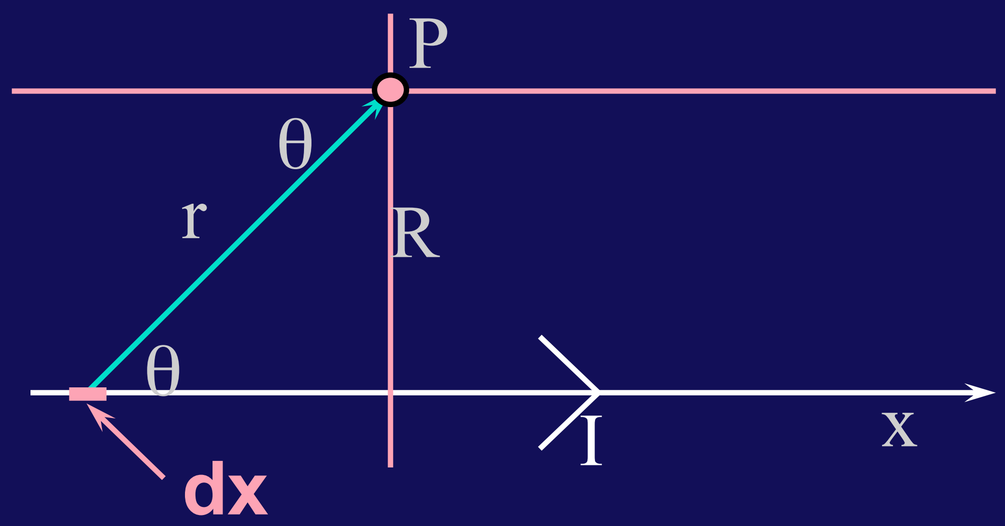
Magnetism

The Law of *Biot-Savart* & RHR



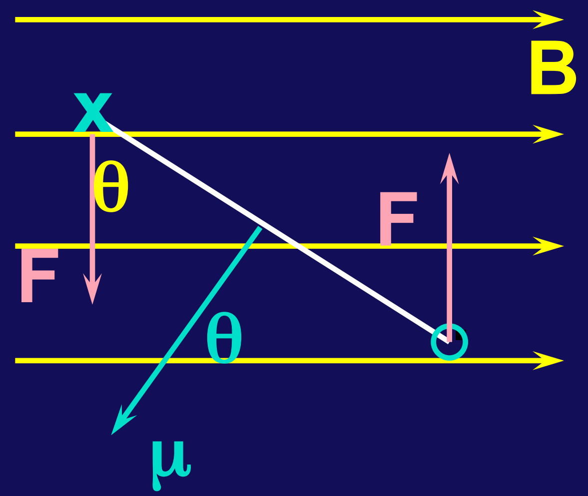
Jean-Baptiste Biot

Félix Savart

Phys 122 Lecture 19
G. Rybka

Recall: Potential Energy of Dipole

- Work required to rotate a current-carrying loop in a magnetic field

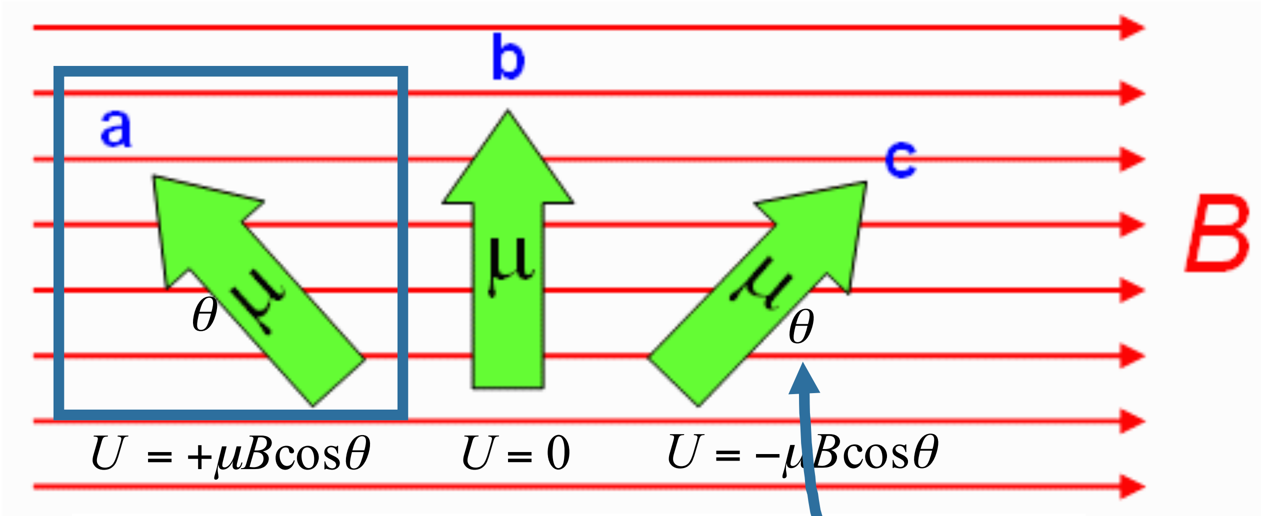


- Potential energy U = - Work done by field leads to

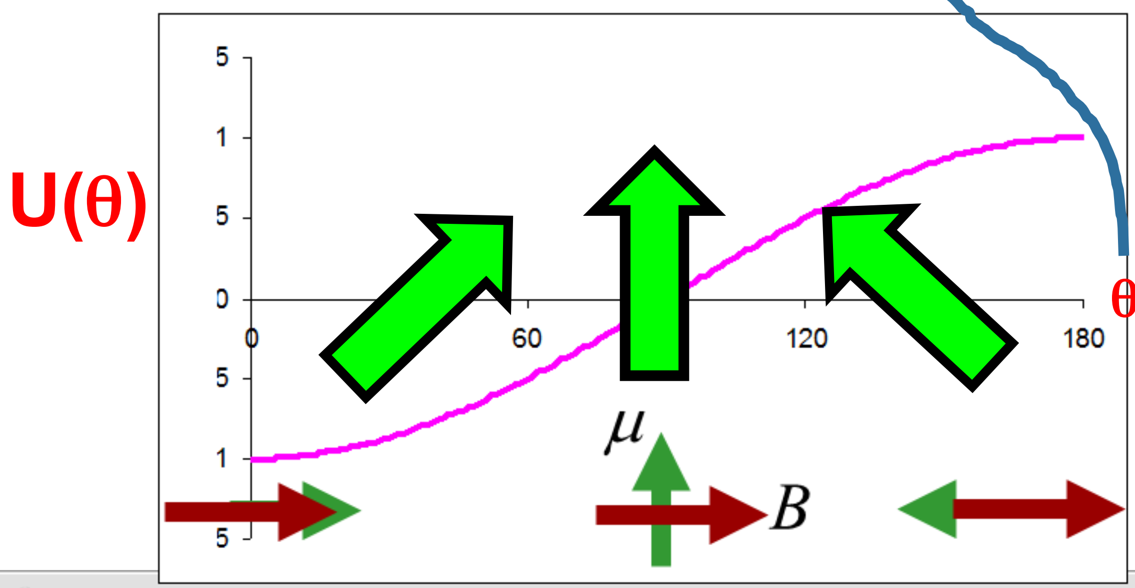
$$U \equiv \int \tau d\theta \quad Þ \quad U = -\vec{\mu} \bullet \vec{B}$$

- With U = 0 at θ = 90° (where torque is maximum)

Plotting $U = -\mu \cdot B$



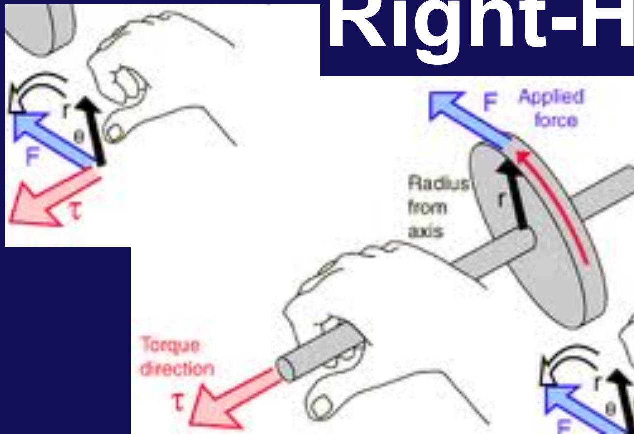
WHICH ORIENTATION HAS THE MOST POTNTIAL ENERGY?



