Our Current Thinking

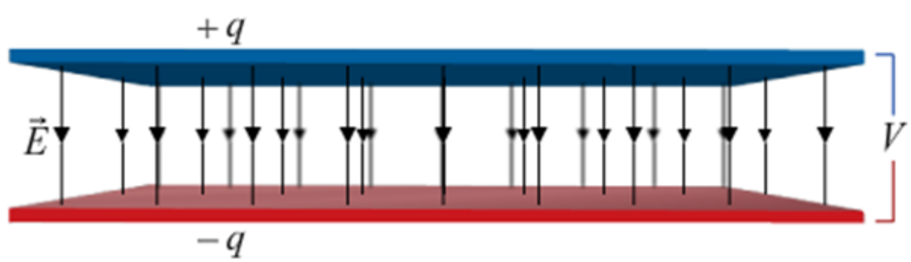
C ≡ Q/V   CV = Q   V = Q/C

Energy Stored in Capacitors

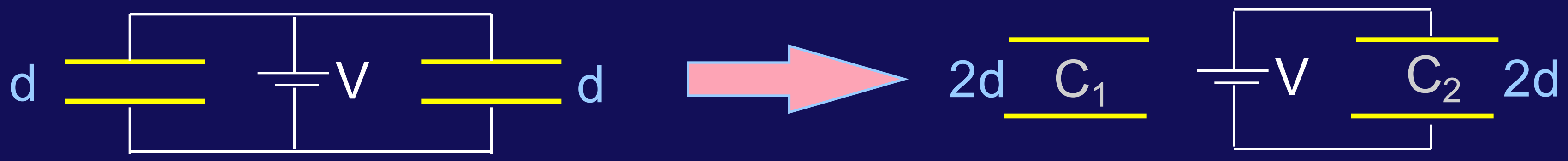
$$U = \frac{1}{2}QV \quad \text{or} \quad U = \frac{1}{2}\frac{Q^2}{C} \quad \text{or} \quad U = \frac{1}{2}CV^2$$

Energy Density

$$u = \frac{1}{2}\varepsilon_o E^2$$



Great Question!



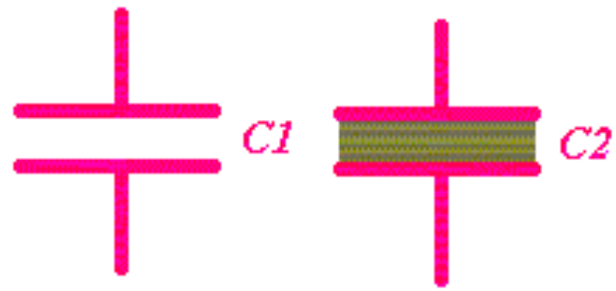
Two capacitors, one with constant charge, one with constant voltage (hooked to a battery). Their separations are doubled; how does potential energy change

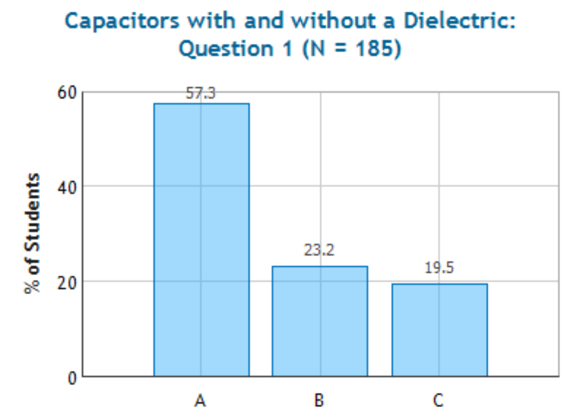
Can I solve this problem by calculating the forces on the plates, and then figuring out how much work I do by moving the plates?

Short answer: Only for the constant charge situation! For the constant voltage situation, your work is both changing the energy in the capacitor and also charging the battery, so not so easy.

CheckPoint 8

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C2 has been charged and disconnected, it is filled with a dielectric.





Compare the voltages of the two capacitors.

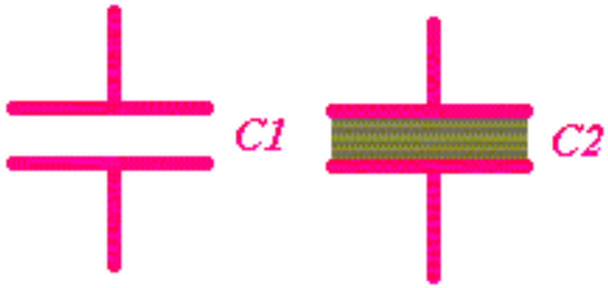
○ $V_1 > V_2$ ○ $V_1 = V_2$ ○ $V_1 < V_2$

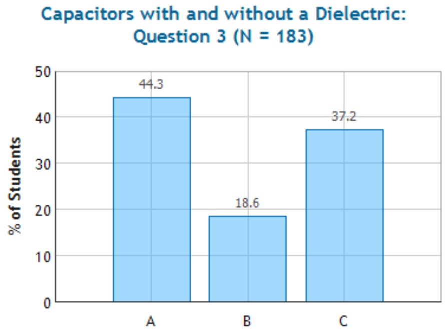
The dielectric increases C; $C_2 > C_1$
Q remains the same
$V_2 = Q/C_2$ so V decreases

Alternately, recall E reduced
→ $E = E_0/\kappa$

CheckPoint 10

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C2 has been charged and disconnected, it is filled with a dielectric.





Compare the potential energy stored by the two capacitors.

A) $U_1 > U_2$ B) $U_1 = U_2$ C) $U_1 < U_2$

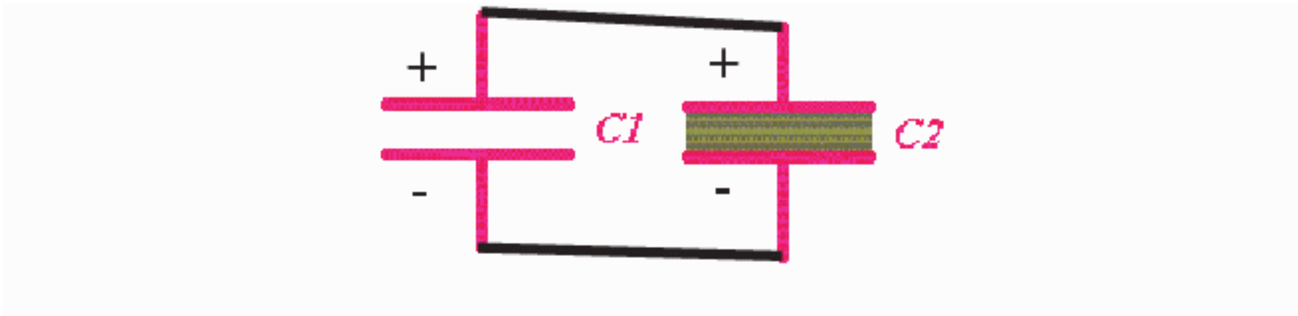
Just learned $V_1 > V_2$ for same Q, so …

Recall $U = \frac{1}{2}QV$ or $U = \frac{1}{2}\frac{Q^2}{C}$ or $U = \frac{1}{2}CV^2$

Also, since Q unchanged, larger C implies lower U

CheckPoint 12

The two capacitors are now connected to each other by wires as shown. How will the charge redistribute itself, if at all?



A. The charges will flow so that the charge on C1 will become equal to the charge on C2.
B. The charges will flow so that the energy stored in C1 will become equal to the energy stored in C2
C. The charges will flow so that the potential difference across C1 will become the same as the potential difference across C2.
D. No charges will flow. The charge on the capacitors will remain what it was before they were connected.

