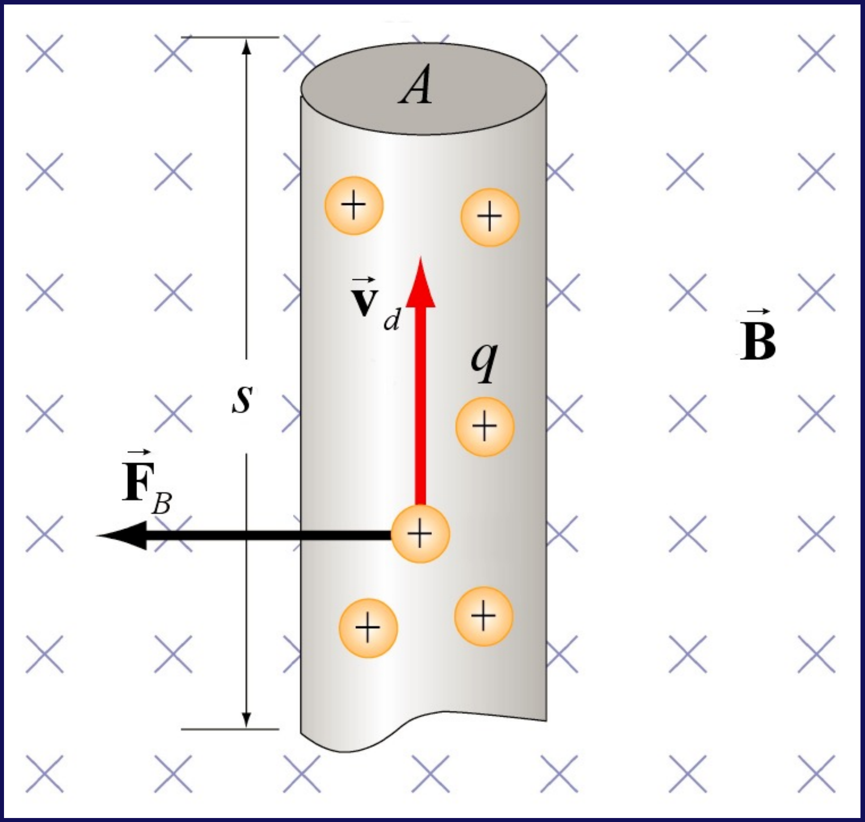
Magnetism

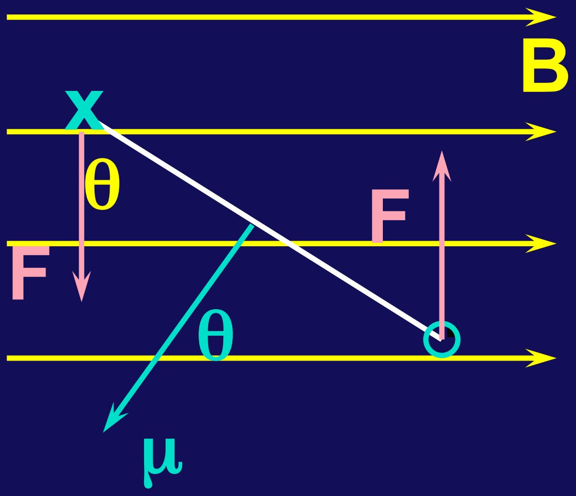
Forces & Magnetic Dipoles



$$\mu = AI$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \bullet \vec{B}$$



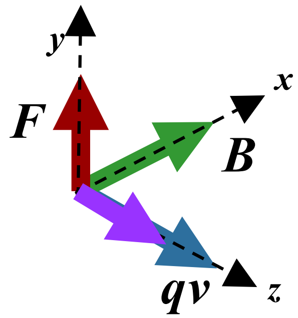
Phys 122 Lecture 18
D. Hertzog

Business

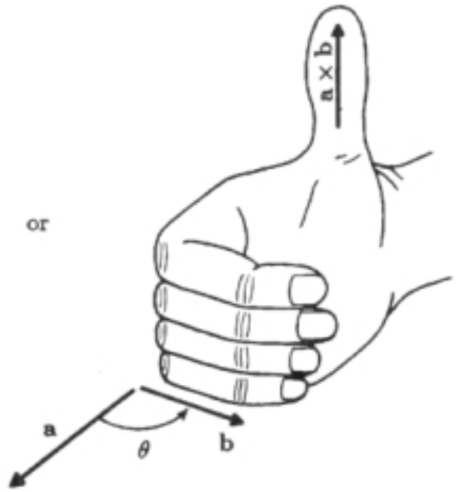
- Regrade requests by 4 pm Friday (no exceptions)
- Solutions/Key posted on home page

Last Time: The Lorentz Force and the RHR

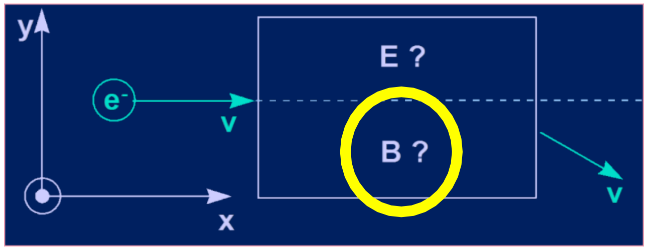
$$\vec{F} = q\vec{v} \times \vec{B}$$



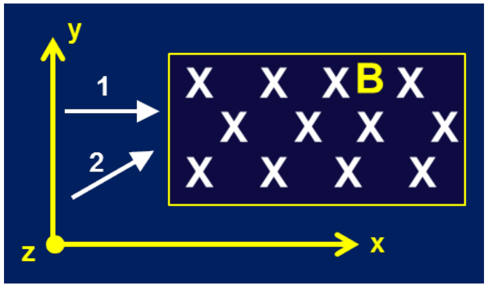
1. Point Fingers in direction of $\vec{v}$
2. Curl all fingers in direction of $\vec{B}$
3. Thumb points in direction of $\vec{F}$



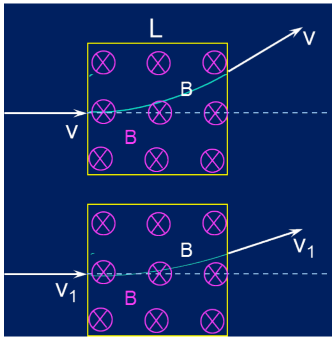
And a few challenging Clickers:



Only B_z since it does not change the speed



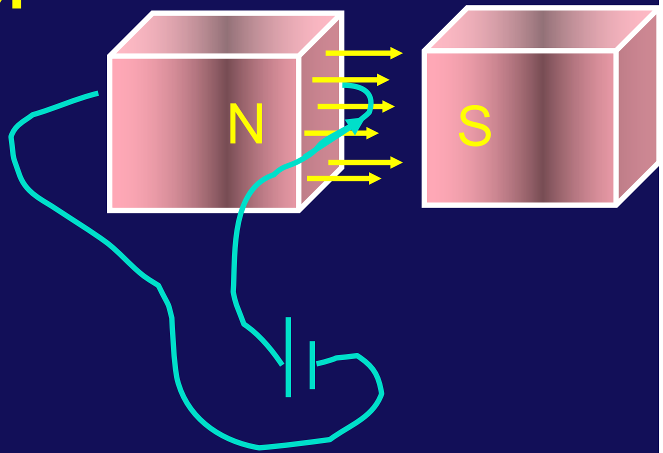
$F_1 = F_2$; both are perpendicular to B



Work done by magnetic field is = 0; F is always perpendicular to dx

Magnetic Force on a Current

- **What is the total force dF on a length dl of current-carrying wire in B field?**
- Current described by:
  - **n** charges **q** per volume
  - Each has velocity= **v**
  - Wire cross-section = **A**.



