Online Appendix for Identifying the Presence and Cause of Fashion Cycles in Data

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A Online Appendix

A.1 Proof of Proposition 1

Consider a non-stationary AR(p) process with roots $1, \frac{1}{c_1}, \ldots, \frac{1}{c_{p-1}}$, where $p \geq 2$ and $0 < c_1, \ldots, c_{p-1} \leq 1$. We know that y_{it} can be expressed as:

$$(1-L)(1-c_1L)\ldots(1-c_{p-1}L)y_{it} = \epsilon_{it} \tag{A-1}$$
$$\Rightarrow E[(1-c_1L)\ldots(1-c_{p-2}L)(1-c_{p-1}L)\Delta y_{it}] = 0 \tag{A-2}$$
$$\Rightarrow E[(1-c_1L)\ldots(1-c_{p-2}L)\Delta y_{it}] = E[c_{p-1}L(1-c_1L)\ldots(1-c_{p-2}L)\Delta y_{it}] \tag{A-3}$$

Since $E[L\Delta y_{it}] = y_{it-1}$, we have:

$$\Rightarrow E\left[\left(\prod_{k=1}^{p-2}(1-c_kL)\right)\Delta y_{it}\right] = c_{p-1}\left(\prod_{k=1}^{p-2}(1-c_kL)\right)\Delta y_{it-1} \tag{A-4}$$

where g If $\left(\prod_{k=1}^{p-2}(1-c_kL)\right)\Delta y_{it-1} \geq 0$, then we have:

$$E[(1-c_1L)\ldots(1-c_{p-2}L)\Delta y_{it}] \geq 0 \tag{A-5}$$

since $c_{p-1} > 0$ and $\left(\prod_{k=1}^{p-2}(1-c_kL)\right)\Delta y_{it-1}$.
If, on the other hand, $\left(\prod_{k=1}^{p-2}(1-c_kL)\right)\Delta y_{it-1} \leq 0$, then we have:

$$E[(1-c_1L)\ldots(1-c_{p-2}L)\Delta y_{it}] \leq 0 \tag{A-6}$$

since $c_{p-1} > 0$ and $\left(\prod_{k=1}^{p-2}(1-c_kL)\right)\Delta y_{it-1} \leq 0$. Q.E.D.

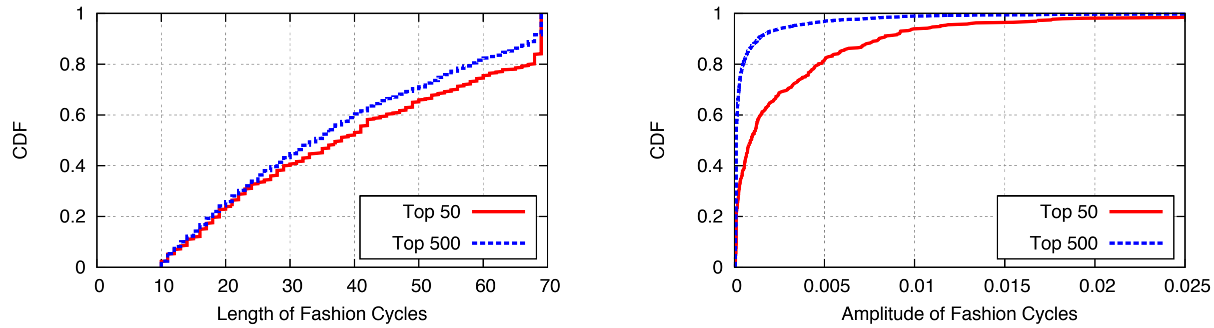

Figure A.1: Distributions of the length and amplitude of cycles for Top50 and Top500 datasets.