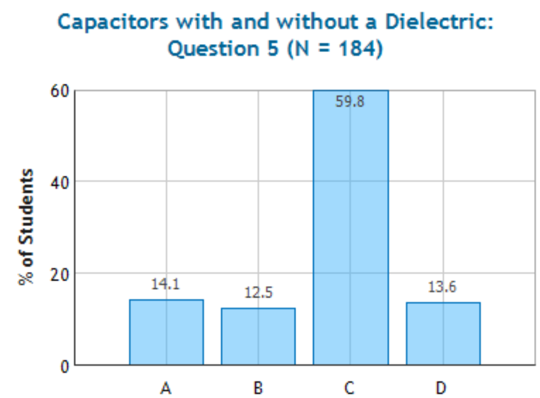
V must be the same !

Q:

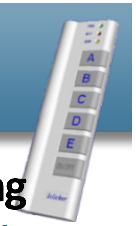
$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad \rightarrow \quad Q_1 = \frac{C_1}{C_2} Q_2$$

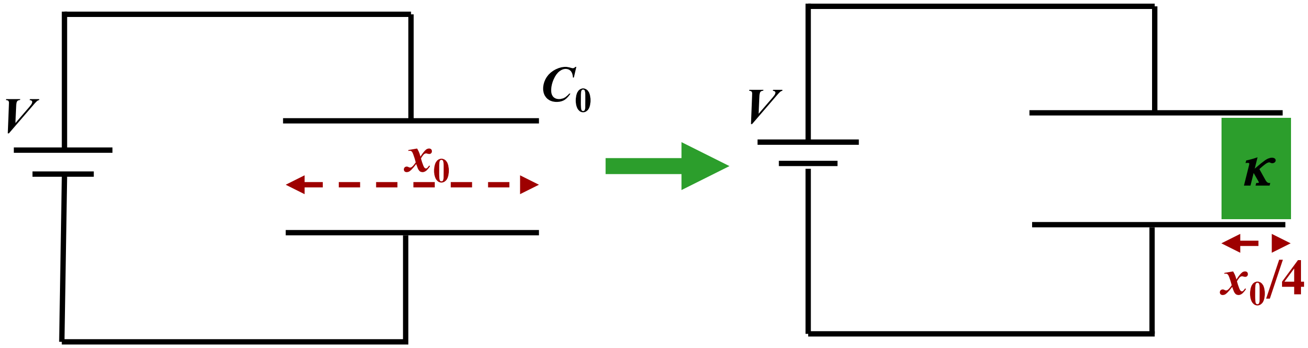
U:

$$U_1 = \tfrac{1}{2} C_1 V^2$$
$$U_2 = \tfrac{1}{2} C_2 V^2 \quad \rightarrow \quad U_1 = \frac{C_1}{C_2} U_2$$



Calculation





An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

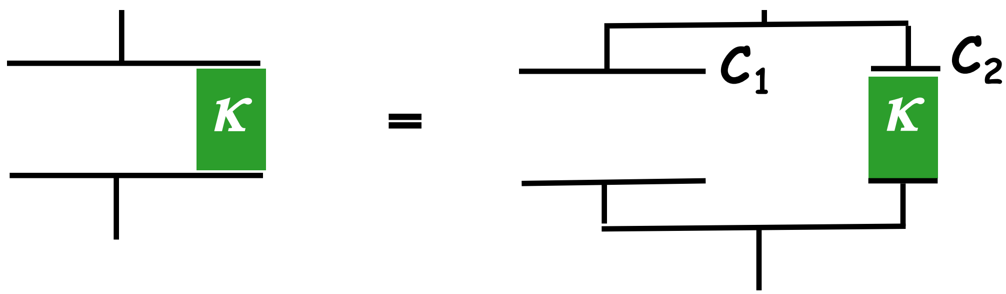
A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Now, Strategic Analysis:

- Calculate new capacitance C
- Apply definition of capacitance to determine Q

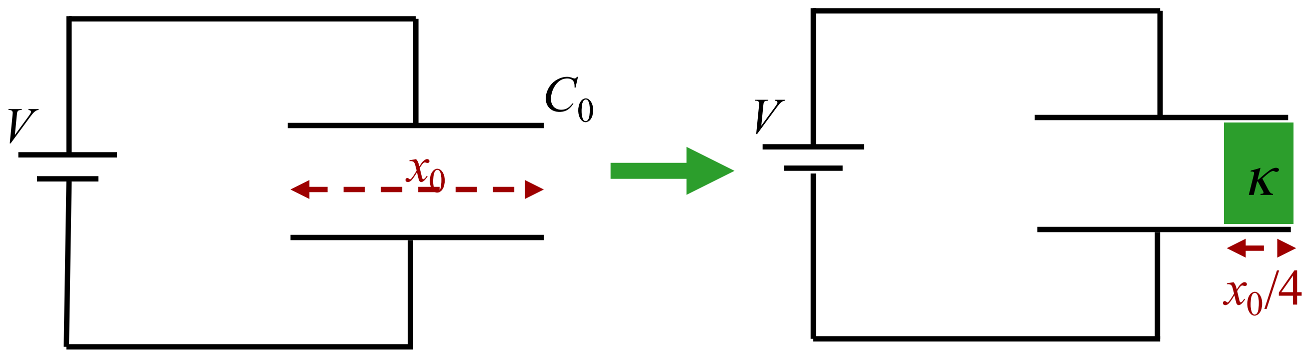
Consider C to be two capacitances, C_1 and C_2, in parallel



Parallel plate capacitor: $C = \varepsilon_0 A/d$

$$A = {}^3/_4 A_0, \quad d = d_0 \;\rightarrow\; C_1 = {}^3/_4(\varepsilon_0 A_0/d_0) \;\rightarrow\; \boxed{C_1 = {}^3/_4 C_0}$$

Clicker and Typical Calculation



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A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

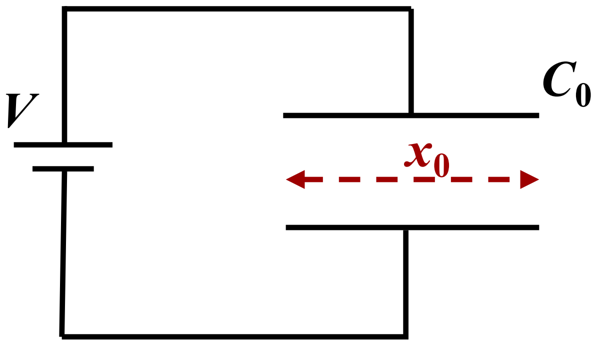
First a Clicker: What changes when the dielectric added?

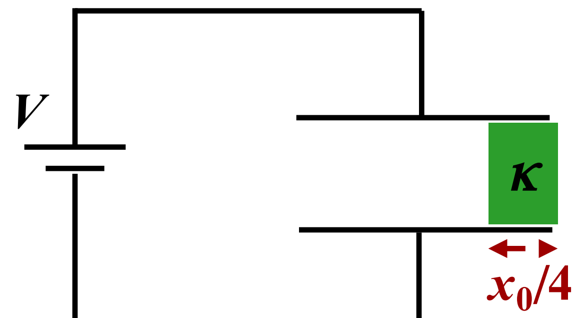
A) Only C B) only Q C) only V D) C and Q E) V and Q

Adding dielectric changes the physical capacitor → **C changes**

V does not change and C changes → **Q changes**

Calculation

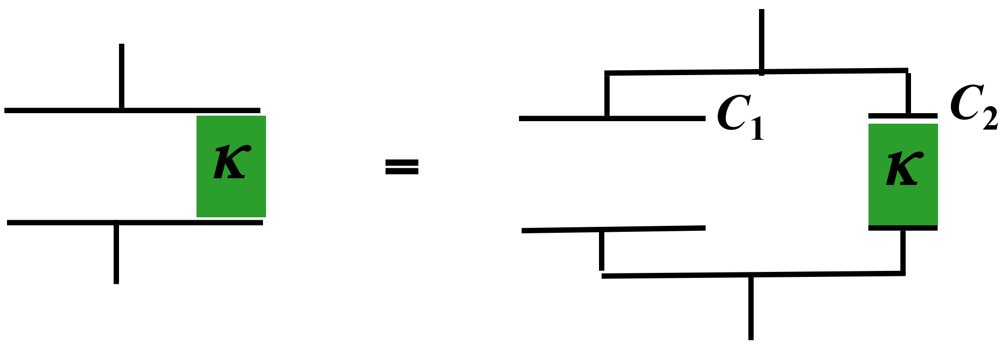




An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?