Force on a single charge = $q\vec{v} \times \vec{B}$

Force on charges moving in line segment dl = $d\vec{F} = q_{tot}(\vec{v} \times \vec{B}) = q(nA)(dL)(\vec{v} \times \vec{B})$

Recall Current = $I = \frac{dq}{dt} = qnAv$ *

Þ $d\vec{F} = Id\vec{l} \times \vec{B}$

For a straight wire of length L carrying a current I, the force on it is therefore :

$$\vec{F} = I\vec{L} \times \vec{B}$$

*Reminder from Prelecture

- What units result when a velocity v is multiplied by an area A? Volume per second
- A simple and useful way to picture this is to think of water flowing in a garden hose. If the speed of the water is known (v = 10 m/s = 1,000 cm/s for example) and the cross-sectional area of the hose is also known (A = 10 cm^2 for example), then the volume of water that travels past any point of the hose is every second is v*A = 10,000 cm^3/s = 10 liters/s. In other words, if we know the velocity of the water and the area of the hose we can easily figure out the volume of water transported by the hose per unit time. If we multiply this by the density of water (i.e., 10 liters/s * 1 kg/liter = 10 kg/s) then we immediately discover how much mass is moving past a point in the hose the hose per unit time.
- This has an exact analogy with current in a wire. Think of the wire as the hose and the current flowing in the wire as the water. The average velocity of the charges v_{avg} times the area of the wire A tells us the volume of the charge carriers that moves through the wire per unit time. If we multiply this by the number density of the charge carriers n we find the number of charge carriers per unit time flowing in the wire, and if we multiply this by the charge carried by each one q we find the charge per unit time flowing in the wire.
- In other words, the current in the wire I is equal to the product $qnAv_{avg}$.

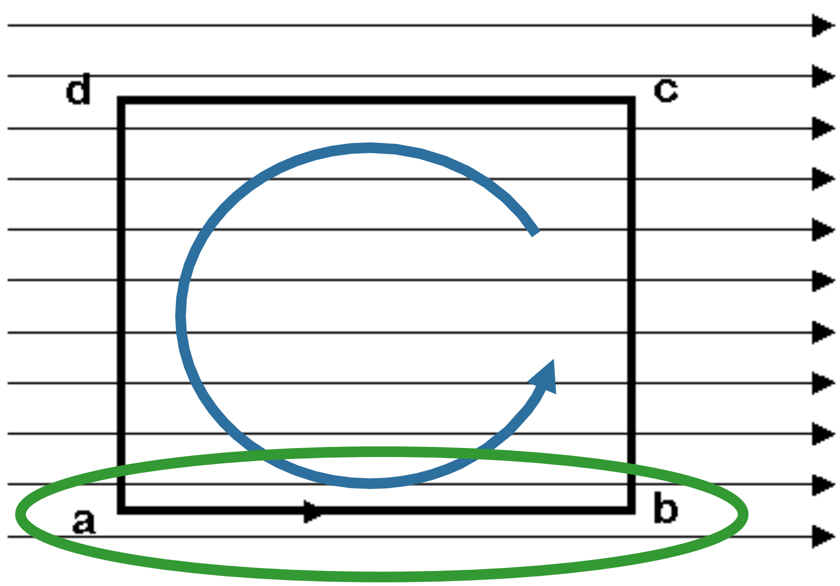
Work and Magnetic Fields

- **Magnetic fields do no work on free charges**
- **Magnetic fields *can* do work on current carrying wires!**

Shout-out Clicker

A square loop of wire is carrying current in the counterclockwise direction. A horizontal uniform magnetic field points to the right

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section a-b of the loop?

A ○ zero

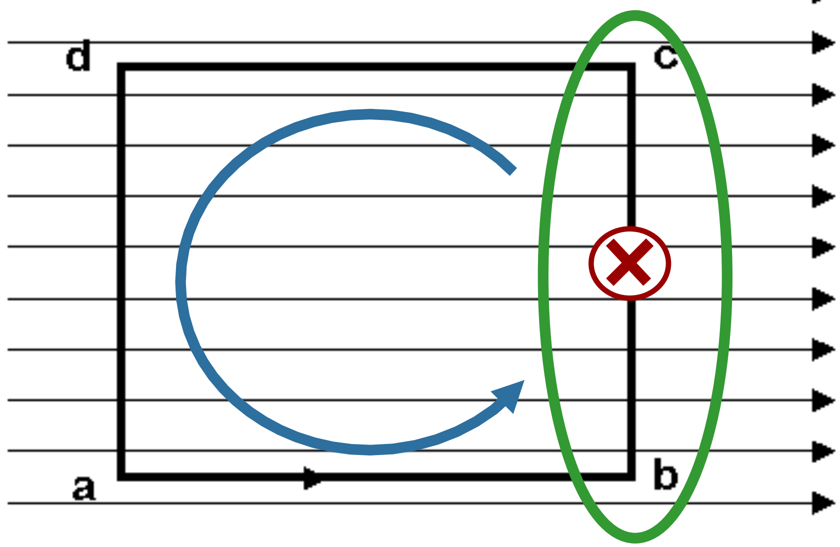
B ○ out of the page

C ○ into the page

Shout-out Clicker

A square loop of wire is carrying current in the counterclockwise direction. A horizontal uniform magnetic field points to the right

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section b-c of the loop?

A ○ zero

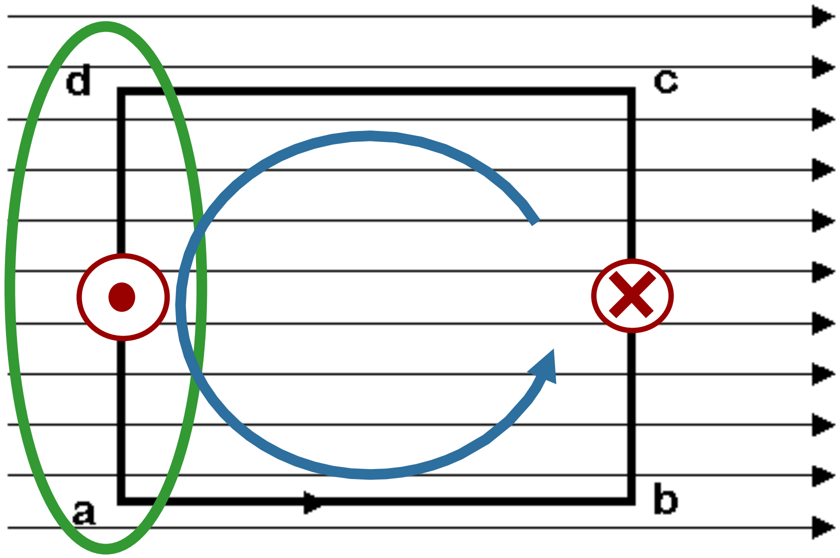
B ○ out of the page

C ○ into the page

Shout-out Clicker

A square loop of wire is carrying current in the counterclockwise direction. A horizontal uniform magnetic field points to the right

$$\vec{F} = I\vec{L} \times \vec{B}$$



What is the force on section d-a of the loop?

