of sampling from the full KDEs, we sample only from the top quartile of reputation values.

[21] As discussed earlier, a marketplace with a healthy reputation system should see a range of prices and reputations, with L-reputation sellers receiving lower prices and H-reputation sellers receiving higher prices. Hence, the reduction in transaction price here does not indicate the failure of the reputation system; rather, it is capturing how buyers' trade off price and reputation.

[22] In our simulation, sellers do not adjust their bids to account for the increase in competition. Intuitively, if we were to allow that, transaction prices would fall even more, thus decreasing revenues even more.

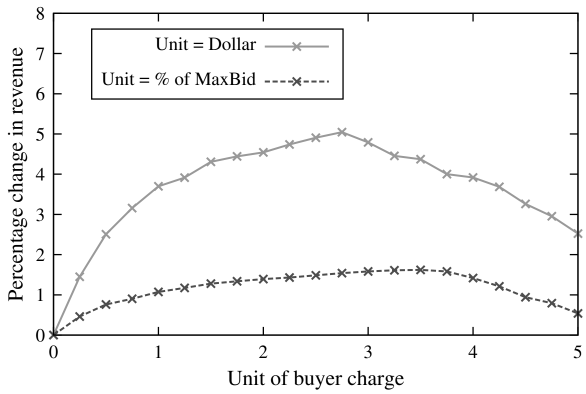
The results here are strikingly different from those in the previous counterfactual (see Figure 12). Here, the number of successful transactions does not increase; i.e., increasing the proportion of H-reputation sellers does not lower the cancellation rate. Although this is surprising at first glance, there is a very simple reason for this—for the replacement bids, we sample bids from the top quartile of the nonparametric distribution of reputation, where prices are also high. Thus the positive correlation between reputation and prices seen in the data is also maintained in the simulation.[23] Hence, from a buyer's perspective, H-reputation sellers are not necessarily "better deals." Thus, the number of transactions does not increase with more H-reputation sellers in the market. However, this is not bad news, because buyers are now replacing L-reputation, L-price sellers with H-reputation, H-price sellers in a bunch of auctions, which increases the transaction prices, and hence the revenues. Replacing 20% of bids with bids from the top quartile of reputation values increases revenue by 3%, while replacing 50% of them increases the revenue by almost 6%. These are significant increases in revenue, and the fact that the number of transactions remains constant implies that these increases directly translate to profits (since transaction costs do not increase).

10.3. Modifications to Buyers' Side: Auction Fees

Finally, we present a series of counterfactual experiments where we modify buyer incentives and examine the impact of these changes on buyer entry, cancellation rates, and revenues. At this point, the site does not charge the buyers any fees for posting auctions nor does it have any membership fees. To evaluate

[23] For all bids (both those from the full distribution and those from the top quartile of reputation), we simulate the geographic region and past interaction indicators using the logit models. So the correlations between all the bid attributes are always preserved.

Figure 13 Impact of Introducing Auction Posting Fees on Site Revenues



whether auction fees can improve site profits, we consider the consequences of two types of auction fees: (a) a fixed fee to post an auction and (b) a percentage of MaxBid.

We present the results from these experiments in Figure 13. Introducing an auction fee has two opposing effects on revenue. On the positive side, the site has a new revenue stream. This is especially useful since it generates revenues from canceled auctions too, which were previously not at all contributing to revenues. On the flip side, auction fees now act as a negative cost on the left-hand side of Equation (34), which is given by $\beta_1 \times \ln(\text{auction fees} + 1)$, where β_1 is our estimate of buyers' price sensitivity. Thus, some buyers who might have entered the market and previously procured from the site now do not even enter the auction. This leads to lower revenues from commissions for the site. We find that these two opposing forces give rise to an inverted U-shaped curve, which has a unique maxima.