A.2 Length and Amplitude Distributions of Cycles

The left panel of Figure A.1 shows the distributions of cycle lengths and cycle amplitude for the Top50 and Top500 datasets. Note that this is a CDF over cycles and not names; some names might have more than one cycle, in which case their data is represented multiple times, and some others might have no cycles in which case they are not represented in the graph. Please see Table 4 for data on the fractions of names with different numbers of cycles. Two patterns emerge from these two figures. First, the cycle length distributions are similar for the two datasets. Second, while the cycle lengths span a large range, the median name enjoys a $\approx$ 35-year cycle. The right panel shows the magnitude distributions for the Top50 and Top500 datasets. Not surprisingly, the distribution for the Top50 dataset first-order stochastically dominates that of the Top500 dataset. Note that 20% of all names in Top50 have an amplitude of 0.005 or more, which implies that these names were chosen by more than 10000 parents per year at the peak of their popularity.

A.3 Sensitivity Analysis to Varying τ and M

No. of cycles	$\tau=4, M=0.00005$		$\tau=5, M=0.00005$		$\tau=4, M=0.000075$		$\tau=4, M=0.0001$	
	Female	Male	Female	Male	Female	Male	Female	Male
0	18.6	25.8	18.0	28.4	21.6	28.4	37.9	37.1
1	54.1	40.3	57.7	43.6	54.1	40.4	41.5	35.6
2	21.0	26.2	20.5	22.9	18.3	24.7	15.9	22.9
3	5.5	7.3	3.5	5.1	5.2	6.2	4.2	4.4
4	0.5	0.4	0.3	0	0.8	0.3	0.5	0
5	0.3	0	0	0	0	0	0	0
Total Percentage	100	100	100	100	100	100	100	100
No. of names	366	275	366	275	366	275	366	275

Table A.1: Percentage of cycles at different values of τ and M in the Top100 dataset.

A.4 Derivation of Aggregate Model from an Individual Level Model

A.4.1 Basic Model

Let individual q's probability of adopting name i at time t be:

$$y_{qit} = const. + \sum_{k=1}^{p} \phi_k y_{ijt-k} + \rho_1 w_{qt} + \rho_2 c_{qt} + \rho_3 w_{qt} y_{it-1} + \rho_4 c_{qt} y_{it-1} + \rho_5 x_{it}^1 + \rho_6 x_{it}^2 + \rho_7 z_i + \tau_{iqt} \quad \text{(B-1)}$$

The interpretation of the variables is similar to that in §7.1.1. The probability that agent q will adopt name i is a function of the past adoptions in her local neighborhood (state), her own wealth and cultural capital (w_{qt} and c_{qt}), interaction effects between her wealth, cultural capital and past adoptions by others, some endogenous factors that affect her affinity for name i (x^1_{ijt} which includes total adoptions), some exogenous time-varying factors (x^2_{it}), time invariant name attributes (z_i), and an unobserved taste for name i (τ_{iqt}).

Summing Equation (B-1) over all potential adopters in state j at time t, and then dividing the resulting equation by the number of potential adopters, we have:

$$\bar{y}_{ijt} = const. + \sum_{k=1}^{p} \phi_k y_{ijt-k} + \rho_1 \bar{w}_{jt} + \rho_2 \bar{c}_{jt} + \rho_3 \bar{w}_{jt} y_{it-1} + \rho_4 \bar{c}_{qt} y_{it-1} + \rho_5 x^1_{it} + \rho_6 x^2_{it} + \rho_7 z_i + \bar{\tau}_{ijt} \quad \text{(B-2)}$$

Here, $\bar{w}_{jt}$ and $\bar{c}_{jt}$ are the mean wealth and cultural capital of state j at time t. $\bar{\tau}_{ijt}$ is the mean unobserved preference of potential adopters in state j, at time t, for name i. This can be rewritten as: $\bar{\tau}_{ijt} = \gamma_{ij} + e_{ijt}$, *i.e.*, we can extract out the mean preferences of residents of state j for name i and write the rest as a mean zero error term that varies with time. With these transformations, Equation (B-2) can be rewritten as:

$$\bar{y}_{ijt} = const. + \sum_{k=1}^{p} \phi_k y_{ijt-k} + \rho_1 \bar{w}_{jt} + \rho_2 \bar{c}_{jt} + \rho_3 \bar{w}_{jt} y_{it-1} + \rho_4 \bar{c}_{qt} y_{it-1} + \rho_5 x^1_{it} + \rho_6 x^2_{it} + \rho_7 z_i + \gamma_{ij} + e_{ijt} \quad \text{(B-3)}$$

This model is analogous to the aggregate model specified in §7.1.1, *i.e.*, all the parameter estimates from this aggregate model can be interpreted as individual level parameters with the right multipliers.