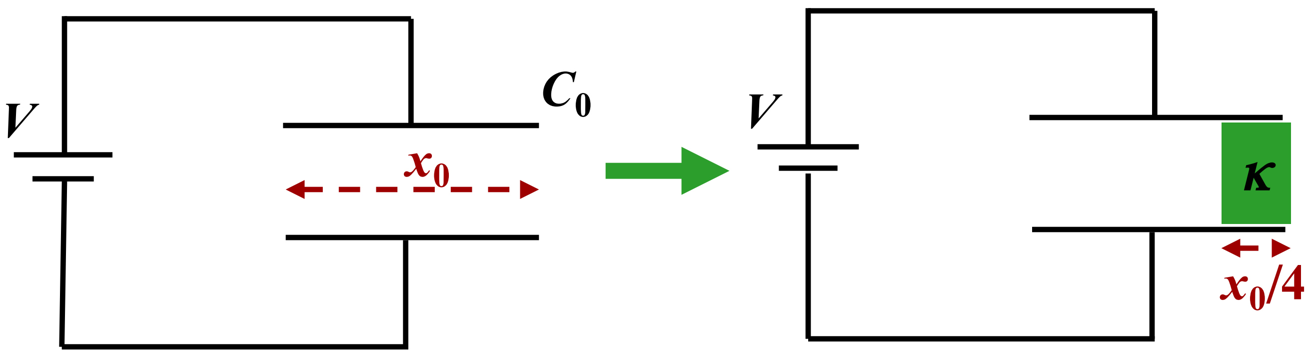
$$C_1 = {}^3/_4 C_0$$

Parallel plate capacitor filled with dielectric: $C = \kappa\varepsilon_0 A/d$

$$A = {}^1/_4 A_0$$
$$d = d_0$$

$$\rightarrow \quad C = \tfrac{1}{4}(\kappa\varepsilon_0 A_0/d_0) \quad \rightarrow \quad C_2 = {}^1/_4\, \kappa C_0$$

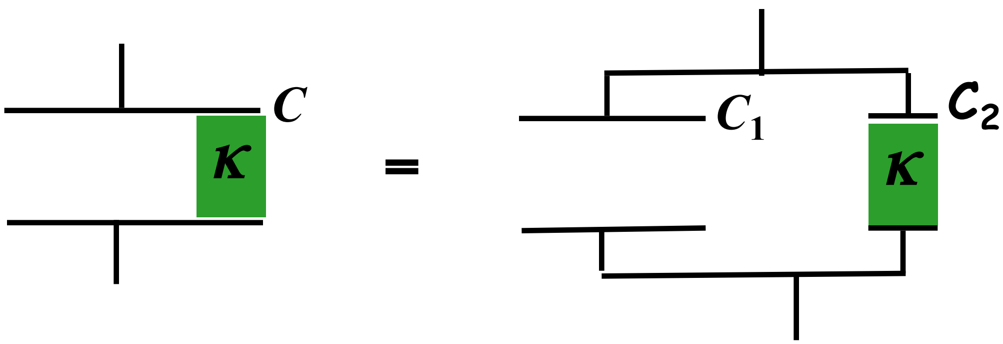
Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

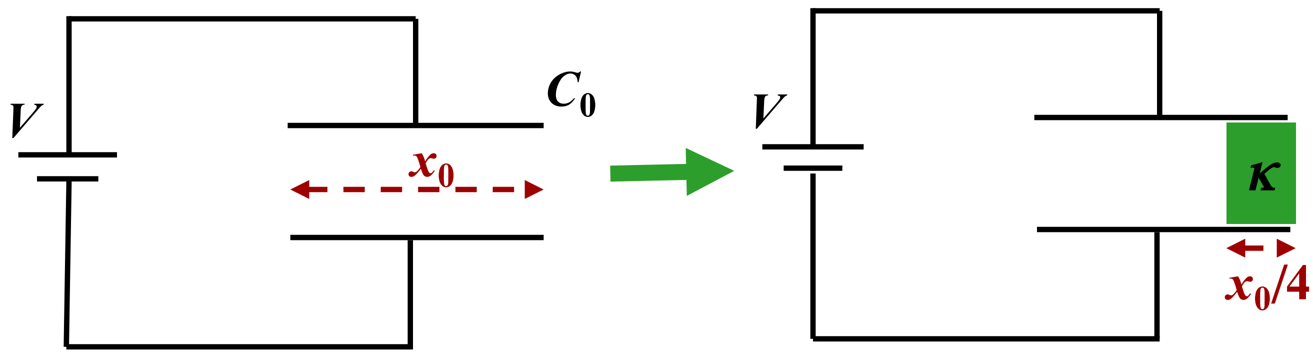


$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa\, C_0$

C = parallel combination of C_1 and C_2: $\quad C = C_1 + C_2$

$$\rightarrow \quad C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$$

Calculation



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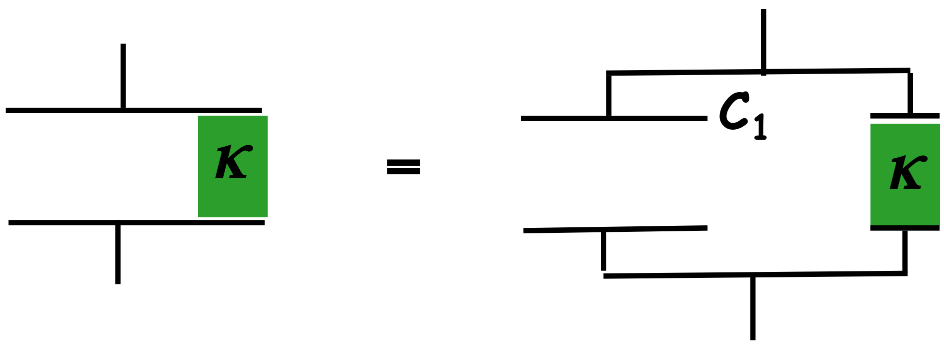


$C_1 = {}^3/_4 C_0$ $\quad$ $C_2 = {}^1/_4\, \kappa\, C_0$

$\rightarrow$ $C = C_0\,({}^3/_4 + {}^1/_4\, \kappa)$

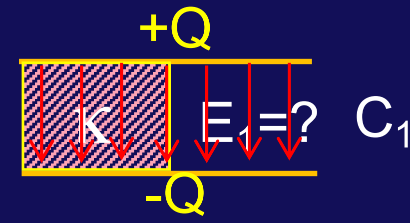
What is Q?

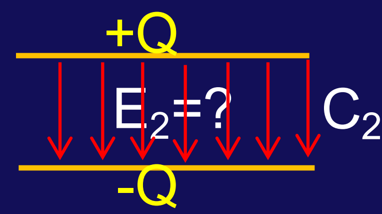
$$C \equiv \frac{Q}{V} \rightarrow Q = VC$$

$$Q_f = VC_0\left(\frac{3}{4} + \frac{1}{4}\kappa\right)$$

Clicker Redux

- Two parallel plate capacitors are identical except C_1 has half of the space between the plates filled with a material of dielectric constant κ.
  - Both capacitors have charge Q
  - Compare E_1, the electric field in the air of C_1, and E_2, the electric field in the air of C_2





(a) $E_1 < E_2$ (b) $E_1 = E_2$ (c) $E_1 > E_2$

- The key here is to realize that the electric field in the air in C_1 must be equal to the electric field in the dielectric in C_1!!
  - The top plane is a conductor → equipotential surface.
  - The bottom plane is a conductor → equipotential surface.
    - V is proportional to E d
  - For this to happen, the charge density on each plane must be non-uniform to create equal electric fields!!
- Since $C_1 > C_2$, for the same charge, $V_1 < V_2$.
- Consequently, $E_1 < E_2$.