Specifically, we find that fixed auction fees consistently dominate auction fees based on the percentage of MaxBids. This is because many of the auctions in our data are low-value auctions; thus fees based on a percentage of MaxBids contribute minimally to revenues. Next, we find that site revenues are maximized at an auction fee of approximately \$2.75. After this point, the revenue loss from lower entry (and hence lost commissions) overwhelms the gains from auction fees. Overall, our findings suggest that the site may benefit from introducing a small fixed fee for its auctions.

11. Conclusion and Future Directions

In this paper, we develop a structural framework of buyer behavior to help researchers and managers estimate the role of seller reputations in reverse auction settings. In our framework, buyers face uncertainty over the number of bids they will receive in the future and the attributes of those bids. Each period, they solve a dynamic programming problem to decide whether to terminate the auction (by choosing a submitted bid or canceling the auction) or to continue waiting for another period. Unlike traditional auctions, in this setting, buyers do not pick the winning bid based on just prices; rather, buyers trade off sellers' reputations, bid prices, other bid attributes, and the cost of waiting and canceling when making their decisions. Our framework is able to correct for dynamic selection using two types of persistent unobserved heterogeneities: bid arrival rates and buyers' unobserved preference for bids. Although persistent unobserved heterogeneity is difficult to handle in large state-space problems such as ours, we use the two-step method recently proposed by Arcidiacono and Miller (2011). In our estimation, we exploit finite dependence to reformulate value functions to improve computational tractability and then employ an EM-like algorithm to accommodate persistent unobservables.

We use our framework to study the role of reputation in online freelance marketplaces—websites that match buyers of electronically deliverable services with sellers or freelancers. Online freelancing has grown tremendously in the last few years, but there exist no research studies of these marketplaces. Our estimation results from a leading online freelance place suggest that buyers are forward looking and that they place significant weight on bidder reputation. We find that not controlling for buyers' intertemporal trade-offs and dynamic selection can considerably bias reputation estimates. Based on our estimates, we present some results on the dollar values of seller reputations and buyer entry probabilities. We also find that the site's reputation system is responsible for over 11% of its revenues. Finally, we provide three broad sets of guidelines to managers of online freelance firms. First, decreasing commission rates uniformly to either attract sellers from other sites or incentivizing all existing sellers to bid more is not a good idea. Second, it is important to incentivize high-reputation sellers alone to bid more and win auctions at higher prices. Third, the introduction of a fixed \$2.75 auction posting fee can increase site revenues by 5%. Although our results are based on a partial equilibrium model, they nevertheless provide the best possible estimates of seller reputations and policy changes in this market.

In sum, our paper makes three key contributions to the literature. First, from a substantive perspective, we quantify the returns to reputation in freelance marketplaces. As far as we know, this is the first paper in marketing to study freelance marketplaces. Second, from a methodological perspective, we provide a dynamic structural framework to model and estimate the value of bidder attributes in reverse auction settings. Our framework is fairly general and can be adapted to a large class of optimal stopping problems. Third, we provide normative guidelines to managers of freelance marketplaces on improving the incentive mechanisms in their websites.

Nevertheless, there remain issues that our paper overlooks that serve as excellent avenues for future research. First, we assume that state transitions are independent of persistent unobservables. In the future, researchers may want to relax this assumption by estimating nonparametric mixture models of state transitions within the EM loop. Second, in this paper, we only look at the demand side of the freelance marketplace. Although it may not be feasible to model a fully two-sided market (which would consist of a game between sellers, as well as a game between buyers and sellers), a supply-side model that explores the primitives of sellers' costs and their bidding strategy would be a good next step. Doing so would allow us to run counterfactuals that take sellers' strategic behavior into consideration and further help managers optimize the commission structure on their websites. See Yoganarasimhan (2013) for recent developments in this area. Third, we only consider buyers' behavior within an auction and ignore interauction dynamics. However, we know from previous research that agents learn about market conditions over time (Crawford and Shum 2005, Narayanan and Manchanda 2009). Models that incorporate learning in this context would be especially useful since they would shed light on how learning affects agents' ability to build and sustain reputation in this marketplace.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mksc.2013.0809.