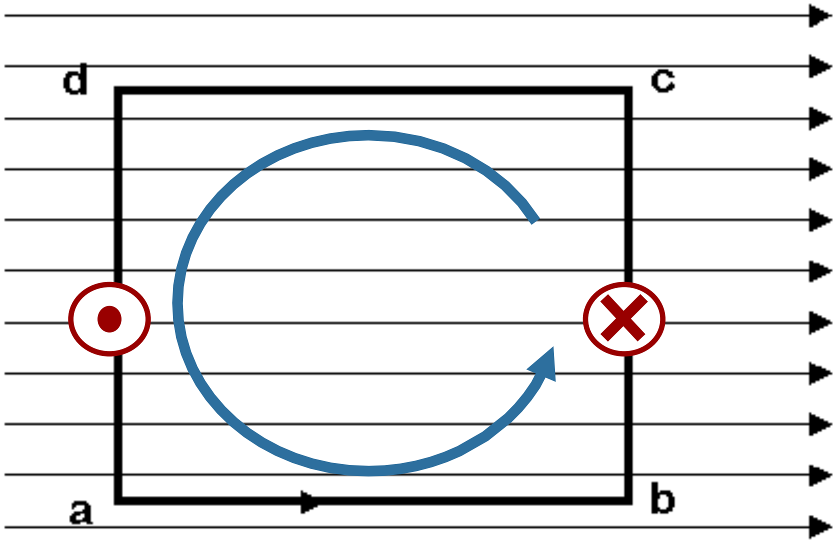
A) Zero

B) Out of the page

C) Into the page

Shout out … CheckPoint 2

A square loop of wire is carrying current in the counterclockwise direction. A horizontal uniform magnetic field points to the right



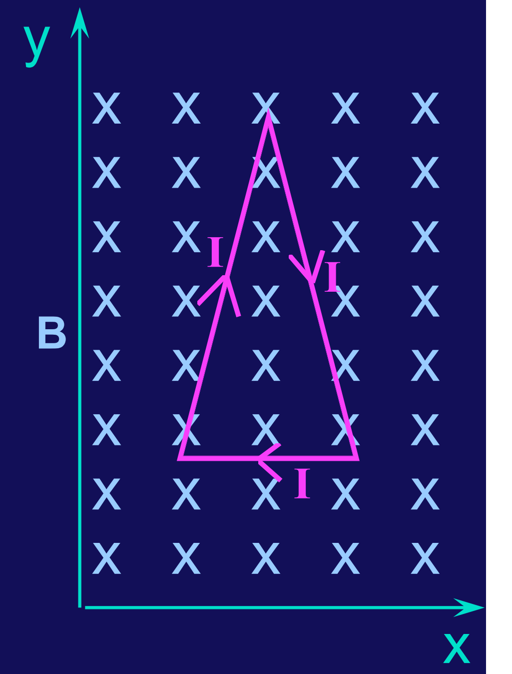
What is the direction of the net force on the loop?

A ○ out of the page

B ○ into the page

C ○ the net force on the loop is zero

(real) Clicker

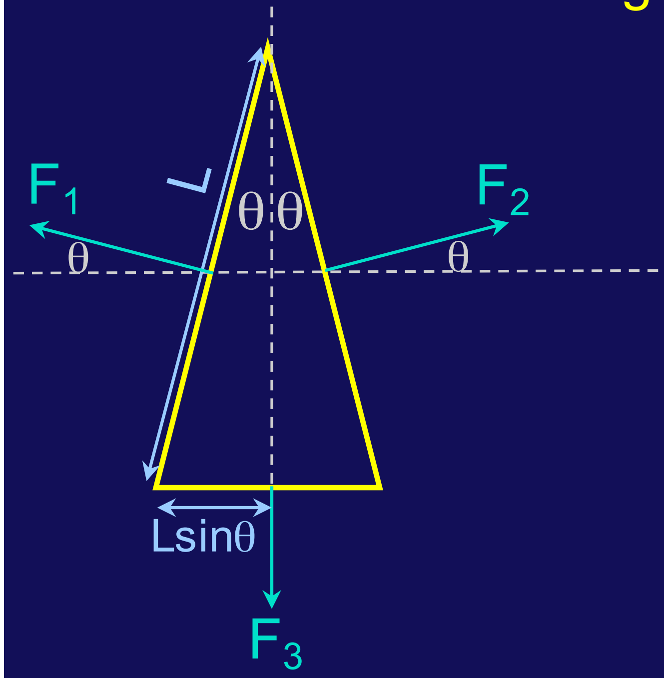


A current I flows in a wire which is formed in the shape of an isosceles triangle as shown. A constant magnetic field exists in the -z direction.

What is F_y, net force on the wire in the y-direction?

(a) $F_y < 0$ (b) $F_y = 0$ (c) $F_y > 0$

- The forces on each segment are determined by: $\vec{F} = I\vec{L} \times \vec{B}$



- From symmetry, $F_x = 0$
- For the y-component:

$$F_{1_y} = F_{2_y} = ILB\sin\theta$$

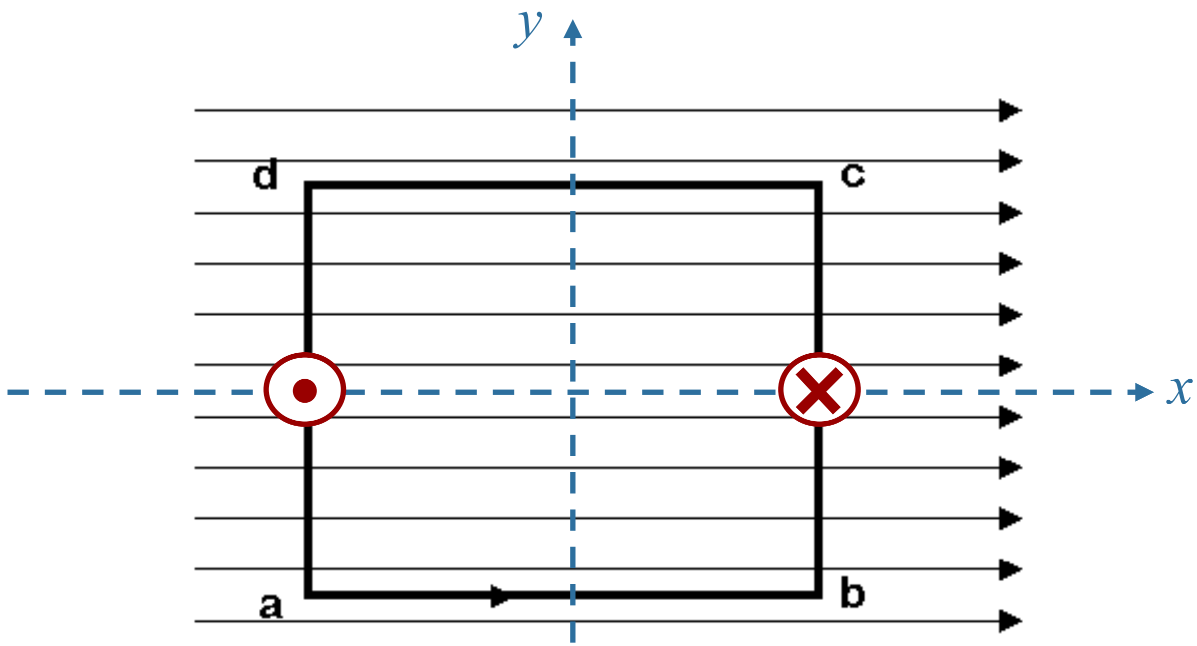
$$F_{3_y} = -2ILB\sin\theta$$

- Therefore:

$$F_y = F_{1y} + F_{2y} + F_{3y} = 0$$

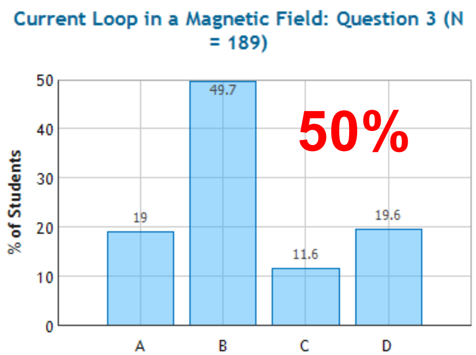
Clicker of a CheckPoint 4

A square loop of wire is carrying current in the counterclockwise direction. A horizontal uniform magnetic field points to the right



