

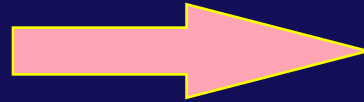
# Review for Exam III

“Leftovers”



RC circuits

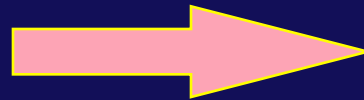
Basic Laws



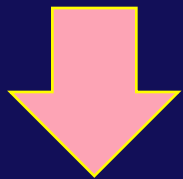
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

+

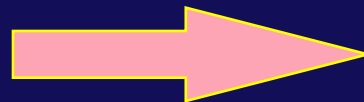
Basic Definitions



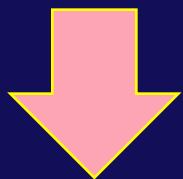
$$U = -\vec{\mu} \cdot \vec{B}$$



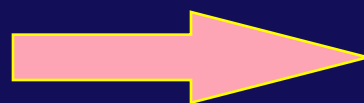
Important Derivations



$$d\vec{F} = I d\vec{l} \times \vec{B}$$

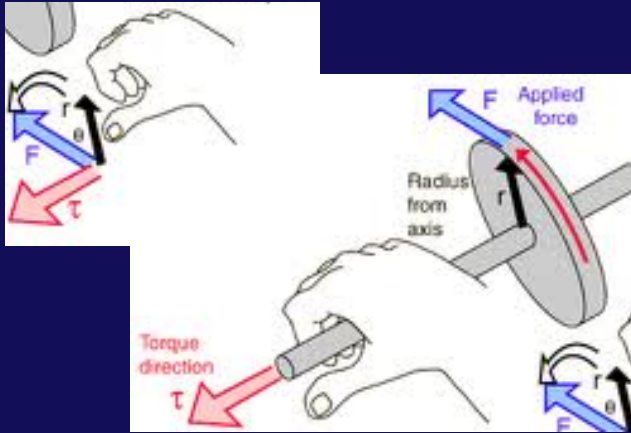


Examples



$$R = \frac{mv}{qB}$$

# Right-Hand-Rules



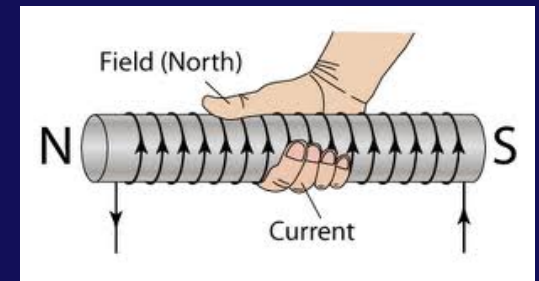
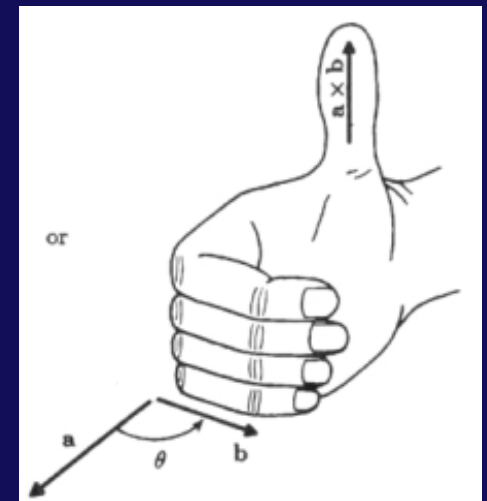
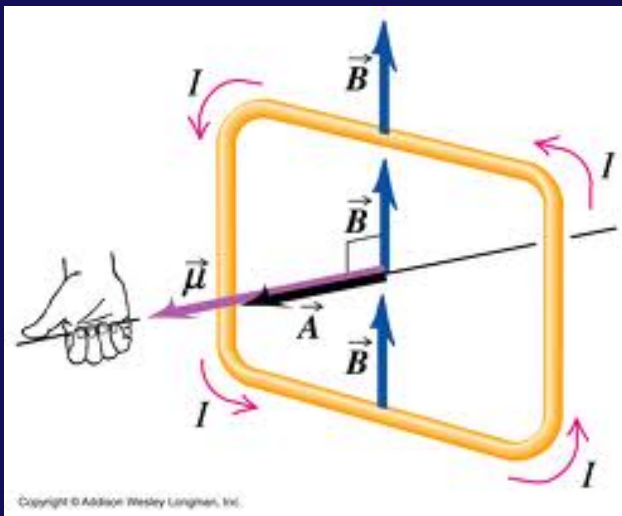
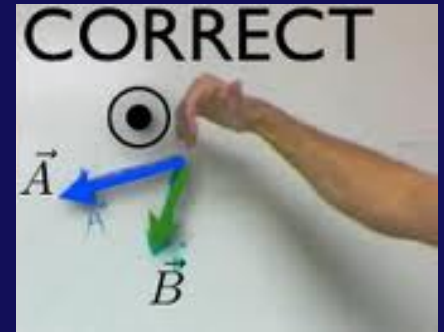
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$\tau = \vec{r} \times \vec{F}$$

$$\tau = \vec{\mu} \times \vec{B}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$



