Web Appendix

A Extensions of the Simultaneous Action Game

A.1 General Linear Combination

We now consider a case where the group outcome is a linear combination of all agents' revealed preferences such that $\bar{a} = \alpha a_i + (1-\alpha)a_j$, where $0 < \alpha < 1$. We can then write the utility as:

$$u_\alpha(x_i, a_i, a_j) = -r(x_i - a_i)^2 - (1-r)(x_i - \alpha a_i - (1-\alpha)a_j)^2 \tag{A1}$$

Because j's preference (x_j) is her private information at the time of choosing the action, i's expected utility from choosing a_i as $EU_\alpha(x_i, a_i) = \frac{\int_{\mathbb{R}} u_\alpha(x_i, a_i, \hat{a}_j) g(x_j) \mathrm{d}x_j}{\int_{\mathbb{R}} g(x_j) \mathrm{d}x_j}$. By differentiating $EU(x_i, a_i)$ and setting it equal to zero at i's equilibrium action $a_i = \hat{a}_i$ gives us:

$$\left.\frac{\partial EU_\alpha(x_i, a_i)}{\partial a_i}\right|_{a_i=\hat{a}_i} = 2r(x_i - \hat{a}_i) + 2(1-r)\alpha \left[x_i - \alpha \hat{a}_i) + (1-\alpha) \int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j\right] = 0. \tag{A2}$$

We can then write the equilibrium action of i as:

$$\hat{a}_i = \frac{r + (1-r)\alpha}{r + (1-r)\alpha^2} x_i - \frac{(1-r)\alpha(1-\alpha)}{r + (1-r)\alpha^2} \int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j \tag{A3}$$

As in the main model, we can easily show that $\int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j = 0$. Therefore, we have:

$$\hat{a}_i = \frac{r + (1-r)\alpha}{r + (1-r)\alpha^2} x_i \tag{A4}$$

We can easily show that $|\hat{a}_i| > x_i$ and $|\alpha \hat{a}_i + (1-\alpha)\hat{a}_j| > \bar{x}$. Thus, we see polarization in individual agents' actions as well as the group outcome.

A.2 Analysis of m-Player Game

A.2.1 Equilibrium of m-Player Game

To derive the Bayesian Nash equilibrium for a m-player simultaneous game, we can write the utility of a player i as:

$$u(x_i, a_{\mathrm{m},i}, a_{\mathrm{m},-i}) = -r(x_i - a_{\mathrm{m},i})^2 - (1-r)\left(x_i - \frac{a_{\mathrm{m},i}}{m} - \frac{\sum_{j=1}^{m-1} a_{\mathrm{m},j}}{m}\right)^2 \tag{A5}$$

To obtain i's expected utility for any $a_{\mathrm{m},i}$ in equilibrium, we can take the expectation of equation (A5) over all the other $m-1$ agents' equilibrium actions:

$$EU_{\mathrm{m}}(x_i, a_{\mathrm{m},i}) = \frac{\int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \int_{\mathbb{R}} (u(x_i, a_{\mathrm{m},i}, \hat{a}_{\mathrm{m},-i}))\, g(x_1) g(x_2) \ldots g(x_{m-1}) \mathrm{d}x_1 \mathrm{d}x_2 \ldots \mathrm{d}x_{m-1}}{\int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \int_{\mathbb{R}} g(x_1) g(x_2) \ldots g(x_{m-1}) \mathrm{d}x_1 \mathrm{d}x_2 \ldots \mathrm{d}x_{m-1}} \tag{A6}$$

Substituting for $u(x_i, a_{\mathrm{m},i}, a_{\mathrm{m},-i})$ from equation (A5) and simplifying, we have:

$$\begin{aligned}
& EU_{\mathrm{m}}\left(x_i, a_{\mathrm{m},i}\right) = -r(x_i - a_{\mathrm{m},i})^2 \qquad \text{(A7)} \\
+ \quad & \frac{2(1-r)}{m}\left(x_i - \frac{a_{\mathrm{m},i}}{m}\right) \frac{\int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \int_{\mathbb{R}} \left(\sum_{j=1}^{m-1} a_{\mathrm{m},j}\right) g(x_1)g(x_2)...g(x_{m-1})\mathrm{d}x_1\mathrm{d}x_2...\mathrm{d}x_{m-1}}{\int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \int_{\mathbb{R}} g(x_1)g(x_2)...g(x_{m-1})\mathrm{d}x_1\mathrm{d}x_2...\mathrm{d}x_{m-1}} \\
- \quad & (1-r)\left[\left(x_i - \frac{a_{\mathrm{m},i}}{m}\right)^2 + \frac{\int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \int_{\mathbb{R}} \left(\sum_{j=1}^{m-1} a_{\mathrm{m},j}\right)^2 g(x_1)g(x_2)...g(x_{m-1})\mathrm{d}x_1\mathrm{d}x_2...\mathrm{d}x_{m-1}}{m^2 \int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \int_{\mathbb{R}} g(x_1)g(x_2)...g(x_{m-1})\mathrm{d}x_1\mathrm{d}x_2...\mathrm{d}x_{m-1}}\right]
\end{aligned}$$

We know that $\int_{\mathbb{R}} g(x)dx = 1$. Also, from i's perspective, all the other agents' types are i.i.d from the distribution $g(x)$. So we can simplify equation (A7) to:

$$\begin{aligned}
EU_{\mathrm{m}}\left(x_i, a_{\mathrm{m},i}\right) = \; & - \; r(x_i - a_{\mathrm{m},i})^2 + 2(1-r)\frac{m-1}{m}\left(x_i - \frac{a_{\mathrm{m},i}}{m}\right)\int_{\mathbb{R}} a_{\mathrm{m},j} g(x_j)\mathrm{d}x_j \qquad \text{(A8)} \\
& - \; (1-r)\left[\left(x_i - \frac{a_{\mathrm{m},i}}{m}\right)^2 + \int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \int_{\mathbb{R}} \left(\sum_{j=1}^{m-1} a_{\mathrm{m},j}\right)^2 g(x_1)g(x_2)...g(x_{m-1})\mathrm{d}x_1\mathrm{d}x_2...\mathrm{d}x_{m-1}\right]
\end{aligned}$$

Taking the F.O.C of Equation (A8) w.r.t $a_{\mathrm{m},i}$ at $\hat{a}_{\mathrm{m},i}$ gives us:

$$\left.\frac{\mathrm{d}EU_{\mathrm{m}}\left(x_i, a_{\mathrm{m},i}\right)}{\mathrm{d}a_{\mathrm{m},i}}\right|_{a_{\mathrm{m},i}=\hat{a}_{\mathrm{m},i}} = 2r(x_i - \hat{a}_{\mathrm{m},i}) - \frac{(1-r)(m-1)}{m^2}\int_{\mathbb{R}} \hat{a}_{\mathrm{m},j} g(x_j)\mathrm{d}x_j + \frac{1-r}{m}\left(x_i - \frac{\hat{a}_{\mathrm{m},i}}{m}\right) = 0 \qquad \text{(A9)}$$

This gives us $\hat{a}_{\mathrm{m},i}$ as:

$$\hat{a}_{\mathrm{m},i} = \frac{\left[rm^2 + (1-r)m\right] x_i - (1-r)(m-1)\int_{\mathbb{R}} \hat{a}_{\mathrm{m},j} g(x_j)\mathrm{d}x_j}{rm^2 + (1-r)} \qquad \text{(A10)}$$

Integrating $\hat{a}_{\mathrm{m},i}$ w.r.t x_i, we have:

$$\int_{\mathbb{R}} \hat{a}_{\mathrm{m},i} g(x_i)\mathrm{d}x_i = \frac{\left[rm^2 + (1-r)m\right]\int_{\mathbb{R}} x_i g(x_i)\mathrm{d}x_i - (1-r)(m-1)\int_{\mathbb{R}} \hat{a}_{\mathrm{m},j} g(x_j)\mathrm{d}x_j \int_{\mathbb{R}} g(x_i)\mathrm{d}x_i}{rm^2 + (1-r)} \qquad \text{(A11)}$$

Since $\int_{\mathbb{R}} g(x_i)\mathrm{d}x_i = 1$ and $\int_{\mathbb{R}} \hat{a}_{\mathrm{m},i} g(x_i)\mathrm{d}x_i = \int_{\mathbb{R}} \hat{a}_{\mathrm{m},j} g(x_j)\mathrm{d}x_j = E(a_m)$ this simplifies to $E(a_m) = E(x)$. Since $E(x) = 0$ in a symmetric distribution, we have:

$$\hat{a}_{\mathrm{m},i} = \frac{rm^2 + (1-r)m}{rm^2 + (1-r)} x_i \qquad \text{(A12)}$$

A.2.2 Comparative Statics in m-Player Game

We now derive the comparative statics regarding the extent of shading $s_{\mathrm{m},i}$.

i) $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}r} = (m-1)\frac{-(1-r)(m^2-1)-[rm^2+(1-r)]}{[rm^2+(1-r)]^2}|x_i|$. This simplifies to $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}r} = -\frac{(m-1)m^2}{[rm^2+(1-r)]^2}|x_i|$. We know that $m, m-1 > 0$, because by definition $m \geq 2$. Also, it is clear that $[rm^2+(1-r)]^2 > 0$ and that $|x_i| \geq 0$. It therefore follows that $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}r} \geq 0$.
ii) $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}|x_i|} = \frac{(1-r)(m-1)}{rm^2+(1-r)}$. Since $0 < r < 1$ and $m \geq 2$, this value is always positive. So $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}|x_i|} > 0$.
iii) $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}m} = (1-r)\frac{1-r(m-1)^2}{[rm^2+(1-r)]^2}$. If $m < \frac{1}{\sqrt{r}}+1$, then $1-r(m-1)^2 > 0$; so in this range $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}m} > 0$. Else if $m > \frac{1}{\sqrt{r}}+1$, then $1-r(m-1)^2 < 0$; so in this range $\frac{\mathrm{d}s_{\mathrm{m},i}}{\mathrm{d}m} < 0$.