Now some new stuff

… but first …

Right-Hand-Rule Pre-Summary

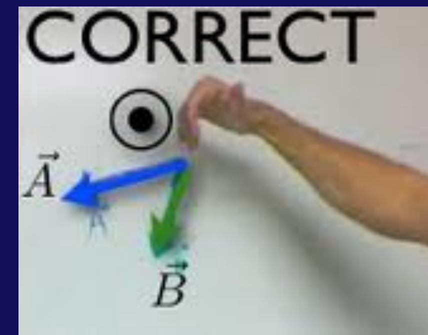


$$\vec{F} = q\vec{v} \times \vec{B}$$

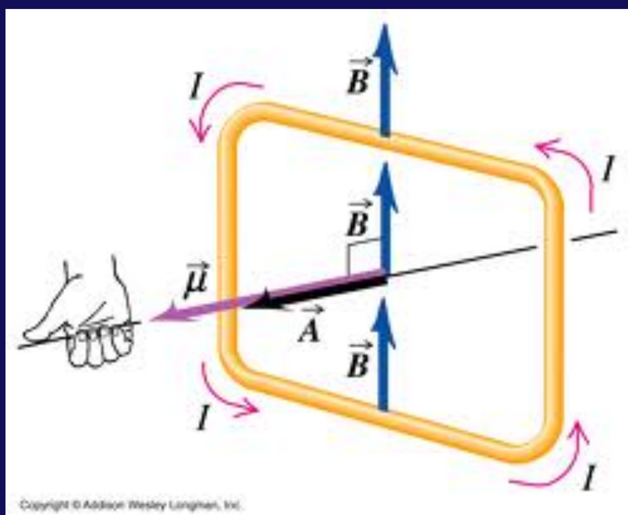
$$\vec{F} = I\vec{L} \times \vec{B}$$

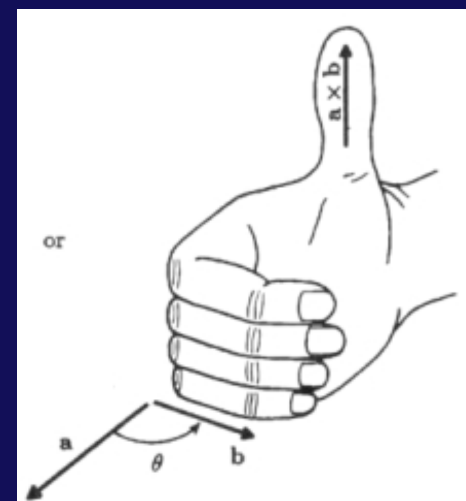
$$\tau = \vec{r} \times \vec{F}$$

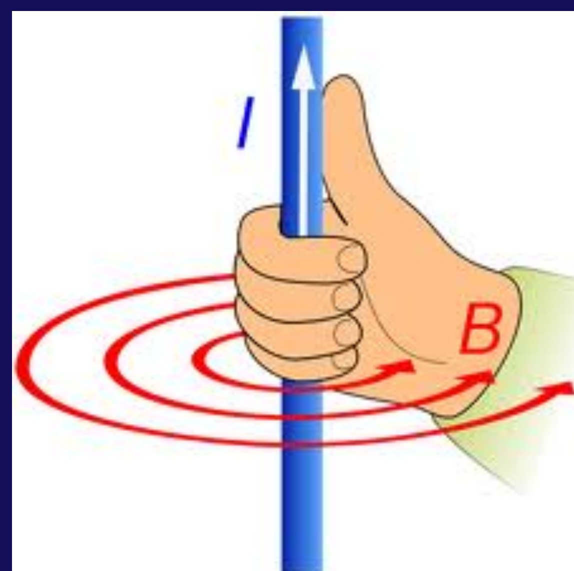
$$\tau = \vec{\mu} \times \vec{B}$$

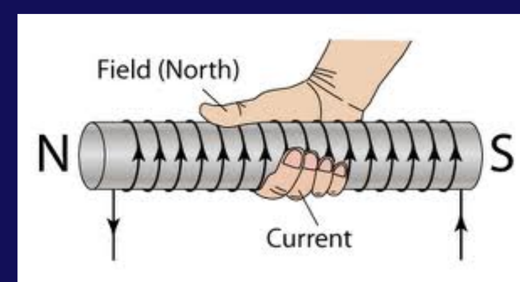


$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$







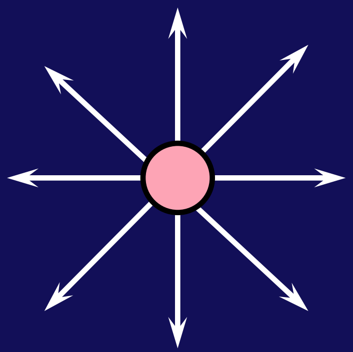


It is going to get easier !!!

Calculation of *Electric* Field

- **Two ways to calculate the Electric Field:**
  - **Coulomb's Law:**

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$



"Brute force"

  - **Gauss' Law**

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = q$$

"High symmetry"

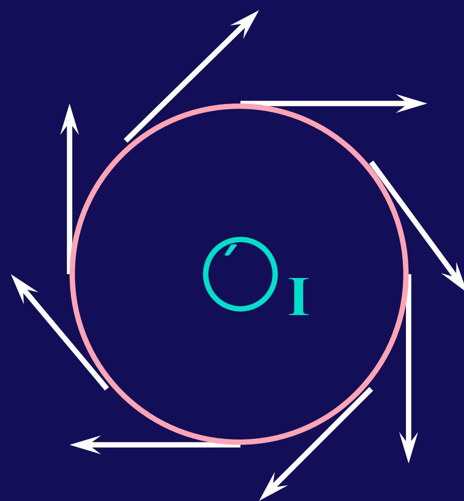
- **What are the analogous equations for the Magnetic Field?**

Calculation of *Magnetic* Field

- **Two ways to calculate the Magnetic Field:**
  - **Biot-Savart Law:**

TODAY

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \vec{r}}{r^3}$$



"Brute force"

  - **Ampere's Law**

NEXT TIME

$$\oint B \bullet dl = \mu_0 I$$

"High symmetry"