A.4.2 Expanded Model with within State Effects

We now expand the above model with within state effects. Let:

- y^{hw}_{jit-1} = the number of high wealth parents who have adopted name i in state j at time $t-1$.
- y^{lw}_{jit-1} = the number of low wealth parents who have adopted name i in state j at time $t-1$.
- y^{hc}_{jit-1} = the number of high culture parents who have adopted name i in state j at time $t-1$.
- y^{lc}_{jit-1} = the number of low culture parents who have adopted name i in state j at time $t-1$.

We now expand Equation (B-4) so that individual q's probability of adopting name i at time t is also affected by the number of high and low types that have adopted the name within state j:

$$\begin{aligned} y_{qit} = const. \quad &+ \quad \sum_{k=1}^{p} \phi_k y_{ijt-k} + \rho_1 w_{qt} + \rho_2 c_{qt} + \rho_3 w_{qt} y_{it-1} + \rho_4 c_{qt} y_{it-1} + \rho_5 x^1_{it} + \rho_6 x^2_{it} + \rho_7 z_i \\ &+ \quad \rho_8 w_{qt} y^{hw}_{jit-1} + \rho_9 w_{qt} y^{lw}_{jit-1} + \rho_{10} c_{qt} y^{hc}_{jit-1} + \rho_{11} c_{qt} y^{lc}_{jit-1} + \tau_{iqt} \end{aligned} \quad \text{(B-4)}$$

Aggregating this over all potential adopters in state j, we have:

$$\begin{aligned} \bar{y}_{ijt} \quad &= \quad const. + \sum_{k=1}^{p} \phi_k y_{ijt-k} + \rho_1 \bar{w}_{jt} + \rho_2 \bar{c}_{jt} + \rho_3 \bar{w}_{jt} y_{it-1} + \rho_4 \bar{c}_{qt} y_{it-1} + \rho_5 x^1_{it} + \rho_6 x^2_{it} + \rho_7 z_i \\ &+ \quad \rho_8 \bar{w}_{jt} y^{hw}_{ijt-1} + \rho_9 \bar{w}_{jt} y^{lw}_{ijt-1} + \rho_{10} \bar{c}_{jt} y^{hc}_{ijt-1} + \rho_{11} \bar{c}_{jt} y^{lc}_{ijt-1} + \gamma_{ij} + e_{ijt} \end{aligned} \quad \text{(B-5)}$$

where $\bar{\tau}_{ijt} = \gamma_{ij} + e_{ijt}$, as before. Thus, the new error-term of the aggregate model can be written as:

$$\epsilon_{ijt} = \rho_8 \bar{w}_{jt} y^{hw}_{ijt-1} + \rho_9 \bar{w}_{jt} y^{lw}_{ijt-1} + \rho_{10} \bar{c}_{jt} y^{hc}_{ijt-1} + \rho_{11} \bar{c}_{jt} y^{lc}_{ijt-1} + e_{ijt} \tag{B-6}$$

First, note that much of the variation in these terms can be extracted out using lagged dependent variables (y_{ijt-1}s) and interaction effects of mean wealth/culture with state-level adoptions ($\bar{w}_{jt} y_{ijt-1}$s). For instance, we can rewrite the above equation as:

$$\epsilon_{ijt} = \rho_8 \bar{w}_{jt} y_{ijt-1} + \bar{w}_{jt}(\rho_9 y^{lw}_{ijt-1} - \rho_8 y^{hw}_{ijt-1}) + \rho_{10} \bar{c}_{jt} y_{ijt-1} + \bar{c}_{jt}(\rho_{10} y^{lc}_{ijt-1} - \rho_{11} y^{hc}_{ijt-1}) + e_{ijt} \tag{B-7}$$

