

# Review for Exam II



- **Topics** from ...
  - Electric Potential Energy: “U”
  - Electric Potential Difference: “V” (or  $\Delta V$ )
- ... to ...
  - Kirchoff’s Loops Laws, Power and Internal Resistance
    - » (and everything in between)
- **No RC circuits this time**

# Today

- **Plan**

- *Wild & fast review of main points from this Unit*
- *Some problems from a different older exam*

- **Suggest that you ...(read it later)**

- *Go back and flip through slides (and Clicker questions) to make sure you understand them.*
- *Look over classic HW problems in SP or Webassign*
- *Make sure you can do the types of calculations on the practice exam.*
  - » *Solutions will be posted*

# What did we learn?

## • Physics & Relations

- Electrical Potential Energy:  $U$  (or  $\Delta U$ )
- Electrical Potential:  $V$  (or  $\Delta V$ )
- Energy density in  $E$  field:  $u = \frac{1}{2} \epsilon_0 E^2$

**Joules**

**Joules / Coulomb**

**Joules / volume**

- $V$  from  $E$ :

$$\Delta V_{a \rightarrow b} = - \int_a^b \vec{E} \cdot d\vec{l}$$

**The Difference matters**

- $E$  from  $V$ :

$$\vec{E} = -\vec{\nabla}V$$

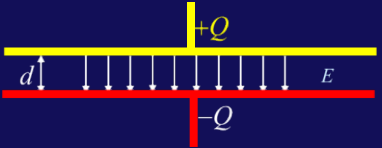
## • Some Consequences

- $E = 0$  inside a conductor (or wires)
- $V = \text{constant}$  in conductor (or wires)
- $E$  field perpendicular to conductor surface
- Equipotential  $V$  values orthogonal to  $E$  field

# What did we learn?

- **Practical Use** (but not new stuff)

- Devices for electronic circuits introduced
- Batteries\*, Resistors, Capacitors, switches, ...

- **Capacitors:**   $E = \frac{\sigma}{\epsilon_0}$   $V = Ed = \frac{Q}{\epsilon_0 A} d$   $\therefore C = \frac{\epsilon_0 A}{d}$

- » Value depends on geometry

- If a dielectric material is between plates,  $C' = \kappa C_0$

- » “in series” same Q on all plates

- » “in parallel” same  $\Delta V$  across elements

- **Resistors: control rate of current:  $I = dq/dt$  in circuits**

- » Value depends on shape and stuff it's made from

- » “Voltage across” or “Voltage Drop” between two ends of a resistor is proportional to the current:  $V = IR$  (it's a difference)

- » “in series” same current goes through both

- » “in parallel” same  $\Delta V$  across elements

\* We often assume that batteries maintain a constant  $\Delta V$  from + to - terminal

**Now, many examples**

# Work to assemble set of charges

$$W = \sum W_i = + \frac{1}{4\pi\epsilon_0} \frac{Q^2}{d}$$

Work to bring in first charge:

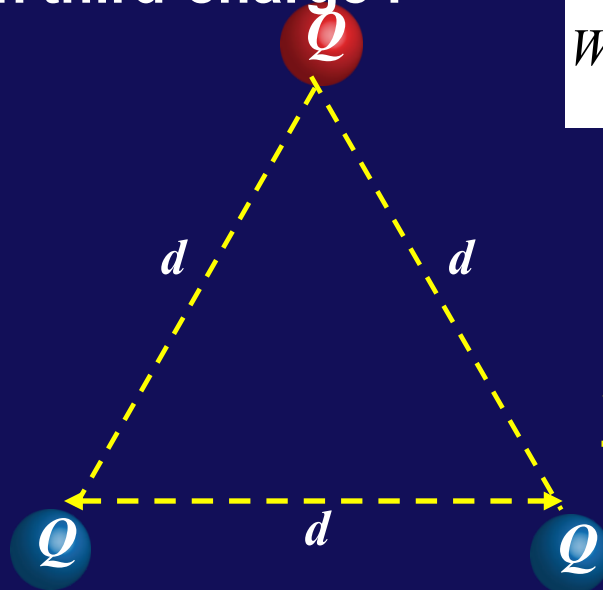
$$W_1 = 0$$

Work to bring in second charge :

$$W_2 = + \frac{1}{4\pi\epsilon_0} \frac{Q^2}{d}$$

Work to bring in third charge :

$$W_3 = + \frac{1}{4\pi\epsilon_0} \frac{Q^2}{d} - \frac{1}{4\pi\epsilon_0} \frac{Q^2}{d} = 0$$