- **These are the analogous equations for the Magnetic Field!**

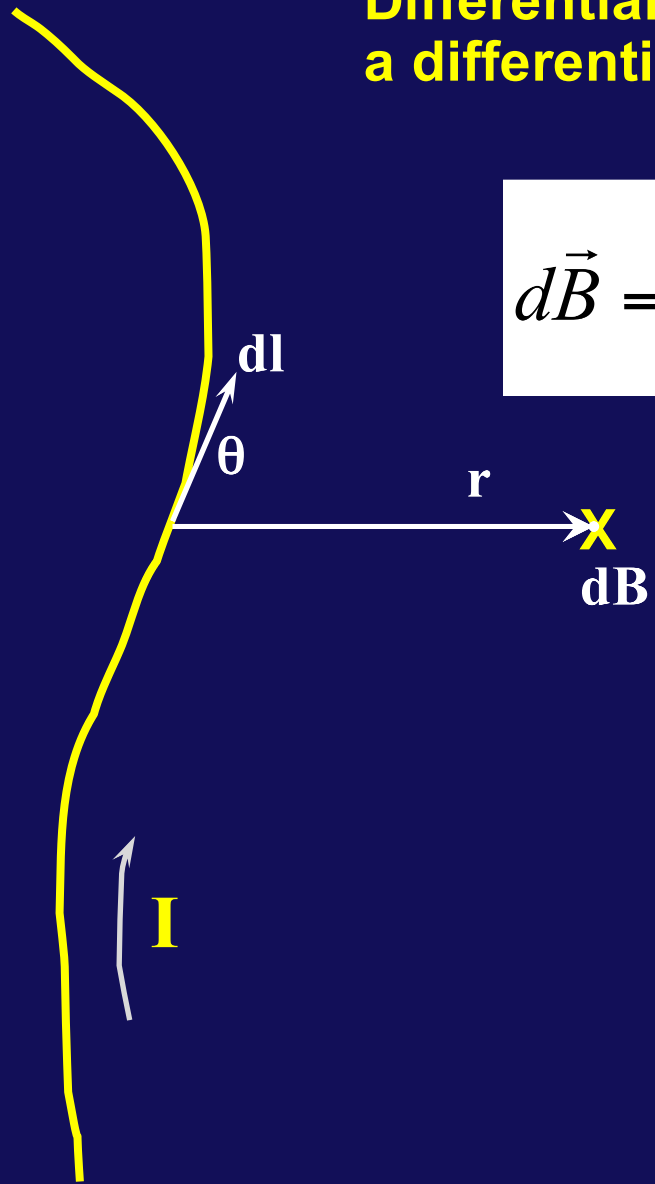
Biot-Savart Law … bits and pieces

Differential magnetic field from a differential current element

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Tm/A}$$

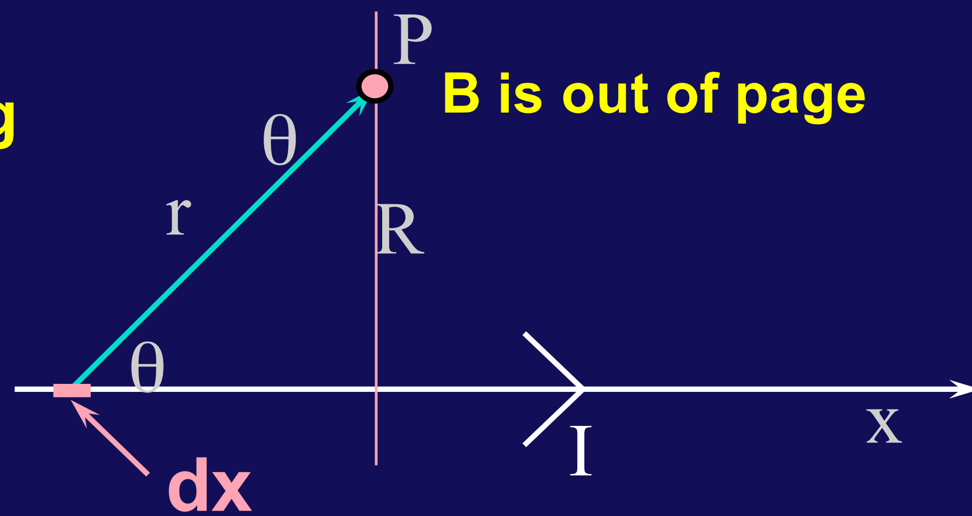
"permeability" of free space,



- Field here?
- Apply RHR from above
- Field points into the page
- Integrate along the whole wire

So, the magnetic field "circulates" around the wire

Magnetic Field of ¥ Straight Wire



- **Calculate field at point P using Biot-Savart Law:**

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{x}\times\vec{r}}{r^3}$$

Which way is B? → RHR

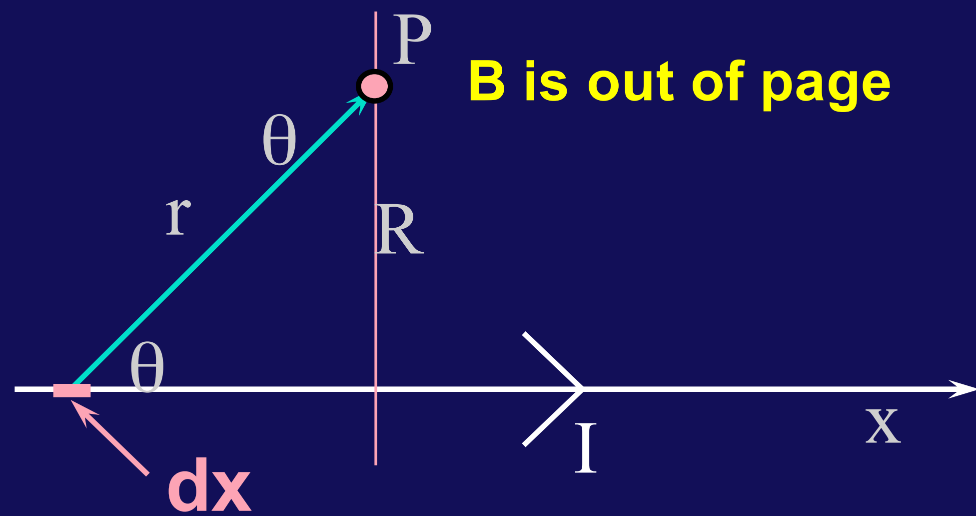