Devices

- Capacitors:

Purpose: store charge (energy).

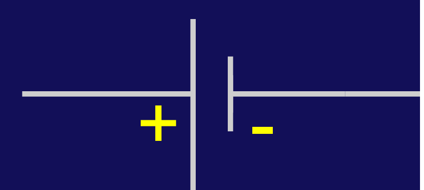
» Several geometries calculated to determine C

» Modified to account for dielectrics: $C = \kappa C_0$

» Effective capacitance of series or parallel combinations

- Batteries (Voltage sources, seats of emf):

Purpose: provide a constant potential difference between 2 points.



OR

» Cannot calculate from first principles

» electrical « chemical energy conversion.

» Non-ideal batteries will be dealt with in terms of an "internal resistance".

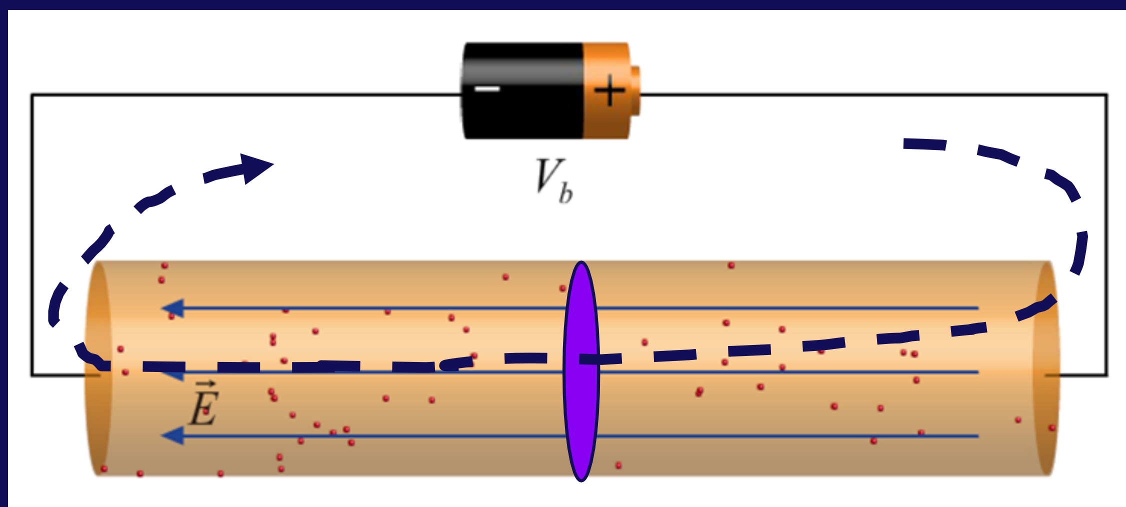
Devices

- Resistors: 

Purpose: limit current in a circuit.

Note: $I = \frac{dq}{dt}$ UNIT: Ampere = A = C/s

- Resistance will depend on geometry and a material's "resistivity"

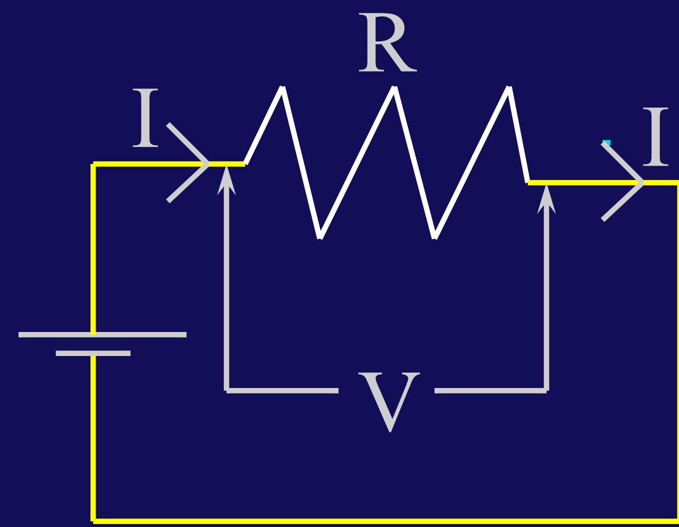


Current Density $J = I / A$

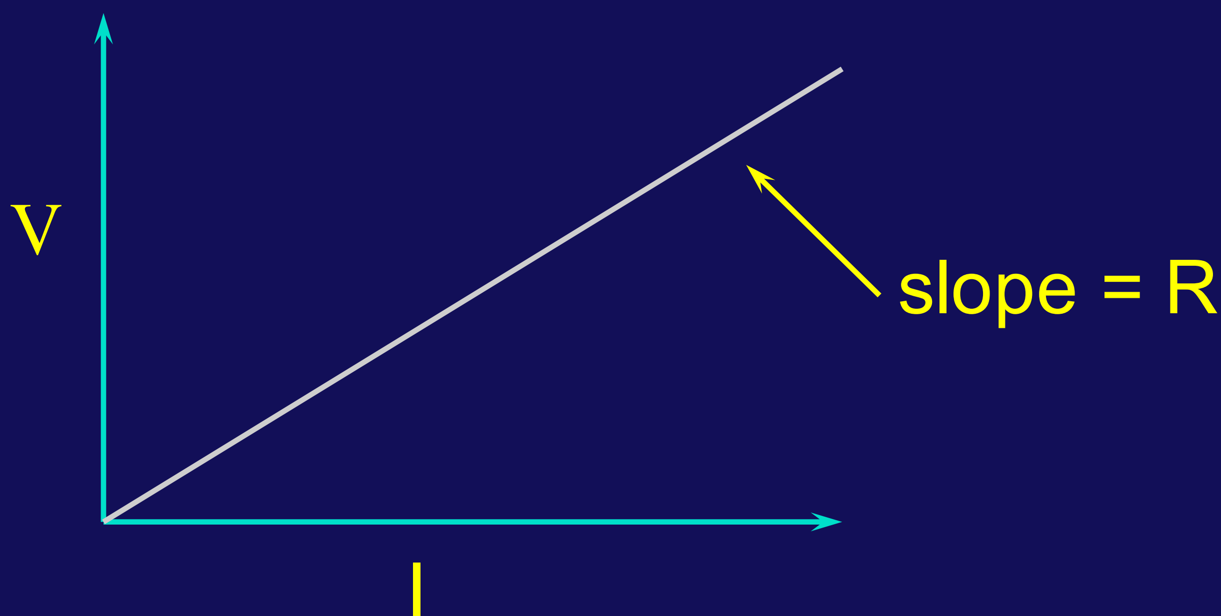
Note: Our CONVENTION for current flow is + to – as dashed line shows as shown nicely in the SP animation, the electrons "flow" opposite

Ohm's Law

- **Demo:**
  - Vary applied voltage V.
  - Measure current I
  - Does ratio (V/I) remain constant?



$$R \equiv \frac{V}{I}$$



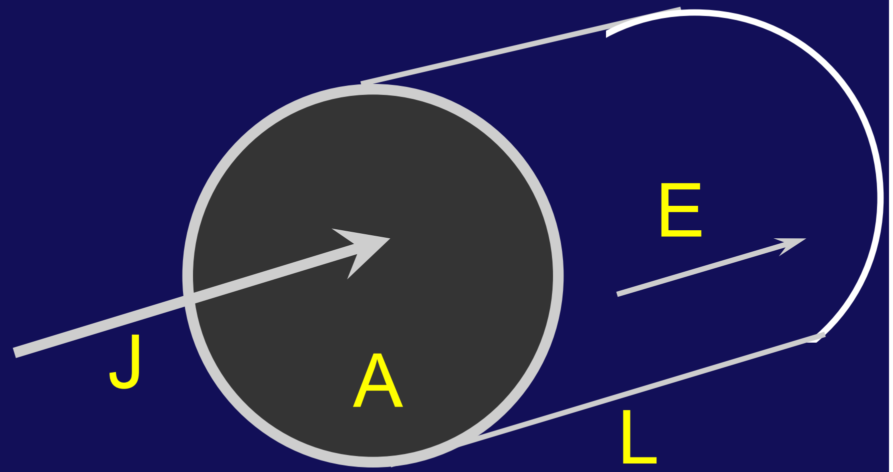
An empirical finding

Resistivity

Property of bulk matter related to resistance: resistivity ρ

$$\rho \equiv \frac{E}{J}$$

E = electric field
J = current density



For uniform case: $J = \frac{I}{A}$ & $V = EL$

Þ $V = EL = \rho JL = \rho \frac{I}{A} L = I\left(\frac{\rho L}{A}\right)$ Þ $R = \rho \frac{L}{A}$