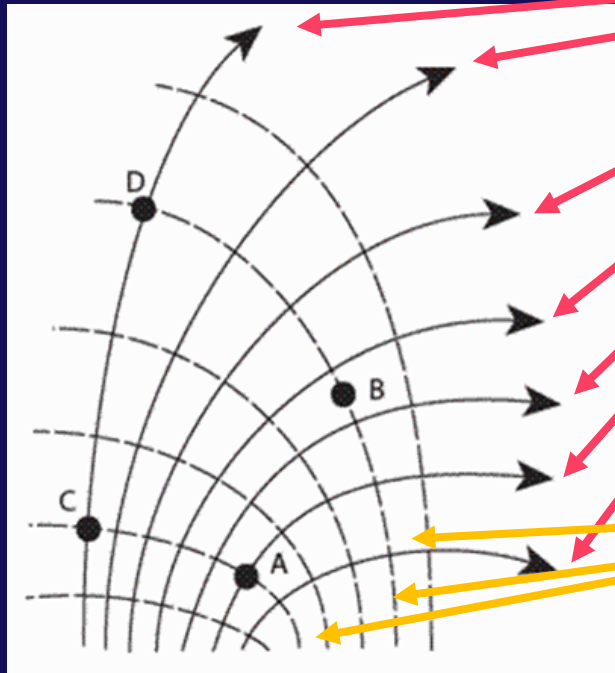
**PE is sum of P.E.s for each pair**

**Sign positive if + is moved toward another + charge**

**Potential energy is negative the Work required to build it**

$$\Delta U = - \frac{1}{4\pi\epsilon_0} \frac{Q^2}{d}$$

# Field Lines and Equipotentials

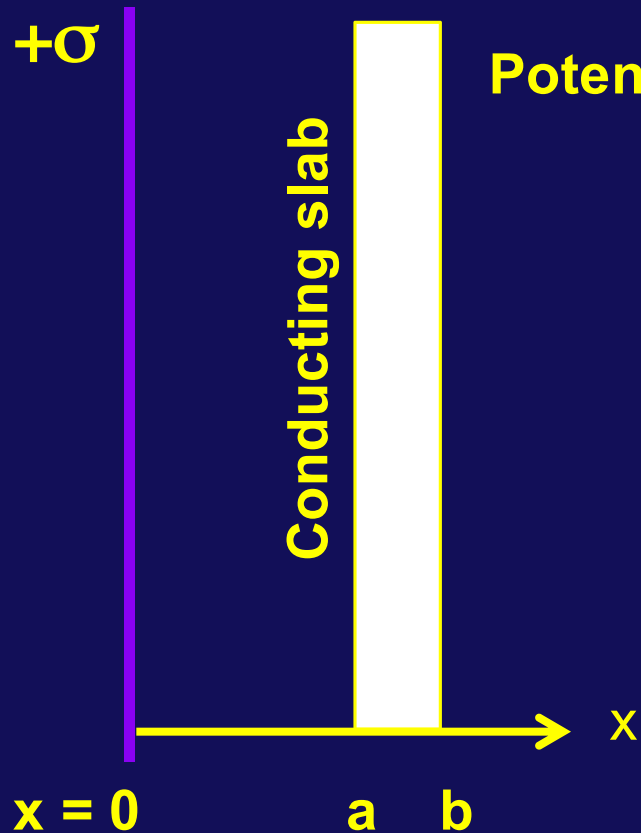


**ELECTRIC FIELD  
LINES!**

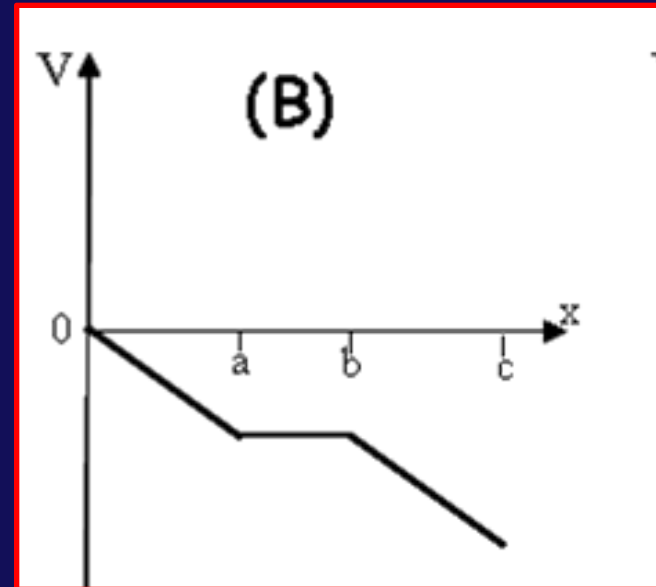
**EQUIPOTENTIALS!**

**Points:**

**Density of E  
Orthogonal E to V  
Work to move along**



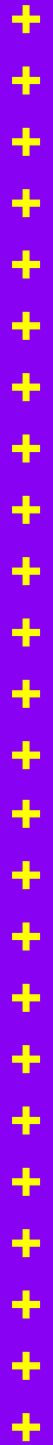
Potential in the positive  $x$  direction