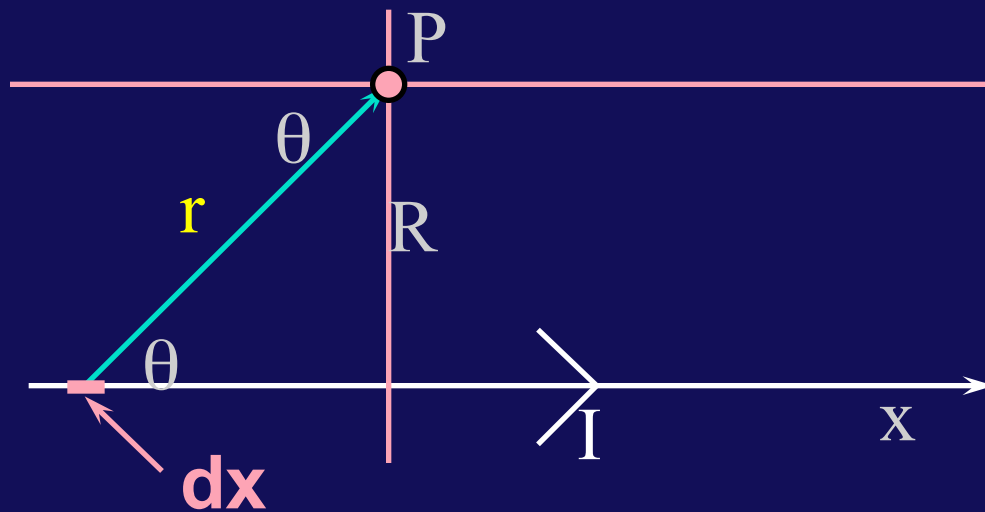
# Basic Laws

- **Biot-Savart Law:**

Direct calculation of B from currents.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$$

*RHR*



We did the integral but it is messy and takes time ... so

# Basic Laws

- **Biot-Savart Law:**

direct calculation of B from currents.

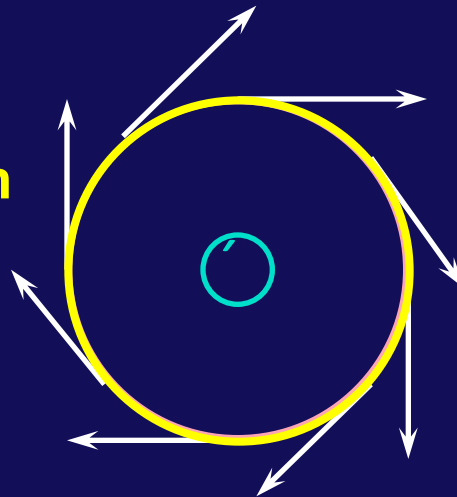
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$$

- **Ampere's Law:**

calculation of B in cases of high symmetry

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

If B “constant” along path  
then integral is just the path



$$B = \frac{\mu_0 I}{2\pi R}$$

# Basic Laws

- Faraday's Law:

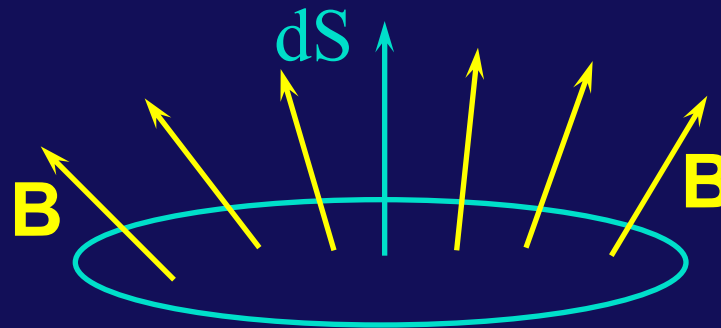
determines the induced E field (or emf) from a changing magnetic flux.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

## Define Flux

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{S}$$

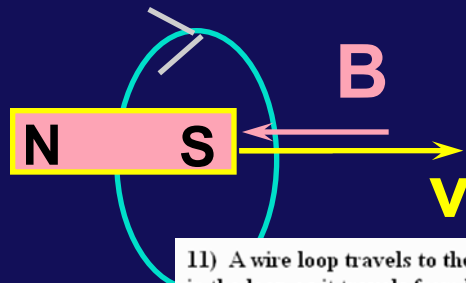
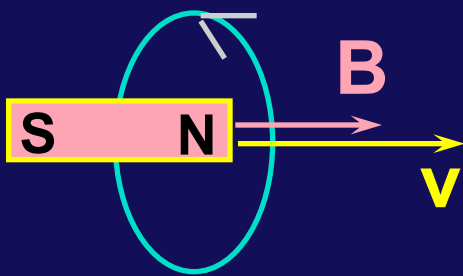


Basic question: Does it change in time (ie, increase or decrease?)