The terms $\rho_8 \bar{w}_{jt} y_{ijt-1}$ and $\rho_{10} \bar{c}_{jt} y_{ijt-1}$ can, of course, be pulled out and used directly in the estimation since they are observables. Thus, the residual error-term is:

$$\epsilon'_{ijt} = \bar{w}_{jt}(\rho_9 y^{lw}_{ijt-1} - \rho_8 y^{hw}_{ijt-1}) + \bar{c}_{jt}(\rho_{10} y^{lc}_{ijt-1} - \rho_{11} y^{hc}_{ijt-1}) + e_{ijt} \tag{B-8}$$

Second, some terms can again be decomposed and written as functions of past state-level adoptions, aggregate wealth and cultural capitals, and their interactions. For example, $y^{lw}_{ijt-1}, y^{hw}_{ijt-1}$ can be written as:

$$y^{lw}_{ijt-1} = \mathcal{F}_{ht}(y_{ijt-1}, \ldots y_{ijt-p-1}, y_{it-2}, x_{it-1}, \bar{w}_{jt-1}, \bar{c}_{jt-1}, z_i, \gamma_{ij}) \tag{B-9}$$

$$y^{hw}_{ijt-1} = \mathcal{F}_{lt}(y_{ijt-1}, \ldots y_{ijt-p-1}, y_{it-2}, x_{it-1}, \bar{w}_{jt-1}, \bar{c}_{jt-1}, z_i, \gamma_{ij}) \tag{B-10}$$

Thus, much of the remaining variation in $y^{lw}_{ijt-1}, y^{hw}_{ijt-1}$ is captured through these lag variables and name-state fixed effects. Third, since many of the instruments in the estimation are for the first-differenced equation, the error-terms used in estimation are $\epsilon'_{ijt} - \epsilon'_{ijt-1}$. It is well-known that first-differencing significantly assuages aggregation issues in models like this by differencing out much of the variation in the error terms, making the first-differenced error terms to be independent of endogenous explanatory variables. Please see Stoker (1993) for details.

Nevertheless, some remnant variation may still remain significant. If so, it will lead to serial correlation in estimated errors (through correlations in adoptions among high and low types across consecutive years). The main advantage of our estimator is that it allows us to test this empirically.

After estimating the model, and obtaining the parameters and error terms, we test for serial correlation in error terms using the Arellano-Bond (2) test. If the test rejects the hypothesis of no serial correlation, then it implies that the presence of within-state effects has invalidated our aggregated social effects. That is, if we find that $E(\widetilde{\epsilon}_{jit} \cdot \widetilde{\epsilon}_{ijt-1}) \neq 0$, where:

$$\begin{aligned} E(\widetilde{\epsilon}_{jit} \cdot \widetilde{\epsilon}_{ijt-1}) = {} & E[\bar{w}_{jt}(\rho_9 y^{lw}_{ijt-1} - \rho_8 y^{hw}_{ijt-1}) + \bar{c}_{jt}(\rho_{10} y^{lc}_{ijt-1} - \rho_{11} y^{hc}_{ijt-1}) + e_{ijt}) \\ & \cdot (\bar{w}_{jt-1}(\rho_9 y^{lw}_{ijt-1} - \rho_8 y^{hw}_{ijt-1}) + \bar{c}_{jt-1}(\rho_{10} y^{lc}_{ijt-1} - \rho_{11} y^{hc}_{ijt-1}) + e_{ijt-1})] \end{aligned} \tag{B-11}$$

then the estimates from the aggregated model are inconsistent. If instead $E(\widetilde{\epsilon}_{jit} \cdot \widetilde{\epsilon}_{ijt-1}) = 0$, then the estimates of state-level social effects are consistent even if we do not have information on within state or more local neighborhood-level effect.

Thus, the presence of local/within-state social effects does not invalidate aggregate-level social effects if the Arellano-Bond (2) test is satisfied.