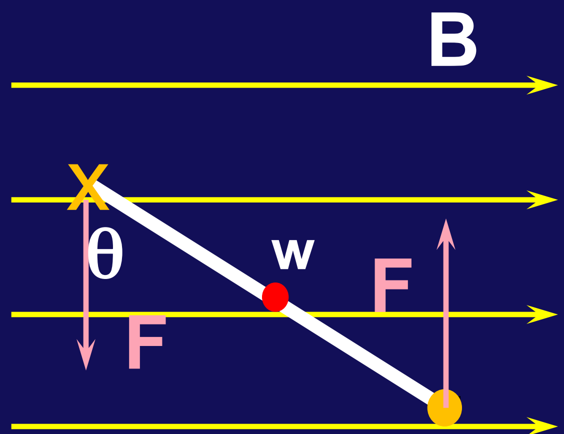
In which direction will the loop rotate?
(assume the z axis is out of the page)

A) Around the x axis
B) Around the y axis
C) Around the z axis
D) It will not rotate



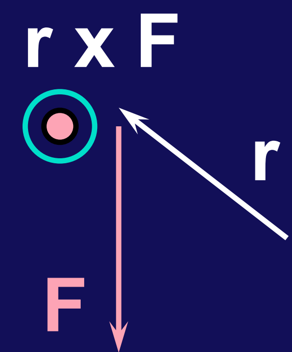
Torque on a loop

- Coil has width w (the side you see) and length L (into the screen).
- Torque is given by: $\vec{\tau} \equiv \vec{r} \times \vec{F}$



þ $\tau = 2\left(\frac{w}{2} F \sin\theta\right)$

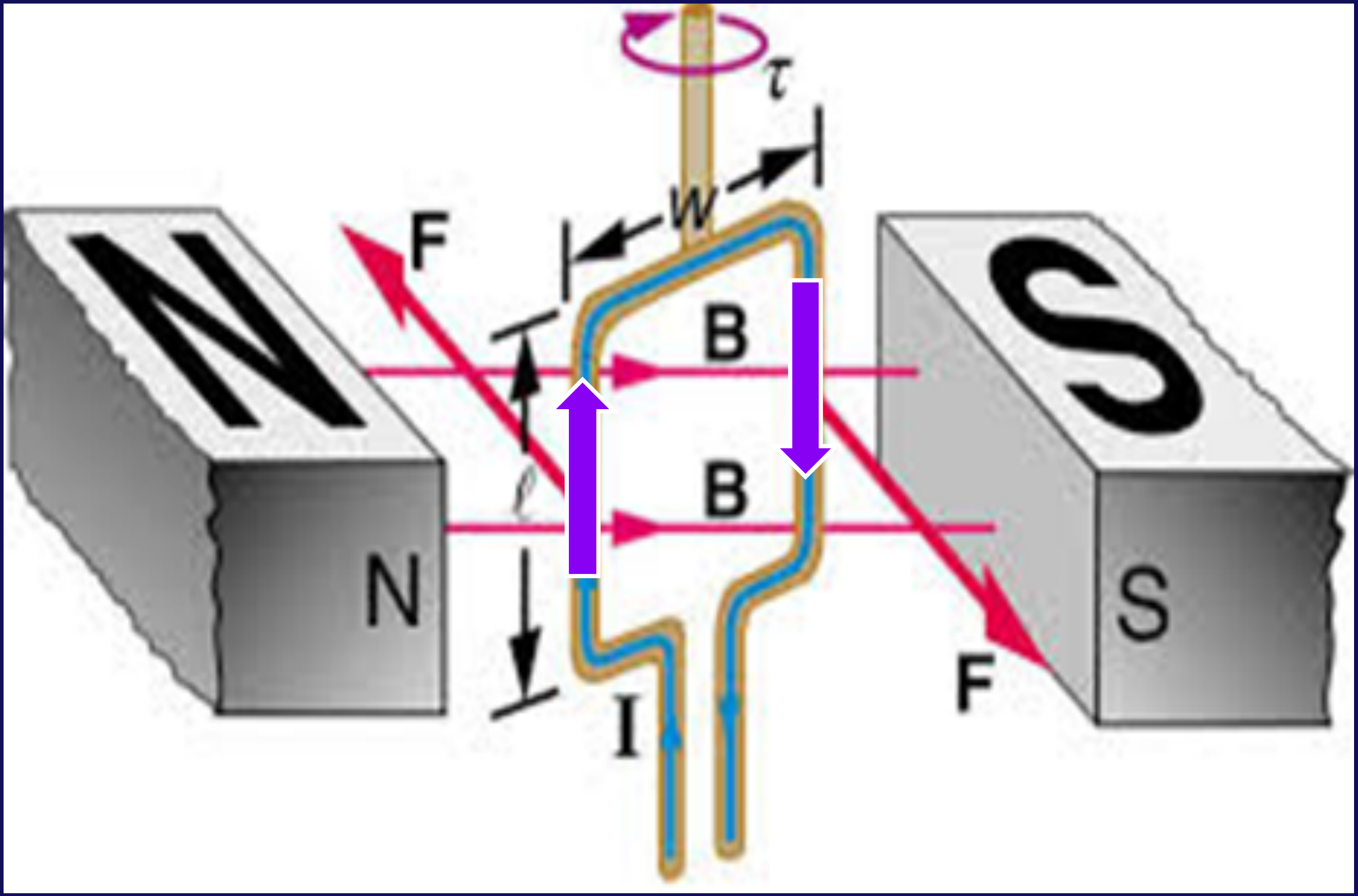
$F = ILB$ þ $\boxed{\tau = IAB \sin\theta}$



where $A = wL$ = area of loop

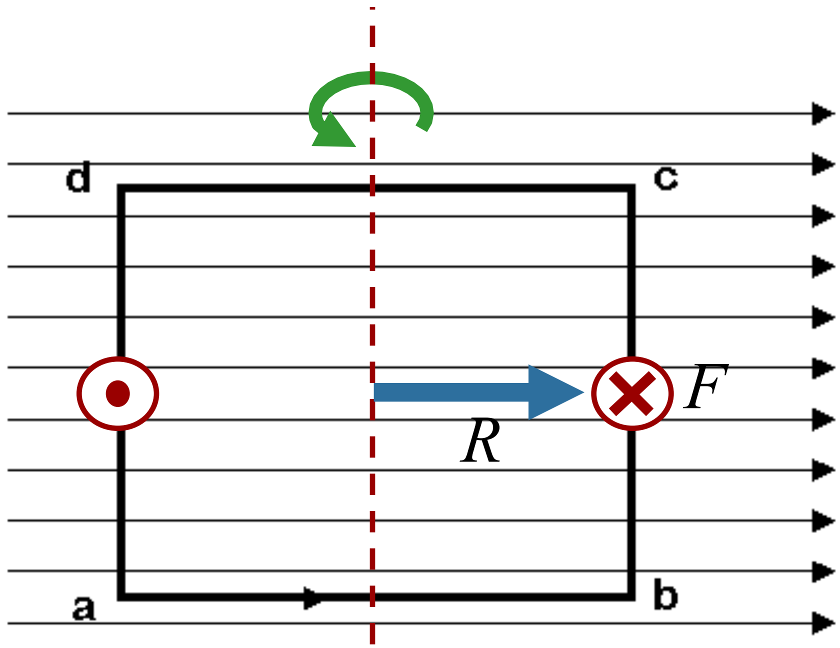
maximum τ occurs when the plane of the loop is parallel to B, which is when θ = 90