$$B = \int dB = \int_{-\infty}^{+\infty} \frac{\mu_0 I}{4\pi}\frac{(dx) r \sin\theta}{r^3}$$

- **Rewrite in terms of R,θ:**

$$r = \frac{R}{\sin\theta} \quad þ \quad \tan\theta = \frac{R}{-x} \quad þ \quad x = -R\cot\theta$$

$$\therefore \quad dx = -R\left(-\frac{1}{\sin^2\theta}\right)d\theta \quad þ \quad \frac{dx(\sin\theta)}{r^2} = \frac{\sin\theta\, d\theta}{R}$$

Magnetic Field of ¥ Straight Wire



$$B = \int_0^{\pi} \frac{\mu_0 I}{4\pi} \frac{d\theta}{R} \sin\theta$$

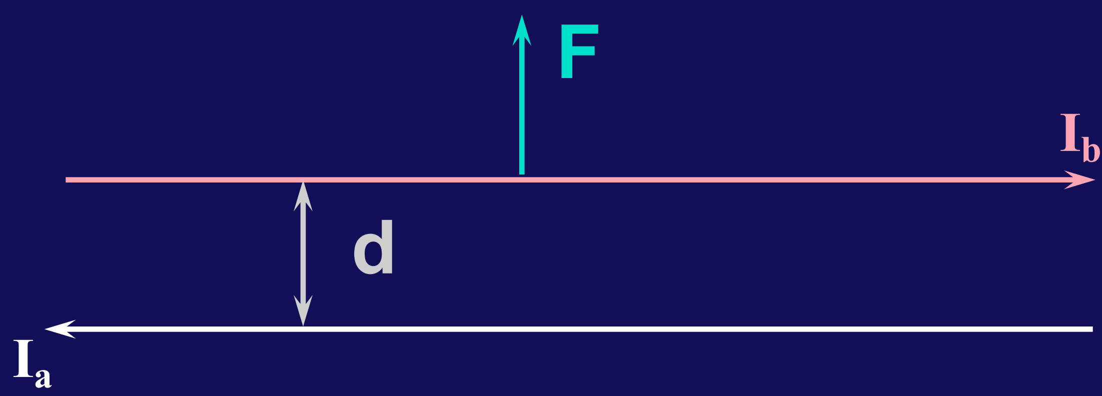
$$B = \frac{\mu_0 I}{4\pi R} \int_0^{\pi} \sin\theta d\theta$$ Þ $$B = \frac{\mu_0 I}{4\pi R} \left[-\cos\theta \right]_0^{\pi}$$

$\therefore$ $$B = \frac{\mu_0 I}{2\pi R}$$

How do we check this result ?