A.5 Additional Results

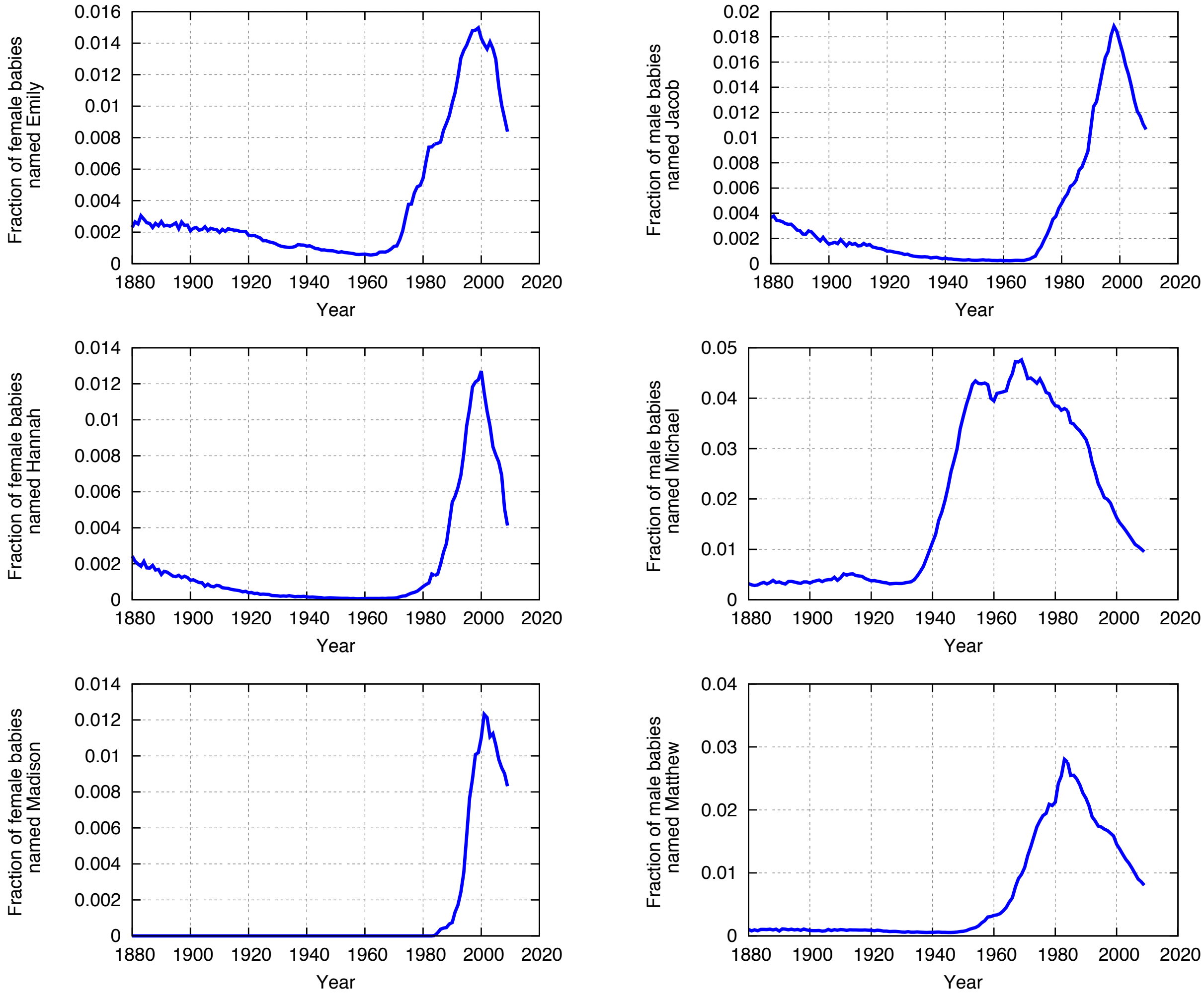

Figure A.2: Popularity Curves of the Top Three Female and Male Baby Names in 2000.