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References

Aguirregabiria V, Mira P (2007) Sequential estimation of dynamic discrete games. *Econometrica* 75(1):1–53.

Aguirregabiria V, Mira P (2010) Dynamic discrete choice structural models: A survey. *J. Econometrics* 156(1):38–67.

Akerlof GA (1970) The market for "lemons": Quality uncertainty and the market mechanism. *Quart. J. Econom.* 84(3):488–500.

Allenby GM, Rossi PE (1998) Marketing models of consumer heterogeneity. *J. Econometrics* 89(1–2):57–78.

Altug S, Miller RA (1998) The effect of work experience on female wages and labour supply. *Rev. Econom. Stud.* 65(1):45–85.

Arcidiacono P (2005) Affirmative action in higher education: How do admission and financial aid rules affect future earnings? *Econometrica* 73(5):1477–1524.

Arcidiacono P, Ellickson PB (2011) Practical methods for estimation of dynamic discrete choice models. *Annual Rev. Econom.* 3(September):363–394.

Arcidiacono P, Miller RA (2011) Conditional choice probability estimation of dynamic discrete choice models with unobserved heterogeneity. *Econometrica* 79(6)1823–1867.

Arcidiacono P, Khwaja A, Ouyang L (2012) Habit persistence and teen sex: Could increased access to contraception have unintended consequences for teen pregnancies? *J. Bus. Econom. Statist.* 30(2):312–325.

Arcidiacono P, Sieg H, Sloan F (2007) Living rationally under the volcano? An empirical analysis of heavy drinking and smoking. *Internat. Econom. Rev.* 48(1)37–65.

Bajari P, Hortaçsu A (2004) Economic insights from Internet auctions. *J. Econom. Literature* 42(2):457–486.

Beresteanu A, Ellickson P, Misra S (2010) The dynamics of retail oligopoly. Working paper, University of Rochester, Rochester, NY.

Berry S, Reiss P (2007) Empirical models of entry and market structure. Armstrong M, Porter R, eds. *Handbook of Industrial Organization*, Vol. 3 (North-Holland, Amsterdam), 1845–1886.

Bradlow ET, Park Y-H (2007) Bayesian estimation of bid sequences in Internet auctions using a generalized record-breaking model. *Marketing Sci.* 26(2):218–229.

Brousseau E, Glachant J, eds. (2002) *The Economics of Contracts: Theories and Applications* (Cambridge University Press, Cambridge, UK).

Cabral LMB, Hortaçsu A (2004) The dynamics of seller reputation: Theory and evidence from eBay. CEPR Discussion Paper 4345, Centre for Economic Policy Research, London.

Carro JM, Mira P (2006) A dynamic model of contraceptive choice of Spanish couples. *J. Appl. Econometrics* 21(7):955–980.

Chintagunta PK, Jain DC, Vilcassim NJ (1991) Investigating heterogeneity in brand preferences in logit models for panel data. *J. Marketing Res.* 28(4):417–428.

Chung DJ, Steenburgh T, Sudhir K (2013) Do bonuses enhance sales productivity? A dynamic structural analysis of bonus-based compensation plans. *Marketing Sci.* Forthcoming.

Crawford GS, Shum M (2005) Uncertainty and learning in pharmaceutical demand. *Econometrica* 73(4):1137–1173.

Cripps MW, Mailath GJ, Samuelson L (2004) Imperfect monitoring and impermanent reputations. *Econometrica* 72(2):407–432.

Dellarocas C (2003) The digitization of word of mouth: Promise and challenges of online feedback mechanisms. *Management Sci.* 49(10):1407–1424.

Dempster AP, Laird NM, Rubin DB (1977) Maximum likelihood from incomplete data via the EM algorithm. *J. Roy. Statist. Soc. Ser. B* 39(1):1–38.

Douceur JR (2002) The Sybil attack. Druschel P, Kaashoek MF, Rowstron AIT, eds. *Revised Papers From the First International Workshop on Peer-to-Peer Systems (IPTPS '01)* (Springer-Verlag, London), 251–260.