Some pictures



CheckPoint 6

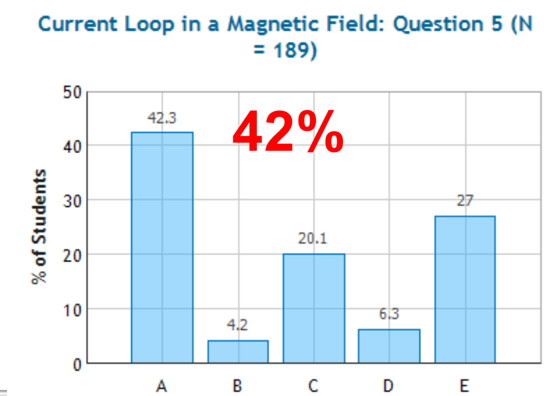
A square loop of wire is carrying current in the counterclockwise direction. A horizontal uniform magnetic field points to the right



$$\vec{\tau} = \vec{R} \times \vec{F}$$

6) What is the direction of the net torque on the loop?

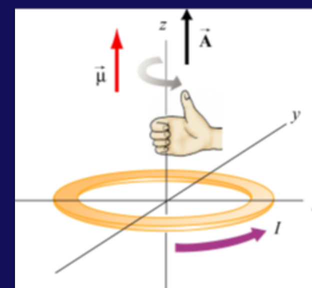
- up
- down
- out of the page
- into the page
- the net torque is zero



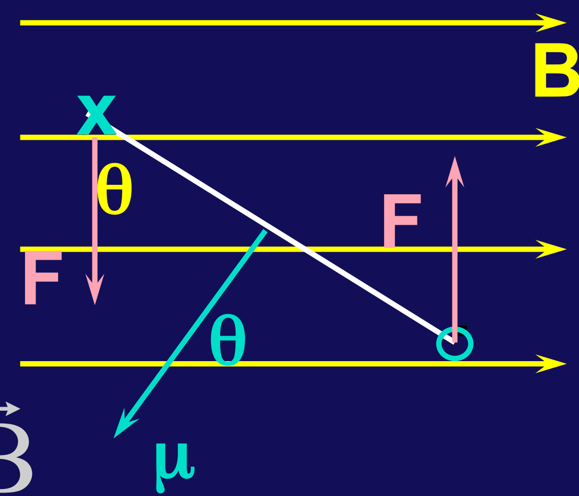
Magnetic Dipole Moment

- **Define magnetic dipole moment of a current loop as:**

magnitude: $\mu = AI$



direction: $\perp$ to plane of the loop in the direction the thumb of right hand points if fingers curl in direction of current.



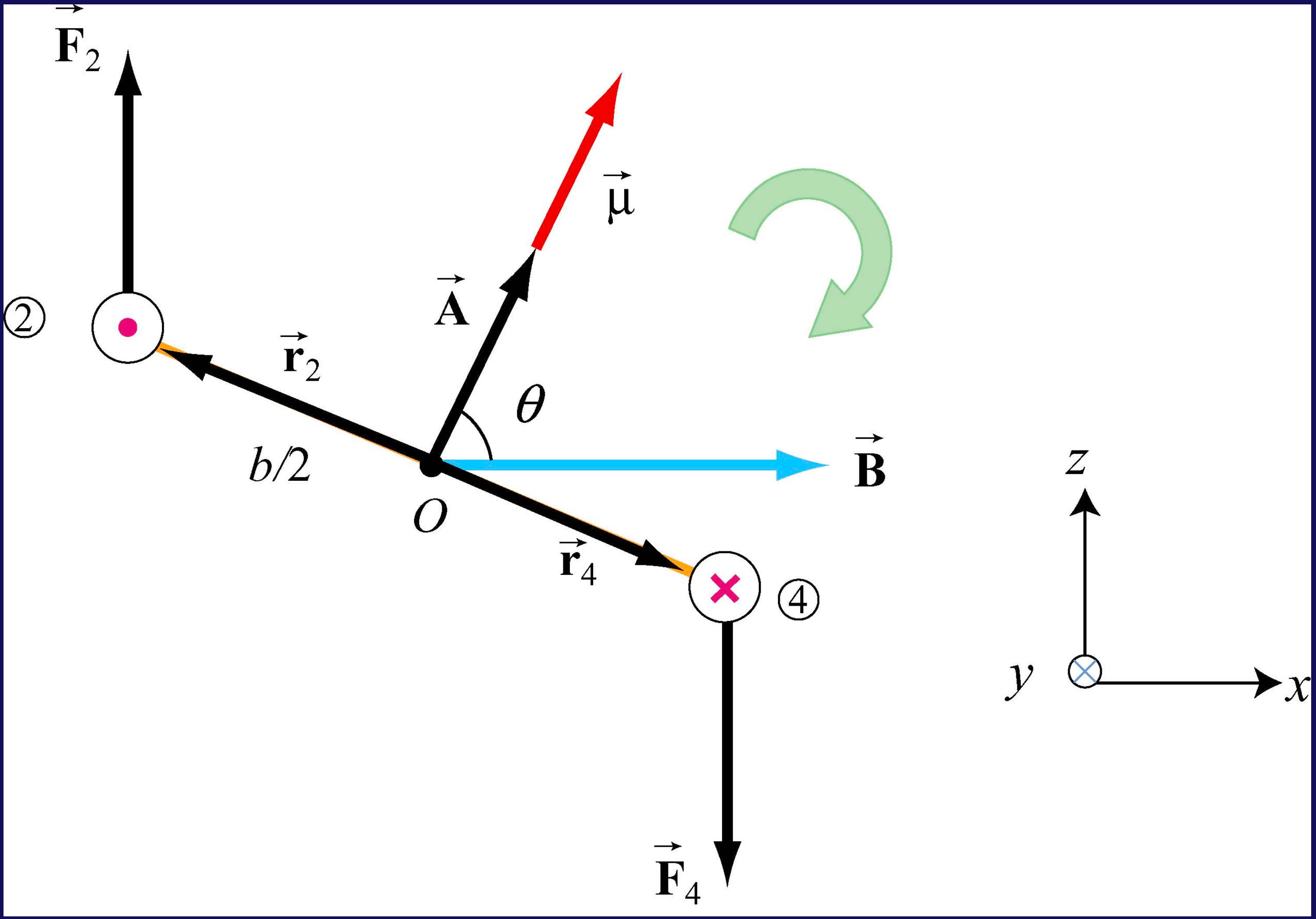
- **Torque on loop is then:**

$\tau = AIB\sin\theta$ Þ $\vec{\tau} = \vec{\mu} \times \vec{B}$

- **Note: if loop consists of N turns,** $\mu = NAI$

Remember this: *The torque always wants to line* $\boldsymbol{\mu}$ *up with* $\boldsymbol{B}$*!*

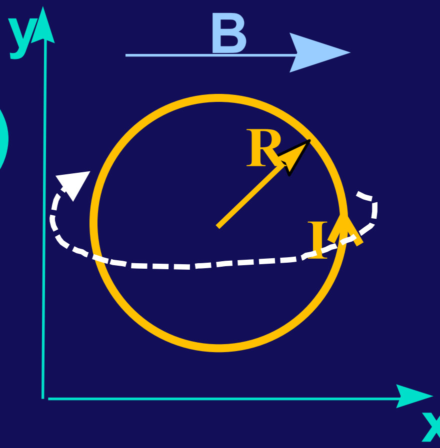
Do we know the directions?



