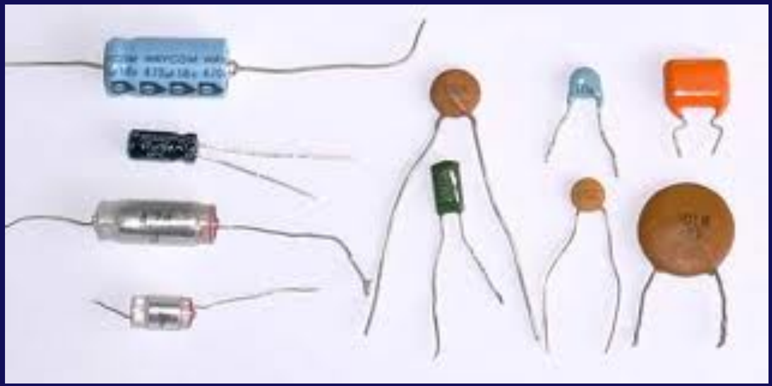
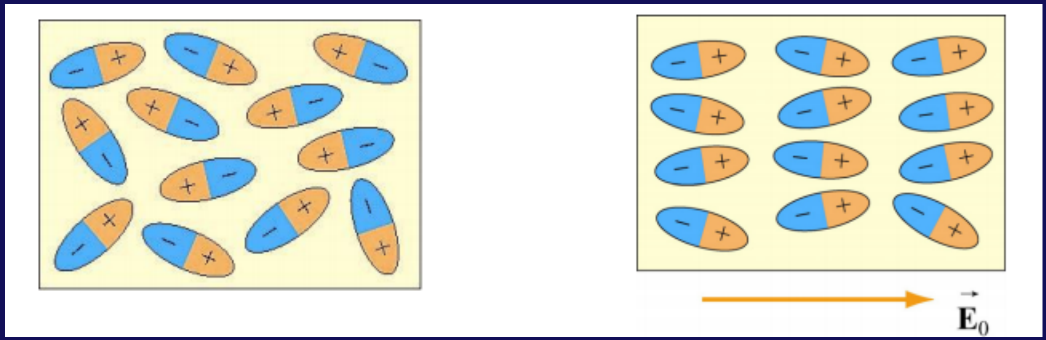
Energy / Dielectrics in Capacitors





Polar dielectrics, like Water …. In an Electric field

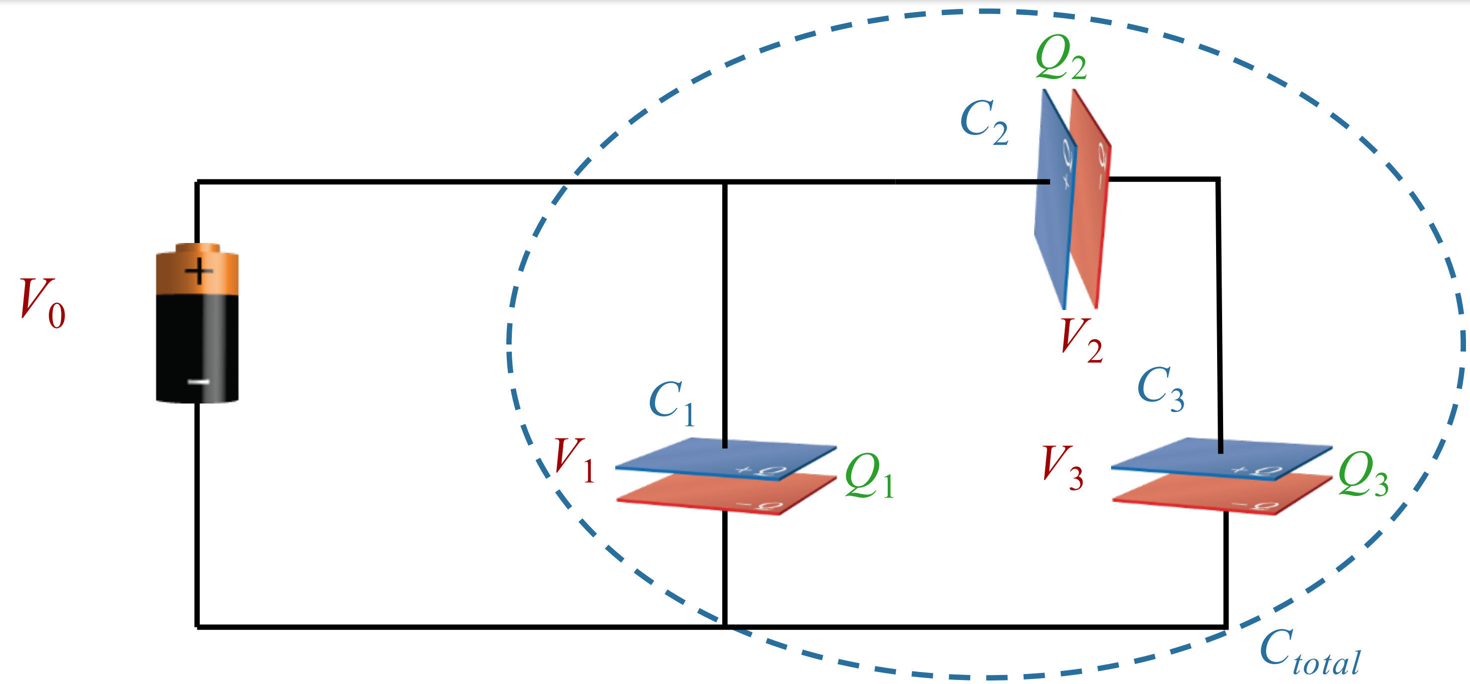
$$C = Q/V$$
$$CV = Q$$
$$V = Q/C$$

Useful stuff

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Phys 122 Lecture 12
G. Rybka