- $E$  is constant between  $x = 0$  and  $x = a$ 
  - $V$  must be a linear function of  $x$ , downward since **sigma is positive**
- $E = 0$  in conductor
  - $V = \text{constant}$
- $E$  has same value to right of conductor
  - $V$  continues downward with same slope

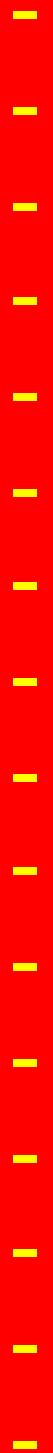


$+\sigma$

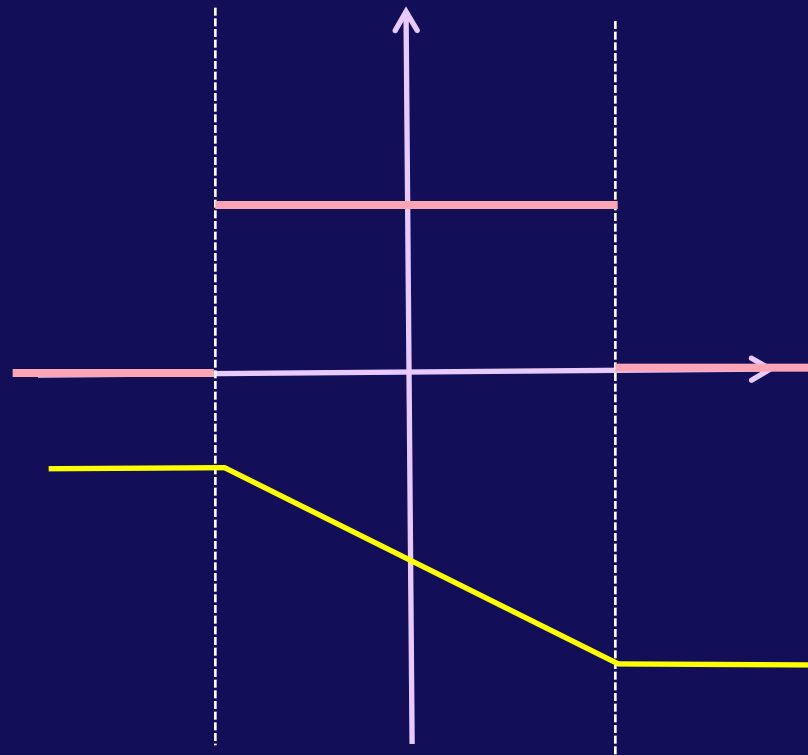


# Toward a capacitor

Two “infinite” oppositely charged parallel plates are located at  $-d$  and  $+d$  on the  $x$  axis. Which graphs best represent the Electric Field and the Potential Difference vs  $x$  ?



$-\sigma$



# E and V with spheres and cylinders

- Determine  $\mathbf{E}(\mathbf{r})$  everywhere from Gauss's Law

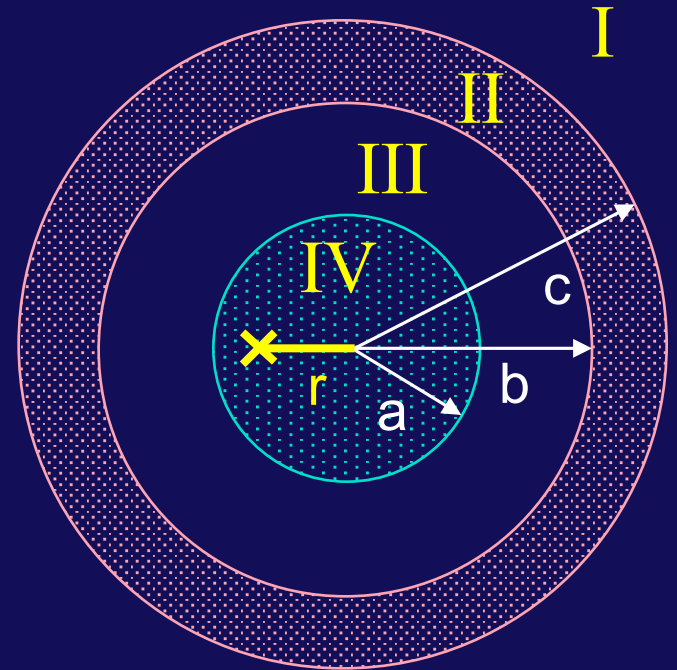
$$\left. \begin{aligned} \vec{E}_I(r) &= \frac{kQ}{r^2} \hat{r} & \vec{E}_{III}(r) &= \frac{kQ}{r^2} \hat{r} \\ \vec{E}_{II}(r) &= 0 & \vec{E}_{IV}(r) &= \frac{kQr}{a^3} \hat{r} \end{aligned} \right\} \text{Depends on problem of course}$$

- Determine  $\Delta V$  for each region by integration

$$V(\mathbf{r}) = V_\infty + \Delta V_{\infty \rightarrow c} + \Delta V_{c \rightarrow b} + \Delta V_{b \rightarrow a} + \Delta V_{a \rightarrow r}$$