A.3 Equilibrium for Sub-group Interactions

The expected utility of agent i from sub-group 1 and agent j in sub-group 2 are given in Equations (6). The first-order condition for agent i is $\frac{\mathrm{d}EU_1^i}{\mathrm{d}a_1^i} = 0$ which can be calculated to be:

$$r(x_1 - a_1^i) + \frac{1-r}{m}\left(x_1 - \frac{a_1^i + \sum\limits_{k_1=1(\neq i)}^{n_1} a_1^{k_1} + \sum\limits_{k_2=1}^{n_2} \int_{\mathbb{R}} a_2^{k_2} dx_2}{m}\right) = 0 \tag{A13}$$

As we are looking for a symmetric in actions with sub-group equilibrium, we can set $a_1^i = a_1^k = \hat{a}_1$, and $a_2^k = \hat{a}_2$, and simplify (A13) to:

$$x_1(r + \frac{1-r}{m}) - \hat{a}_1(r + \frac{n_1(1-r)}{m^2}) - \frac{(1-r)n_2}{m^2}\int_{\mathbb{R}} \hat{a}_2 g(x_2)dx_2 = 0 \tag{A14}$$

Thus the equilibrium action $\hat{a}_1$ for agents from sub-group 1 is:

$$\hat{a}_1 = x_1\left(\frac{m(mr+(1-r))}{m^2r + n_1(1-r)}\right) - \frac{(1-r)n_2}{m^2r + n_1(1-r)}\int_{\mathbb{R}} \hat{a}_2 g(x_2)dx_2 \tag{A15}$$

Integrating the equilibrium action over the entire distribution of x_1 we can get:

$$\int_{\mathbb{R}} \hat{a}_1 g(x_1)dx_1 = E(x_1)\left(\frac{m(mr+(1-r))}{m^2r + n_1(1-r)}\right) - \frac{(1-r)n_2}{m^2r + n_1(1-r)}\int_{\mathbb{R}} \hat{a}_2 g(x_2)dx_2 \tag{A16}$$

By noting that $E(x_1) = 0$ for a symmetric distribution, we have that $\int_{\mathbb{R}} \hat{a}_1 g(x_1)dx_1 = \int_{\mathbb{R}} \hat{a}_2 g(x_2)dx_2$. Therefore the equilibrium $\hat{a}_1$ is $\hat{a}_1 = x_1\left(\frac{m(mr+(1-r))}{m^2r+n_1(1-r)}\right)$. Using similar analysis for sub-group 2 we can also derive $\hat{a}_2 = x_2\left(\frac{m(mr+(1-r))}{m^2r+n_2(1-r)}\right)$.

A.4 Analysis of Constrained Choice Game

For a m-player simultaneous choice where preferences are drawn from $U[-1,1]$ and actions are curtailed between $[-1,1]$, there exists a unique Perfect Bayesian equilibrium, where player i chooses action $\hat{a}_{\mathrm{c},i} = min\{\frac{rm^2+(1-r)m}{rm^2+(1-r)}x_i, 1\}$ if $x_i \geq 0$, and $\hat{a}_{\mathrm{c},i} = max\{\frac{rm^2+(1-r)m}{rm^2+(1-r)}x_i, -1\}$ if $x_i < 0$.

We now present the detailed proof for the above statement. Let $\hat{a}_{\mathrm{c},i}$ be i's equilibrium action, and let $A_{\mathrm{c},i}$ be i's optimal action. It therefore follows that $\hat{a}_{\mathrm{c},i} = min\{A_{\mathrm{c},i}, 1\}$ if $A_{\mathrm{c},i} \geq 0$ and $\hat{a}_{\mathrm{c},i} = max\{A_{\mathrm{c},i}, -1\}$ if $A_{\mathrm{c},i} < 0$. Player i's optimal action $A_{\mathrm{c},i}$ can be derived using the same steps those used in Web Appendix A.2.1. We therefore have:

$$A_{\mathrm{c},i} = \frac{\left[2rm^2 + 2(1-r)m\right] x_i - (m-1) \int_{-1}^{1} \hat{a}_{\mathrm{c},j} \mathrm{d}x_j}{2rm^2 + 2(1-r)} \tag{A17}$$

Note that $A_{\mathrm{c},i}$ is monotonically increasing in x_i. Hence, if at some point $x_i = q_2 > 0$, $A_{\mathrm{c},i} = 1$, then for all $x_i > q_2$, $A_{\mathrm{c},i} > 1$, which implies that $\hat{a}_{\mathrm{c},i} = 1$. Similarly, if at some point $x_i = q_1 < 0$, $A_{\mathrm{c},i} = -1$, then for all $x_i < q_2$, $A_{\mathrm{c},i} < -1$, which implies that $\hat{a}_{\mathrm{c},i} = -1$. Therefore, we can express $\int_{-1}^{1} \hat{a}_{\mathrm{c},i} \mathrm{d}x_i$ as follows:

$$\int_{-1}^{1} \hat{a}_{\mathrm{c},i} \mathrm{d}x_i = \int_{-1}^{q_1} -1 \mathrm{d}x_i + \int_{q_1}^{q_2} A_{\mathrm{c},i} \mathrm{d}x_i + \int_{q_2}^{1} 1 \mathrm{d}x_i \tag{A18}$$

Substituting for $A_{\mathrm{c},i}$ from Equation (A17) and integrating, we have:

$$\int_{-1}^{1} \hat{a}_{\mathrm{c},i} \mathrm{d}x_i = -(1+q_1) + \frac{1}{2} \frac{rm^2 + (1-r)m}{rm^2 + (1-r)} \left(q_2^2 - q_1^2\right) - (q_2 - q_1) \frac{m-1}{2rm^2 + 2(1-r)} \int_{-1}^{1} \hat{a}_{\mathrm{c},j} + (1-q_2) \tag{A19}$$

We also know that $A_{\mathrm{c},i}$ is just equal to -1 at q_1 and $A_{\mathrm{c},i}$ is just equal to 1 at q_2. So, we have:

$$-1 = \frac{\left[2rm^2 + 2(1-r)m\right] q_1 - (m-1) \int_{-1}^{1} \hat{a}_{\mathrm{c},j} \mathrm{d}x_j}{2rm^2 + 2(1-r)} \tag{A20}$$

$$1 = \frac{\left[2rm^2 + 2(1-r)m\right] q_2 - (m-1) \int_{-1}^{1} \hat{a}_{\mathrm{c},j} \mathrm{d}x_j}{2rm^2 + 2(1-r)} \tag{A21}$$

Next, we multiply Equation (A20) with q_1 and Equation (A21) with q_2 and subtract the latter from the former. This gives us:

$$(q_2 - q_1) \frac{m-1}{2rm^2 + 2(1-r)} \int_{-1}^{1} \hat{a}_{\mathrm{c},j} \mathrm{d}x_j = (q_2^2 - q_1^2) \frac{2rm^2 + 2(1-r)m}{2rm^2 + 2(1-r)} - (q_1 + q_2) \tag{A22}$$

Next, we substitute the L.H.S of Equation (A22) in Equation (A19) and obtain:

$$\int_{-1}^{1} \hat{a}_{\mathrm{c},i} \mathrm{d}x_i = -\frac{1}{2} \frac{rm^2 + (1-r)m}{rm^2 + (1-r)} \left(q_2^2 - q_1^2\right) \tag{A23}$$

Adding Equation (A20) to Equation (A21) gives us:

$$(q_1 + q_2) \frac{rm^2 + (1-r)m}{rm^2 + (1-r)} = \frac{(m-1)}{rm^2 + (1-r)} \int_{-1}^{1} \hat{a}_{\mathrm{c},j} \mathrm{d}x_j \tag{A24}$$

Substituting the L.H.S into Equation (A23) gives us:

$$\int_{-1}^{1} \hat{a}_{c,i} \mathrm{d}x_i = -\frac{q_2 - q_1}{2} \frac{(m-1)}{rm^2 + (1-r)} \int_{-1}^{1} \hat{a}_{c,j} \mathrm{d}x_j \tag{A25}$$

If $q_2 - q_1 = 0$, then we directly have: $\int_{-1}^{1} \hat{a}_{c,j} \mathrm{d}x_j = 0$. Else if $q_2 - q_1 \neq 0$, it still follows that $\int_{-1}^{1} \hat{a}_{c,i} \mathrm{d}x_i = \int_{-1}^{1} \hat{a}_{c,j} \mathrm{d}x_j = 0$. Hence the optimal response of agent i is:

$$A_{c,i} = \frac{rm^2 + (1-r)m}{rm^2 + (1-r)} x_i \tag{A26}$$

It is clear that $\frac{rm^2+(1-r)m}{rm^2+(1-r)} > 0$. So the equilibrium response of agent i is given by $\hat{a}_{c,i} = min\{\frac{rm^2+(1-r)m}{rm^2+(1-r)} x_i, 1\}$ if $x_i \geq 0$ and $\hat{a}_{c,i} = max\{\frac{rm^2+(1-r)m}{rm^2+(1-r)} x_i, -1\}$ if $x_i < 0$.