We ended last time looking at this clicker



Which of the following is NOT necessarily true:

A) $V_0 = V_1$ — **C_1 is "in parallel" with battery (same V)**

B) $C_{total} > C_1$ — **C_1 is "in parallel" with C_{23}; $C_{total} = C_1 + C_{23}$**

C) $V_2 = V_3$ — **Q: When might this be true?**

D) $Q_2 = Q_3$ — **Q_2 and Q_3 same since capacitors in series**

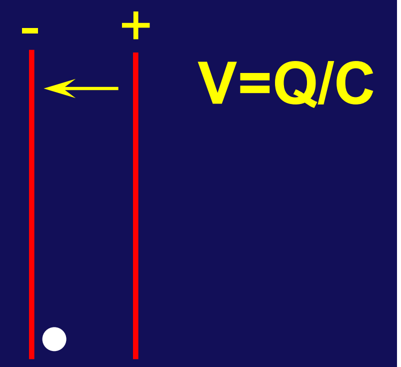
E) $V_1 = V_2 + V_3$ — **C_1 in parallel with combination of C_2 and C_3 ; same ΔV**

Reminder: Energy of a Capacitor

- We obtain the energy stored in a charged capacitor by
  - Calculating the work provided (usually by a battery) to charge a capacitor to +/- Q:

- Incremental work dW needed to add dq to capacitor at voltage V:

$$dW = dq(V) = dq\left(\frac{q}{C}\right)$$



- The total work W to charge to Q is then given by:

$$W = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}$$

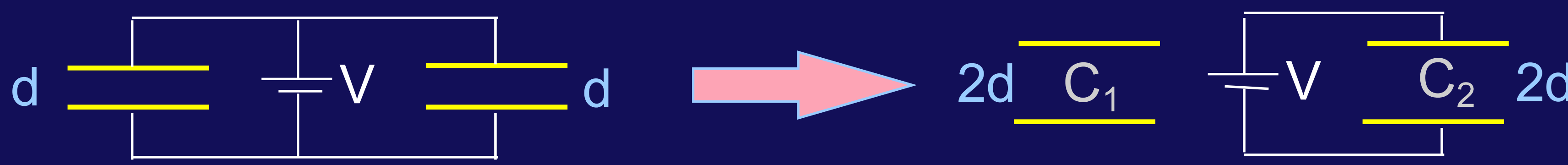
We will use U as in Potential Energy

- Three equivalent forms:

$$W = U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Clicker

- Two identical parallel plate capacitors are connected to a battery.
  - C_1 is then disconnected from the battery and the separation between the plates of both capacitors is doubled.



  - What is the relation between U_1, the energy stored in C_1, and U_2, the energy stored in C_2?

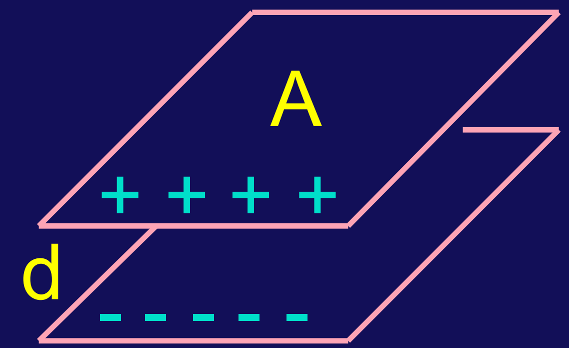
(a) $U_1 < U_2$ (b) $U_1 = U_2$ (c) $U_1 > U_2$

- After …
  - The charge on C_1 has not changed. $U = \frac{1}{2} Q^2/C$; C smaller; U up
  - The voltage on C_2 has not changed. $U = \frac{1}{2} CV^2$; C smaller; U down

- Or, think that the work done to separate the plates with fixed charge in C1 meant takes more Energy

- And, Energy in C_2 decreased. Charge must leave in order to reduce the electric field so that the potential difference remains the same.

Questions



- Suppose the capacitor shown here is charged to Q and then the battery disconnected.
- Now suppose I pull the plates further apart so that the final separation is d_1.
- How do the quantities Q, U, C, E, V change?

- Q: remains the same.. no way for charge to leave.
- U: increases.. add energy to system by separating
- C: decreases.. Separation d increases; C A/d
- E: remains the same.. depends only on charge density
- V: increases.. equal to E x d, and d increases

- How much do these quantities change

answers:

$$U_1 = \frac{d_1}{d} U \qquad C_1 = \frac{d}{d_1} C \qquad V_1 = \frac{d_1}{d} V$$

Dielectrics

- Empirical observation:

Yes, it's an insulator

Inserting a non-conducting material between the plates of a capacitor changes the VALUE of the capacitance.