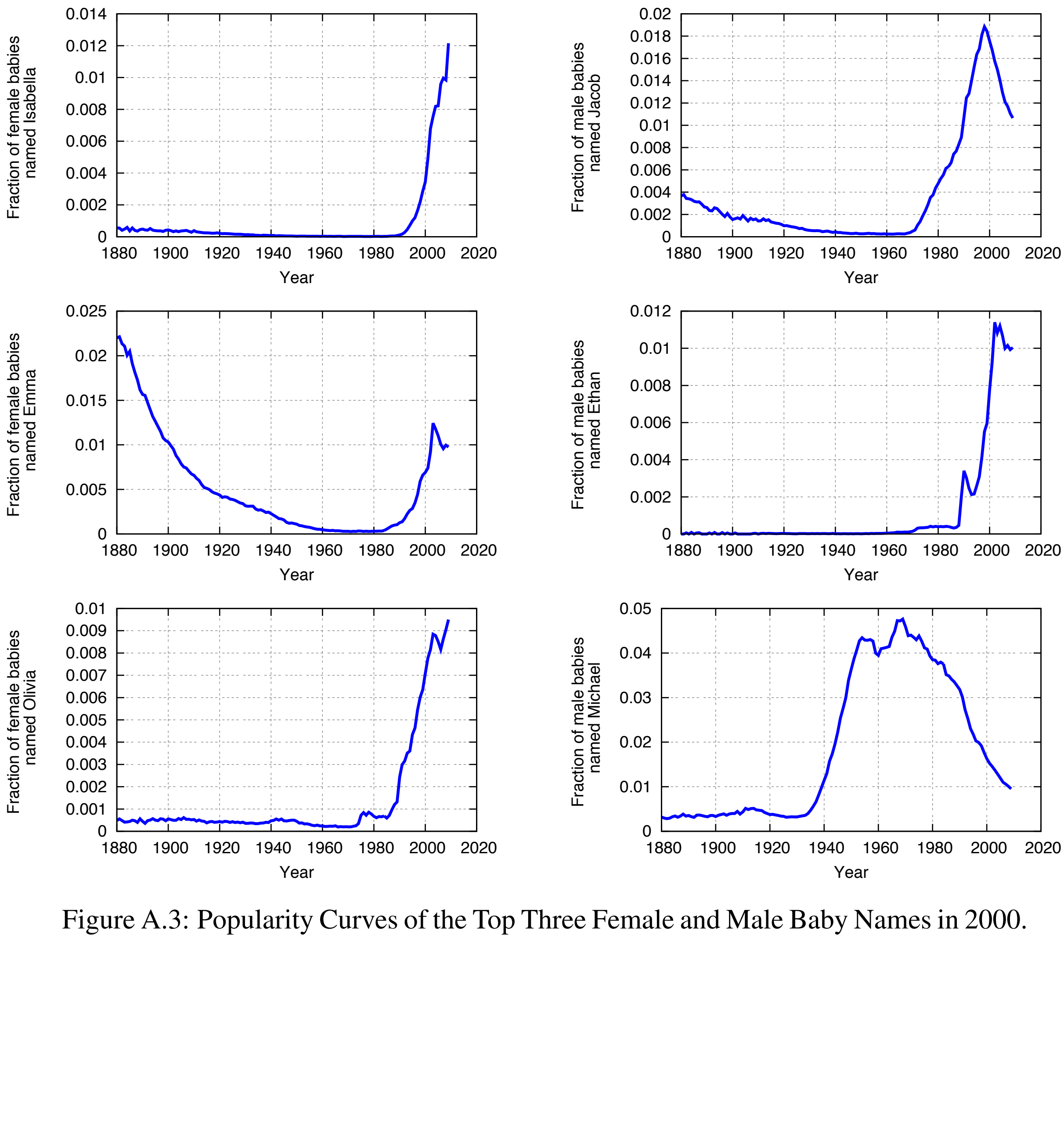

Figure A.3: Popularity Curves of the Top Three Female and Male Baby Names in 2000.