A.5 Asymmetric Distribution

We now consider a game where the distribution of types, $g(x)$, is asymmetric, i.e., $E(x) \neq 0$. To solve for the equilibrium in this case, we start with the general equation considered in Equation (4):

$$\hat{a}_i = \frac{2(1+r)}{1+3r} x_i - \frac{(1-r)}{(1+3r)} \int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j$$

As usual, integrating i's equilibrium action $\hat{a}_i$ over the entire range of x_i gives us:

$$\int_{\mathbb{R}} \hat{a}_i g(x_i) \mathrm{d}x_i = \frac{2(1+r)}{1+3r} \int_{\mathbb{R}} x_i g(x_i) \mathrm{d}x_i - \frac{(1-r)}{(1+3r)} \int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j \int_{\mathbb{R}} g(x_i) \mathrm{d}x_i \tag{A27}$$

Unlike the symmetric distribution case, here we know that $E(x_i) = \int_{\mathbb{R}} x_i g(x_i) \mathrm{d}x_i \neq 0$. But we can re-arrange the terms to write $\int_{\mathbb{R}} \hat{a}_i g(x_i) \mathrm{d}x_i$ as follows:

$$\int_{\mathbb{R}} \hat{a}_i g(x_i) \mathrm{d}x_i = E(x_i) \tag{A28}$$

We can now substitute the above equivalence in Equation (4) to derive the a_i as follows:

$$\hat{a}_i = \frac{2(1+r)}{1+3r} x_i - \frac{(1-r)}{1+3r} E(x_i) \tag{A29}$$

A.6 Partial Knowledge

A.6.1 Same-side Leaning Agents

First, we consider the case where both agents i and j lean on the same side. Without loss of generality, let both agents be drawn from $\mathbb{R}_+$. Then, we have:

$$\hat{a}_i = \frac{2(1+r)}{1+3r} x_i - \frac{(1-r)}{(1+3r)} \int_{\mathbb{R}_+} \hat{a}_j g(x_j) \mathrm{d}x_j \tag{A30}$$

Integrating i's equilibrium action $\hat{a}_i$ over the entire range of x_i gives us:

$$\int_{\mathbb{R}_+} \hat{a}_i g(x_i)\mathrm{d}x_i = \frac{2(1+r)}{1+3r}\int_{\mathbb{R}_+} x_i g(x_i)\mathrm{d}x_i - \frac{(1-r)}{(1+3r)}\int_{\mathbb{R}_+} \hat{a}_j g(x_j)\mathrm{d}x_j \int_{\mathbb{R}_+} g(x_i)\mathrm{d}x_i \quad \text{(A31)}$$

Since we know that $x_i \in \mathbb{R}_+$, $\int_{\mathbb{R}_+} g(x_i)\mathrm{d}x_i = 1$. This gives us:

$$\int_{\mathbb{R}_+} \hat{a}_i g(x_i)\mathrm{d}x_i = \int_{\mathbb{R}_+} x_i g(x_i)\mathrm{d}x_i \quad \text{(A32)}$$

Substituting the above equivalence into Equation (A33), we have:

$$\hat{a}_i = \frac{2(1+r)}{1+3r}x_i - \frac{(1-r)}{(1+3r)}\int_{\mathbb{R}_+} x_j g(x_j)\mathrm{d}x_j \quad \text{(A33)}$$

A.6.2 Opposite Leaning Agents

Next, we consider the case where i knows that j is on the opposite side (and vice-versa). Without loss of generality, let i be drawn from $\mathbb{R}_+$ and j be drawn from $\mathbb{R}_-$. Then, we have:

$$\hat{a}_i = \frac{2(1+r)}{1+3r}x_i - \frac{(1-r)}{(1+3r)}\int_{\mathbb{R}_-} \hat{a}_j g(x_j)\mathrm{d}x_j \quad \text{(A34)}$$

Integrating i's equilibrium action $\hat{a}_i$ over the entire range of x_i gives us:

$$\int_{\mathbb{R}_+} \hat{a}_i g(x_i)\mathrm{d}x_i = \frac{2(1+r)}{1+3r}\int_{\mathbb{R}_+} x_i g(x_i)\mathrm{d}x_i - \frac{(1-r)}{(1+3r)}\int_{\mathbb{R}_-} \hat{a}_j g(x_j)\mathrm{d}x_j \int_{\mathbb{R}_+} g(x_i)\mathrm{d}x_i \quad \text{(A35)}$$

Since we know that $x_i \in \mathbb{R}_+$, $\int_{\mathbb{R}_+} g(x_i)\mathrm{d}x_i = 1$. This gives us:

$$\int_{\mathbb{R}_+} \hat{a}_i g(x_i)\mathrm{d}x_i = \frac{2(1+r)}{1+3r}\int_{\mathbb{R}_+} x_i g(x_i)\mathrm{d}x_i - \frac{(1-r)}{(1+3r)}\int_{\mathbb{R}_-} \hat{a}_j g(x_j)\mathrm{d}x_j \quad \text{(A36)}$$

Similarly, for j, we can write:

$$\int_{\mathbb{R}_-} \hat{a}_j g(x_j)\mathrm{d}x_j = \frac{2(1+r)}{1+3r}\int_{\mathbb{R}_-} x_j g(x_j)\mathrm{d}x_j - \frac{(1-r)}{(1+3r)}\int_{\mathbb{R}_+} \hat{a}_i g(x_i)\mathrm{d}x_i \quad \text{(A37)}$$

Because the distribution $g(x)$ is symmetric, we know that $\int_{\mathbb{R}_+} x_i g(x_i) = -\int_{\mathbb{R}_-} x_j g(x_j)$. Therefore, adding up Equations (A36) and (A37), we have:

$$\int_{\mathbb{R}_+} \hat{a}_i g(x_i)\mathrm{d}x_i = -\int_{\mathbb{R}_-} \hat{a}_j g(x_j)\mathrm{d}x_j \quad \text{(A38)}$$

We can thus write $\int_{\mathbb{R}_-} \hat{a}_j g(x_j)\mathrm{d}x_j$ as follows:

$$\int_{\mathbb{R}_-} \hat{a}_j g(x_j)\mathrm{d}x_j = \frac{(1+r)}{2r}\int_{\mathbb{R}_-} x_j g(x_j)\mathrm{d}x_j \quad \text{(A39)}$$

Substituting this in Equation (A34), we have:

$$\hat{a}_i = \frac{2(1+r)}{1+3r}x_i - \frac{(1-r)(1+r)}{2r(1+3r)}\int_{\mathbb{R}_-} x_j g(x_j)\mathrm{d}x_j \quad \text{(A40)}$$

A.7 Game with Disclosure

Consider a two-stage game such that in Stage 1, agents have the opportunity to simultaneously reveal their type (which is verifiable). Then in Stage 2, the players simultaneously choose an action. There are four possible equilibria of this game: (D,D), (ND, ND), (D, ND), and (ND, ND), where D stands for Disclosure and ND stands for Non-Disclosure. The (ND, ND) equilibrium is equivalent to the baseline game considered in the main analysis, where agents' types are private information.

We first consider a (ND, ND) equilibrium and see whether agent i has the incentive to deviate. Suppose that i deviates are reveals her type and j plays the equilibrium strategy of not disclosing her type x_j. Then we can derive the optimal actions of j in this case as:

$$\hat{a}_j = \frac{2(1+r)}{1+3r}x_j - \frac{(1-r)}{(1+3r)}\hat{a}_i \tag{A41}$$

Note that there is no integral over a_i since j does not have uncertainty on i's type anymore. Next, we can derive a_i as:

$$\hat{a}_i = \frac{2(1+r)}{1+3r}x_i - \frac{(1-r)}{(1+3r)}\int_{\mathbb{R}} \hat{a}_j g(x_j)\mathrm{d}x_j \tag{A42}$$

Substituting a_j from Equation (A41) into the above equation, we have:

$$\hat{a}_i = \frac{1+3r}{4r}x_i \tag{A43}$$

Now we can write the expected utility of i when she plays the equilibrium strategy as:

$$EU_{ND,ND}(x_i, a_i) = -\frac{r(1-r)^2}{(1+3r)^2}x_i^2 - \frac{(1-r)4r^2}{(1+3r)^2}x_i^2 - \frac{(1-r)(1+r)^2}{(1+3r)^2}\int_{\mathbb{R}} x_j^2 g(x_j)\mathrm{d}x_j. \tag{A44}$$

Similarly, we can write the expected utility of i when she deviates as:

$$EU_{D,ND}(x_i, a_i) = -\frac{(1-r)^2}{16r}x_i^2 - \frac{(1-r)}{4}x_i^2 - \frac{(1-r)(1+r)^2}{(1+3r)^2}\int_{\mathbb{R}} x_j^2 g(x_j)\mathrm{d}x_j. \tag{A45}$$

Comparing Equations (A44) and (A45), we can see that $EU_{ND,ND}(x_i, a_i) > EU_{D,ND}(x_i, a_i)$ if:

$$\begin{aligned}
& -\frac{r(1-r)^2}{(1+3r)^2}x_i^2 - \frac{(1-r)4r^2}{(1+3r)^2}x_i^2 > \frac{(1-r)^2}{16r}x_i^2 - \frac{(1-r)}{4}x_i^2 \\
\Rightarrow\quad & -\frac{r(1-r)}{(1+3r)^2} - \frac{4r^2}{(1+3r)^2} > -\frac{(1-r)}{16r} - \frac{1}{4} \\
\Rightarrow\quad & -\frac{r}{(1+3r)} > -\frac{1+3r}{16r} \\
\Rightarrow\quad & -16r^2 + (1+3r)^2 > 0 \\
\Rightarrow\quad & (1+7r)(1-r) > 0
\end{aligned} \tag{A46}$$

The above inequality is always true. Hence, we know that agent i has no incentive to deviate from the (ND, ND) equilibrium. Based on the same inequality, we also know that both (D, ND) and (ND, D) cannot be equilibrium outcomes since the agent who is disclosing her type will always benefit from deviating and choosing to not disclose her type.

A.8 Alternative Preferences

We first consider the case where agents care about choosing an action that is close to the mean preferences of the group. That is, suppose that the agent's utility can be expressed as:

$$u_1(x_i, a_i, a_j) = -r(x_i - a_i)^2 - (1-r)(x_i - \bar{x})^2 \tag{A47}$$

Taking the expectation, i's expected utility from choosing a_i in this setting can be written as $EU_1(x_i, a_i) = \frac{\int_{\mathbb{R}} u_1(x_i, a_i, \hat{a}_j) g(x_j) \mathrm{d}x_j}{\int_{\mathbb{R}} g(x_j) \mathrm{d}x_j}$. By differentiating $EU_1(x_i, a_i)$ and setting it equal to zero at i's equilibrium action $a_i = \hat{a}_i$ gives us:

$$\left.\frac{\partial EU_1(x_i, a_i)}{\partial a_i}\right|_{a_i = \hat{a}_i} = 2r(x_i - \hat{a}_i) - 2(1-r)\left[\hat{a}_i - \frac{x_i}{2} - \frac{1}{2}\int_{\mathbb{R}} x_j g(x_j) \mathrm{d}x_j\right] = 0 \tag{A48}$$