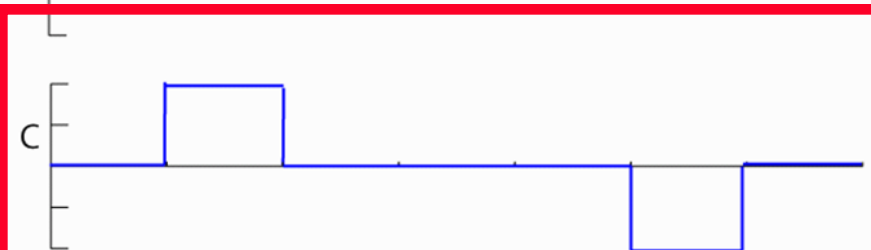
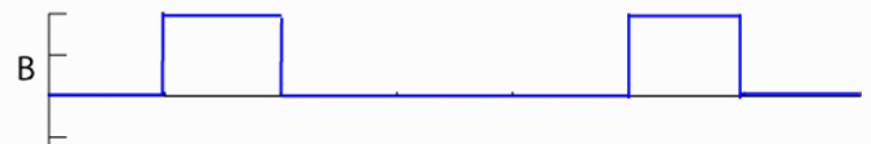
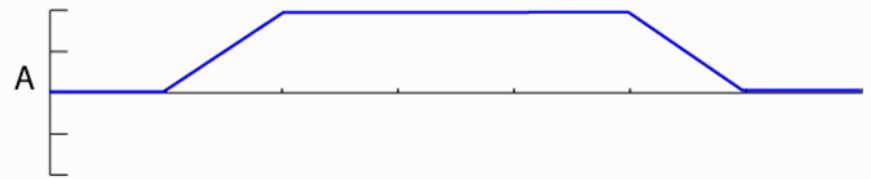
You will have to figure that out

# Lenz's Law

The induced current **will appear in such a direction that it opposes the change in flux that produced it.**

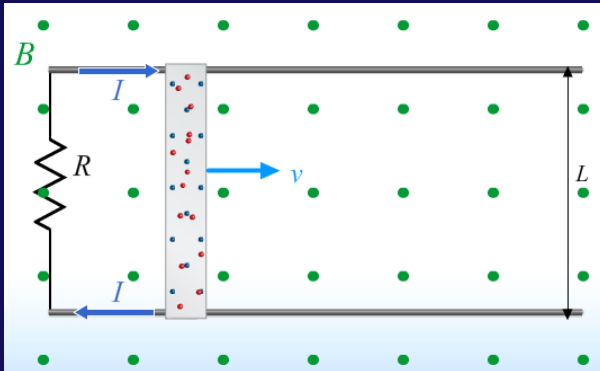


11) A wire loop travels to the right at a constant velocity. Which plot best represents the induced current in the loop as it travels from left of the region of magnetic field, through the magnetic field, and then entirely out of the field on the right side.

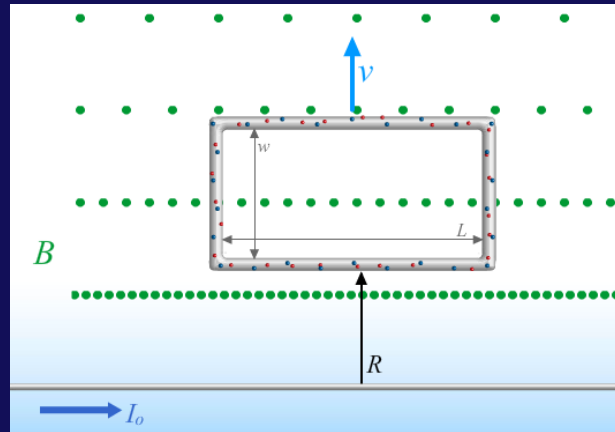


# Examples

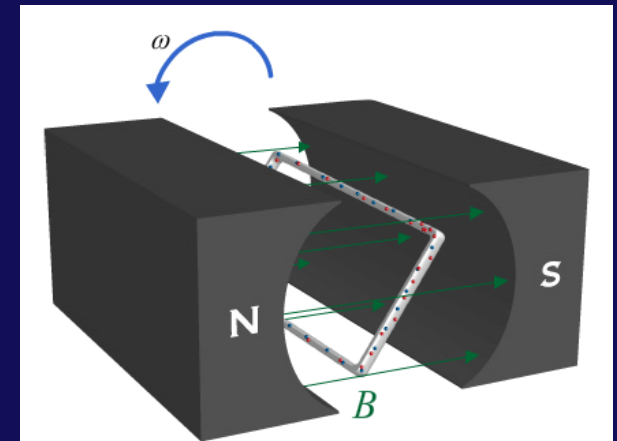
EMF Observed when  $d\Phi/dt$  is non zero  
Direction from Lenz's Law



Change Area of loop



Change magnetic field through loop



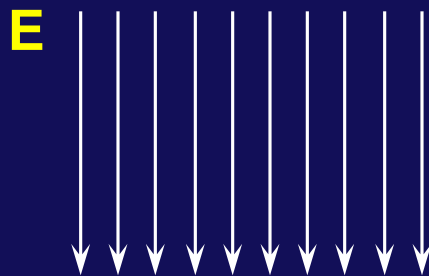
Change orientation of loop relative to  $B$