- Definition:

The dielectric constant of a material is the ratio of the capacitance when filled with the dielectric to that without it. i.e.

$$\kappa = \frac{C}{C_0}$$

  – κ is always > 1 (e.g., glass = 5.6; water = 78)

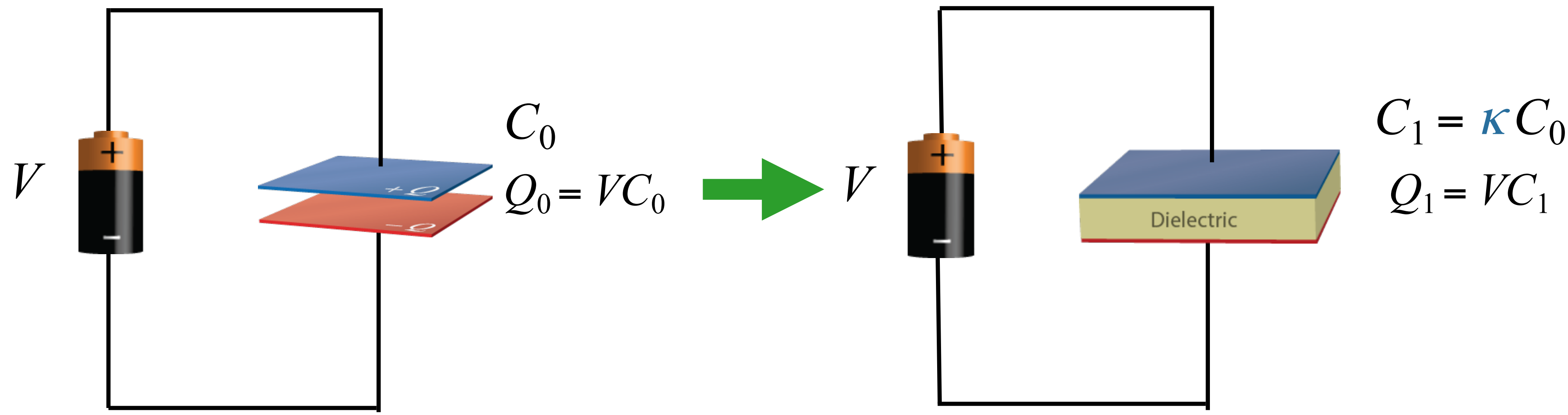
Dielectric Constants Of Various Materials

Material	Min.	Max.
Air	1	1
Amber	2.6	2.7
Asbestos fiber	3.1	4.8
Bakelite	5	22
Barium Titanate	100	1250
Beeswax	2.4	2.8
Cambric	4	4
Carbon Tetrachloride	2.17	2.17
Celluloid	4	4
Cellulose Acetate	2.9	4.5
Durite	4.7	5.1
Ebonite	2.7	2.7
Epoxy Resin	3.4	3.7
Ethyl Alcohol	6.5	25
Fiber	5	5
Formica	3.6	6
Glass	3.8	14.5
Glass Pyrex	4.6	5
Gutta Percha	2.4	2.6
Isolantite	6.1	6.1
Kevlar	3.5	4.5
Lucite	2.5	2.5
Mica	4	9
Micarta	3.2	5.5
Mycalex	7.3	9.3
Neoprene	4	6.7

Material	Min.	Max.
Nylon	3.4	22.4
Paper	1.5	3
Paraffin	2	3
Plexiglass	2.6	3.5
Polycarbonate	2.9	3.2
Polyethylene	2.5	2.5
Polyimide	3.4	3.5
Polystyrene	2.4	3
Porcelain	5	6.5
Quartz	5	5
Rubber	2	4
Ruby Mica	5.4	5.4
Selenium	6	6
Shellac	2.9	3.9
Silicone	3.2	4.7
Slate	7	7
Soil dry	2.4	2.9
Steatite	5.2	6.3
Styrofoam	1.03	1.03
Teflon	2.1	2.1
Titanium Dioxide	100	100
Vaseline	2.16	2.16
Vinylite	2.7	7.5
Water distilled	34	78
Waxes, Mineral	2.2	2.3
Wood dry	1.4	2.9

Dielectrics

By adding a dielectric you are just making a new capacitor with <u>larger</u> capacitance (factor of κ)

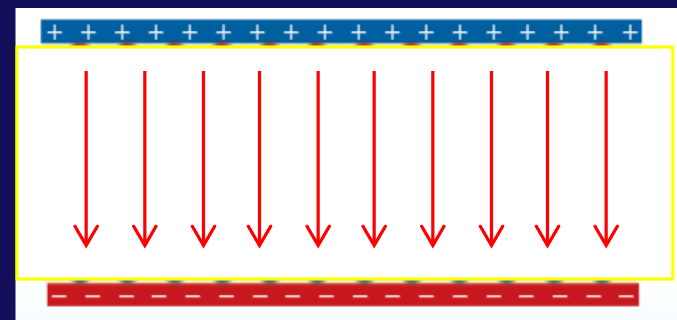


- **A good thing because …**
  - **It is hard to make "big" capacitors with just air gaps**
  - **Permits more energy to be stored than otherwise**

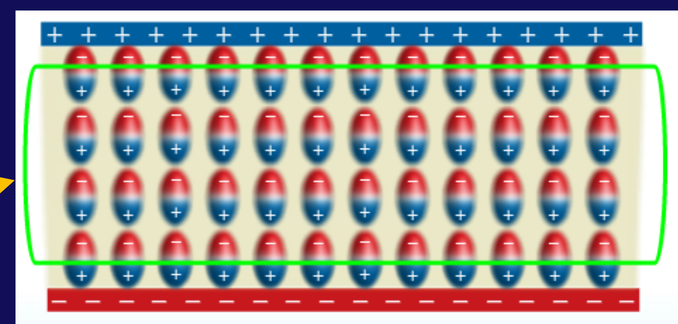
What is going on ?