This in turn simplifies to:

$$\hat{a}_i = \frac{(1+r)}{2} x_i \tag{A49}$$

Next, consider the second alternative preference structure, where agents care about choosing an action that is close to the group's decision ($\bar{a}$). We can write agent i's utility in this case as:

$$u_2(x_i, a_i, a_j) = -r(x_i - a_i)^2 - (1-r)(a_i - \bar{a})^2 \tag{A50}$$

Taking the expectation, i's expected utility from choosing a_i in this setting can be written as $EU_2(x_i, a_i) = \frac{\int_{\mathbb{R}} u_2(x_i, a_i, \hat{a}_j) g(x_j) \mathrm{d}x_j}{\int_{\mathbb{R}} g(x_j) \mathrm{d}x_j}$. By differentiating $EU_2(x_i, a_i)$ and setting it equal to zero at i's action $\hat{a}_i$, we have:

$$\left.\frac{\partial EU_2(x_i, a_i)}{\partial a_i}\right|_{a_i = \hat{a}_i} = 2r(x_i - \hat{a}_i) - \frac{(1-r)}{2}\left[\hat{a}_i - \int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j\right] = 0 \tag{A51}$$

Re-arranging the terms, we have:

$$\hat{a}_i = \frac{4r}{(1+3r)} x_i + \frac{(1-r)}{(1+3r)} \int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j \tag{A52}$$

Taking the integral of the above equation with respect to the distribution of x_i, we have:

$$\int_{\mathbb{R}} \hat{a}_i g(x_i) \mathrm{d}x_i = \frac{4r}{(1+3r)} \int_{\mathbb{R}} x_i g(x_i) \mathrm{d}x_i + \frac{(1-r)}{(1+3r)} \int_{\mathbb{R}} \hat{a}_j g(x_j) \mathrm{d}x_j \tag{A53}$$

Since $\int_{\mathbb{R}} x_i g(x_i) \mathrm{d}x_i = 0$, we have $\int_{\mathbb{R}} \hat{a}_i g(x_i) \mathrm{d}x_i = 0$. This gives us the equilibrium action of i as:

$$\hat{a}_i = \frac{4r}{(1+3r)} x_i \tag{A54}$$

B Equilibrium of the Exogenous Sequential Game

Let i and j be the first and second players, respectively. The second player j's equilibrium action has already been derived in the main text (see section **??**), and is equal to:

$$\hat{a}_{\mathrm{x}2,j} = \frac{2(1+r)}{1+3r} x_j - \frac{(1-r)}{1+3r} a_{\mathrm{x}1,i} \tag{A55}$$

So we now derive the first player's optimal action $\hat{a}_{\mathrm{x}1,i}$. In equilibrium, i's utility from choosing action $a_{\mathrm{x}1,i}$ is given by:

$$u\left(x_i, a_{\mathrm{x}1,i}, \hat{a}_{\mathrm{x}2,j}\right) = -r(x_i - a_{\mathrm{x}1,i})^2 - (1-r)\left(x_i - \bar{a}_{\mathrm{x}}\right)^2 \tag{A56}$$

where $\bar{a}_{\mathrm{x}} = \frac{a_{\mathrm{x}1,i} + \hat{a}_{\mathrm{x}2,j}}{2}$. We know that when i chooses $a_{\mathrm{x}1,i}$, in response, j chooses $\hat{a}_{\mathrm{x}2,j} = \frac{2(1+r)}{1+3r} x_j - \frac{(1-r)}{1+3r} a_{\mathrm{x}1,i}$. This in turn gives us $\bar{a}_{\mathrm{x}}$ as:

$$\bar{a}_{\mathrm{x}} = \frac{(1+r)x_j + 2r a_{\mathrm{x}1,i}}{1+3r} \tag{A57}$$

Substituting this value of $\bar{a}_{\mathrm{x}}$ into Equation (A56), we have:

$$\begin{aligned} u\left(x_i, a_{\mathrm{x}1,i}, \hat{a}_{\mathrm{x}2,j}\right) = -r(x_i - a_{\mathrm{x}1,i})^2 \quad &- \quad (1-r)\left[\left(x_i - \frac{2r}{1+3r} a_{\mathrm{x}1,i}\right)^2 + \left(\frac{1+r}{1+3r} x_j\right)^2\right] \\ &+ \quad 2(1-r)\left[\left(x_i - \frac{2r}{1+3r} a_{\mathrm{x}1,i}\right)\left(\frac{1+r}{1+3r} x_j\right)\right] \end{aligned} \tag{A58}$$

Hence, the expected utility of agent i in from choosing action $a_{\mathrm{x}1,i}$ in period 1 is:

$$EU_{\mathrm{x}1}\left(x_i, a_{\mathrm{x}1,i}\right) = \frac{\int_{\mathbb{R}} u\left(x_i, a_{\mathrm{x}1,i}, \hat{a}_{\mathrm{x}2,j}\right) g(x_j) \mathrm{d}x_j}{\int_{\mathbb{R}} g(x_j) \mathrm{d}x_j} \tag{A59}$$

We can simplify the above as follows:

$$\begin{aligned} EU_{\mathrm{x}1}\left(x_i, a_{\mathrm{x}1,i}\right) = \quad &- \quad r(x_i - a_{\mathrm{x}1,i})^2 + 2(1-r)\left(x_i - \frac{2r}{1+3r} a_{\mathrm{x}1,i}\right)\left(\frac{1+r}{1+3r}\right) \int_{\mathbb{R}} x_j g(x_j) \mathrm{d}x_j \\ &- \quad (1-r)\left[\left(x_i - \frac{2r}{1+3r} a_{\mathrm{x}1,i}\right)^2 + \left(\frac{1+r}{1+3r}\right)^2 \int_{\mathbb{R}} x_j^2 g(x_j) \mathrm{d}x_j\right] \end{aligned} \tag{A60}$$

Computing the integrals in Equation (A60), we have:

$$EU_{\mathrm{x1}}\left(x_i, a_{\mathrm{x1},i}\right) = -r\left(x_i - a_{\mathrm{x1},i}\right)^2 - (1-r)\left[\left(x_i - \frac{2r}{1+3r}a_{\mathrm{x1},i}\right)^2 + \left(\frac{1+r}{1+3r}\right)^2 \int_{\mathbb{R}} x_j^2 g(x_j)\mathrm{d}x_j\right] \tag{A61}$$

Solving the first-order condition for agent i's action gives us the equilibrium choice $\hat{a}_{\mathrm{x1},i}$:

$$\hat{a}_{\mathrm{x1},i} = \frac{(1+3r)(3+r)}{(1+3r)^2 + 4r(1-r)}x_i \tag{A62}$$

Thus, given x_i and x_j, there exists a unique exogenous sequential choice equilibrium, where $\hat{a}_{\mathrm{x1},i} = \frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}x_i$ and $\hat{a}_{\mathrm{x2},j} = \frac{2(1+r)x_j-(1-r)a_{\mathrm{x1},i}}{1+3r}$.

C Proof of Proposition 2

Recall that the mean of the equilibrium outcome for the exogenous sequential game can be expressed as:

$$\bar{a}_{\mathrm{x}} = \frac{\hat{a}_{\mathrm{x1},i} + \hat{a}_{\mathrm{x2},j}}{2} = \frac{2ra_{\mathrm{x1},i} + (1+r)x_j}{1+3r} = \frac{2r(3+r)}{(1+3r)^2 + 4r(1-r)}x_i + \frac{1+r}{1+3r}x_j \tag{A63}$$

Next we show that $0 < k_1(r), k_2(r) < 1$.

- $k_1(r) < 1$ if $\frac{(1+3r)(1-r)}{(1+3r)^2+4r(1-r)} < 1 \Rightarrow (1+3r)(1-r) < (1+3r)^2 + 4r(1-r) \Rightarrow -4r(1+3r) < 4r(1-r)$, which is always true if $0 < r < 1$. Hence $k_1(r) < 1$.
- $k_2(r) < 1$ if $\frac{(1+3r)\left[(1+3r)^2+16r\right]}{[(1+3r)^2+4r(1-r)][2(1+r)+(1+3r)]} < 1, \Rightarrow 4r(1+3r)(3+r) < 2(1+3r)^2(1+r) + 4r(1-r)(1+r) \Rightarrow -(1+3r)(1-r)^2 < 4r(1-r)(1+r)$, which is always true since $0 < r < 1$.

Further, $k_1(r), k_2(r), k_3(r) > 0$ since all the terms in their numerators and denominators are positive. Thus $0 < k_1(r), k_2(r), k_3(r) < 1$.

a) First, we compare $\bar{a}_{\mathrm{x}}$ with $\bar{x}$. Without loss of generality, let $x_i \geq 0$.

- Polarization – $|\bar{a}_{\mathrm{x}}| > |\bar{x}|$ and $\bar{a}_{\mathrm{x}}\bar{x} > 0$.
  - First, consider the case where $x_j > k_1(r)x_i$. In this case, $\bar{a}_{\mathrm{x}}\bar{x} > 0$ because both $\bar{a}_{\mathrm{x}} > 0$ and $\bar{x} > 0$. The condition $|\bar{a}_{\mathrm{x}}| > |\bar{x}|$ therefore simplifies to $\bar{a}_{\mathrm{x}} > \bar{x}$. This can be expressed as:

$$\frac{2r(3+r)}{(1+3r)^2 + 4r(1-r)}x_i + \frac{1+r}{1+3r}x_j > \frac{1+r}{1+3r}x_i + \frac{1+r}{1+3r}x_j$$
$$\Rightarrow \quad x_j > \frac{(1+3r)(1-r)}{(1+3r)^2 + 4r(1-r)}x_i \tag{A64}$$

    where the multiplier of x_i is $k_1(r) = \frac{(1+3r)(1-r)}{(1+3r)^2+4r(1-r)}$. By definition,(A64) is satisfied.

- Second, consider the case where $-x_i < x_j \leq k_1(r)x_i$. Here, $\bar{x} > 0$. So the condition $|\bar{a}_\text{x}| > |\bar{x}|$ simplifies to $\bar{a}_\text{x} < \bar{x}$. From the previous case, we know that $\bar{a}_\text{x} > \bar{x}$ if and only if $x_j > k_1(r)x_i$. This is not possible here since by definition $x_j \leq k_1(r)x_i$.
- Third, consider the case where $x_j = -x_i$. Here $\bar{x} = 0$. So, we cannot have $\bar{a}_\text{x}\bar{x} > 0$.
- Fourth, consider the case where $x_j < -x_i$. Here, $\bar{x} < 0$. Since we require $\bar{a}_\text{x}\bar{x}$ to be greater than zero, it follows that $\bar{a}_\text{x} < 0$. So the condition $|\bar{a}_\text{x}| > |\bar{x}|$ simplifies to $\bar{a}_\text{x} < \bar{x} \Rightarrow x_j < k_1(r)x_i$, which is always true since by definition $x_j < -x_i$.