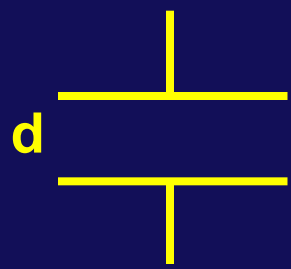
- When plotting, recall

- E can be discontinuous (e.g,  $E = 0$  in the conductors)
- V must be continuous (and have a  $V = 0$  defined somewhere)

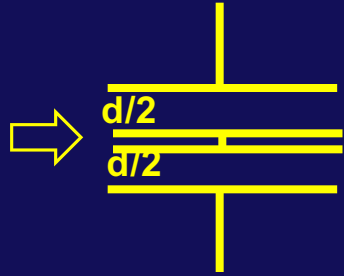


(Let's assume just positive charges here)

# Capacitors in Series and Parallel & Dielectrics

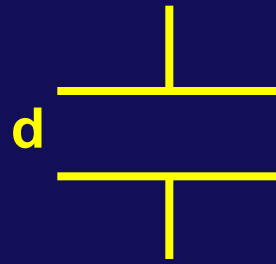


$C_0$

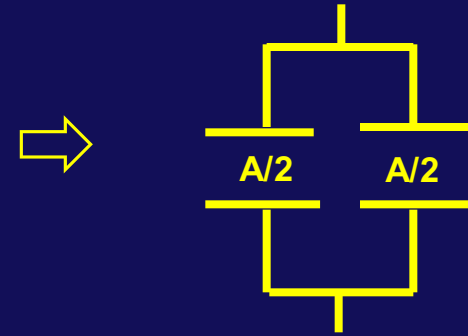


$$C' = \left( \frac{1}{2C_0} + \frac{1}{2C_0} \right)^{-1} = C_0$$

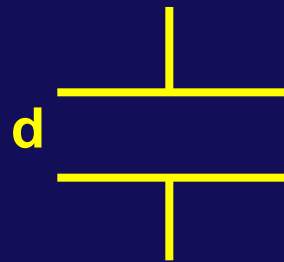
Or, two in series with  $d/2$  separation ☺



$C_0$

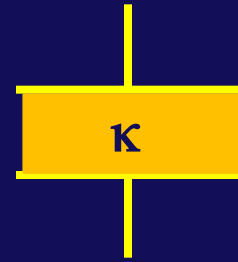


$$C' = C_0/2 + C_0/2 = C_0$$



$C_0$

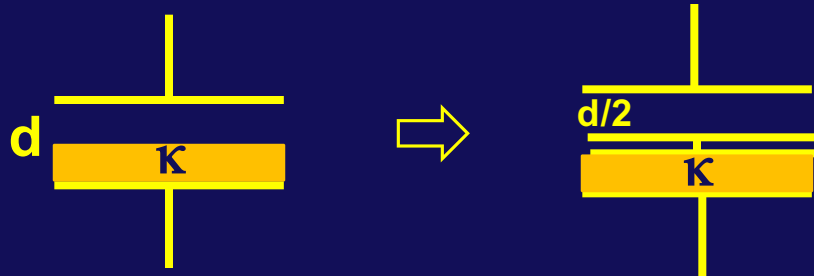
original



$C' = \kappa C_0$

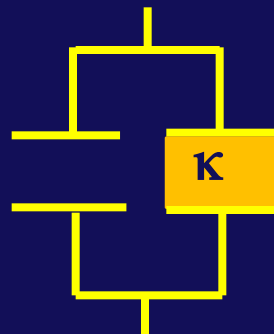
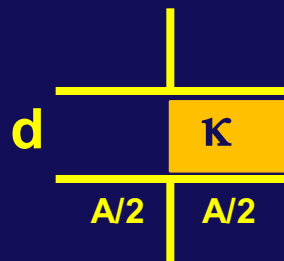
adding dielectric

# Capacitors in Series and Parallel & Dielectrics



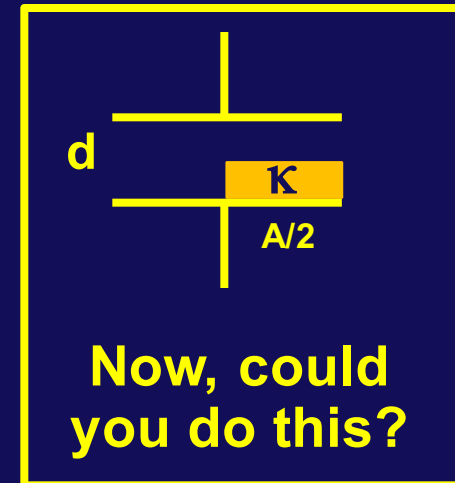
What is this?

Think about this



What is this?