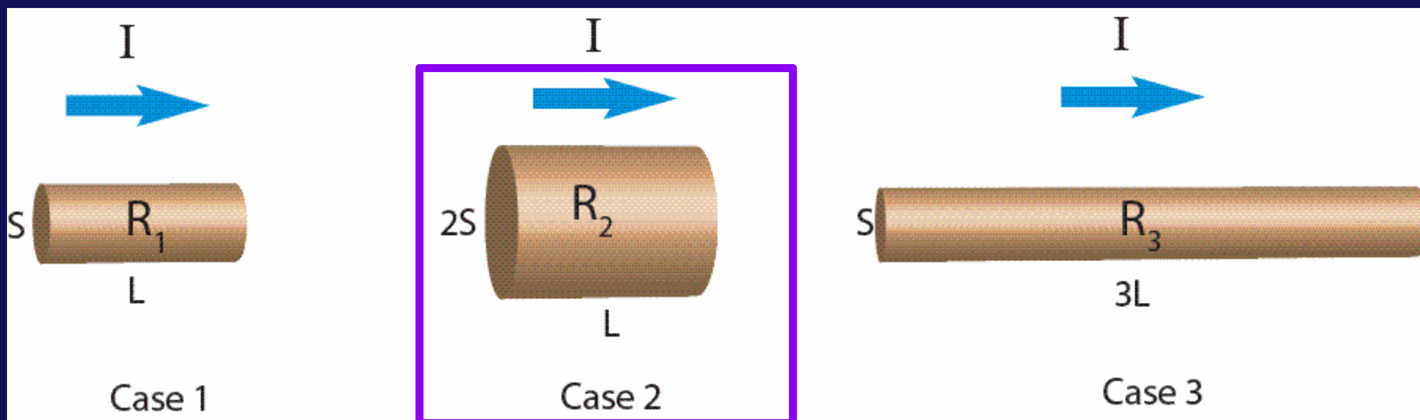
The resistance is a property that belongs only to the device

eg, for a copper wire, ρ ~ 10^{-8} Ω-m, 1 mm radius, 1 m long, → R » 0.01Ω

checkpoint

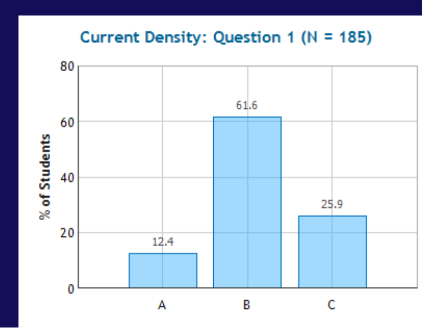
The SAME amount of current I passes through three different resistors.

- R_2 has twice the cross-sectional area and the same length as R_1,
- R_3 is three times as long as R_1 but has same cross-sectional area as R_1.



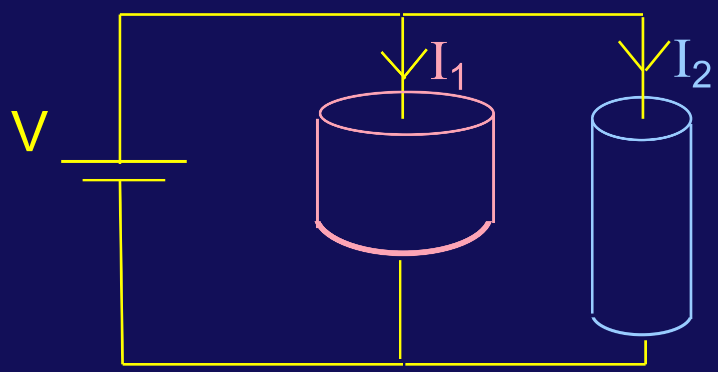
In which case is the CURRENT DENSITY through the resistor the smallest?

$$J = \frac{I}{A}$$



Clicker

- Two cylindrical resistors, R_1 and R_2, are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1.
  - These resistors are then connected to a battery V as shown:



  - What is the relation between I_1, the current flowing in R_1, and I_2, the current flowing in R_2?

(a) $I_1 = I_2$ (b) $I_1 = 2I_2$ (c) $I_1 = 4I_2$ (d) $I_1 = 8I_2$

The resistivity ρ is the same
Resistances are:

$$R_2 = \rho\frac{L_2}{A_2} = \rho\frac{2L_1}{(A_1/4)} = 8\rho\frac{L_1}{A_1} = 8R_1$$

The resistors have the same voltage across them; therefore

$$I_2 = \frac{V}{R_2} = \frac{V}{8R_1} = \frac{1}{8}I_1$$

(brief) Electric Power

"The resistor network problem was tough! Also power, where did it come from? It felt like it needed to be covered more in this prelecture."

- **Suppose a charge *dq* moves through potential difference *V*.**
  - Its Potential Energy changes by $dU = dq\, V$
  - The Rate of Energy Change (gain or loss) is:

$$P \equiv \frac{dU}{dt} = \frac{dqV}{dt} = IV$$

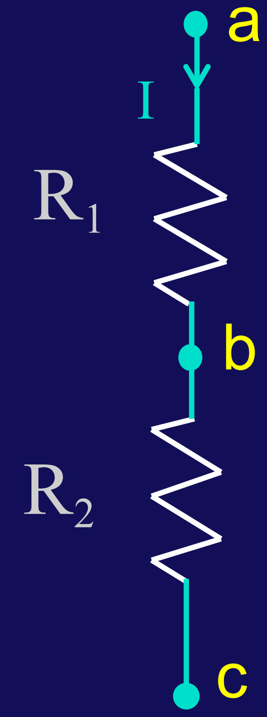
  - Applies to any electrical device that can deliver, store or use electrical energy.
  - Units = J / s Watts (W)
  - Applies at any instant of time, but also can be averaged over a time interval, "the average power"

CLICKER

How is it that a Constant *E*- Field Produces Constant Velocity of Moving Charges?

A. Constant Force Usually Means Constant Velocity.
B. Newton's Laws Do Not Apply to Charged Particles.
C. Electrons Accelerate Randomly, So Average Acceleration = 0
D. "Drag" Force Exists, So the Net Force on Each Charge = 0
E. None of the Above.

Resistors in Series



The Voltage “drops”:

$$V_a - V_b = IR_1 \qquad V_b - V_c = IR_2$$

$$V_a - V_c = I(R_1 + R_2)$$

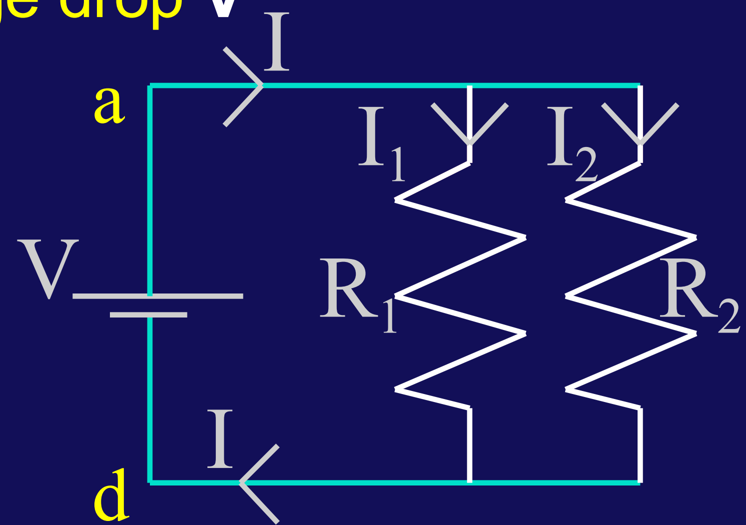
Whenever devices are in SERIES, the current is the same through both !