Hence, polarization occurs in the first and third cases, $i.e.$, when $x_j > k_1(r)x_i$ or when $x_j < -x_i$.

- Reverse Polarization – $|\bar{a}_\text{x}| > |\bar{x}|$ and $\bar{a}_\text{x}\bar{x} \leq 0$.
  - First, consider the case where $x_j > -k_2(r)x_i$. Here both $\bar{a}_\text{x} > 0$ and $\bar{x} > 0$ since $k_2(r) < 1$. So it cannot be that $\bar{a}_\text{x}\bar{x} \leq 0$. Therefore, this case is ruled out.
  - Second, consider the case where $-x_i \leq x_j < -k_2(r)x_i$. Here $\bar{x} \geq 0$. So the condition $|\bar{a}_\text{x}| \geq |\bar{x}|$ simplifies to $\bar{a}_\text{x} \leq -\bar{x}$. This can be expressed as:

$$\frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i + \frac{1+r}{1+3r}x_j < -\frac{1+r}{1+3r}x_i - \frac{1+r}{1+3r}x_j$$
$$\Rightarrow \quad x_j < \frac{(1+3r)\left[(1+3r)^2+16r\right]}{\left[(1+3r)^2+4r(1-r)\right]\left[2(1+r)+(1+3r)\right]}x_i \tag{A65}$$

  where the multiplier of x_i is labeled $k_2(r) = \frac{(1+3r)\left[(1+3r)^2+16r\right]}{\left[(1+3r)^2+4r(1-r)\right]\left[2(1+r)+(1+3r)\right]}$. Since by definition $x_j < -k_2(r)x_i$, condition (A65) is always satisfied.
  - Third, consider the case where $x_j < -x_i$. Here $\bar{x} < 0$. So the condition $|\bar{a}_\text{x}| \geq |\bar{x}|$ simplifies to $\bar{a}_\text{x} \geq \bar{x}$, which we know is the same as $x_j \geq k_1(r)x_i > 0$. However, this is not possible since by definition $x_j < -x_i$.

  Hence, Reverse Polarization only occurs when $-x_i \leq x_j < -k_2(r)x_i$.
- Moderation – $|\bar{a}_\text{x}| \leq |\bar{x}|$.
  - First, consider the case where $x_j < -x_i$. Here, $\bar{x} \leq 0$. So the condition $|\bar{a}_\text{x}| \leq |\bar{x}|$ simplifies to $-\bar{x} \leq \bar{a}_\text{x} \leq \bar{x}$. If $-\bar{x} \leq \bar{a}_\text{x}$, it then follows that $x_j \geq k_2(r)x_i \geq 0$, which is impossible since by definition $x_j < -x_i$.
  - Second, consider the case where $-x_i \leq x_j < -k_2(r)x_i$. Here, $\bar{x} \geq 0$. So the condition $|\bar{a}_\text{x}| \leq |\bar{x}|$ simplifies to $-\bar{x} \leq \bar{a}_\text{x} \leq \bar{x}$. We know that this condition can be expressed as

$-k_2(r)x_i \leq x_j \leq k_1(r)x_i$. This is not possible, since by definition $x_j < -k_2(r)x_i$.

- Third, consider the case where $-k_2(r)x_i \leq x_j \leq k_1(r)x_i$. Here, $\bar{x} > 0$. So the condition $|\bar{a}_{\mathrm{x}}| \leq |\bar{x}|$ simplifies to $\bar{x} \leq \bar{a}_{\mathrm{x}} \leq -\bar{x}$. This in turn can be expressed as $-k_2(r)x_i \leq x_j \leq k_1(r)x_i$, which we know is true by definition.
- Fourth, consider the case where $x_j > k_1(r)x_i$. Here, $\bar{x} > 0$. So the condition $|\bar{a}_{\mathrm{x}}| \leq |\bar{x}|$ simplifies to $\bar{x} \leq \bar{a}_{\mathrm{x}} \leq -\bar{x}$. This in turn can be expressed as $-k_2(r)x_i \leq x_j \leq k_1(r)x_i$, which cannot be true, since by definition $x_j > k_1(r)x_i$.

Hence, moderation occurs only when $-k_2(r)x_i \leq x_j \leq k_1(r)x_i$.

Proof of Proposition 3

1. First, we show that, for $x_j > 0$, $\hat{a}_{\mathrm{x}2,j} \geq \hat{a}_j$ if $x_i \leq 0$, and $\hat{a}_{\mathrm{x}2,j} < \hat{a}_j$ if $x_i > 0$.

   (a) Let $x_i \leq 0$

   $\hat{a}_{\mathrm{x}2,j} - \hat{a}_j$ can be simplified to $\hat{a}_{\mathrm{x}2,j} - \hat{a}_j = -\frac{(1-r)}{1+3r}\hat{a}_{\mathrm{x}1,i}$. We know that $\hat{a}_{\mathrm{x}1,i} = \mu_x(r)x_i \leq 0$ because $\mu_x(r) > 0$, $x_i \leq 0$. It therefore follows that $-\frac{(1-r)}{1+3r}\hat{a}_{\mathrm{x}1,i} \geq 0 \Rightarrow \hat{a}_{\mathrm{x}2,j} \geq \hat{a}_j$.

   (b) Let $x_i > 0$

   As before $\hat{a}_{\mathrm{x}2,j} - \hat{a}_j = -\frac{(1-r)}{1+3r}\hat{a}_{\mathrm{x}1,i}$. However, here $\hat{a}_{\mathrm{x}1,i} = \mu_x(r)x_i > 0$ because $\mu_x(r), x_i > 0$. Hence it follows that $\hat{a}_{\mathrm{x}2,j} - \hat{a}_j < 0 \Rightarrow \hat{a}_{\mathrm{x}2,j} < \hat{a}_j$.

2. Next, we show that $|\hat{a}_{\mathrm{x}1,i}| \geq |\hat{a}_i|$ and $\frac{\mathrm{d}|\hat{a}_{\mathrm{x}1,i}|}{\mathrm{d}r} < 0$.

   To show that $|\hat{a}_{\mathrm{x}1,i}| \geq |\hat{a}_i|$, we need to show that $\mu_{\mathrm{x}}(r) \geq \mu(r)$.

$$\begin{aligned} \mu_{\mathrm{x}}(r) - \mu(r) &= \frac{(1+3r)(3+r)}{(1+3r)^2 + 4r(1-r)} - \frac{2(1+r)}{1+3r} \\ &= \frac{(1-r)^3}{(1+3r)[(1+3r)^2 + 4r(1-r)]} > 0 \text{ if } r > 0 \end{aligned} \tag{A66}$$

   Therefore, $\mu_{\mathrm{x}}(r) \geq \mu(r) \Rightarrow |\hat{a}_{\mathrm{x}1,i}| \geq |\hat{a}_i|$.

   The derivative of $\hat{a}_{\mathrm{x}1,i}$ w.r.t r can be calculated and simplified to $\frac{\mathrm{d}\hat{a}_{\mathrm{x}1,i}}{\mathrm{d}r} = -\frac{4(5r^2+6r+5)}{[(1+3r)^2+4r(1-r)]^2}x_i$. Since $r > 0$, it follows that $\frac{\mathrm{d}\hat{a}_{\mathrm{x}1,i}}{\mathrm{d}r} < 0$ $\square$

D Proof of Proposition 4

First, we show that $0 < k_3(r) < 1$. This is true if $\frac{(1+3r)^3+8r(1-r)(1+r)}{2(1+r)[(1+3r)^2+4r(1-r)]} < 1 \Rightarrow (1+3r)^3 < 2(1+r)(1+3r)^2, \Rightarrow r < 1$, which we know is always true. Further, $k_3(r) > 0$ since all the terms in their numerators and denominators are positive. Thus $0 < k_3(r) < 1$.

Next, we compare $\bar{a}_{\mathrm{x}}$ with $\bar{a}$. Without loss of generality, let $x_i \geq 0$.

- $|\bar{a}_{\mathrm{x}}| > |\bar{a}|$ and $\bar{a}_{\mathrm{x}}\bar{x} > 0$.
  - First, consider the case where $x_j \geq -x_i$. Here, $\bar{x} \geq 0$. So the condition $|\bar{a}_{\mathrm{x}}| > |\bar{a}|$ simplifies to $\bar{a}_{\mathrm{x}} > \bar{a} \Rightarrow \frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i > \frac{1+r}{1+3r}x_i$, which is impossible since $0 < r < 1$ and $x_i \geq 0$.
  - Second, consider the case where $-x_j < x_i$. Here $\bar{x} < 0$. So the condition $|\bar{a}_{\mathrm{x}}| > |\bar{a}|$ simplifies to $\bar{a}_{\mathrm{x}} < \bar{a} \Rightarrow, \frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i < \frac{1+r}{1+3r}x_i$, which is always true for $0 < r < 1$, $x_i \geq 0$.

  Hence, for $-x_j < x_i$, the mean outcome in the exogenous sequential game is more polarized than that in the simultaneous game, and this polarization is in the same direction as $\bar{x}$.
- $|\bar{a}_{\mathrm{x}}| > |\bar{a}|$ and $\bar{a}_{\mathrm{x}}\bar{x} \leq 0$.
  - First, consider the case where $x_j < -x_i$. Then $\bar{a}, \bar{x} < 0$. So for $\bar{a}_{\mathrm{x}}\bar{x} \leq 0$ to be true, we require $\bar{a}_{\mathrm{x}} \leq 0$, which is not possible since $\bar{a}_{\mathrm{x}} < \bar{x} < 0$.
  - Second, consider the case where $-x_i \leq x_j < -k_3(r)x_i$. Here, $\bar{x} \geq 0$ and the condition $|\bar{a}_{\mathrm{x}}| > |\bar{a}|$ simplifies to $\bar{a}_{\mathrm{x}} > -\bar{a}$. This can be expressed as:

$$\frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i < -\frac{1+r}{1+3r}x_i$$
$$\Rightarrow \quad x_j < \frac{(1+3r)^3+8r(1-r)(1+r)}{2(1+r)\left[(1+3r)^2+4r(1-r)\right]}x_i \tag{A67}$$

    where the multiplier of x_i is labeled $k_3(r) = \frac{(1+3r)^3+8r(1-r)(1+r)}{2(1+r)[(1+3r)^2+4r(1-r)]}x_i$. Since by definition, $x_j < k_3(r)x_i$, condition (A67) is always satisfied.
  - Third, consider the case where $x_j \geq k_3(r)x_i$. Then, $\bar{x} \geq 0$ and the $|\bar{a}_{\mathrm{x}}| > |\bar{a}|$ simplifies to $\bar{a}_{\mathrm{x}} < -\bar{a}$. However, from the second case, we know that this condition can only be satisfied when $x_j < k_3(r)x_i$, which cannot hold here, since by definition $x_j \geq k_3(r)x_i$.