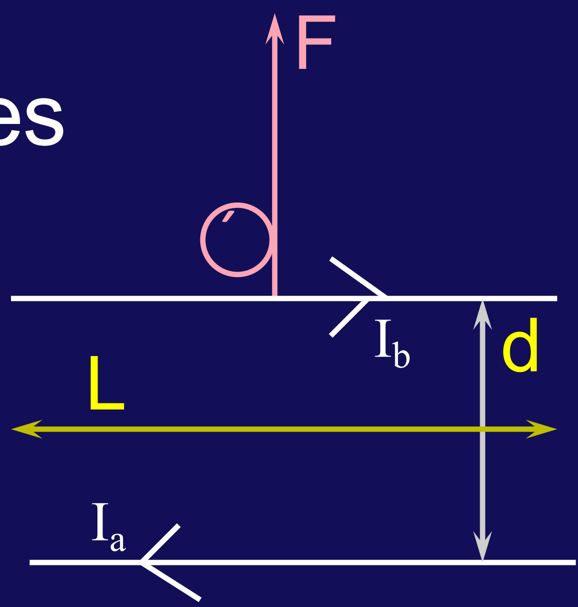
i.e. that we expect B *from wire to be proportional to* I/R ?

- Measure FORCE on current-carrying wire due to the magnetic field PRODUCED by ANOTHER current carrying wire!



- How does force depend on currents and separation?

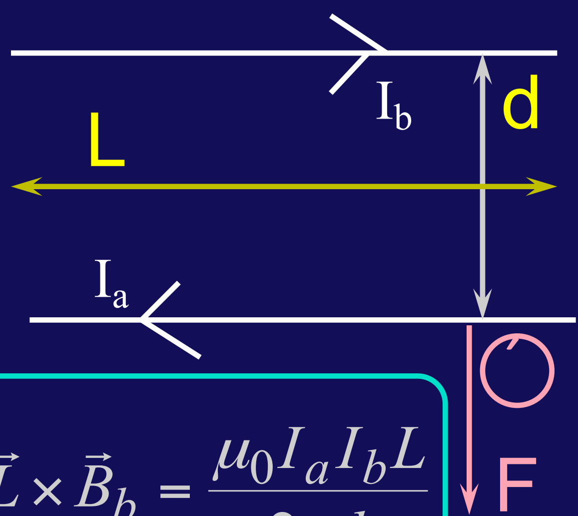
Force on 2 Parallel Current-Carrying Wires



- Calculate force on length L of wire b due to field of wire a:

  The field at b due to a is given by:

$$B_a = \frac{\mu_0 I_a}{2\pi d}$$ Þ Force on b = $$\vec{F}_b = I_b \vec{L} \times \vec{B}_a = \frac{\mu_0 I_a I_b L}{2\pi d}$$



- Calculate force on length L of wire a due to field of wire b:

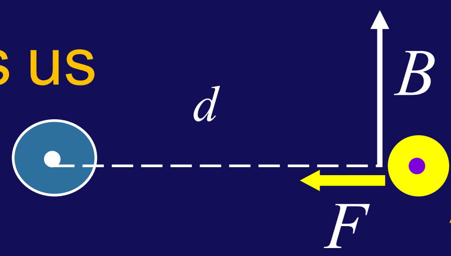
  The field at a due to b is given by:

$$B_b = \frac{\mu_0 I_b}{2\pi d}$$ Þ Force on a = $$\vec{F}_a = I_a \vec{L} \times \vec{B}_b = \frac{\mu_0 I_a I_b L}{2\pi d}$$

Force Between Current-Carrying Wires

$$\vec{F}_{12} = I_2 \vec{L} \times \vec{B} \quad \Rightarrow \quad F_{12} = I_2 L \cdot \frac{\mu_o}{2\pi d} I_1$$

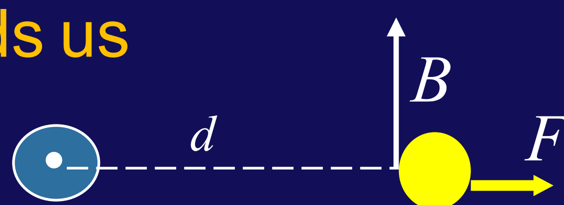
I towards us



Another I towards us

Conclusion: Currents in same direction attract!

I towards us

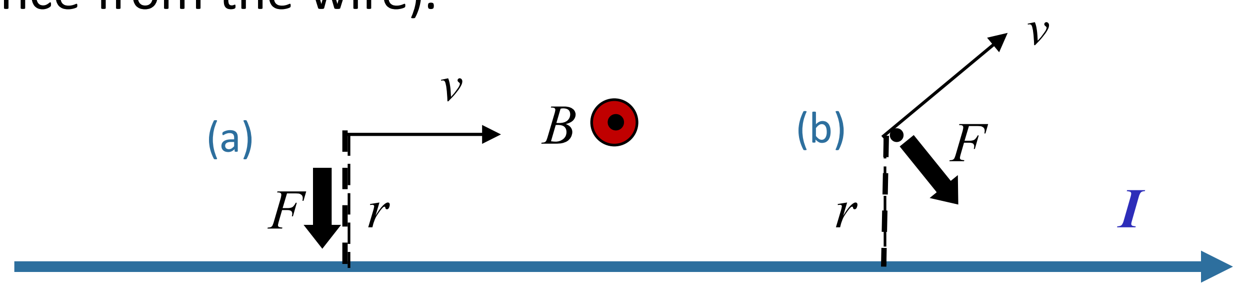


Another I away from us

Conclusion: Currents in opposite direction repel!

Clicker

A long straight wire is carrying current from left to right. Two identical charges are moving with equal speed. Compare the magnitude of the force on charge a moving directly to the right, to the magnitude of the force on charge b moving up and to the right **at the instant shown** (i.e. same distance from the wire).