Parallel Plate Example

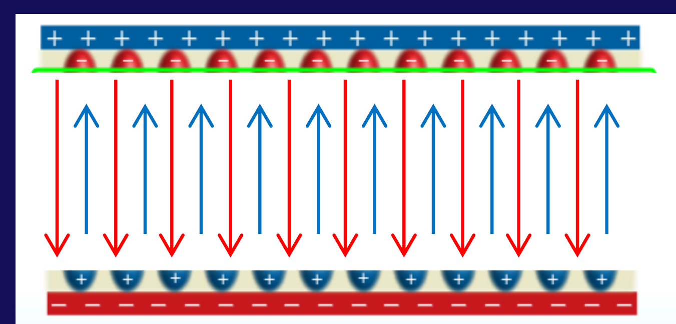
- Charge a parallel plate capacitor filled with vacuum (air) to potential difference V_0.
- $Q = C_0 V_0$ is deposited on each plate.
- $E_0 = \sigma/\varepsilon_0$



- Insert material with dielectric constant κ .
  - Charge Q remains constant
  - Induced dipoles in material align
  - Bulk middle is neutral



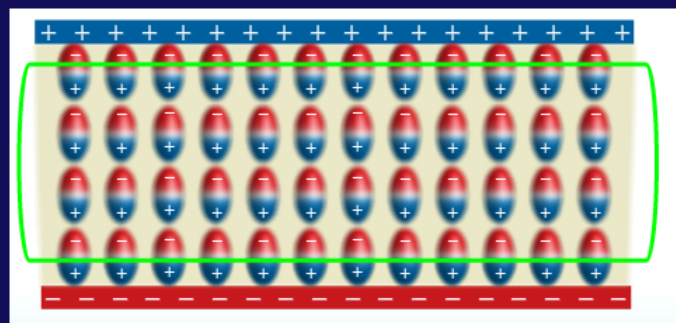
  - Effective opposite E field added to original field gives SMALLER net E' between plates: **$E' = E_0 / \kappa$**



- $V' = E'd$ → V' smaller for same Q.
  - $C = Q/V$ → $C' = \kappa C_0$

What about GAUSS' LAW?

- How can field decrease if charge remains the same??
- Answer: the dielectric becomes polarized in the presence of the field due to Q.



$$E = \frac{E_0}{\kappa}$$

  - **The molecules partially align with the field.**
  - **The field inside the dielectric (from the dipoles) opposes the original field and is responsible for the reduction in the effective field**

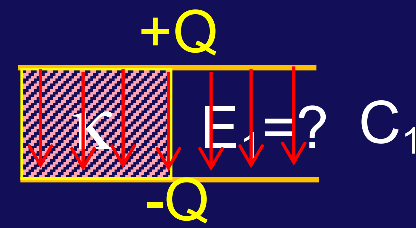
Rewrite Gauss' Law in presence of a dielectric:

$$\varepsilon_0 \oint \kappa \left(\frac{\vec{E}_0}{\kappa} \right) \bullet d\vec{S} = q \quad \Rightarrow \quad \varepsilon_0 \oint \kappa \vec{E} \bullet d\vec{S} = q$$

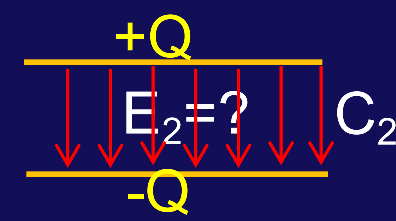
This form of Gauss' Law can be used in vacuum or dielectric alike where q represents the "free" charge.

Clicker

- Two parallel plate capacitors are identical except C_1 has half of the space between the plates filled with a material of dielectric constant κ.



  - Both capacitors have charge Q
  - Compare E_1, the electric field in the air of C_1, and E_2, the electric field in the air of C_2

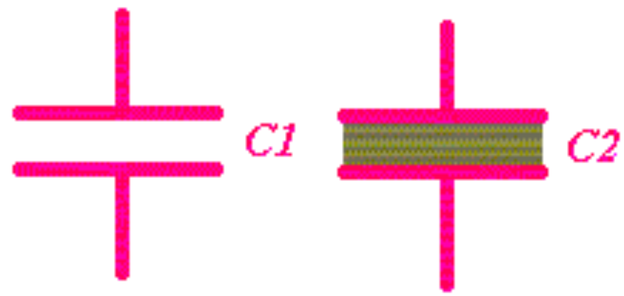


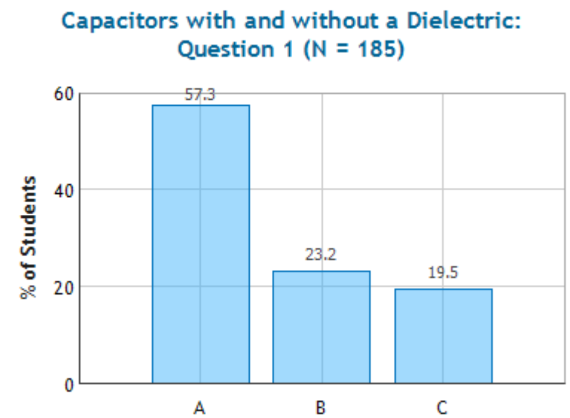
(a) $E_1 < E_2$ (b) $E_1 = E_2$ (c) $E_1 > E_2$