This reduces the circuit to:

Hence: $$R_{effective} = (R_1 + R_2)$$

Resistors in Parallel

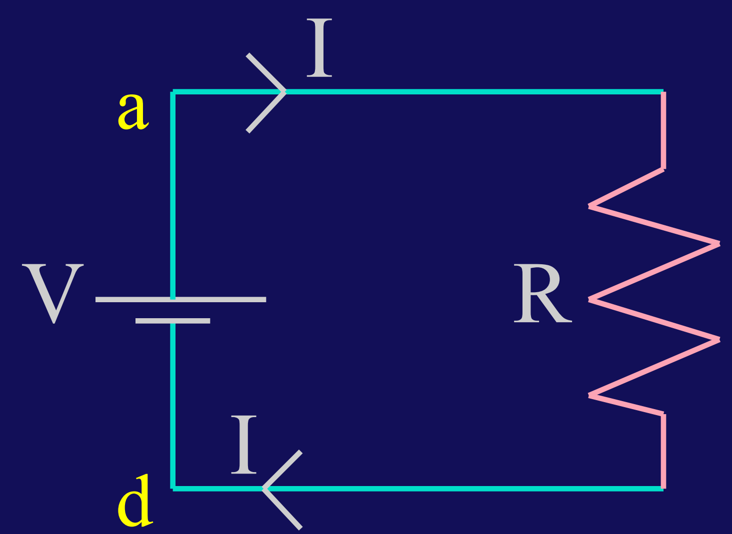
- Devices in parallel have the same voltage drop **V**
- But current through R_1 is <u>not</u> **I**
  - Call it I_1.
  - Similarly, current through R_2 is I_2.



- Current is conserved at junction!

$\rightarrow I = I_1 + I_2$

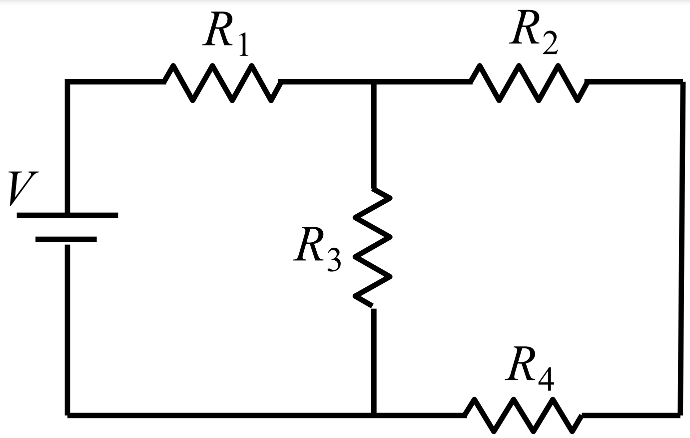
Þ $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$



Þ $$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Analog: Water Flow in Pipes
(Current is like gallons/second of flow)
(Voltage is like pressure)

Calculation with Clickers



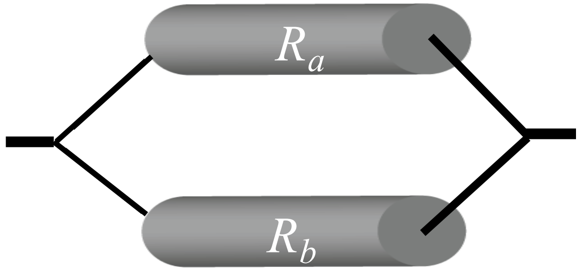
In the circuit shown: $V = 18V$,
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances: R_1 and R_2 are connected:

A) in series B) in parallel C) neither in series nor in parallel

Parallel Combination

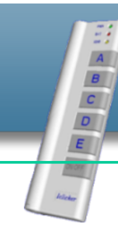


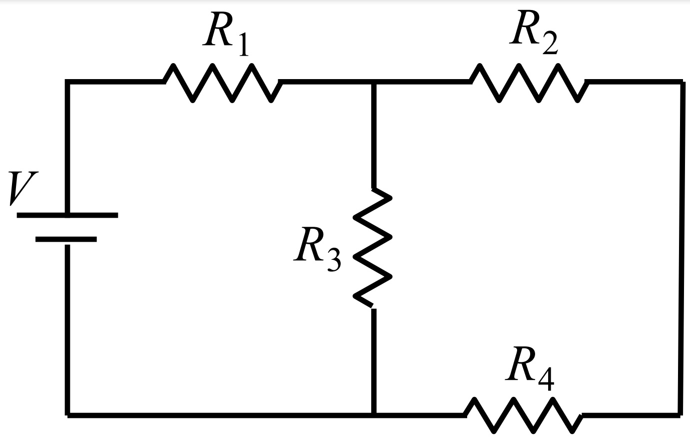
Parallel: Can make a loop that contains only those two resistors

Series Combination

Series : Every loop with resistor 1 also has resistor 2.

Shout-out "clicker"





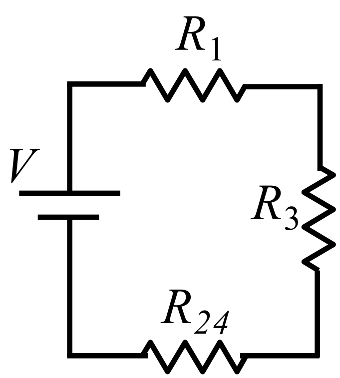
In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

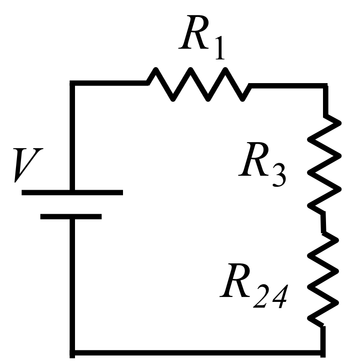
What is V_2, the voltage across R_2?

R_2 and R_4 are connected in series (R_{24}) which is connected in parallel with R_3

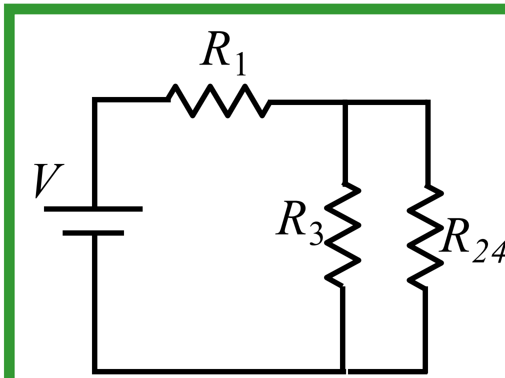
Redraw using the equivalent resistor R_{24} = series combination of R_2 and R_4.



(A)

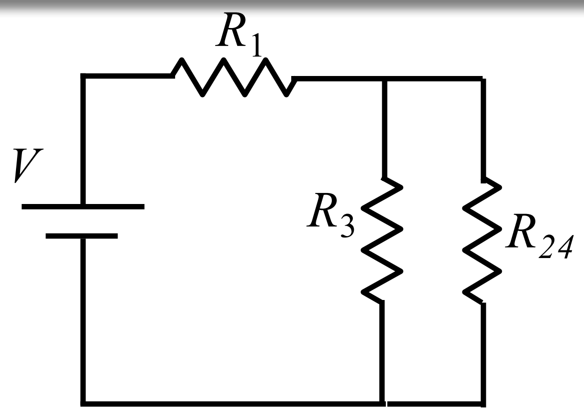


(B)



(C)

Resume Calculation and Click with Numbers



In the circuit shown: $V = 18V$,
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

R_2 and R_4 are connected in series = R_{24}
R_3 and R_{24} are connected in parallel = R_{234}

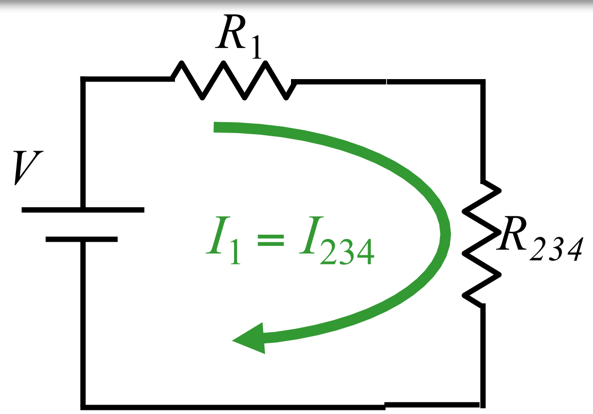
What is the value of R_{234}?