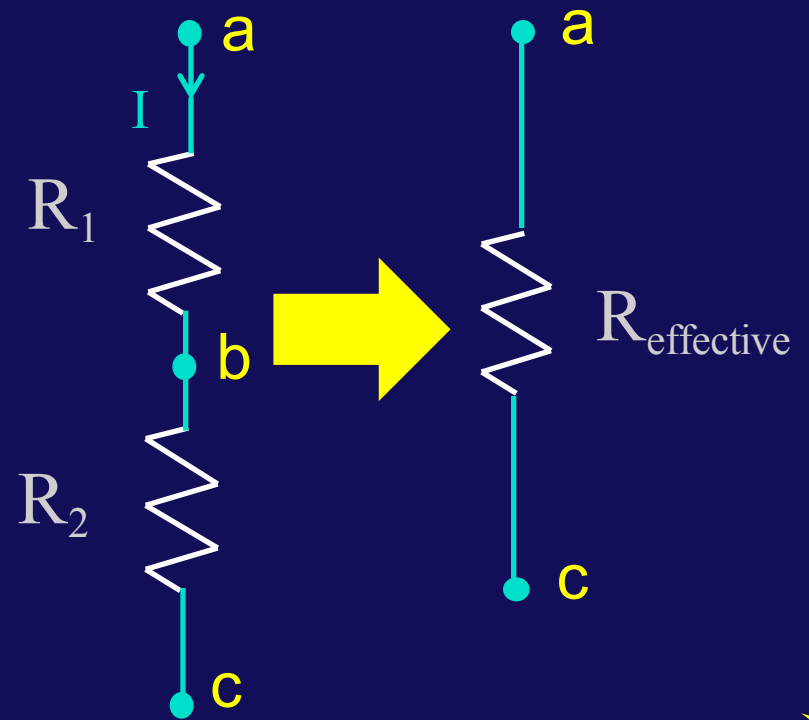
Think about this



# Resistors in Series & Parallel

The current is the same for devices in SERIES,

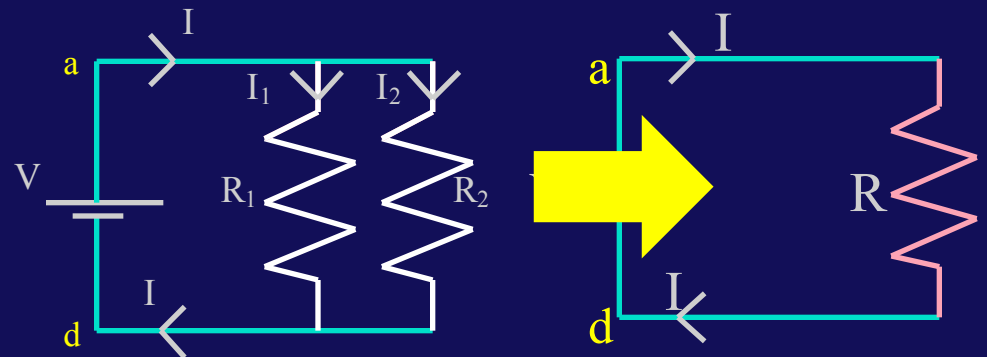
$$R_{\text{effective}} = (R_1 + R_2)$$



The voltage drop is the same for devices in PARALLEL

$\rho$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



# Energy and Power in Devices

Energy is stored in the electric field between plates of a Capacitor

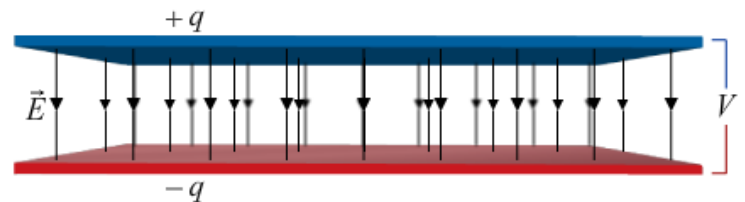
$$U = \frac{1}{2} QV$$

or

$$U = \frac{1}{2} \frac{Q^2}{C}$$

or

$$U = \frac{1}{2} CV^2$$



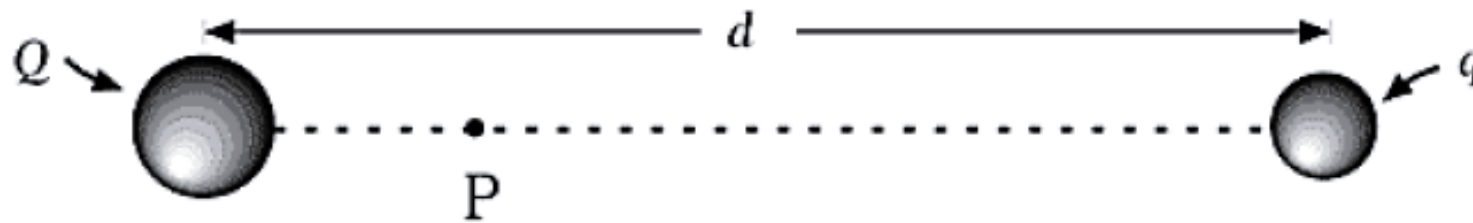
$$P = VI$$

$$P = (IR)I = I^2 R$$

Rate of Energy expended in a resistor with current running through

$$\frac{\text{Joule}}{\text{Coulomb}} \times \frac{\text{Coulomb}}{\text{second}} = \frac{\text{J}}{\text{s}} = \text{Watt}$$