Variable		Model N2 Estimate	Model N2 Std. Err.	Model N3 Estimate	Model N3 Std. Err.
Lagged	n_{ijt-1}	0.9946^{***}	0.2017×10^{-2}	0.9958^{***}	0.2648×10^{-2}
Dep. Var.	Δn_{ijt-1}	$0.3144\times10^{-1***}$	0.8188×10^{-2}	$0.6441\times10^{-1***}$	0.1144×10^{-1}
	Δn_{ijt-2}	0.1398^{***}	0.6794×10^{-2}	0.1475^{***}	0.8648×10^{-2}
	Δn_{ijt-3}	0.1237^{***}	0.4429×10^{-2}	0.1202^{***}	0.5617×10^{-2}
	Δn_{ijt-4}	$0.8733\times10^{-1***}$	0.4787×10^{-2}	$0.8458\times10^{-1***}$	0.6648×10^{-2}
Name	l_i	$0.7061\times10^{-1***}$	0.1506×10^{-1}	0.1604^{***}	0.2810×10^{-1}
Char.	bib_i	$0.2479\times10^{-2***}$	0.1237×10^{-2}	$0.5869\times10^{-2***}$	0.2050×10^{-2}
	s_i	-0.2858^{***}	0.3995×10^{-1}	-0.5447^{***}	0.7676×10^{-1}
Cultural	c_{jt}	$0.3851\times10^{-1***}$	0.1620×10^{-1}	$0.5425\times10^{-1**}$	0.3062×10^{-1}
Capital	$c_{jt}.n_{it-1}$	$-0.2410\times10^{-4***}$	0.6070×10^{-5}	$-0.2290\times10^{-4***}$	0.7860×10^{-5}
Economic	w_{jt}	$-0.1674\times10^{-1***}$	0.9275×10^{-3}	$-0.2679\times10^{-1***}$	0.1827×10^{-2}
Capital	$w_{jt}.n_{it-1}$	$0.8630\times10^{-5***}$	0.3210×10^{-6}	$0.9260\times10^{-5***}$	0.4140×10^{-6}
Other	n_{it-1}	$0.1766\times10^{-2***}$	0.1880×10^{-3}	$0.1097\times10^{-2***}$	0.2297×10^{-3}
	n_{it-2}	$-0.5974\times10^{-2***}$	0.1451×10^{-3}	$-0.5616\times10^{-2***}$	0.1845×10^{-3}
	Γ_{sijt}	$0.8720\times10^{-5***}$	0.2530×10^{-5}	0.5700×10^{-5}	0.4880×10^{-5}
	Const.	$0.8079\times10^{1***}$	0.4777	$0.1286\times10^{2***}$	0.9775
AR-Bond (2) Test		Do not reject		Do not reject	
Test stat (p-value)		-1.3415 (0.1798)		-0.9758 (0.3292)	
No. of names, states, years		641, 50, 34		361, 49, 34	
Dataset used		Top100		Top50	
Diff. Eqn. Instr.	GMM	L(2/4)$[n_{ijt},\Delta n_{ijt},n_{it},c_{jt}.n_{it-1},w_{jt}.n_{it-1}]$			
	Standard	$\Delta[\Gamma_{sijt},c_{jt},w_{jt}]$			
Level Eqn. Instr	GMM	L2 $\Delta[n_{ijt},\Delta n_{ijt},n_{it},c_{jt}.n_{it-1},w_{jt}.n_{it-1}]$			
	Standard	$[s_i,l_i,bib_i,\Gamma_{sijt},c_{jt},w_{jt}]$			
Note: $^{***}\Rightarrow p\leq0.01$, $^{**}\Rightarrow p\leq0.05$, $^{*}\Rightarrow p\leq0.1$					

Table A.2: Interacting wealth and cultural capital with past adoptions. Dependent variable is n_{ijt}.