- The key here is to realize that the electric field in the air in C_1 must be equal to the electric field in the dielectric in C_1!!
  - The top plane is a conductor → equipotential surface.
  - The bottom plane is a conductor → equipotential surface.
    - V is proportional to E d
  - For this to happen, the charge density on each plane must be non-uniform to create equal electric fields!!
- Since $C_1 > C_2$, for the same charge, $V_1 < V_2$.
- Consequently, $E_1 < E_2$.

CheckPoint 8

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C2 has been charged and disconnected, it is filled with a dielectric.





Compare the voltages of the two capacitors.

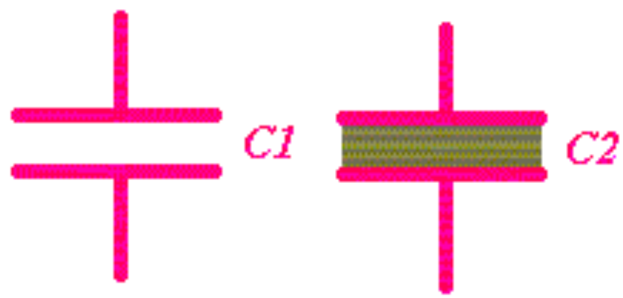
○ $V_1 > V_2$ ○ $V_1 = V_2$ ○ $V_1 < V_2$

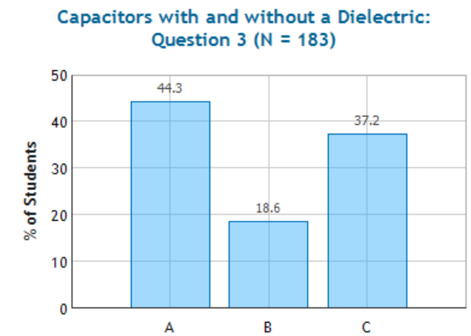
The dielectric increases C; $C_2 > C_1$
Q remains the same
$V_2 = Q/C_2$ so V decreases

Alternately, recall E reduced
→ $E = E_0/\kappa$

CheckPoint 10

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C2 has been charged and disconnected, it is filled with a dielectric.





Compare the potential energy stored by the two capacitors.

A) $U_1 > U_2$ B) $U_1 = U_2$ C) $U_1 < U_2$

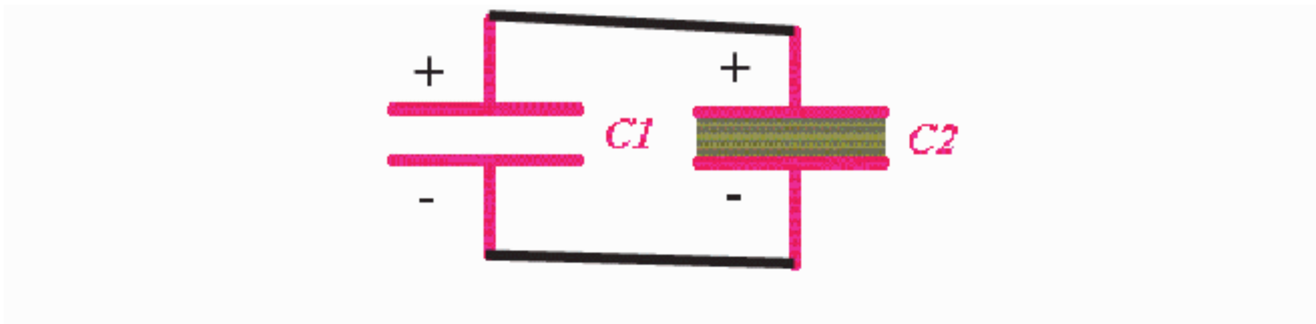
Just learned $V_1 > V_2$ for same Q, so …

Recall $U = \frac{1}{2}QV$ or $U = \frac{1}{2}\frac{Q^2}{C}$ or $U = \frac{1}{2}CV^2$

Also, since Q unchanged, larger C implies lower U

CheckPoint 12

The two capacitors are now connected to each other by wires as shown. How will the charge redistribute itself, if at all?



A. The charges will flow so that the charge on C1 will become equal to the charge on C2.

B. The charges will flow so that the energy stored in C1 will become equal to the energy stored in C2

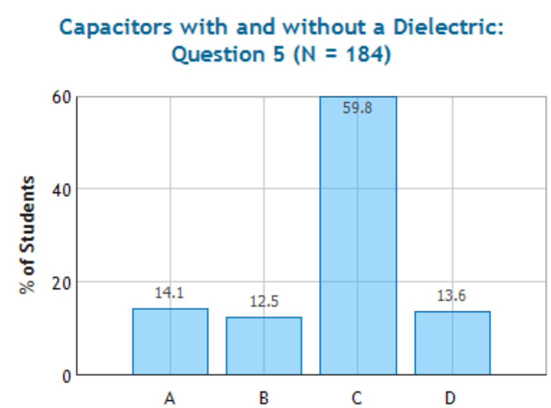
C. The charges will flow so that the potential difference across C1 will become the same as the potential difference across C2.