**Now some problems from the past**



1. (4 pts.) Charges  $Q$  and  $q$  ( $Q > q$ ), separated by a distance  $d$ , produce a potential  $V_P = 0$  at point  $P$ . This means that

- A) no force is acting on a test charge placed at point  $P$ .
- B)  $Q$  and  $q$  must have the same sign.
- C) the electric field must be zero at point  $P$ .
- D) the net work in bringing  $Q$  to distance  $d$  from  $q$  is zero.
- E) the net work needed to bring a charge from infinity to point  $P$  is zero.

- Take  $V = 0$  at infinity
- Then,  $\Delta U = q\Delta V = 0 = -W$  to go to point  $P$



2. (4 pts.) A charge of  $5.0 \mu\text{C}$  is located in a uniform electric field of intensity  $3.5 \times 10^5 \text{ N/C}$ . How much work is required to move this charge at constant speed 50 cm along a path making an angle of  $33^\circ$  with respect to the electric field direction?



A) 0.16 J

B) 0.27 J

C) 0.54 J

D) 0.73 J

E) 7.3 mJ

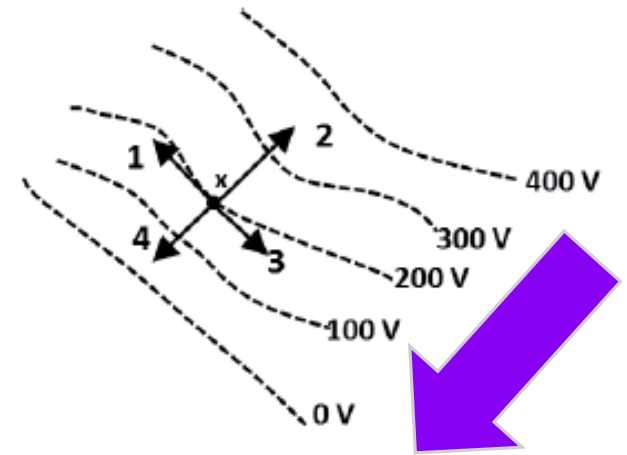
- $W = \int qE \cdot dl$

- $\rightarrow W = q \int E \cdot dl = E \cos \Theta \int dl$

- $\rightarrow W = qLE \cos \Theta$

4. (3 pts.) The vector that best represents the direction of the electric field at point  $x$  on the 200 V equipotential line is

- A) 1
- B) 2
- C) 3
- D) 4**
- E) None of these is correct.

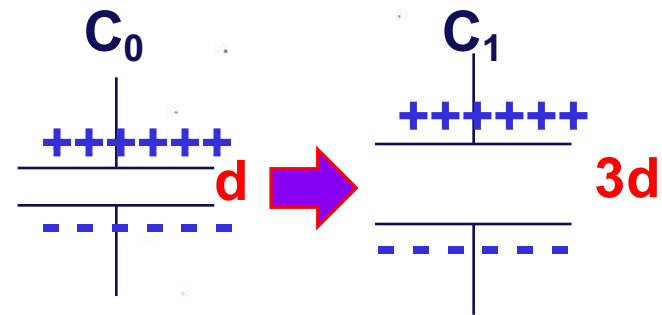


- $\vec{E} = -\vec{\nabla}V$
- **E then points toward DECREASING potential**

Two plates of a parallel plate capacitor are separated by distance  $d$ , and each has area  $A$ . The capacitor is charged to a potential difference  $V_0$  and then disconnected from the voltage source. The plates are then pulled apart until the separation becomes  $3d$ .

4. What is the **new potential difference** between the plates of the capacitor? (Express your answer in terms of the original potential difference,  $V_0$ .)

- A.  $V_0$
- B.  $3V_0$
- C.  $9V_0$
- D.  $0.33V_0$
- E.  $0.11V_0$



1.  $C_0 = A\epsilon/d$
2.  $C_1 = A\epsilon/3d$
  
3.  $V_0 = Q/C_0$
4.  $V_1 = Q/C_1$       (same Q)
  
5.  $V_1 = 3V_0$

5. How much work was required in order to change the plate separation from  $d$  to  $3d$ ?  
**Looking for the *difference* in stored energy**