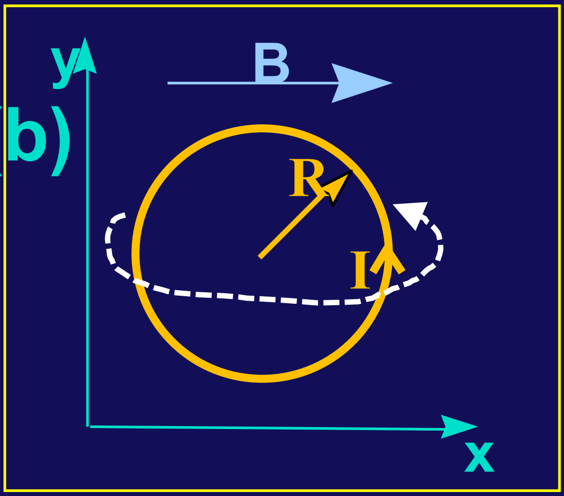
Clicker

A circular loop of radius R carries current I. A constant magnetic field B exists in the +x direction. Initially the loop is in the x-y plane.

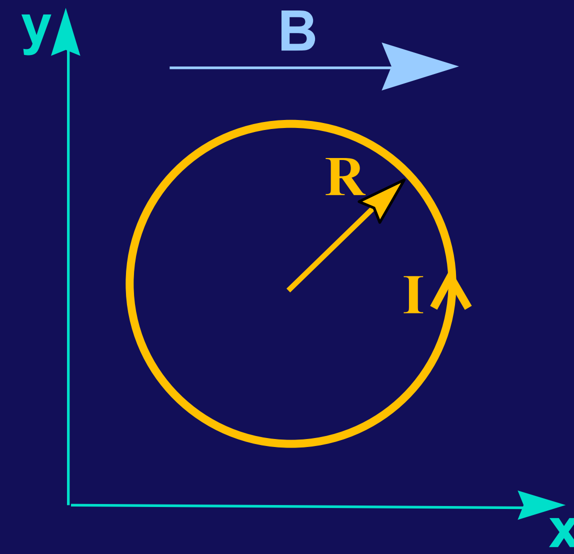
– How will the coil rotate?



(a)



(b)



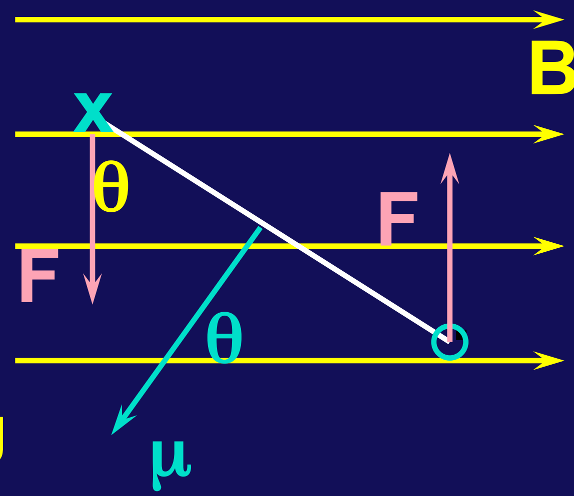
(c) It will not rotate

- The coil will rotate if the torque on it is non-zero: $\vec{\tau} = \vec{\mu} \times \vec{B}$

- The magnetic moment μ is in +z direction.
- Therefore the torque τ is in the +y direction.
- Therefore the loop will rotate as shown in (b).

Potential Energy of Dipole

- **Work required to change the orientation of a magnetic dipole in the presence of a magnetic field.**



- **Define potential energy U (with zero at position of max torque) corresponding to this work.**

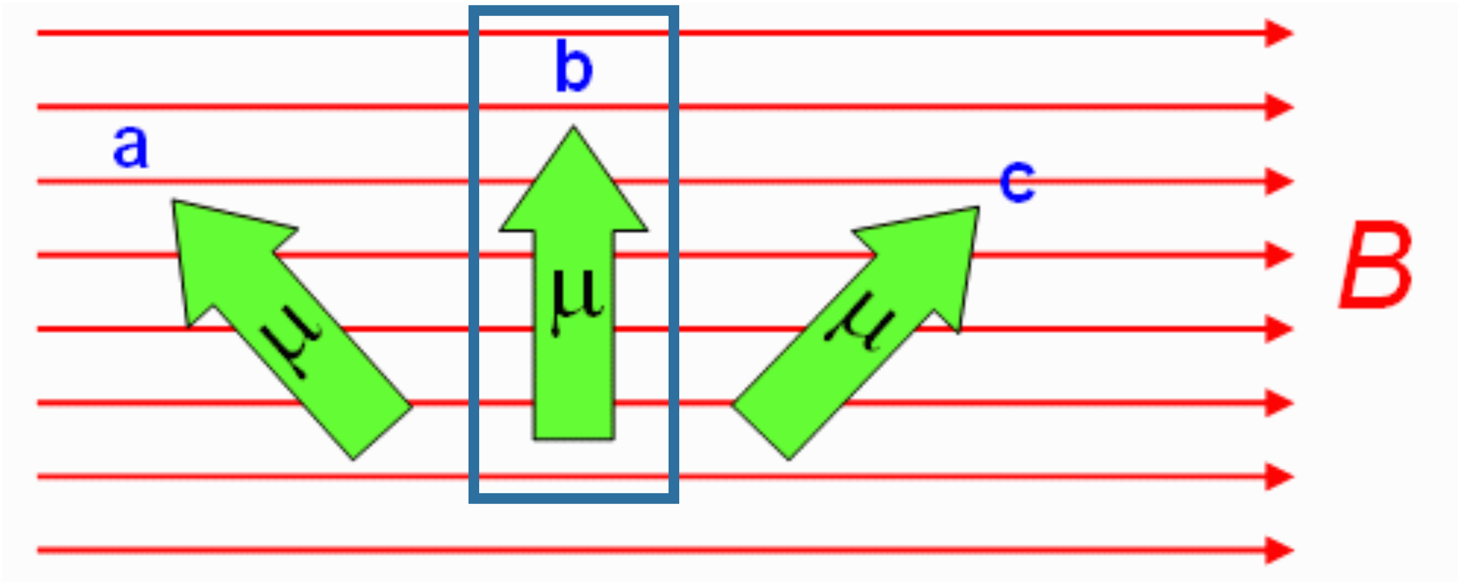
$$U \equiv \int \tau d\theta$$

Þ

$$U = -\vec{\mu} \bullet \vec{B}$$

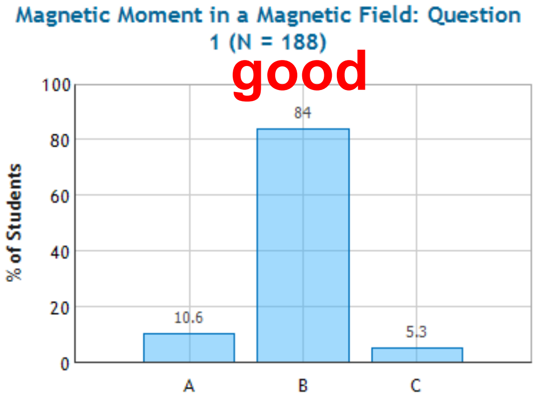
CheckPoint 8

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. Which orientation results in the largest magnetic torque on the dipole ?