  Hence, for $-x_i \leq x_j < -k_3(r)x_i$, the mean outcome in the exogenous sequential game is more extreme than that in the simultaneous game, but in the direction opposite to that indicated by the mean preference $\bar{x}$.

- $|\bar{a}_{\mathrm{x}}| \leq |\bar{a}|$.
    - First, consider the case where $x_j \geq -k_3(r)x_i$. Here, $\bar{a}_{\mathrm{x}}, \bar{x}, \bar{a} \geq 0$. So the condition $|\bar{a}_{\mathrm{x}}| \leq |\bar{a}|$ simplifies to $\bar{a}_{\mathrm{x}} \leq \bar{a} \Rightarrow$, $\frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i \leq \frac{1+r}{1+3r}x_i$, which is always true for $0 < r < 1$, $x_i \geq 0$.
    - Second, consider the case where $-x_i \leq x_j < k_3(r)x_i$. Here also $\bar{x}, \bar{a} \geq 0$. So the condition $|\bar{a}_{\mathrm{x}}| \leq |\bar{a}|$ simplifies to $-\bar{a} \leq \bar{a}_{\mathrm{x}} \leq \bar{a}$. From the case before, we know that $\bar{a}_{\mathrm{x}} \leq \bar{a}$ for $x_i \geq 0$. However, to ensure that $-\bar{a} \leq \bar{a}_{\mathrm{x}}$, we need $x_j \geq k_3(r)x_i$, which cannot be true since by definition $-x_i \leq x_j < k_3(r)x_i$.
    - Third, consider the case where $x_j < -x_i$. Here, $\bar{x}, \bar{a} < 0$. So the condition $|\bar{a}_{\mathrm{x}}| \leq |\bar{a}|$ simplifies to $\bar{a} \leq \bar{a}_{\mathrm{x}} \leq -\bar{a}$. The condition $\bar{a}_{\mathrm{x}} \geq \bar{a}$ reduces to $x_j \geq -k_3(r)x_i$, which we know is not possible since $x_j < -x_i$ and $0 < k_3(r) < 1$.

Hence, for $x_j \geq -k_3(r)x_i$, the mean outcome in the exogenous sequential game is less extreme (moderate) compared to that in the simultaneous game.

E Proof of Proposition 5

The expected utility of the first player i in equilibrium is given by (11). Substituting for $\hat{a}_{\mathrm{x}1,i}$ gives us:

$$\begin{aligned} EU_{\mathrm{x}1}\left(x_i, \hat{a}_{\mathrm{x}1,i}\right) = & - r\left(x_i - \frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}x_i\right)^2 \\ & - (1-r)\left[\left(x_i - \frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i\right)^2 + \frac{(1+r)^2}{3(1+3r)^2}\int_{\mathbb{R}} x_j^2 g(x_j)\mathrm{d}x_j\right] \end{aligned} \tag{A68}$$

This in turn simplifies to:

$$EU_{\mathrm{x}1}\left(x_i, \hat{a}_{\mathrm{x}1,i}\right) = -\frac{(1-r)(1+r)^2}{(1+3r)^2+4r(1-r)}x_i^2 - \frac{(1-r)(1+r)^2}{(1+3r)^2}\int_{\mathbb{R}} x_j^2 g(x_j)\mathrm{d}x_j \tag{A69}$$

Next, consider the a priori expected utility of player j, in equilibrium. It is obtained by integrating the utility of the second player over the range of x_i. That is, $EU_{\mathrm{x}2}\left(x_j, \hat{a}_{\mathrm{x}2,j}\right) = \int_{\mathbb{R}} u\left(x_j, \hat{a}_{\mathrm{x}2,j}, \hat{a}_{\mathrm{x}1,i}\right) g(x_i)\mathrm{d}x_i$. Substituting for $\hat{a}_{\mathrm{x}2,j}$ as $\frac{2(1+r)}{1+3r}x_j - \frac{(1-r)}{1+3r}a_{\mathrm{x}1,i}$, we have:

$$\begin{aligned} EU_{\mathrm{x}2}\left(x_j, \hat{a}_{\mathrm{x}2,j}\right) = & - r\int_{\mathbb{R}}\left(x_j - \frac{2(1+r)}{1+3r}x_j - \frac{1-r}{1+3r}\hat{a}_{\mathrm{x}1,i}\right)^2 g(x_i)\mathrm{d}x_i \\ & - (1-r)\int_{\mathbb{R}}\left(x_j - \frac{1+r}{1+3r}x_j - \frac{2r}{1+3r}\hat{a}_{\mathrm{x}1,i}\right)^2 g(x_i)\mathrm{d}x_i \end{aligned} \tag{A70}$$

Substituting for $\hat{a}_{\text{x1},i}$ and integrating, we have:

$$EU_{\text{x2}}\left(x_j,\hat{a}_{\text{x2},j}\right) = -\frac{r(1-r)}{1+3r}x_i^2 - \frac{r(1-r)(1+3r)(3+r)^2}{\left[(1+3r)^2+4r(1-r)\right]^2}\int_{\mathbb{R}} x^2 g(x)\mathrm{d}x \tag{A71}$$

Next, we compare the difference in the expected utilities for a specific player i, where $D_{\text{x}}(x_i) = EU_{\text{x1}}\left(x_i,\hat{a}_{\text{x1},i}\right) - EU_{\text{x2}}\left(x_i,\hat{a}_{\text{x2},i}\right)$. We can show that $D_{\text{x}}(x_i) \leq 0$ for all i if the following two conditions are satisfied:

$$\frac{r(1-r)(1+3r)(3+r)^2}{\left[(1+3r)^2+4r(1-r)\right]^2} > -\frac{(1-r)(1+r)^2}{(1+3r)^2} \tag{A72}$$

and

$$-\frac{r(1-r)}{1+3r} \geq -\frac{(1-r)(1+r)^2}{(1+3r)^2+4r(1-r)} \tag{A73}$$

First, consider the inequality (A72), which can be simplified to:

$$(1+r)^2\left[(1+3r)^2+4r(1-r)\right]^2 > r(3+r)^2(1+3r)^3 \tag{A74}$$

This in turn simplifies to:

$$\begin{aligned}&(1+3r)^3\left[(1+r)^2(1+3r)-r(3+r)^2\right]+(1+r)^2\left[16r^2(1-r)^2+8r(1-r)(1+3r)^2\right]>0\\ \Rightarrow\quad &16r^2(1+r)^2(1-r)^2+(1+3r)^2(1-r)\left[2r(1+r)^2+(1+3r)(1-r)\right]>0\end{aligned} \tag{A75}$$

Since both the terms in the R.H.S of inequality (A75) are non-negative, the inequality is always true. Therefore, (A72) is always true. Next, consider the inequality (A73), which can be expressed as:

$$\frac{(1-r)(1+r)^2}{(1+3r)^2+4r(1-r)}x_i^2 \geq \frac{r(1-r)}{1+3r}x_i^2 \tag{A76}$$

This is true if:

$$\begin{aligned}&(1-r)\left[(1+r)^2(1+3r)-r[(1+3r)^2+4r(1-r)]\right]x_i^2 \geq 0\\ \Rightarrow\quad &(1-r)^2\left[2r^2+5r+1\right]x_i^2 \geq 0\end{aligned} \tag{A77}$$

We know that $x_i^2 \geq 0$ and that both $(1-r)^2$ and $2r^2+5r+1$ are positive for $0<r<1$. Hence this inequality is always true too. Further, since both (A72) and (A73) are always true, it follows that $EU_{\text{x2}}\left(x_i,\hat{a}_{\text{x2},i}\right) > EU_{\text{x1}}\left(x_i,\hat{a}_{\text{x1},i}\right)$.

b) Now we prove the second part of the Proposition. Let $x_i > 0$, then:

$$\frac{\mathrm{d}D_{\text{x}}(x_i)}{\mathrm{d}x_i} = 2x_i(1-r)\left[\frac{r}{1+3r} - \frac{(1+r)^2}{(1+3r)^2+4r(1-r)}\right] \tag{A78}$$

Since we have already shown that (A73) is true, we know that $(1-r)\left[\frac{r}{1+3r} - \frac{(1+r)^2}{(1+3r)^2+4r(1-r)}\right] > 0$. It therefore follows that $2x_i(1-r)\left[\frac{r}{1+3r} - \frac{(1+r)^2}{(1+3r)^2+4r(1-r)}\right] \leq 0$. Hence, $\frac{\mathrm{d}D_{\mathrm{x}}(x_i)}{\mathrm{d}x_i} \leq 0$. Similar proof applies for $x_i < 0$. $\square$

F Proof of Proposition 6

The solutions for the optimal actions for periods 3 and 4 are analogous to that in the exogenous sequential choice game and are outlined in the main text. Below, we derive the players' optimal action for the first two periods.