Dubé J-P, Hitsch GJ, Jindal P (2012) Estimating durable goods adoption decisions from stated choice data. Working paper, University of Chicago, Chicago.

Duin RPW (1976) On the choice of smoothing parameters for parzen estimators of probability density functions. *IEEE Trans. Comput.* 25(November):1175–1179.

Eaton DH (2002) Valuing information: Evidence from guitar auctions on eBay. Working paper, Murray State University, Murray, KY.

Ellickson PB, Misra S (2011) Estimating discrete games. *Marketing Sci.* 30(6):997–1010.

Ellickson PB, Misra S, Nair HS (2012) Repositioning dynamics and pricing strategy. *J. Marketing Res.* 49(6):750–772.

Fang H, Wang Y (2013) Estimating dynamic discrete choice models with hyperbolic discounting, with an application to mammography decisions. Working paper, University of Pennsylvania, Philadelphia.

Finger SR (2008) Research and development competition in the chemicals industry. Working paper, Duke University, Durham, NC.

Frederick S, Loewenstein G, ODonoghue T (2002) Time discounting and time preference: A critical review. *J. Econom. Literature* 40(2):351–401.

Friedman EJ, Resnick P (2001) The social cost of cheap pseudonyms. *J. Econom. Management Strategy* 10(2):173–199.

Gowrisankaran G, Rysman M (2012) Dynamics of consumer demand for new durable goods. *J. Political Econom.* 120(8): 1173–1219.

Gray AG, Moore AW (2003) Rapid evaluation of multiple density models. *Workshop Artificial Intelligence Statist. (AISTATS), Key West, FL.*

Hartmann W (2006) Intertemporal effects of consumption and their implications for demand elasticity estimates. *Quant. Marketing Econom.* 4(4):325–349.

Holmstrom B (1999) Managerial incentive problems: A dynamic perspective. *Rev. Econom. Stud.* 66(1):169–182.

Hong H, Shum M (2006) Using price distributions to estimate search costs. *RAND J. Econom.* 37(2):257–275.

Hotz JV, Miller RA (1993) Conditional choice probabilities and the estimation of dynamic models. *Rev. Econom. Stud.* 60(3): 497–529.

Ihler A (2003) Kernel density estimation toolbox for MATLAB. Accessed October 9, 2013, http://www.ics.uci.edu/~ihler/code/kde.html.

Jin GZ, Kato A (2002) Blind trust online: Experimental evidence from baseball cards. Working paper, University of Maryland, College Park.

Jøsang A, Ismail R, Boyd C (2007) A survey of trust and reputation systems for online service provision. *Decision Support Systems* 43(2):618–644.

Kalyanam K, McIntyre S (2001) Returns to reputation in online auction markets. Working paper, Santa Clara University, Santa Clara, CA.

Kamakura WA, Russell GJ (1989) A probabilistic choice model for market segmentation and elasticity structure. *J. Marketing Res.* 26(4):379–390.

Kasahara H, Shimotsu K (2009) Nonparametric identification of finite mixture models of dynamic discrete choices. *Econometrica* 77(1):135–175.

Kim JB, Albuquerque P, Bronnenberg BJ (2010) Online demand under limited consumer search. *Marketing Sci.* 29(6):1001–1023.

Koulayev S (2009) Estimating demand in search markets: The case of online hotel bookings. Working Paper 09-16, Federal Reserve Bank of Boston, Boston.

Kozierok C (2011) FreelancingGuide.Net: Online freelancing guide. Accessed October 9, 2013, http://www.freelancingguide.net/index.htm.

Lacity MC, Rottman JW (2008) *Offshore Outsourcing of IT Work: Client and Supplier Perspectives* (Palgrave Macmillan, Basingstoke, UK).

Lucking-Reiley D, Bryan D, Prasad N, Reeves D (2007) Pennies from eBay: The determinants of price in online auctions. *J. Indust. Econom.* 55(2):223–233.