A) $|F_a| > |F_b|$

B) $|F_a| = |F_b|$

C) $|F_a| < |F_b|$

$$\vec{F} = q\vec{v} \times \vec{B}$$

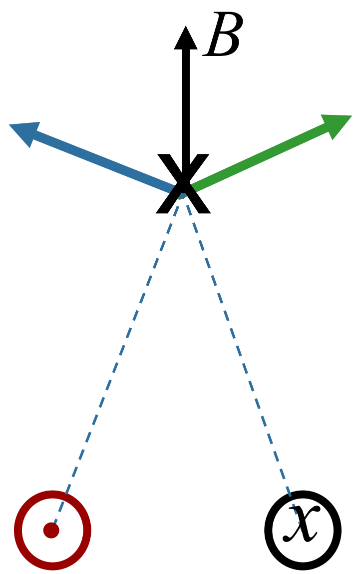
$$\left|\vec{F}\right| = qvB\sin\theta$$

Same q, $|v|$, B and $\theta\,(=90)$

But, Forces are in different directions

Clicker

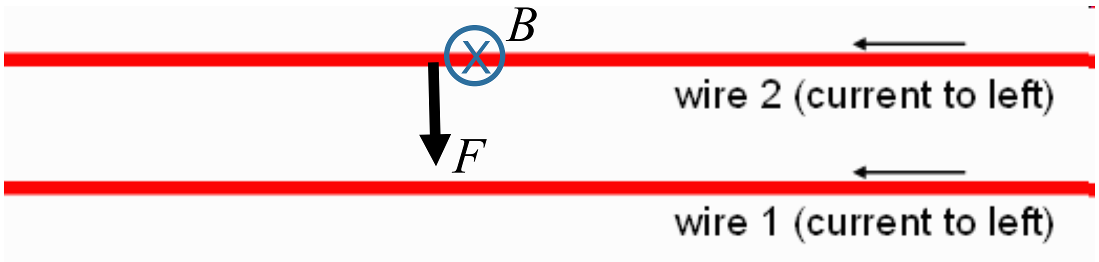
Two long wires carry opposite current



What is the direction of the magnetic field above, and midway between the two wires carrying current – at the point marked "X"?

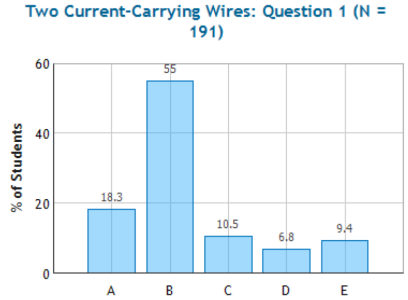
A) Left B) Right C) Up D) Down E) Zero

CheckPoint 2 & 4



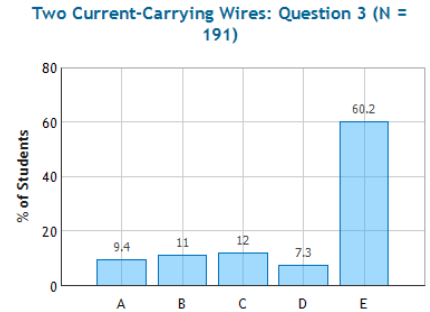
What is the direction of the force on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero



What is the direction of the torque on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero



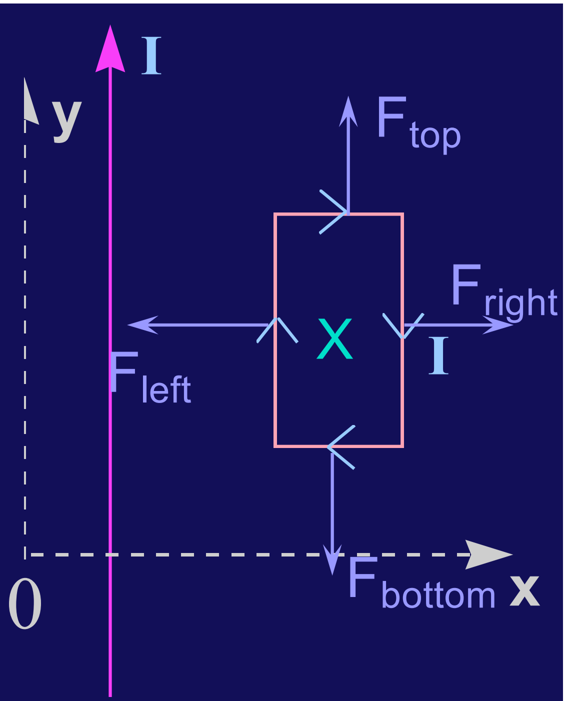
Uniform force at every segment of wire

↓

No torque about any axis

Clicker

- A current **I** flows in the positive **y** direction in an infinite wire; a current **I** also flows in the loop as shown in the diagram.
  - What is F_x, net force on the loop in the x-direction?



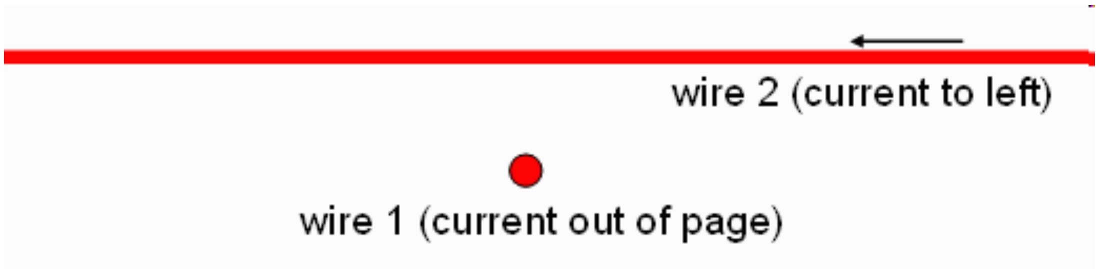
(a) $F_x < 0$

(b) $F_x = 0$

(c) $F_x > 0$

- Recall from a previous CLICKER that the net force on a current loop in a *constant magnetic field* was zero.
- However, the magnetic field here is not a constant field
- The direction of the magnetic field at the current loop is in the -z direction.
- The forces on the top and bottom segments of the loop DO indeed cancel!!
- The forces on the left and right segments of the loop DO NOT cancel!!
  - The left segment of the loop is in a larger magnetic field.
  - Therefore, $\mathbf{F_{left} > F_{right}}$