Period 2 – A player who has lost the auction makes no decisions in period 2. So we only consider the actions of a player who won the auction in period 1. Suppose player j bids according to the symmetric bidding function $\beta(\cdot)$, then player i belief upon winning is that player j must belong to a some symmetric region W for her to have won the auction. In that case, i's expected utility from speaking first is:

$$\begin{aligned} EU_{\mathrm{n1}}\left(x_i, a_{\mathrm{n1},i}\right) = & - r(x_i - a_{\mathrm{n1},i})^2 - (1-r)\left[\left(x_i - \frac{2r}{1+3r}a_{\mathrm{n1},i}\right)^2 + \left(\frac{1+r}{1+3r}\right)^2 \frac{\int_W x_j^2 g(x_j)\mathrm{d}x_j}{\int_W g(x_j)\mathrm{d}x_j}\right] \\ & - 2\frac{1+r}{1+3r}\left(x_i - \frac{2r}{1+3r}a_{\mathrm{n1},i}\right)\frac{\int_W x_j g(x_j)\mathrm{d}x_j}{\int_W g(x_j)\mathrm{d}x_j} \end{aligned} \tag{A79}$$

The last term vanishes because W is symmetric around zero. So:

$$\begin{aligned} EU_{\mathrm{n1}}\left(x_i, a_{\mathrm{n1},i}\right) &= -r(x_i - a_{\mathrm{n1},i})^2 - (1-r)\left[\left(x_i - \frac{2r}{1+3r}a_{\mathrm{n1},i}\right)^2 + \left(\frac{1+r}{1+3r}\right)^2 \frac{\int_W x_j^2 g(x_j)\mathrm{d}x_j}{\int_W g(x_j)\mathrm{d}x_j}\right] \\ &= -\frac{(1-r)(1+r)^2}{(1+3r)^2+4r(1-r)}x_i^2 - \frac{(1-r)(1+r)^2}{(1+3r)^2}\frac{\int_W x_j^2 g(x_j)\mathrm{d}x_j}{\int_W g(x_j)\mathrm{d}x_j} \end{aligned} \tag{A80}$$

Similarly, simplifying i's expected utility from speaking second gives us:

$$EU_{\mathrm{n2}}\left(x_i, a_{\mathrm{n2},i}\right) = -\frac{r(1-r)}{(1+3r)}\left[x_i^2 + \left(\frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}\right)^2 \int_W g(x_j)x_j^2 \mathrm{d}x_j\right] \tag{A81}$$

i prefers to speak second if:

$$EU_{\mathrm{n2}}\left(x_i, a_{\mathrm{n2},i}\right) > EU_{\mathrm{n1}}\left(x_i, a_{\mathrm{n1},i}\right) \tag{A82}$$

Let $A(r) = \frac{(1-r)(1+r)^2}{(1+3r)^2+4r(1-r)}$, $B(r) = \frac{(1-r)(1+r)^2}{(1+3r)^2}$, $C(r) = \frac{r(1-r)}{(1+3r)}$, and $D(r) = \frac{r(1-r)}{(1+3r)}\left(\frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}\right)^2$. It is trivial to show that the multiplier of x_i^2 in $EU_{\mathrm{n2}}\left(x_i, a_{\mathrm{n2},i}\right)$ is always greater than that in

$EU_{\mathrm{n1}}(x_i, a_{\mathrm{n1},i}) \Rightarrow A(r) > C(r)$. Similarly, we can show that the multiplier of $\int_W x_j^2 g(x_j)\mathrm{d}x_j$ in $EU_{\mathrm{n2}}(x_i, a_{\mathrm{n2},i})$ is also greater than that in $EU_{\mathrm{n1}}(x_i, a_{\mathrm{n1},i}) \Rightarrow B(r) > D(r)$. Therefore, in a symmetric bidding equilibrium, for all x_is, the expected value from speaking second is higher than that from speaking first. Therefore, upon winning the auction, all types will choose to speak second.

Period 1 At the beginning of period 1, before placing her bid, player i knows that if she wins, she will choose to go second and if player j wins, she will go first. Hence, her expected utility from choosing a bid b_i and then choosing the optimal action in the subsequent periods is

$$EU(x_i, b_i) = \frac{\int_{L_i} u(x_i, \hat{a}_{\mathrm{n1},i}, \hat{a}_{\mathrm{n2},j}) g(x_j)\mathrm{d}x_j + \int_{W_i} \left[u(x_i, \hat{a}_{\mathrm{n1},j}, \hat{a}_{\mathrm{n2},i}) - b_i\right] g(x_j)\mathrm{d}x_j}{\int_{\mathbb{R}} g(x_j)\mathrm{d}x_j} \tag{A83}$$

where $j \in W_i$ for i to win the auction and $j \in L_i$ for her to lose the auction, if she chooses a bid b_i. This simplifies to:

$$\begin{aligned} 2EU(x_i, b_i) = & - A(r)x_i^2 \int_{L_i} g(x_j)\mathrm{d}x_j - B(r) \int_{L_i} x_j^2 g(x_j)\mathrm{d}x_j \\ & - C(r)x_i^2 \int_{W_i} g(x_j)\mathrm{d}x_j - D(r) \int_{W_i} x_j^2 g(x_j)\mathrm{d}x_j - \int_{W_i} b_i g(x_j)\mathrm{d}x_j \\ = & - 2x_i^2 + [A(r) - C(r)]x_i^2 \int_{W_i} g(x_j)\mathrm{d}x_j + [B(r) - D(r)] \int_{W_i} x_j^2 g(x_j)\mathrm{d}x_j - \int_{W_i} b_i g(x_j)\mathrm{d}x_j \end{aligned} \tag{A84}$$

Now consider any two types x' and x'' and a bidding function $\beta(\cdot)$. In equilibrium, x' can do no better by playing x'''s strategy $\beta(x'')$ over her own strategy $\beta(x')$ and vice-versa. That is:

$$EU(x', \beta(x')) \geq EU(x', \beta(x'')) \tag{A85}$$

$$EU(x'', \beta(x'')) \geq EU(x', \beta(x'')) \tag{A86}$$

Substituting the simplified expressions for the expected utilities into the above inequalities and adding them up gives us:

$$[A(r) - C(r)]\left(x'^2 - x''^2\right)\left(\int_{W_1} g(x_j)\mathrm{d}x_j - \int_{W_2} g(x_j)\mathrm{d}x_j\right) \geq 0 \tag{A87}$$

We know that $A(r) - C(r) > 0$. So if $x'^2 > x''^2$, then for the above inequality to hold, we require that $\int_{W_1} g(x_j)\mathrm{d}x_j - \int_{W_2} g(x_j)\mathrm{d}x_j \geq 0 \Rightarrow$ the region over which a player wins upon bidding $\beta(x')$ is greater than that over which she wins when she bids $\beta(x'')$. In other words, the equilibrium bidding strategies are monotonically increasing in $|x|$. Further, following the technique as that outlined in

Fudenberg and Tirole (1991) p. 217, we can show strict monotonicity, $i.e.$, if $|x'| > |x''|$, then $\beta(x') > \beta(x'')$.

Now that we have shown that the bidding strategies are monotonically increasing in $|x|$, for the specific bid b_i by player i (when the other player uses the bidding function $\beta(\cdot)$), we re-write Equation (A84) as:

$$EU(x_i, b_i) = -2x_i^2 + 2[A(r) - C(r)]x_i^2 \int_0^{\beta^{-1}(b_i)} g(x)\mathrm{d}x + 2[B(r) - D(r)] \int_0^{\beta^{-1}(b_i)} x^2 g(x)\mathrm{d}x - 2\int_0^{\beta^{-1}(b_i)} b_i g(x)\mathrm{d}x \tag{A88}$$

Further, we specify the following expression for the derivatives:

$$\frac{\mathrm{d}\left[\int_0^{\beta^{-1}(b_i)} F(x)\right]}{\mathrm{d}b_i}\mathrm{d}x = F(V)\frac{\mathrm{d}V}{\mathrm{d}b_i} \tag{A89}$$

where $V = \beta^{-1}(b_i)$. To obtain the equilibrium bidding function, we can calculate the F.O.C of Equation (A88) as $\left.\frac{\mathrm{d}EU(x_i,b_i)}{\mathrm{d}b_i}\right|_{b_i=\hat{b}_i} = 0$. This simplifies to:

$$[A(r) - C(r)]\, x_i^2 \frac{g(\beta^{-1}(\hat{b}_i))}{\beta'(\beta^{-1}(\hat{b}_i))} + [B(r) - D(r)]\, \frac{g(\beta^{-1}(\hat{b}_i)) \left[\beta^{-1}(\hat{b}_i)\right]^2}{\beta'(\beta^{-1}(\hat{b}_i))} - \left[\hat{b}_i \frac{g(\beta^{-1}(\hat{b}_i))}{\beta'(\beta^{-1}(\hat{b}_i))} + \int_0^{\beta^{-1}(\hat{b}_i)} g(x)\mathrm{d}x\right] = 0$$

In equilibrium $\hat{b}_i = \beta(x_i)$ and so $\beta^{-1}(\hat{b}_i) = x_i$. So the above equation simplifies to:

$$[A(r) - C(r) + B(r) - D(r)]\, x_i^2 g(\beta^{-1}(\hat{b}_i)) = \frac{\mathrm{d}\left[\beta(x_i) \int_0^{x_i} g(x)\mathrm{d}x\right]}{\mathrm{d}x_i} \tag{A90}$$

Integrating this from 0 to x_i, we have:

$$\begin{aligned} \hat{\beta} = \beta(x_i) &= [A(r) - C(r) + B(r) - D(r)] \left[\frac{\int_0^{x_i} x^2 g(x)\mathrm{d}x}{\int_0^{x_i} g(x)\mathrm{d}x}\right] \\ &= f(r)\left[\frac{\int_0^{x_i} x^2 g(x)\mathrm{d}x}{\int_0^{x_i} g(x)\mathrm{d}x}\right], \end{aligned} \tag{A91}$$

where $f(r) = A(r) - C(r) + B(r) - D(r)$ is the multiplier of $\left[\frac{\int_0^{x_i} x^2 g(x)\mathrm{d}x}{\int_0^{x_i} g(x)\mathrm{d}x}\right]$. Since $A(r) - C(r) > 0$ and $B(r) - D(r) > 0$, $f(r)$ is positive, which recovers the assumption that the bidding function is symmetric around zero. $\square$

G Social Welfare Derivations and Comparisons

G.1 Welfare Under First-Best Planner's Choice

The social planner's choice involve agents choosing their true preferences as their actions, $i.e.$, $a_i = x_i$ and $a_j = x_j$, and $\bar{a} = \frac{x_i + x_j}{2}$. Then:

$$\begin{aligned} W_{FB}(x_i, x_j, a_i, a_j) &= W_{FB}(x_i, x_j) \\ &= -r(x_i - a_i)^2 - (1-r)(x_i - \bar{a})^2 - r(x_j - a_j)^2 - (1-r)(x_j - \bar{a})^2 \\ &= -(1-r)\frac{(x_i - x_j)^2}{2} \end{aligned} \tag{A92}$$