Magnac T, Thesmar D (2002) Identifying dynamic discrete choice models. *Econometrica* 70(2):801–816.

Mankiw NG, Swagel P (2006) The politics and economics of offshore outsourcing. Working Paper 12398, National Bureau of Economic Research, Cambridge, MA.

McFadden D (1978) Modelling the choice of residential location. Karlvist A, Lundqvist L, Snickars F, Weibull J, eds. *Spatial Interaction Theory and Planning Models*, Vol. 673 (North-Holland, Amsterdam), 75–96.

McLachlan G, Peel D (2004) *Finite Mixture Models* (Wiley-Interscience, New York).

Melnik MI, Alm J (2002) Does a seller's ecommerce reputation matter? Evidence from eBay auctions. *J. Indust. Econom.* 50(3):337–349.

Misra S, Nair H (2011) A structural model of sales-force compensation dynamics: Estimation and field implementation. *Quant. Marketing Econom.* 9(3):211–257.

Morgan M (2011) The freelance marketplace annual review, 2010. Report, WhichLance.com. http://www.whichlance.com/outsource-resource/market-review/doc_download/1-freelance-market-annual-review-2010.

Murphy A (2013) A dynamic model of housing supply. Working paper, Washington University in St. Louis, St. Louis.

Narayanan S, Manchanda P (2009) Heterogeneous learning and the targeting of marketing communication for new products. *Marketing Sci.* 28(3):424–441.

Park WG (2008) International patent protection: 1960–2005. *Res. Policy* 37(4):761–766.

Park Y-H, Bradlow ET (2005) An integrated model for bidding behavior in Internet auctions: Whether, who, when, and how much. *J. Marketing Res.* 42(4):470–482.

Resnick P, Zeckhauser R (2002) Trust among strangers in Internet transactions: Empirical analysis of eBay's reputation system. Baye MR, ed. *The Economics of the Internet and E-Commerce* Advances in Applied Microeconomics, Vol. 11 (Elsevier Science, Oxford, UK), 127–158.

Rezende L (2008) Econometrics of auctions by least squares. *J. Appl. Econometrics* 23(7):925–948.

Rothwell G, Rust J (1997) On the optimal lifetime of nuclear power plants. *J. Bus. Econom. Statist.* 15(2):195–208.

Rust J (1987) Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher. *Econometrica* 55(5):999–1033.

Rust J (1994) Structural estimation of Markov decision processes. Engle RE, McFadden D, eds. *Handbook of Econometrics*, Vol. 4 (North-Holland, Amsterdam), 3082–3139.

Rust J, Phelan C (1997) How social security and medicare affect retirement behavior in a world of incomplete markets. *Econometrica* 65(4):781–832.

Silverman BW (1986) *Density Estimation for Statistics and Data Analysis* (Monographs on Statistics and Applied Probability, Vol. 26) (Chapman & Hall, Boca Raton, FL).

Weitzman ML (1979) Optimal search for the best alternative. *Econometrica* 47(3):641–654.

Winkelmann R (2003) *Econometric Analysis of Count Data* (Springer-Verlag, Berlin).

Yao S, Mela CF (2008) Online auction demand. *Marketing Sci.* 27(5):861–885.

Yao S, Mela CF, Chiang J, Chen Y (2012) Determining consumers' discount rates with field studies. *J. Marketing Res.* 49(6):822–841.

Yoganarasimhan H (2013) Estimation of beauty contest auctions. Working paper, University of California, Davis, Davis.

Zeithammer R (2006) Forward–looking bidding in online auctions. *J. Marketing Res.* 43(3):462–476.

Zeithammer R (2007) Optimal selling in dynamic auctions: Adaptation versus commitment. *Marketing Sci.* 26(6):859–867.

Zeithammer R, Adams A (2010) The sealed-bid abstraction in online auctions. *Marketing Sci.* 29(6):964–987.