# Definitions & Derivations

• **Lorentz Force:**  $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

*RHR*

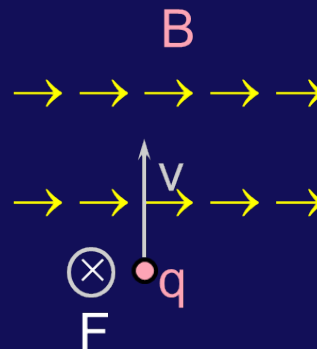
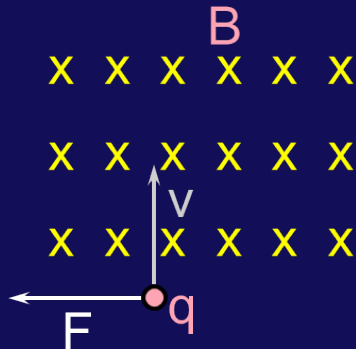
Force in an electric field



Force in a magnetic field



**Examples with Magnetic Field only and a Moving charge:**



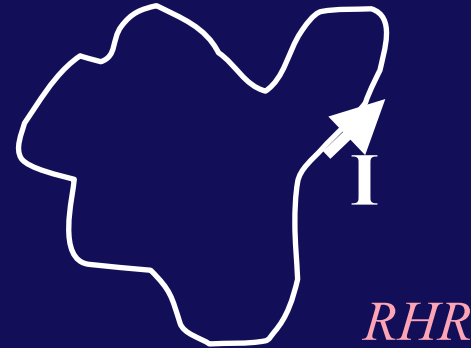


# Definitions & Derivations

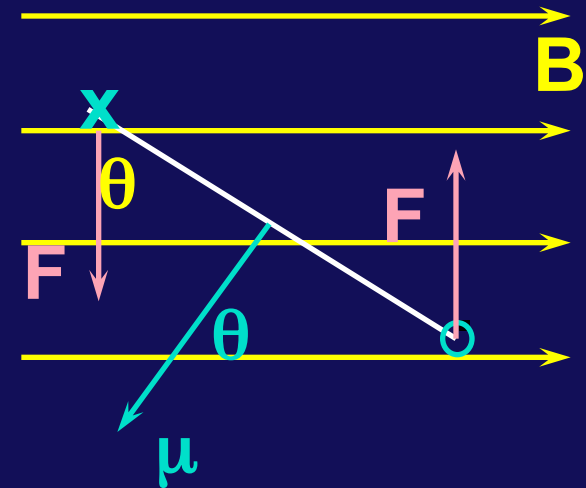
## Magnetic Dipole Moment

**Magnitude:**

$$\mu = AI$$



**Direction:**  $\perp$  to plane of the loop in the direction the thumb of right hand points if fingers curl in direction of current.



- Torque on loop is then:**

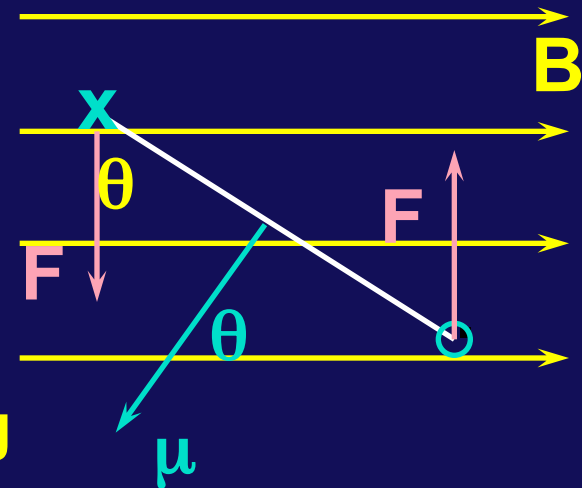
$$\tau = AIB \sin\theta \quad \text{or} \quad \vec{\tau} = \vec{\mu} \times \vec{B}$$

- Note: if loop consists of N turns,  $\mu = NAI$**

Remember this: *The torque always wants to line  $\mu$  up with  $B$ !*

# Potential Energy of Dipole

- **Work is required to change the orientation of a magnetic dipole in the presence of a magnetic field.**
- **Define potential energy  $U$  (with zero at position of max torque) corresponding to this work.**



$$U \equiv \int \tau d\theta$$

↳

$$U = -\vec{\mu} \cdot \vec{B}$$

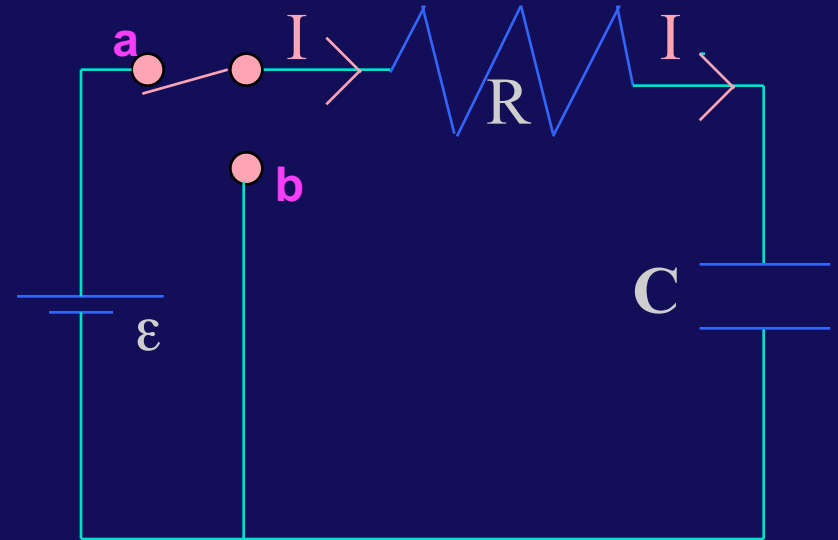
Remember this: *The torque always wants to line  $\mu$  up with  $B$  ... which minimizes the potential energy!*

# Examples with RC circuits

Loop rule !