D. No charges will flow. The charge on the capacitors will remain what it was before they were connected.

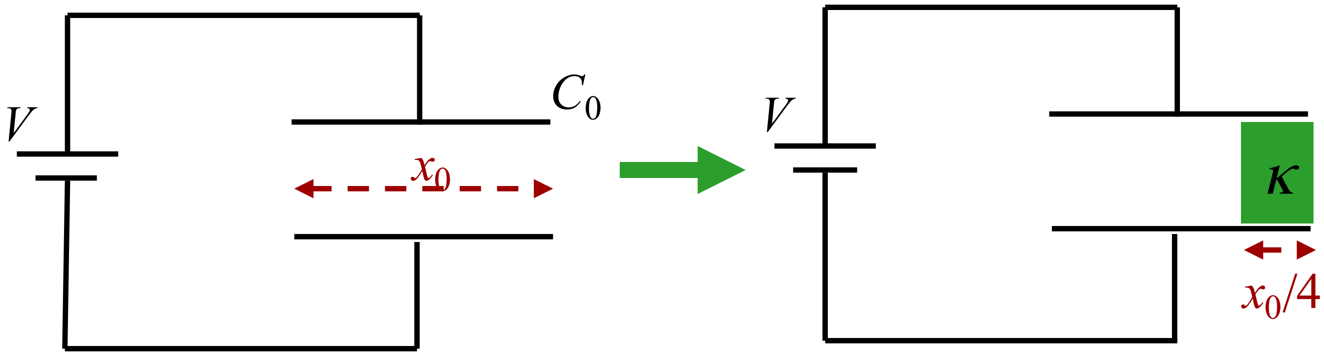
V must be the same !

Q: $$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \;\rightarrow\; Q_1 = \frac{C_1}{C_2} Q_2$$

U: $$U_1 = \tfrac{1}{2} C_1 V^2 \qquad U_2 = \tfrac{1}{2} C_2 V^2 \;\rightarrow\; U_1 = \frac{C_1}{C_2} U_2$$



Clicker and Typical Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

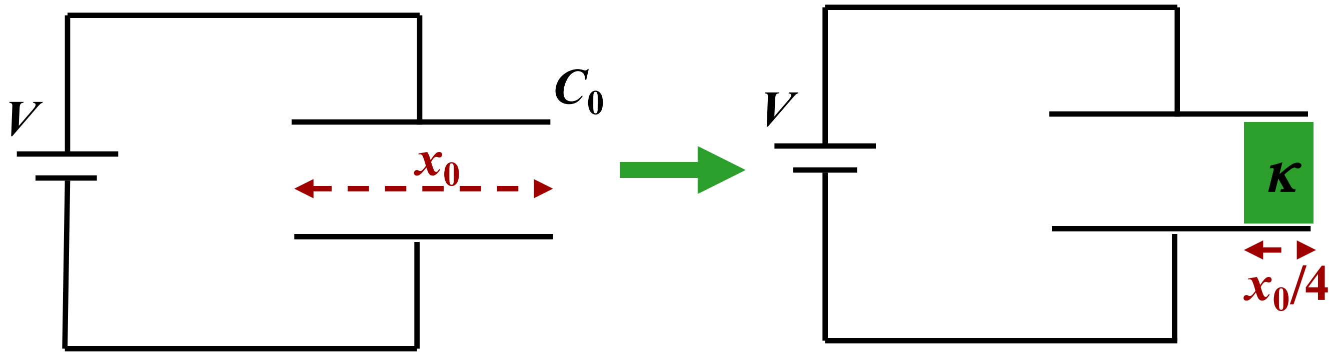
First a Clicker: What changes when the dielectric added?

A) Only C B) only Q C) only V D) C and Q E) V and Q

Adding dielectric changes the physical capacitor → **C changes**

V does not change and C changes → **Q changes**

Calculation



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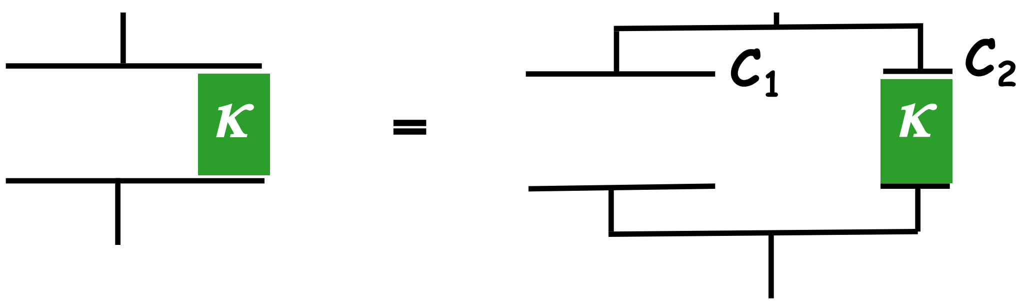
Now, Strategic Analysis:

- Calculate new capacitance C
- Apply definition of capacitance to determine Q

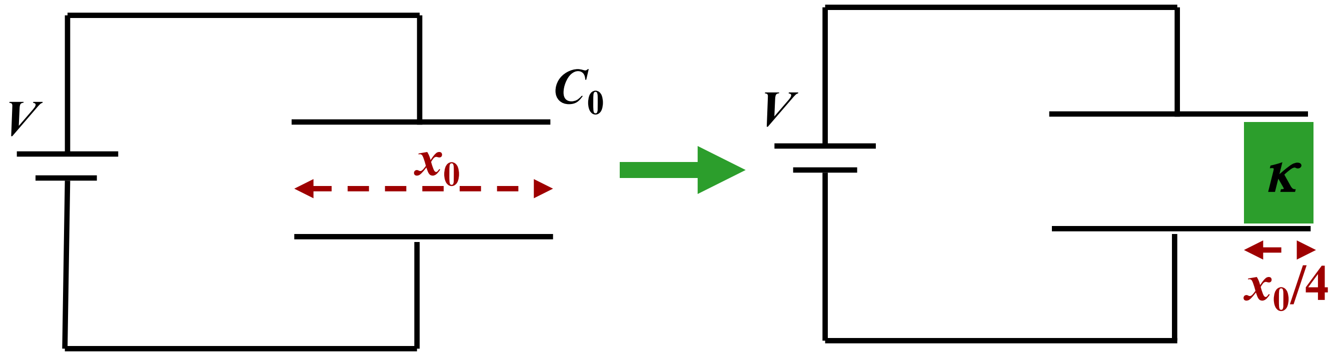
Consider C to be two capacitances, C_1 and C_2, in parallel



Parallel plate capacitor: $C = \varepsilon_0 A/d$

$$A = {}^3/_4 A_0,\quad d = d_0 \;\rightarrow\; C_1 = {}^3/_4(\varepsilon_0 A_0/d_0) \;\rightarrow\; \boxed{C_1 = {}^3/_4 C_0}$$

Calculation



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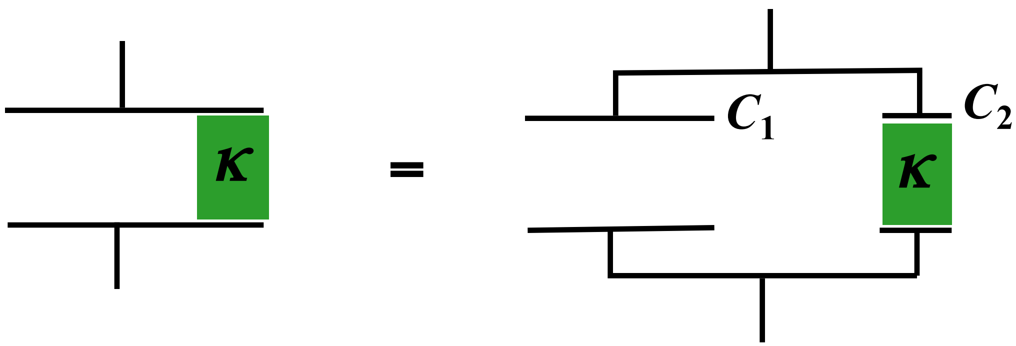
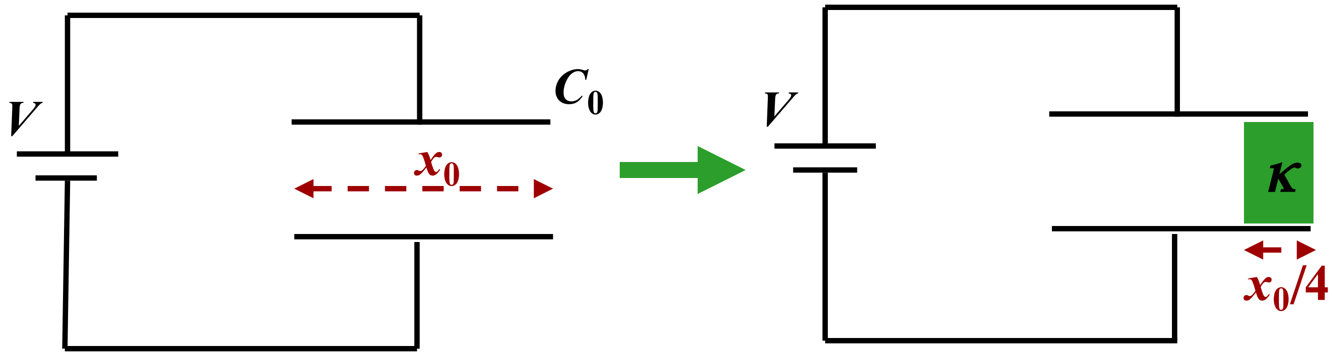


$$C_1 = {}^3/_4 C_0$$

Parallel plate capacitor filled with dielectric: $C = \kappa\varepsilon_0 A/d$

$$A = {}^1/_4 A_0$$
$$d = d_0$$
$$\rightarrow \quad C = \tfrac{1}{4}(\kappa\varepsilon_0 A_0/d_0) \quad \rightarrow \quad C_2 = {}^1/_4\, \kappa C_0$$