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Biggest when $\vec{\mu} \perp \vec{B}$



Magnetic Field can do Work on Current

$$W = \int \tau \, d\theta$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin(\theta)$$

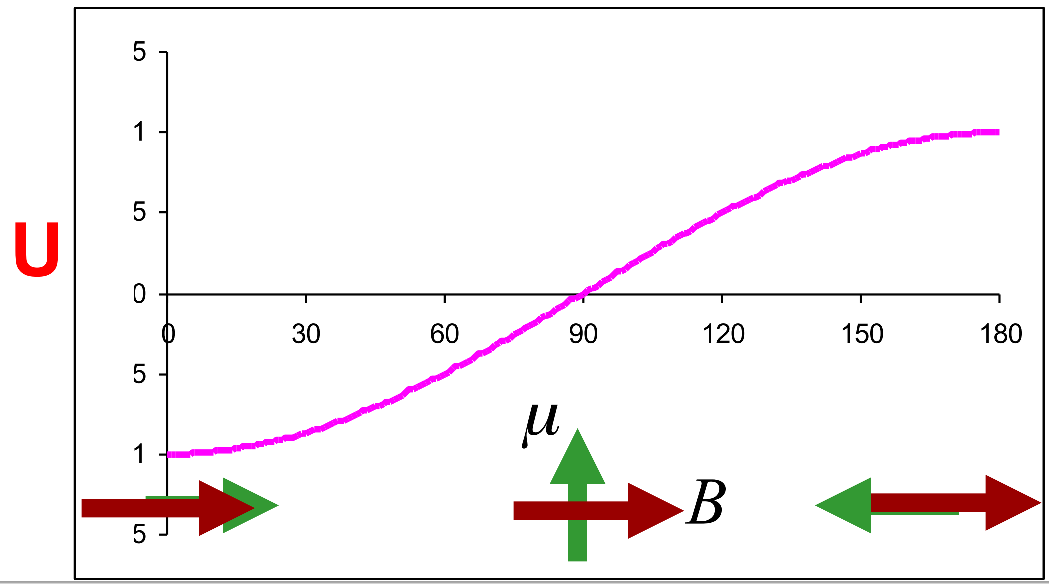
" – " sign in integration because direction of rotation is decreases θ

$$W = \int_{90}^{\theta} - \mu B \sin(\theta) \, d\theta = \mu B \cos(\theta) = \vec{\mu} \cdot \vec{B}$$

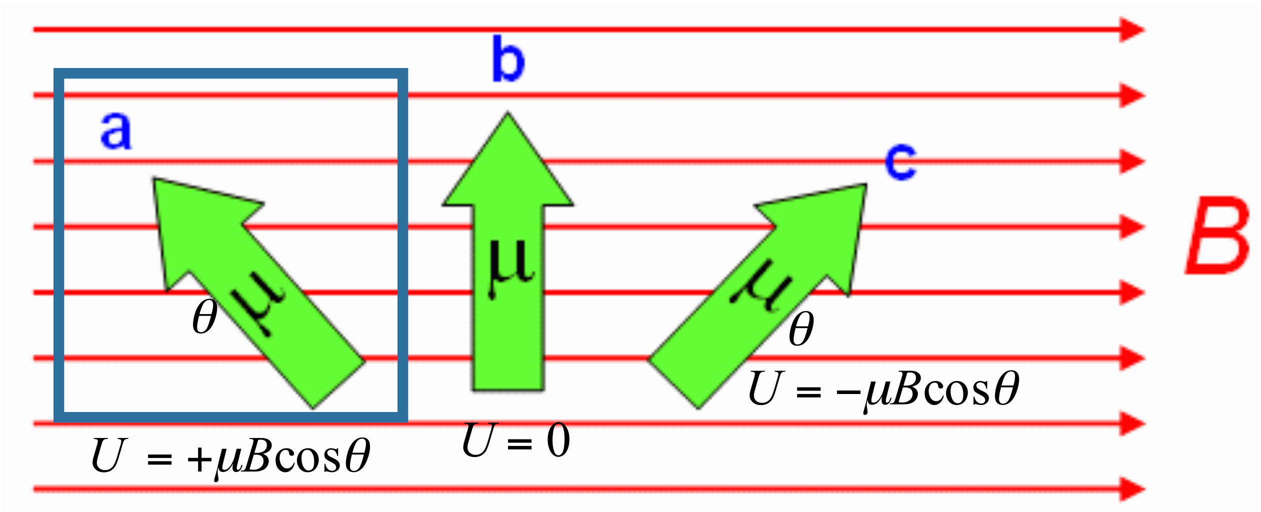
$$\Delta U = -W$$

Defined $\mathbf{U = 0}$ at position of maximum torque

$$U \equiv -\vec{\mu} \cdot \vec{B}$$

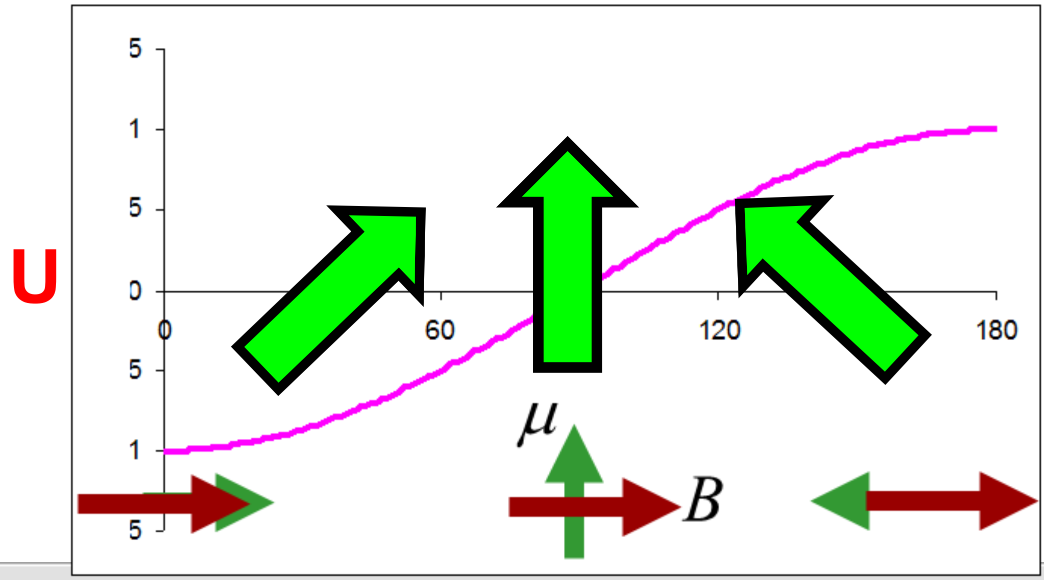


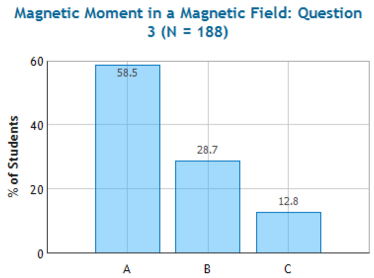
CheckPoint 10



Which orientation has the most potential energy?