A) $R_{234} = 1\ \Omega$ **B) $R_{234} = 2\ \Omega$** C) $R_{234} = 4\ \Omega$ D) $R_{234} = 6\ \Omega$

R_2 and R_4 in series ➔ $R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$ ➔ $1/R_{234} = (1/3) + (1/6) = (3/6)\ \Omega^{-1}$ ➔ $R_{234} = 2\ \Omega$

Calculation



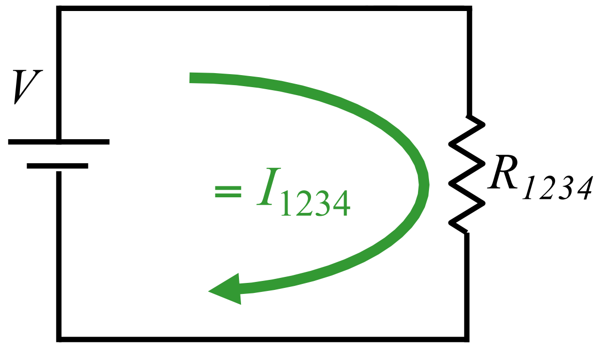
In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

R_1 and R_{234} are in series. $R_{1234} = 1 + 2 = 3\ \Omega$

Our next task is to calculate the total current in the circuit

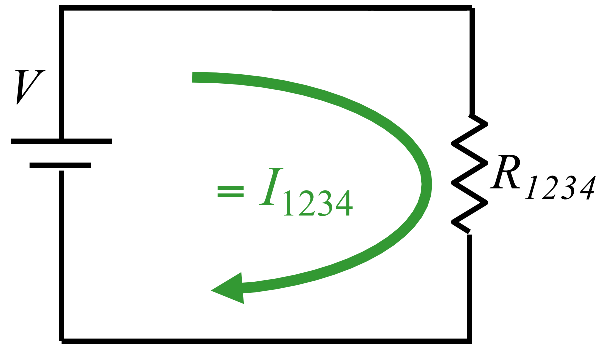


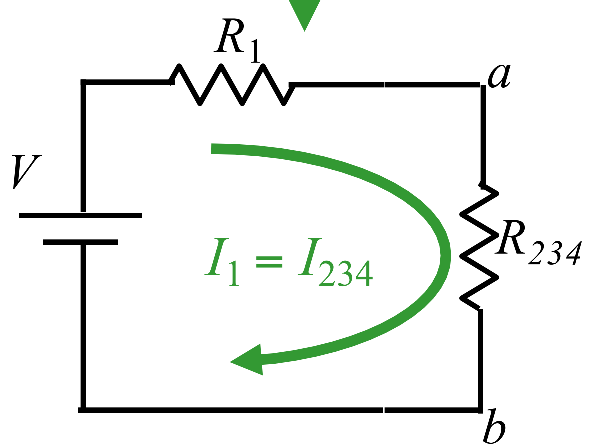
Ohm's Law tells us: $I_{1234} = V/R_{1234}$

$= 18 / 3$

$= 6$ Amps

Shout-out Clicker … Calculation





In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

$I_{234} = I_{1234}$ Since R_1 in series with R_{234}

$V_{234} = I_{234}\, R_{234}$

$= 6 \text{ x } 2$

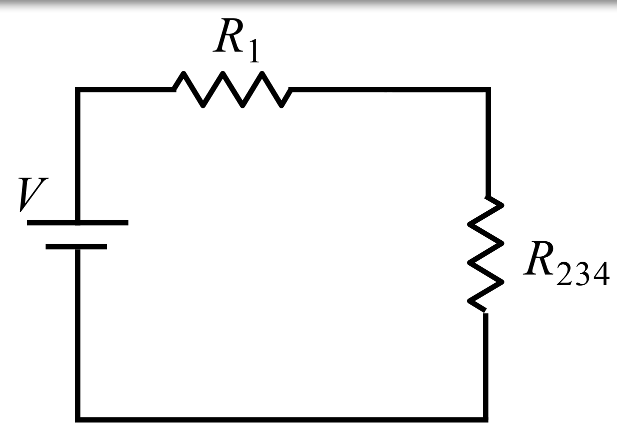
$= 12$ Volts

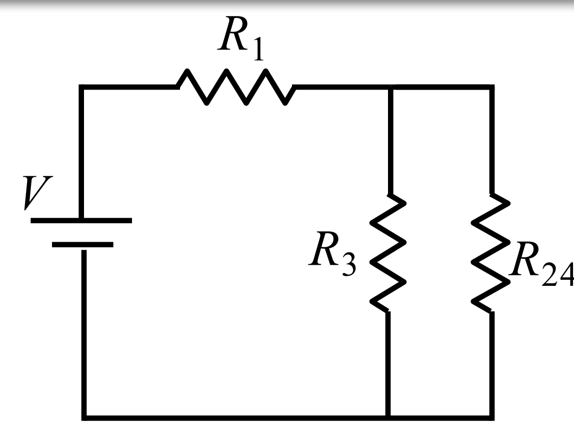
What is V_{ab}, the voltage across R_{234} ?

$I_{1234} = 6$ Amps

A) $V_{ab} = 1\ V$ B) $V_{ab} = 2\ V$ C) $V_{ab} = 9\ V$ D) $V_{ab} = 12\ V$ E) $V_{ab} = 16\ V$

More shout outs …Calculation





$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$I_{1234} = 6$ Amps
$I_{234} = 6$ Amps
$V_{234} = 12V$
What is V_2?

Which of the following are true?

A) $V_{234} = V_{24}$ B) $I_{234} = I_{24}$ C) Both A+B D) None

R_3 and R_{24} were combined in parallel to get R_{234} → Voltages are same!

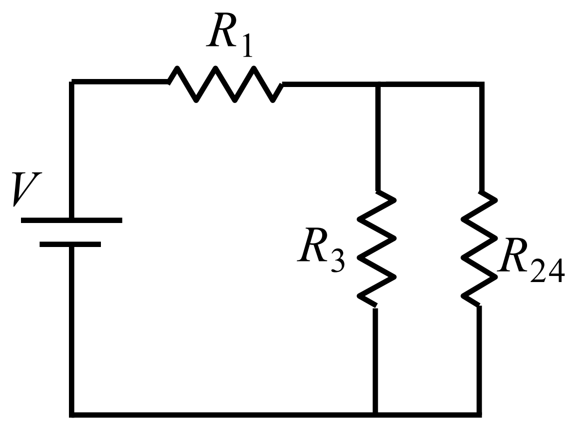
Ohm's Law

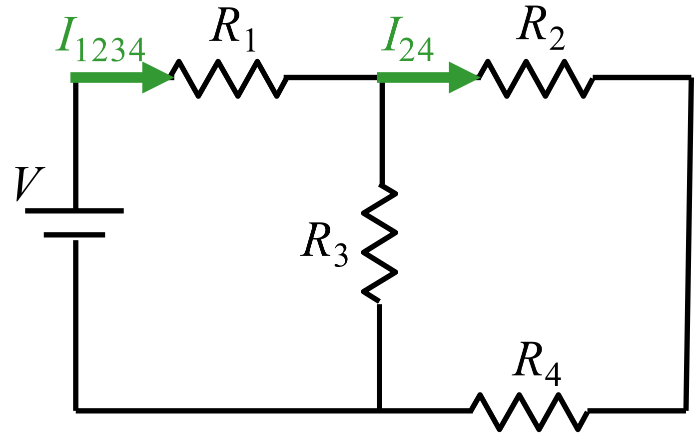
$$I_{24} = V_{24} / R_{24}$$

$$= 12 / 6$$

$$= 2 \text{ Amps}$$

And one more … Calculation





$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega.$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$I_{1234} = 6$ Amps
$I_{234} = 6$ Amps
$V_{234} = 12V$
$V_{24} = 12V$
$I_{24} = 2$ Amps
What is V_2?