Then the expected welfare is given by:

$$\begin{aligned} EW_P &= \frac{\int_{\mathbb{R}}\int_{\mathbb{R}} W_1(x_i, x_j) g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j}{\int_{\mathbb{R}}\int_{\mathbb{R}} g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j} \\ &= -\frac{(1-r)}{2\int_{\mathbb{R}}\int_{\mathbb{R}} g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j} \left[\int_{\mathbb{R}}\int_{\mathbb{R}} (x_i^2 + x_j^2 - 2x_i x_j) g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j \right] \end{aligned}$$

We know that $\int_{\mathbb{R}}\int_{\mathbb{R}} g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j = 1$ and $\int_{\mathbb{R}}\int_{\mathbb{R}} x_i x_j g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j = 0$ because $g(\cdot)$ is a symmetric distribution. Therefore, EW_1 simplifes to:

$$\begin{aligned} EW_{FB} &= \frac{-(1-r)}{2}\int_{\mathbb{R}}\int_{\mathbb{R}} \left[x_i^2 + x_j^2\right] g(x_i) g(x_j) \mathrm{d}x_j \mathrm{d}x_j \\ &= -(1-r)\int_{\mathbb{R}} x^2 g(x) \mathrm{d}x \end{aligned} \tag{A93}$$

G.2 Welfare in the Simultaneous Game

In the simultaneous game, the actions are $a_i = \frac{2(1+r)}{1+3r} x_i$, $a_j = \frac{2(1+r)}{1+3r} x_j$, and the mean action is $\bar{a} = \frac{(1+r)}{1+3r}(x_i + x_j)$. Substituting this in the welfare function we can obtain:

$$\begin{aligned} W_s(x_i, x_j, a_i, a_j) &= W_s(x_i, x_j) \\ &= -\frac{r(1-r)^2}{(1+3r)^2}(x_i^2 + x_j^2) - \frac{1-r}{(1+3r)^2}\left[2rx_i - (1+r)x_j\right]^2 \\ &\quad -\frac{1-r}{(1+3r)^2}\left[2rx_j - (1+r)x_i\right]^2 \qquad \text{(A94)} \\ &= -\frac{r(1-r)^2}{(1+3r)^2}(x_i^2 + x_j^2) \\ &\quad -\frac{1-r}{(1+3r)^2}\left[4r^2(x_i^2 + x_j^2) + (1+r)^2(x_i^2 + x_j^2) - 8r(1+r)x_i x_j\right] \end{aligned}$$

The expected welfare is then given by:

$$EW_s = \frac{\int_{\mathbb{R}}\int_{\mathbb{R}} W_s(x_i,x_j)g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j}{\int_{\mathbb{R}}\int_{\mathbb{R}} g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j}$$

As before, $\int_{\mathbb{R}}\int_{\mathbb{R}} g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j = 1$ and $\int_{\mathbb{R}}\int_{\mathbb{R}} x_i x_j g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j = 0$. So:

$$EW_s = -\frac{2(1-r)}{(1+3r)^2}[r(1-r)+4r^2+(1+r)^2]\int_{\mathbb{R}} x^2 g(x)\mathrm{d}x \qquad \text{(A95)}$$

G.3 Welfare in the Exogenous Sequential Choice Game

Without loss of generality, assume that i speaks first and j speaks second. Then, we have the actions of the two agents as $a_i = \frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}x_i$, $a_j = \frac{2(1+r)}{1+3r}x_j - \frac{1-r}{1+3r}a_i$, and the mean action as $\bar{a} = \frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i + \frac{1+r}{1+3r}x_j$. Using these, we can further derive the following expressions:

$$\begin{aligned}
x_i - a_i &= -\frac{2(1-r)(1+r)}{(1+3r)^2+4r(1-r)}x_i \\
x_j - a_j &= -\frac{1-r}{1+3r}\left[x_j - \frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}x_i\right] \\
x_i - \bar{a} &= \frac{(1+3r)(1+r)}{(1+3r)^2+4r(1-r)}x_i - \frac{1+r}{1+3r}x_j \\
x_j - \bar{a} &= \frac{2r}{1+3r}x_j - \frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i
\end{aligned}$$

Substituting the above terms in the welfare equation, we have:

$$\begin{aligned}
W_x(x_i,x_j,a_i,a_j) &= W_x(x_i,x_j) \\
&= -\frac{4r(1-r)^2(1+r)^2}{((1+3r)^2+4r(1-r))^2}x_i^2 - (1-r)\left[\frac{(1+3r)(1+r)}{(1+3r)^2+4r(1-r)}x_i - \frac{1+r}{1+3r}x_j\right]^2 \\
&\quad -\frac{r(1-r)^2}{(1+3r)^2}\left[x_j - \frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}x_i\right]^2 \\
&\quad -(1-r)\left[\frac{2r}{1+3r}x_j - \frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i\right] \qquad \text{(A96)}
\end{aligned}$$

As before, the expected welfare is given by:

$$EW_x = \frac{\int_{\mathbb{R}}\int_{\mathbb{R}} W_3(x_i,x_j)g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j}{\int_{\mathbb{R}}\int_{\mathbb{R}} g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j}$$

As before, this implies that the integrals of the $x_i x_j$ terms are canceled out. Further, and $\int_{\mathbb{R}}\int_{\mathbb{R}} g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j = 1$.

$$
\begin{aligned}
EW_x = & - \frac{4r(1-r)^2(1+r)^2}{\left[(1+3r)^2+4r(1-r)\right]^2}\int_{\mathbb{R}}\int_{\mathbb{R}} x_i^2 g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j \\
& - \frac{r(1-r)^2}{(1+3r)^2}\int_{\mathbb{R}}\int_{\mathbb{R}}\left[x_j^2+\frac{(1+3r)^2(3+r)^2}{\left[(1+3r)^2+4r(1-r)\right]^2}x_i^2\right] g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j \\
& - (1-r)\int_{\mathbb{R}}\int_{\mathbb{R}}\left[\frac{(1+3r)^2(1+r)^2+4r^2(3+r)^2}{\left[(1+3r)^2+4r(1-r)\right]^2}x_i^2+\frac{(1+r)^2+4r^2}{(1+3r)^2}x_j^2\right] g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j
\end{aligned}
$$

This simplifies to:

$$
\begin{aligned}
EW_x = & -\frac{(1-r)\left[(1+r)^2+4r^2+r(1-r)\right]}{3(1+3r)^2}\int_{\mathbb{R}} x^2 g(x)\mathrm{d}x \\
& - \left[\frac{(1-r)\left[4r(1-r)(1+r)^2+4r^2(3+r)^2+(1+3r)^2(1+r)^2+r(1-r)(3+r)^2\right]}{3\left[(1+3r)^2+4r(1-r)\right]^2}\right]\int_{\mathbb{R}} x^2 g(x)\mathrm{d}x
\end{aligned}
\quad \text{(A97)}
$$

G.4 Welfare in the Endogenous Sequential Choice Game

As before, assume that i speaks first and j speaks second. Recall that the players' actions here are the same as that in the exogenous sequential choice game. However, we know that $|x_i| < |x_j|$. So while the welfare equation remains the same, the integrations regions are different. Thus, we have:

$$
\begin{aligned}
W_n(x_i,x_j,a_i,a_j) &= W_n(x_i,x_j) \\
&= -\frac{4r(1-r)^2(1+r)^2}{((1+3r)^2+4r(1-r))^2}x_i^2-(1-r)\left[\frac{(1+3r)(1+r)}{(1+3r)^2+4r(1-r)}x_i-\frac{1+r}{1+3r}x_j\right]^2 \\
&\quad -\frac{r(1-r)^2}{(1+3r)^2}\left[x_j-\frac{(1+3r)(3+r)}{(1+3r)^2+4r(1-r)}x_i\right]^2 \\
&\quad -(1-r)\left[\frac{2r}{1+3r}x_j-\frac{2r(3+r)}{(1+3r)^2+4r(1-r)}x_i\right]
\end{aligned}
\quad \text{(A98)}
$$

and the expected welfare is:

$$
EW_n = \frac{\iint\limits_{R1\cup R2} W_4(x_i,x_j)g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j}{\iint\limits_{R1\cup R2} g(x_i)g(x_j)\mathrm{d}x_i\mathrm{d}x_j} \quad \text{(A99)}
$$

where the two regions R1 and R2 are defined as follows:

$$
\begin{aligned}
R_1 &\equiv x_i \in [0,\infty], \quad x_j \in [x_i,\infty]\cup[-\infty,-x_i] \\
R_2 &\equiv x_i \in [-\infty,0], \quad x_j \in [-x_i,\infty]\cup[-\infty,x_i]
\end{aligned}
$$

As before, we can show that the integral of the $x_i x_j$ terms over $R_1 \cup R_2$ is zero. So we now consider the integrals of the x_j^2 and x_i^2 terms. Since inference on j's type is conditional on i, we first integrate over j's type. Because of the symmetry of the distrbution, it is easy to show that:

$$\iint_{R1 \cup R2} x_i^2 g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j = 4 \int_0^\infty x^2 g(x)(1 - G(x)) \mathrm{d}x \tag{A100}$$

$$\iint_{R1 \cup R2} g(x_i) g(x_j) \mathrm{d}x_i \mathrm{d}x_j = 4 \int_0^\infty g(x)(1 - G(x)) \mathrm{d}x \tag{A101}$$

Substituting these expressions back in the expected welfare function, we have:

$$\begin{aligned} EW_n = \quad & \left(-\frac{(1-r)\left[(1+r)^2 + 4r^2 + r(1-r)\right]}{(1+3r)^2} \right. \\ - \quad & \left. \frac{(1-r)\left[4r(1-r)(1+r)^2 + 4r^2(3+r)^2 + (1+3r)^2(1+r)^2 + r(1-r)(3+r)^2\right]}{3\left[(1+3r)^2 + 4r(1-r)\right]^2} \right) \\ & \cdot \left(\frac{\int_0^\infty x^2 g(x)(1-G(x))\mathrm{d}x}{\int_0^\infty x^2 g(x)(1-G(x))\mathrm{d}x} \right) \end{aligned} \tag{A102}$$