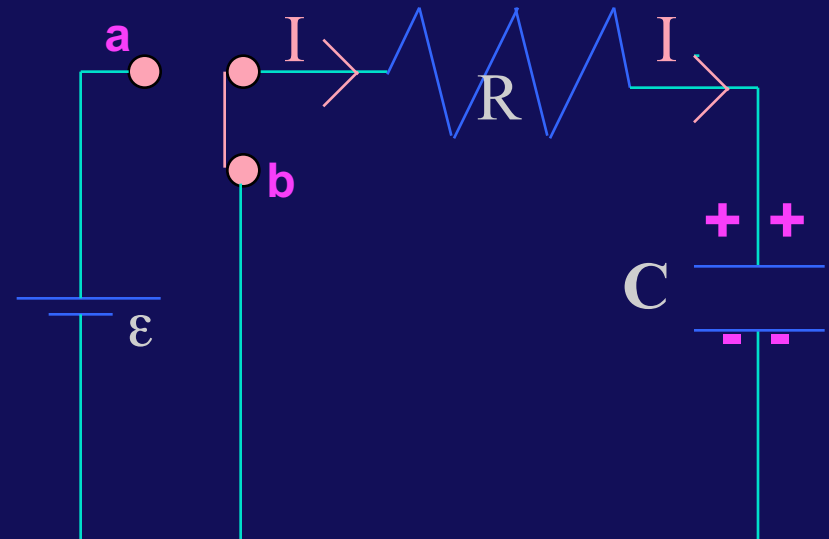
- Charging a capacitor:

$$\varepsilon = R \frac{dq}{dt} + \frac{q}{C}$$

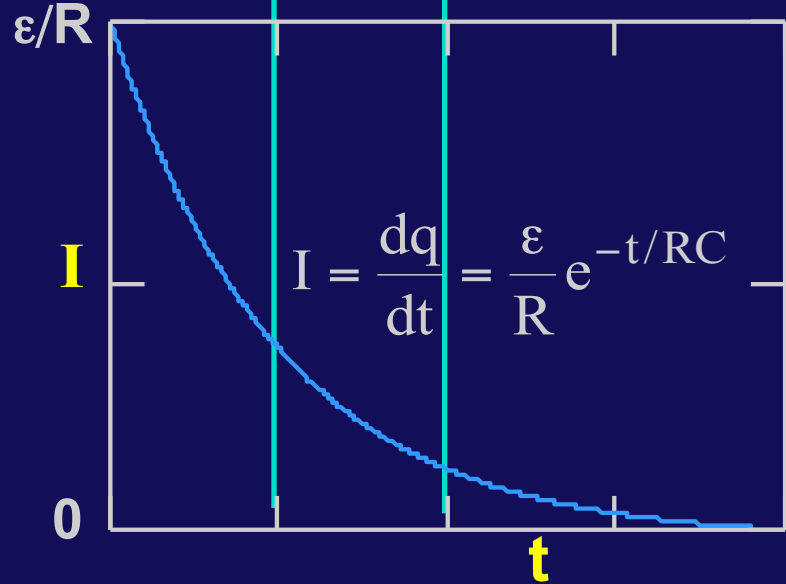
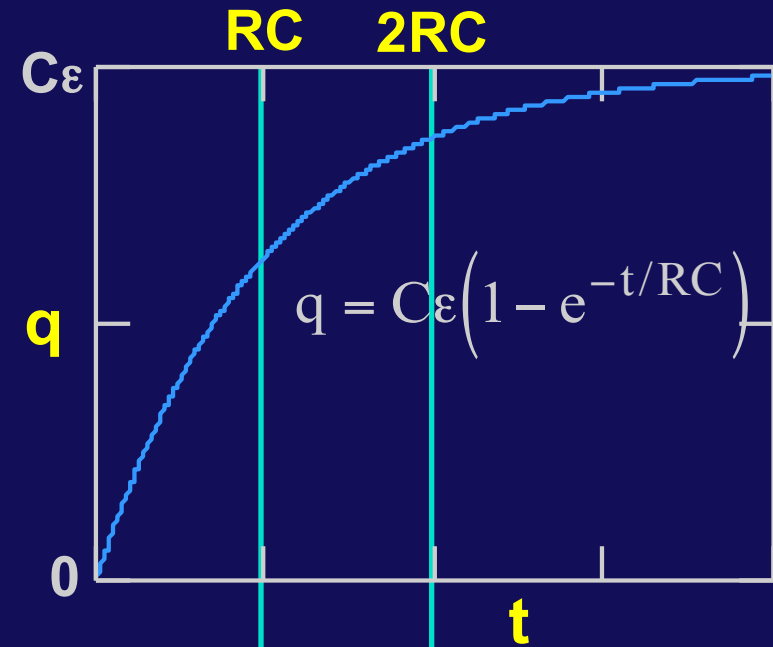