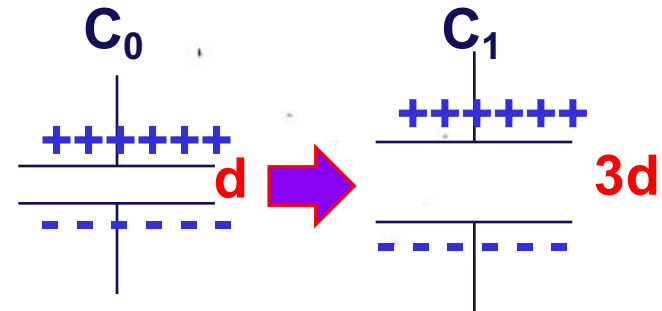
A.  $\epsilon_0 A V_0 / d$

B.  $\epsilon_0 A V_0 / 2d$

C.  $\epsilon_0 A V_0^2 / d$

D.  $\epsilon_0 A^2 V_0 / d$

E.  $\epsilon_0 A^2 V_0^2 / 2d$



1.  $U_0 = \frac{1}{2} Q^2/C_0$  is original stored energy

2.  $U_1 = \frac{1}{2} Q^2/C_1$  is final stored energy

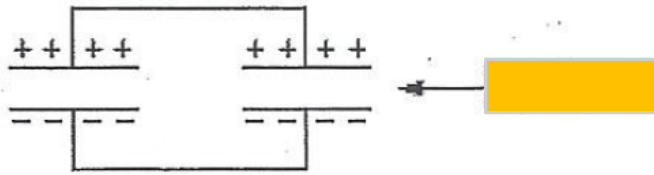
3.  $C_1 = C_0/3$  (see previous problem)

4.  $U_1 = 3 U_0$

5.  $U_1 - U_0$  is the work done (conservative force)

6.  $U_1 - U_0 = 2U_0 = 2 (\frac{1}{2} CV^2) = \epsilon AV^2/d$

6. Two capacitors each have two conducting plates of surface area  $A$  and an air gap width  $d$ . They are connected in parallel, as shown in the figure, and each has a charge  $Q$  on the positively charged plate. A slab that has a width  $d$ , area  $A$ , and a dielectric constant  $\kappa$  is then inserted between the plates of *one* of the capacitors. Find the new charge  $Q'$  on the positively charged plate of the capacitor with the dielectric. Assume that electrostatic equilibrium has been reestablished.



A.  $\kappa Q / (1 + \kappa)$

B.  $2\kappa Q / (1 + \kappa)$

C.  $\kappa Q / (1 + \kappa)^2$

D.  $\kappa Q / (1 + 2\kappa)$

E.  $2\kappa Q / (1 + 2\kappa)$

**After insertion of dielectric,  $Q$  redistributed, but the total  $Q$  is the same**

1.  $Q_L + Q_R = 2Q$  (they both had  $Q$ )

2.  $C_R = \kappa C_L$  (dielectric)

3.  $V_L = V_R$  (in parallel)

4.  $Q_L = C_L V$  &

5.  $Q_R = C_R V = \kappa C_L V = \kappa Q_L$

6.  $Q_R = \kappa Q_L = \kappa(2Q - Q_R)$

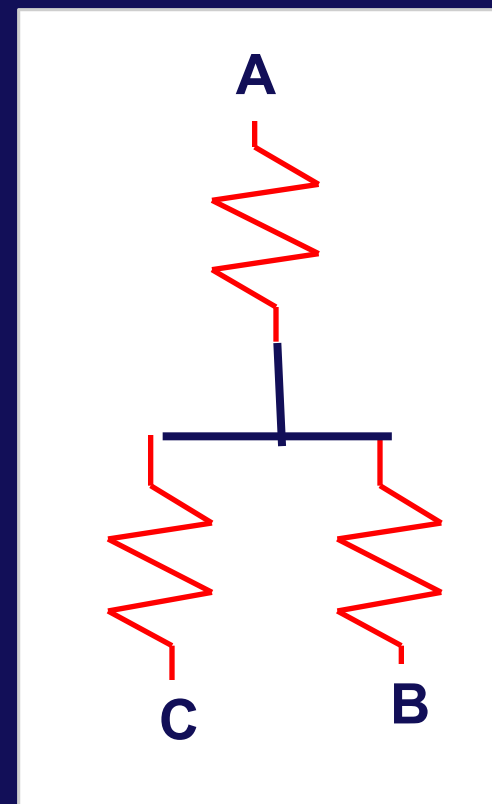
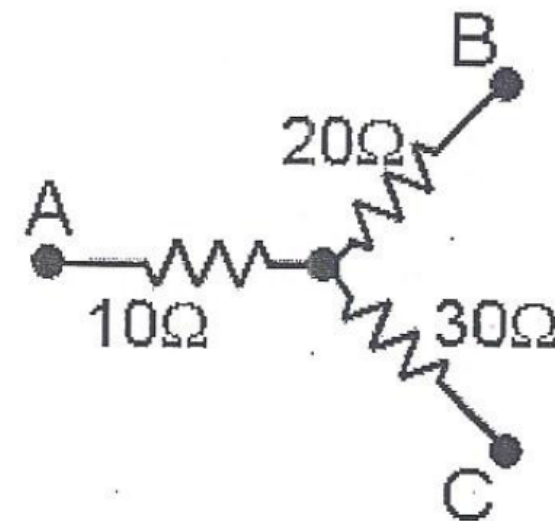
7.  $Q_R(1 + \kappa) = \kappa 2Q$