Variable		Model P3		Model P4	
		Estimate	Std. Err.	Estimate	Std. Err.
Lagged Dep. Var.	n_{ijt-1}	0.9943^{***}	0.2052×10^{-2}	0.9965^{***}	0.2372×10^{-2}
	Δn_{ijt-1}	$0.3056\times10^{-1***}$	0.8447×10^{-2}	$0.6405\times10^{-1***}$	0.1241×10^{-1}
	Δn_{ijt-2}	0.1412^{***}	0.6291×10^{-2}	0.1497^{***}	0.8586×10^{-2}
	Δn_{ijt-3}	0.1242^{***}	0.4170×10^{-2}	0.1216^{***}	0.5043×10^{-2}
	Δn_{ijt-4}	$0.8726\times10^{-1***}$	0.5232×10^{-2}	$0.8503\times10^{-1***}$	0.6330×10^{-2}
Name Char.	l_i	$0.4663\times10^{-1***}$	0.1550×10^{-1}	0.1105^{***}	0.2983×10^{-1}
	bib_i	$0.2126\times10^{-2*}$	0.1221×10^{-2}	$0.4570\times10^{-2**}$	0.2198×10^{-2}
	s_i	-0.2762^{***}	0.4047×10^{-1}	-0.4922^{***}	0.7896×10^{-1}
Cultural Capital	c_{jt}	$0.3711\times10^{-1***}$	0.1643×10^{-1}	$0.7730\times10^{-1***}$	0.2902×10^{-1}
	$c_{jt}.n_{it-1}$	$-0.2250\times10^{-4**}$	0.6160×10^{-5}	$-0.2690\times10^{-4**}$	0.7410×10^{-5}
	d^c_{it-1}	$0.1836\times10^{3***}$	0.4352×10^{2}		
	$d^c_{it-1,t-2}$			$0.1216\times10^{3***}$	0.4749×10^{2}
Economic Capital	w_{jt}	$-0.1693\times10^{-1***}$	0.9309×10^{-3}	$-0.2832\times10^{-1***}$	0.16449×10^{-2}
	$w_{jt}.n_{it-1}$	$0.8740\times10^{-5***}$	0.3220×10^{-6}	$0.9320\times10^{-5***}$	0.3750×10^{-6}
	d^w_{it-1}	$0.2223\times10^{3***}$	0.5183×10^{2}		
	$d^w_{it-1,t-2}$			$0.2520\times10^{3***}$	0.6142×10^{2}
Other	n_{it-1}	$0.1754\times10^{-2***}$	0.1873×10^{-3}	$0.9326\times10^{-3***}$	0.2371×10^{-3}
	n_{it-2}	$-0.5983\times10^{-2***}$	0.1455×10^{-3}	$-0.5499\times10^{-2***}$	0.1884×10^{-3}
	Γ_{s_ijt}	0.9170×10^{-5}	0.2540×10^{-5}	0.5430×10^{-5}	0.4620×10^{-5}
	Const.	$0.8217\times10^{1***}$	0.4788	$0.1385\times10^{2***}$	0.8605
AR-Bond (2) Test		Do not reject		Do not reject	
Test stat (p-value)		-1.5606 (0.1186)		-0.9662 (0.3340)	
Diff. Eqn. Instr.	GMM	L(2/4) $[n_{ijt},\Delta n_{ijt},n_{it},c_{jt}.n_{it-1},w_{jt}.n_{it-1}]$		L(2/4) $[n_{ijt},\Delta n_{ijt},n_{it},c_{jt}.n_{it-1},w_{jt}.n_{it-1}]$	
	Standard	$\Delta[\Gamma_{s_ijt},c_{jt},w_{jt},d^c_{it-1},d^w_{it-1}]$		$\Delta[\Gamma_{s_ijt},c_{jt},w_{jt},d^c_{it-1,t-2},d^w_{it-1,t-2}]$	
Level Eqn. Instr.	GMM	L2$\Delta[n_{ijt},\Delta n_{ijt},n_{it},c_{jt}.n_{it-1},w_{jt}.n_{it-1}]$		L2$\Delta[n_{ijt},\Delta n_{ijt},n_{it},c_{jt}.n_{it-1},w_{jt}.n_{it-1}]$	
	Standard	$[s_i,l_i,bib_i,\Gamma_{s_ijt},c_{jt},w_{jt},d^c_{it-1},d^w_{it-1}]$		$[s_i,l_i,bib_i,\Gamma_{s_ijt},c_{jt},w_{jt},d^c_{it-1,t-2},d^w_{it-1,t-2}]$	
No. of names, states, years		641, 50, 34		361, 50, 34	
Dataset used		Top100		Top50	
Note: $^{***}\Rightarrow p\leq 0.01$, $^{**}\Rightarrow p\leq 0.05$, $^{*}\Rightarrow p\leq 0.1$					

Table A.3: Impact of adoption by high and low types (contd.); dependent variable is n_{ijt}.

References

T. Stoker. Empirical Approaches to the Problem of Aggregation over Individuals. *Journal of Economic Literature*, pages 1827–1874, 1993.