- Discharging a capacitor:

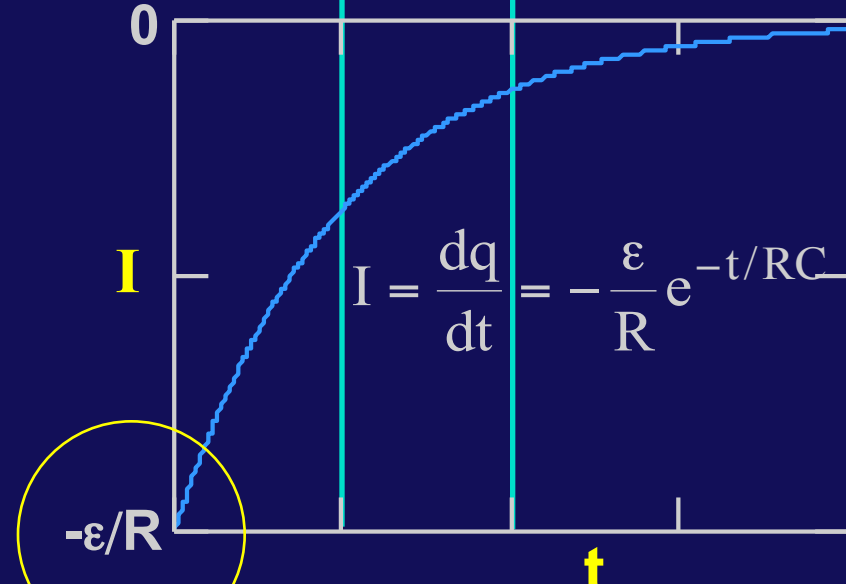
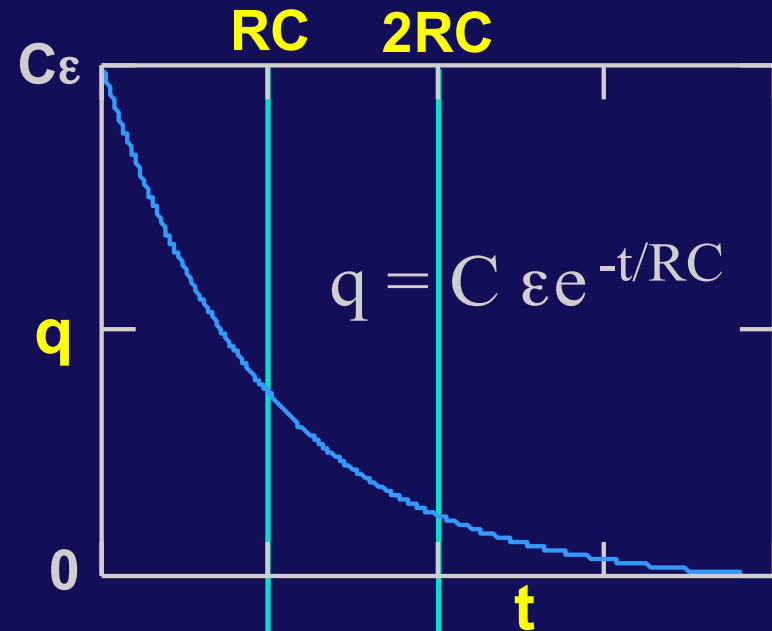
$$R \frac{dq}{dt} + \frac{q}{C} = 0$$