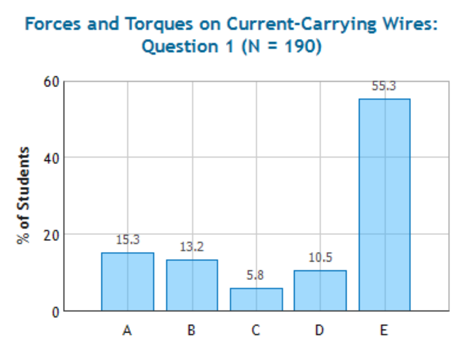
CheckPoint 7:



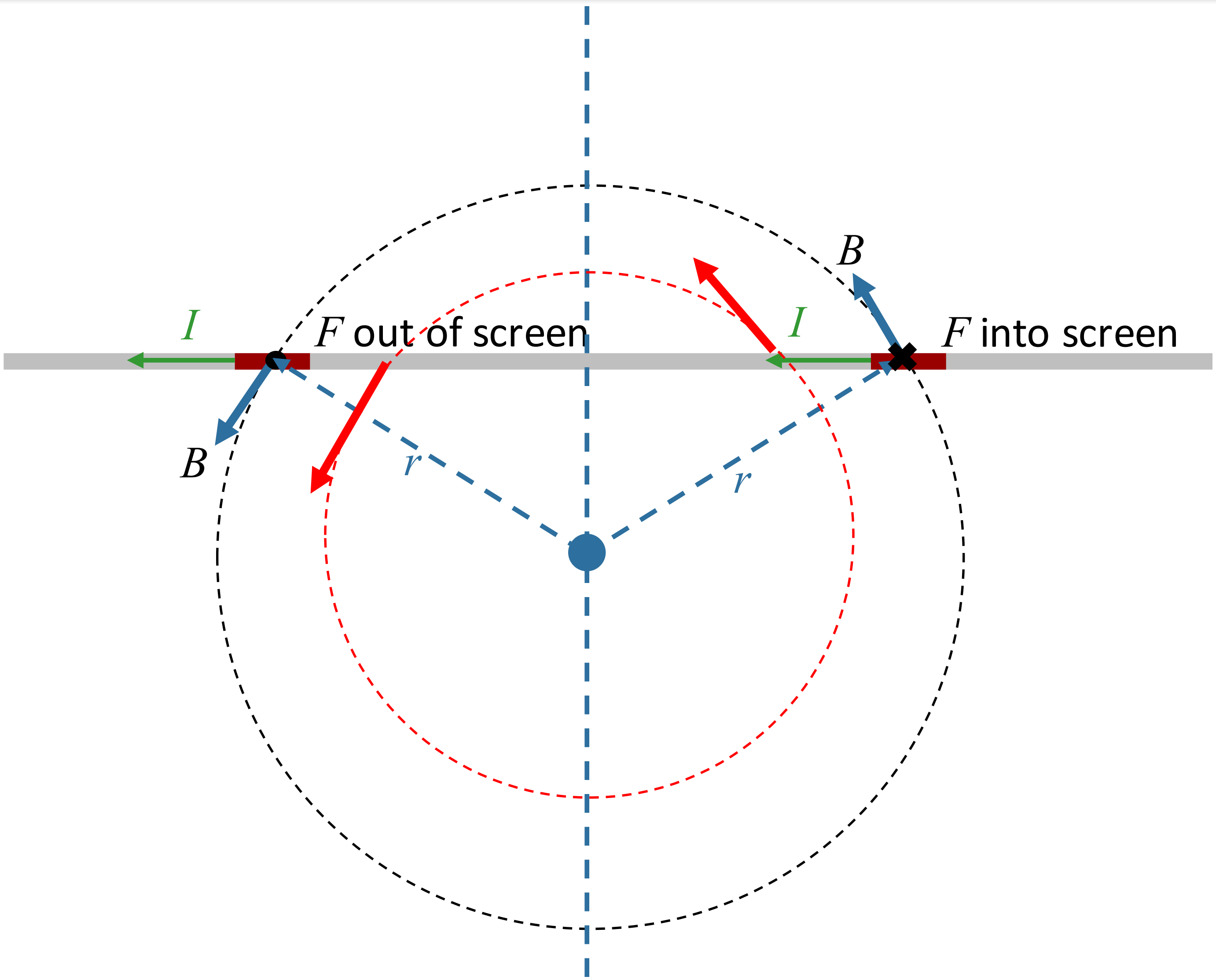
What is the direction of the net force on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero

WHY?
DRAW PICTURE!



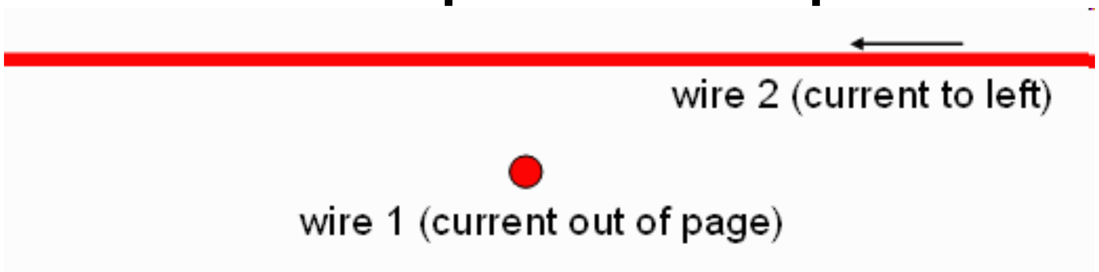
Consider Force on Symmetric Segments

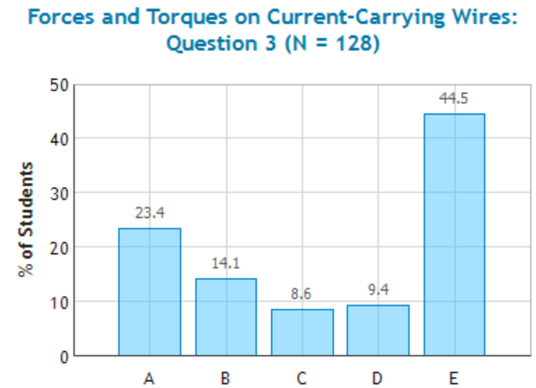


Net Force is Zero!