Which of the following are true?

A) $V_{24} = V_2$ B) $I_{24} = I_2$ C) Both A+B D) None

R_2 and R_4 where combined in series to get R_{24} → Currents are same!

The Problem Can Now Be Solved!

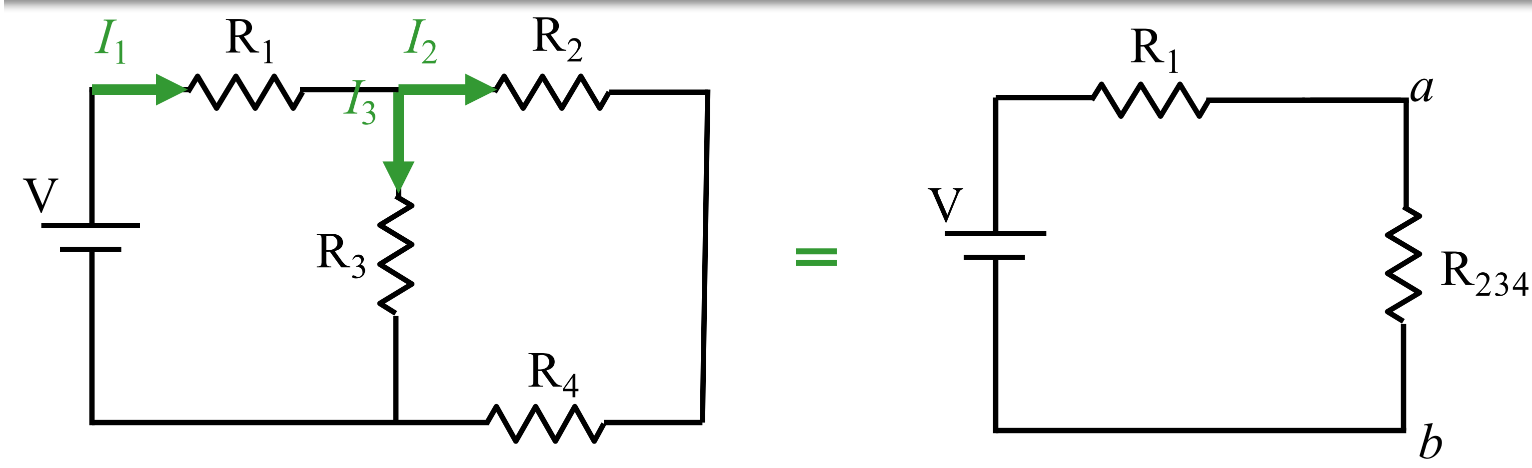
Ohm's Law

$V_2 = I_2 \ R_2$

$= \ 2 \text{ x } 2$

$= 4$ Volts!

And Some Quick Follow-Ups



$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$V_{234} = 12V$
$V_2 = 4V$
$I_{1234} = 6$ Amps

What is I_3 ?

A) $I_3 = 2\ A$ B) $I_3 = 3\ A$ C) $I_3 = 4\ A$

$V_3 = V_{234} = 12V$ → $I_3 = V_3/R_3 = 12V/3\Omega = 4A$

What is I_1 ?

We know $I_1 = I_{1234} = 6\ A$

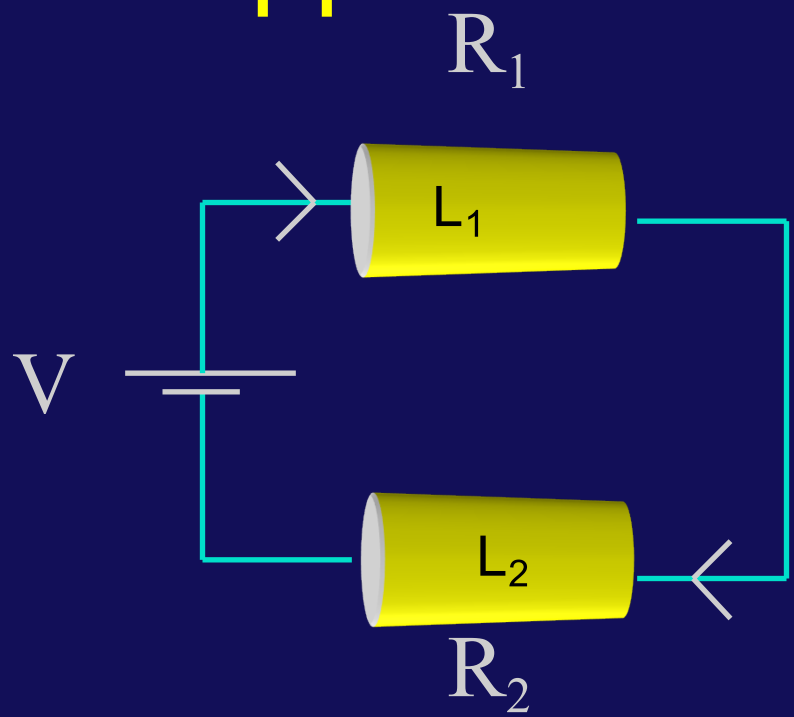
NOTE: $I_2 = V_2/R_2 = 4/2 = 2\ A$ → $I_1 = I_2 + I_3$ Make Sense?

Another (intuitive) way... In Appendix

Consider two cylindrical resistors with lengths L_1 and L_2

$$R_1 = \rho \frac{L_1}{A}$$

$$R_2 = \rho \frac{L_2}{A}$$



Put them together, end to end to make a longer one...

$$R_{effective} = \rho \frac{L_1 + L_2}{A} = R_1 + R_2$$

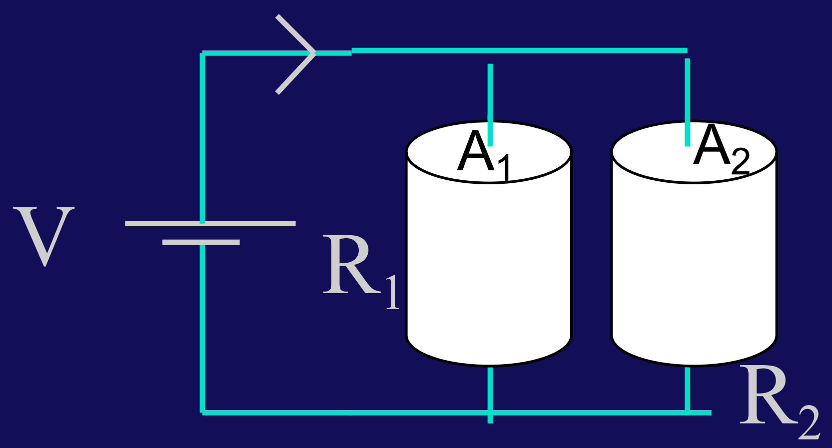
$$\boxed{R = R_1 + R_2}$$

Another (intuitive) way…In Appendix

Consider two cylindrical resistors with cross-sectional areas A_1 and A_2

$$R_1 = \rho \frac{L}{A_1}$$

$$R_2 = \rho \frac{L}{A_2}$$



Put them together, side by side … to make one "fatter"one,

$$R_{effective} = \frac{\rho L}{(A_1 + A_2)} \quad \Longrightarrow \quad \frac{1}{R_{effective}} = \frac{A_1}{\rho L} + \frac{A_2}{\rho L} = \frac{1}{R_1} + \frac{1}{R_2}$$

Þ

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$