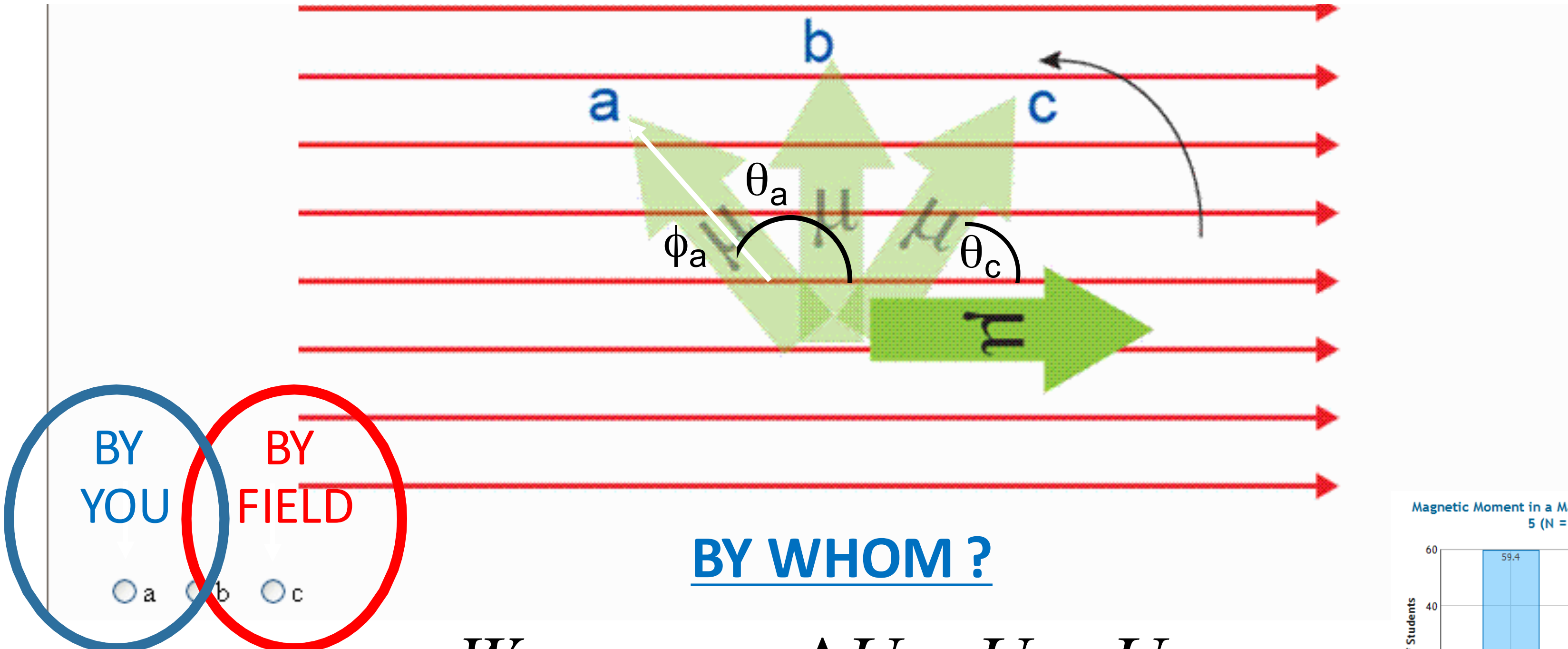
$$U = -\vec{\mu}\cdot\vec{B}$$



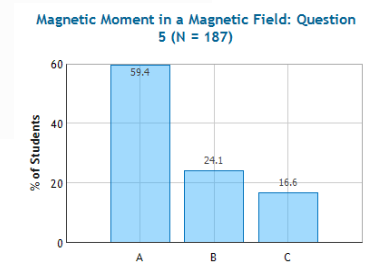


CheckPoint 12

In order to rotate a horizontal magnetic dipole to the three postions shown, which one requires the most work done by the magnetic field?



BY WHOM ?



$$W_{by\_field} = -\Delta U = U_i - U_f$$

$$U = -\vec{\mu} \cdot \vec{B}$$

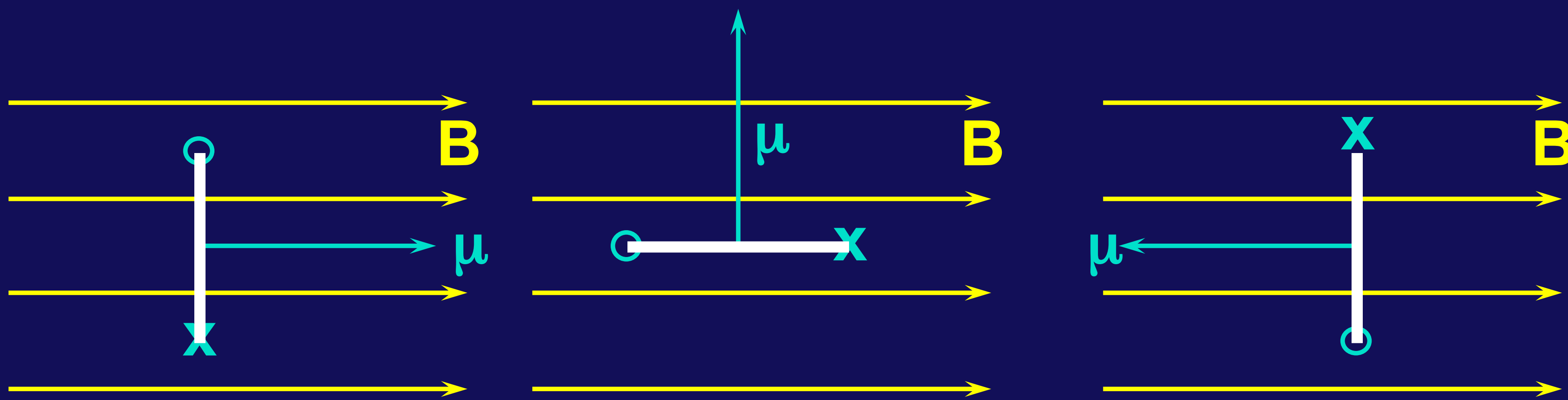
C): $W_{by\_field} = -\mu B - (-\mu B \cos\theta_c) = -\mu B(1 - \cos\theta_c)$

B): $W_{by\_field} = -\mu B - 0 = -\mu B$

A): $W_{by\_field} = -\mu B - (-\mu B \cos\theta_a) = -\mu B(1 + \cos\phi_a)$

(similar) Potential Energy of Dipole

(this is an important slide)



$= 0$	$= \mu B$ ⊗	$\tau = 0$
$\theta = 0$	$\theta = 90$	$\theta = 180$
$U = -\mu B$	$U = 0$	$U = \mu B$
(minimum)		(maximum)