8.  $Q_R = 2\kappa Q / (1 + \kappa)$

8. The voltages at point A, B, and C, are 10V, 6V, and 5V, respectively. What is the current through the 10Ω resistor?

- (a) 0.2 Amps
- (b) 0.4
- (c) 0.6
- (d) 0.8
- (e) 1.2

**First thought: What is this?  
Redraw it perhaps ?**



8. The voltages at point A, B, and C, are 10V, 6V, and 5V, respectively. What is the current through the 10Ω resistor?

- (a) 0.2 Amps
- (b) 0.4
- (c) 0.6
- (d) 0.8
- (e) 1.2

**Now, do Kirchoff's Loops and Junction**

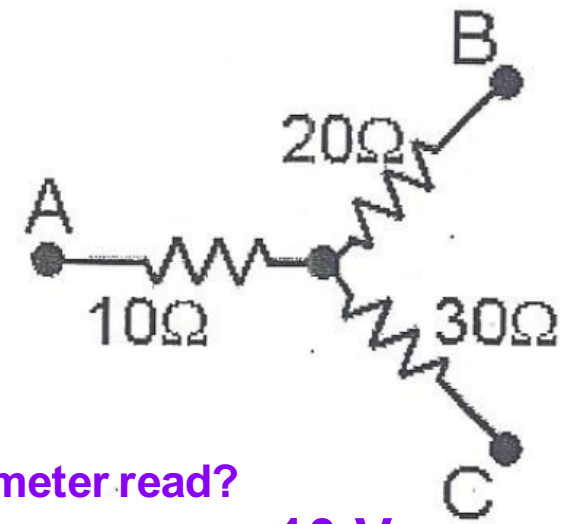
1.  $I_1(10) + I_2(30) - 5 = 0$
2.  $I_1(10) + I_3(20) - 4 = 0$
3.  $I_1 = I_2 + I_3$

4. Now solve for currents

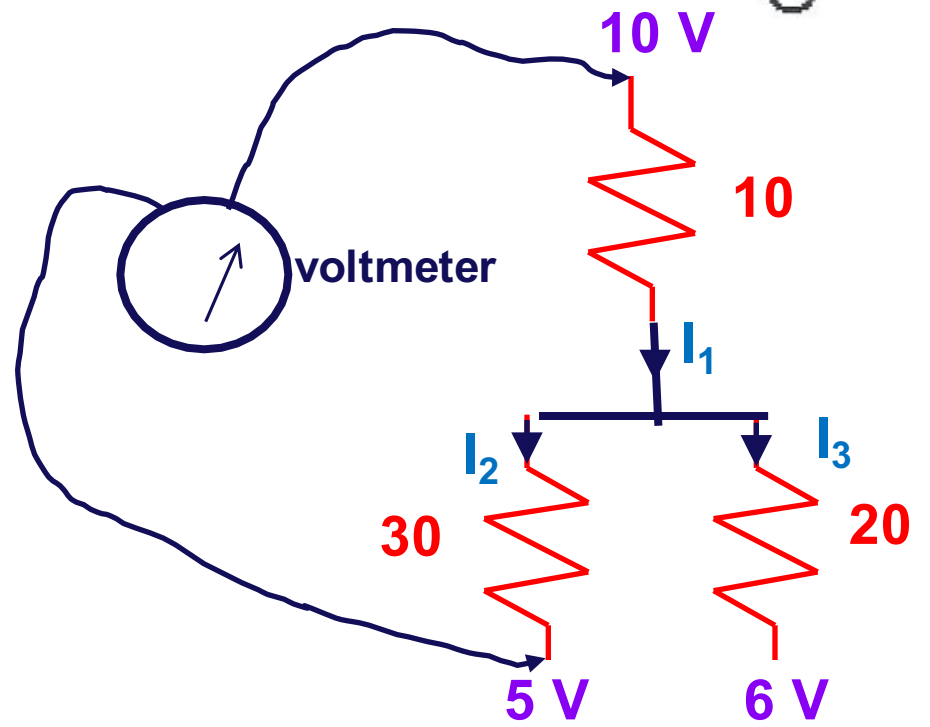
5. Soln:  $I_1 = 0.2 \text{ A}$

6. Make sense?

1. V at node is  $10 - 2 = 8 \text{ V}$
2.  $V_{30} = 3 \text{ V}; I_2 = 0.1 \text{ A}$
3.  $V_{20} = 2 \text{ V}; I_3 = 0.1 \text{ A}$
4. The currents add up
5. The voltages add up



What will voltmeter read?



**Good luck**