# Charging



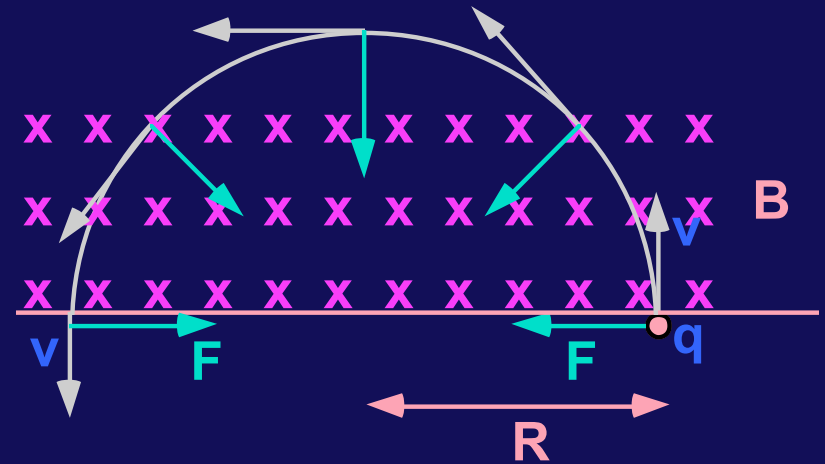
# Discharging



# Examples

- **Orbit of charged particle in uniform magnetic field:**

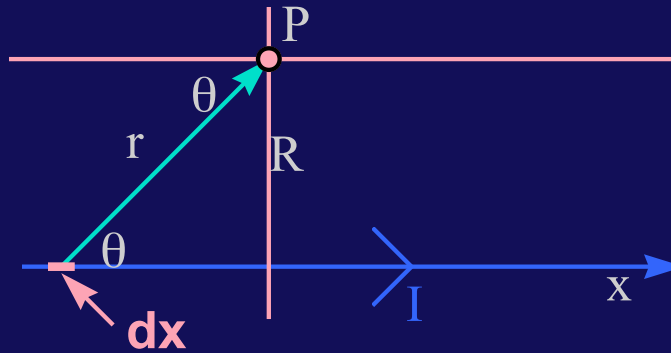
$$R = \frac{mv}{qB}$$



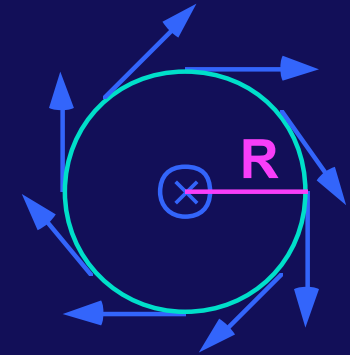
# Examples

- B for straight wire:**

$$B = \frac{\mu_0 I}{2\pi R}$$



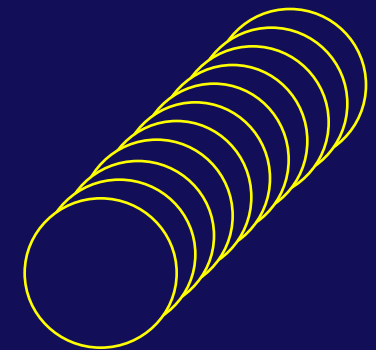
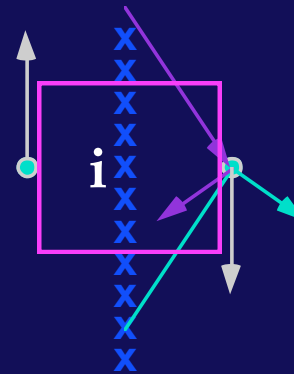
**(Biot-Savart)**



**(Ampere)**

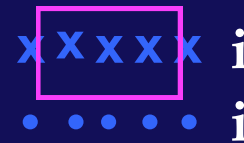
- B for ∞ current sheet:**

$$B = \frac{\mu_0 ni}{2}$$



- B for ∞ solenoid:**

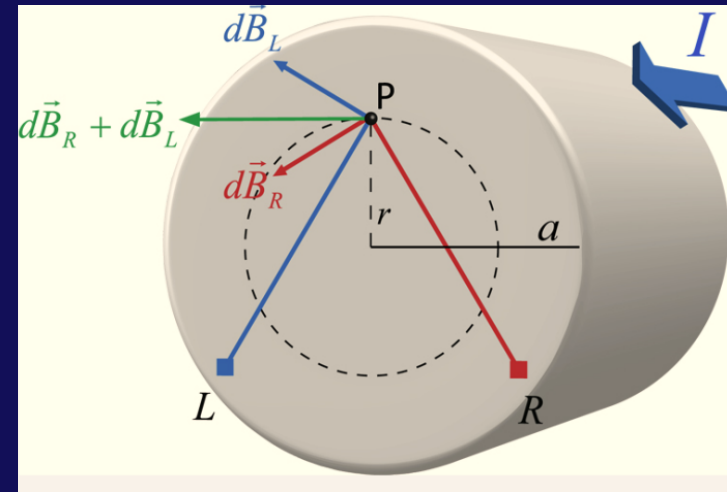
$$B = \mu_0 ni$$



# Examples: B from long straight wire

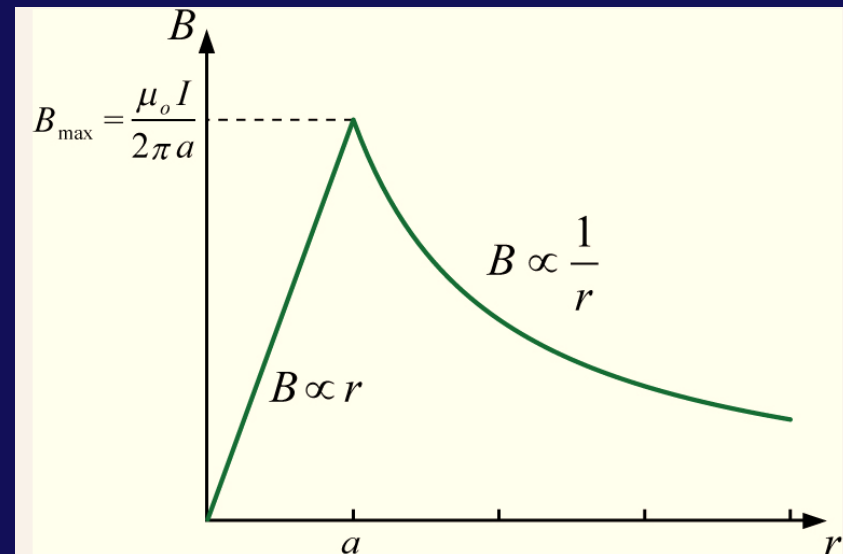
- Inside the wire: ( $r < a$ )