CheckPoint 9: we are basically guessing

What about the torque? … same picture

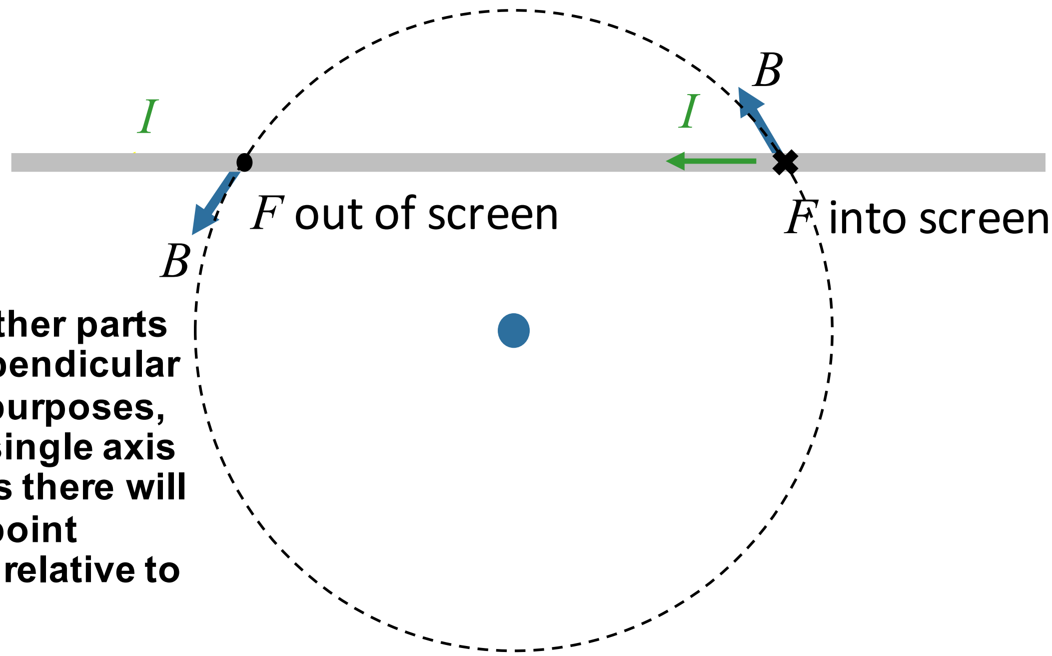




A) Up B) Down C) Into Screen D) Out of screen E) Zero

One Response: I do not understand torque on wires in scenarios such as this one. Or really at all.

Another: While the magnetic forces on further parts of wire 2 (those which are not directly perpendicular to wire 1) add up to zero for translational purposes, they point in opposite directions about a single axis of rotation (directly above wire 1), and thus there will be a net torque on wire 2. The torque will point upwards due to the direction of the forces relative to the distance they are from the axis.



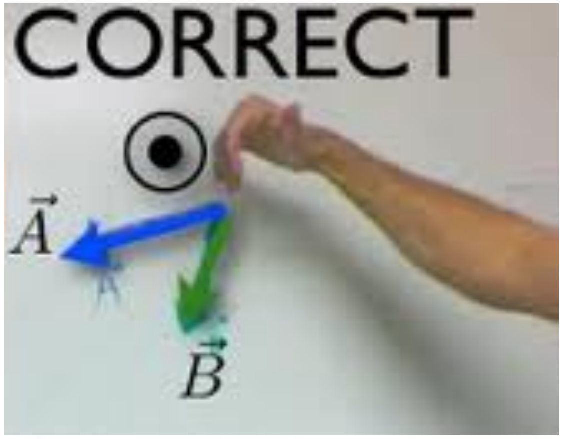
Right Hand Rule Review

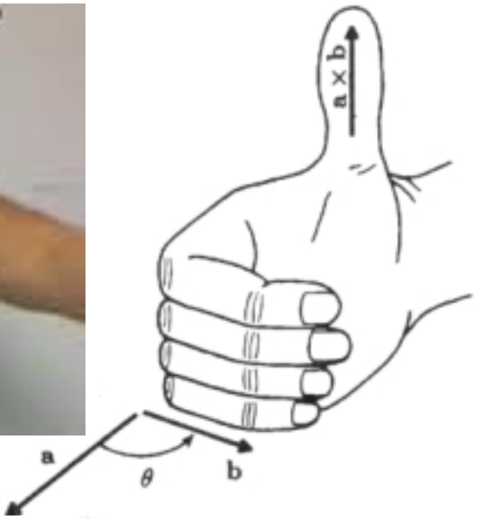
1. ANY CROSS PRODUCT

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = I\vec{L} \times \vec{B}$$

$$\tau = \vec{r} \times \vec{F} \qquad \tau = \vec{\mu} \times \vec{B}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$



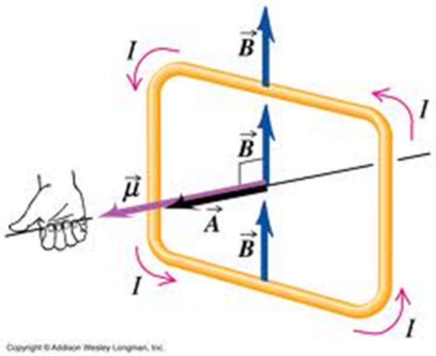


Right-hand rule

2. Direction of Magnetic Moment

Fingers: Current in Loop

Thumb: Magnetic Moment



3. Direction of Magnetic Field from Wire

Fingers: Magnetic Field

Thumb: Current