$$B = \frac{\mu_0 I}{2\pi a^2} r$$



- Outside the wire: ( $r > a$ )

$$B = \frac{\mu_0 I}{2\pi r}$$



# Examples

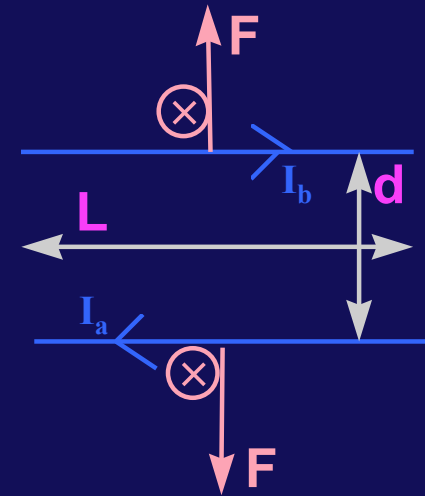
- Force on current carrying wire in a magnetic field:

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

*RHR*

- Force on parallel current-carrying conductors of length  $L$ :

$$\vec{F}_b = I_b \vec{L} \times \vec{B}_a = \frac{\mu_0 I_a I_b L}{2\pi d}$$



- Currents in opposite directions

$\Rightarrow$  repulsive force.

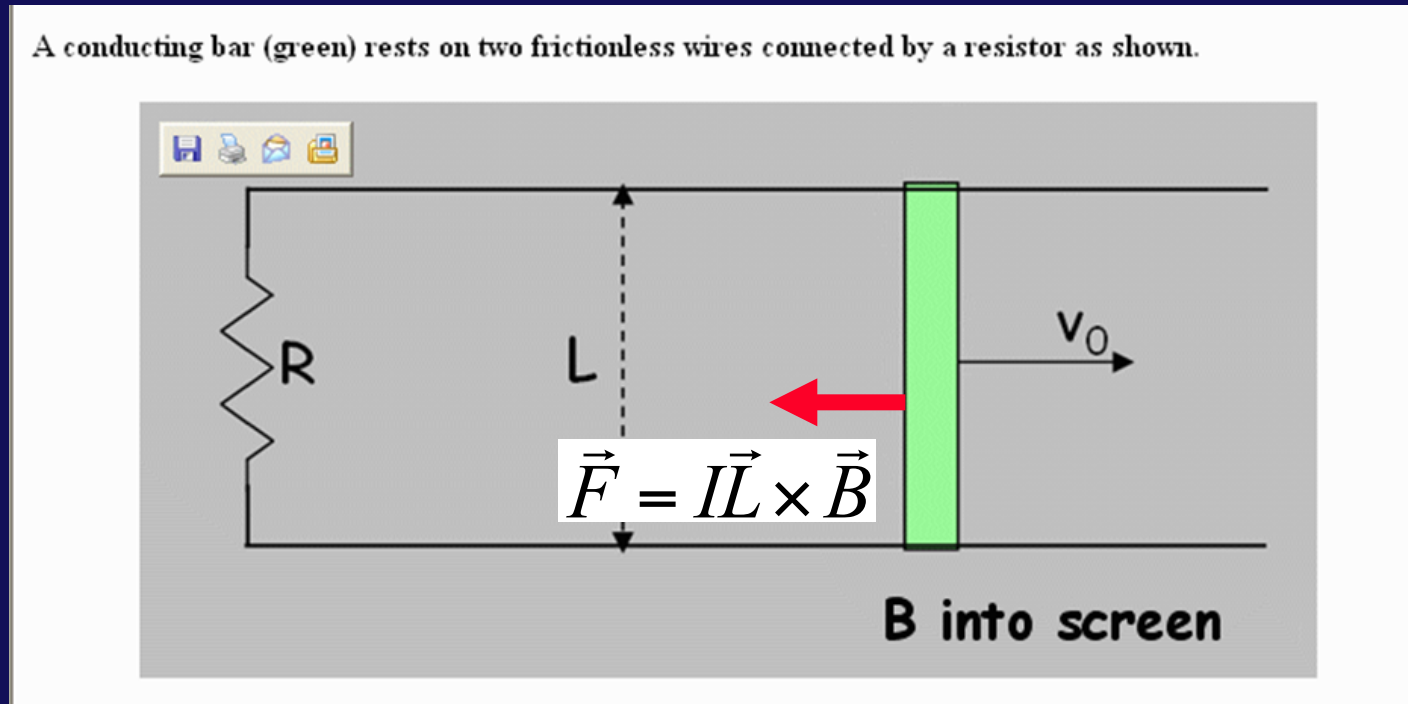
- Currents in same direction

$\Rightarrow$  attractive force.



# Examples

- **Current induced by pulling coil through magnetic field:**



The current through this bar results in a force on the bar

**Is the flux changing?**

**If so, is it increasing? Decreasing?**

**How can you express this quantitatively?**

$$I = \frac{\varepsilon}{R} = \frac{1}{R} \frac{d\Phi}{dt} = \frac{LBv}{R}$$

$$P = Fv = \left( \frac{vBL}{R} \right) LBv = I^2 R$$