Calculation



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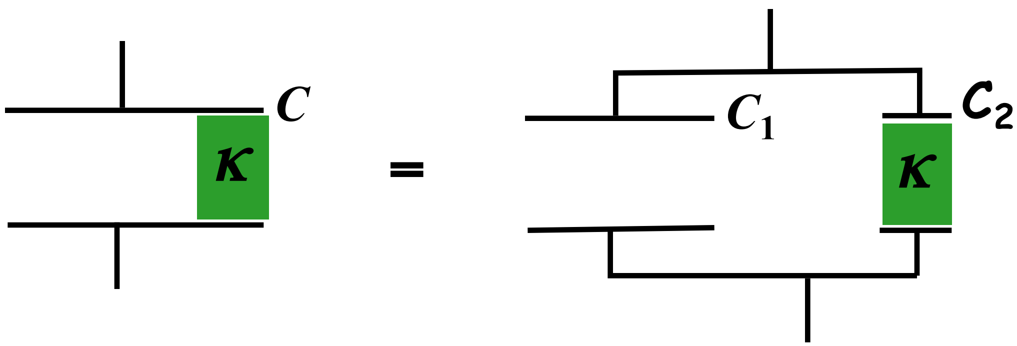
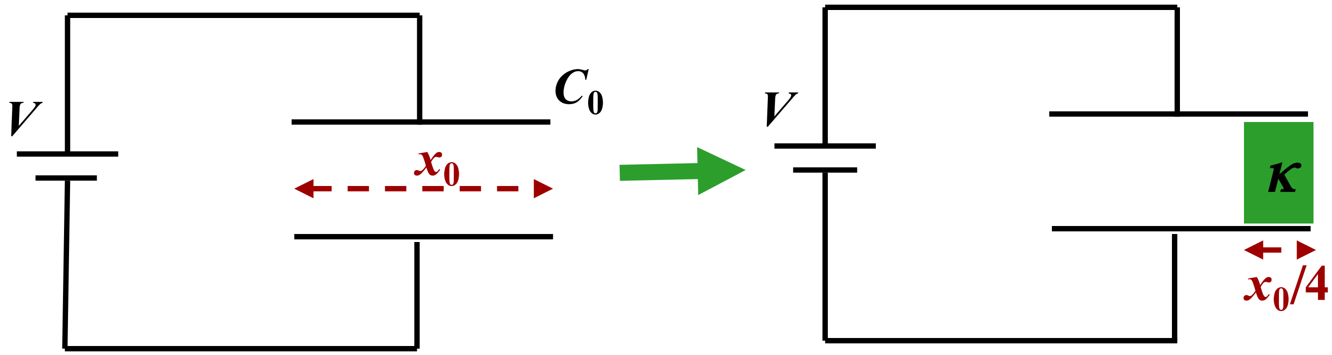


$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa\, C_0$

C = parallel combination of C_1 and C_2: $\quad C = C_1 + C_2$

$$\rightarrow \quad C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$$

Calculation



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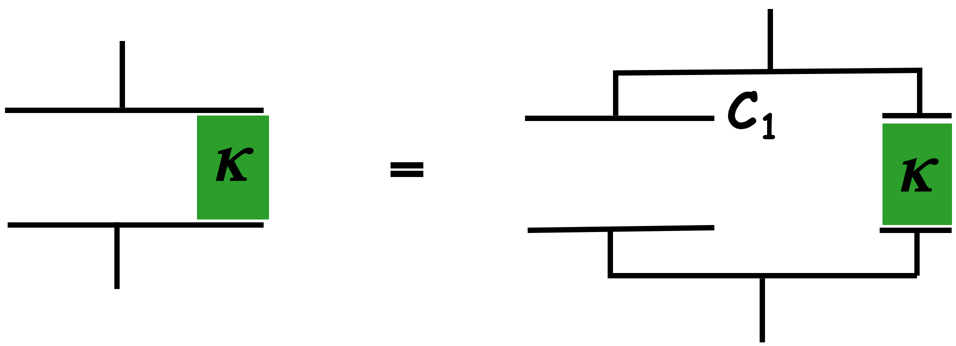


$C_1 = {}^3/_4 C_0$ $\quad$ $C_2 = {}^1/_4\, \kappa\, C_0$

$\rightarrow$ $C = C_0\,({}^3/_4 + {}^1/_4\, \kappa)$

What is Q?

$$C \equiv \frac{Q}{V} \rightarrow Q = VC$$

$$Q_f = VC_0\left(\frac{3}{4} + \frac{1}{4}\kappa\right)$$

Reminder: Where is the Energy Stored?

- Answer: in the Electric field itself

- Consider energy stored by a constant field in a parallel plate capacitor:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{(A\varepsilon_0 / d)}$$

- The Electric field is given by:

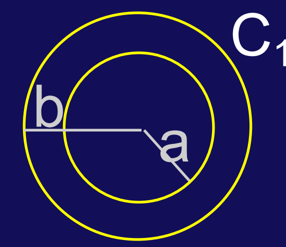
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \Rightarrow \quad U = \frac{1}{2}E^2\varepsilon_0 Ad$$

- The energy density u in the field is given by:

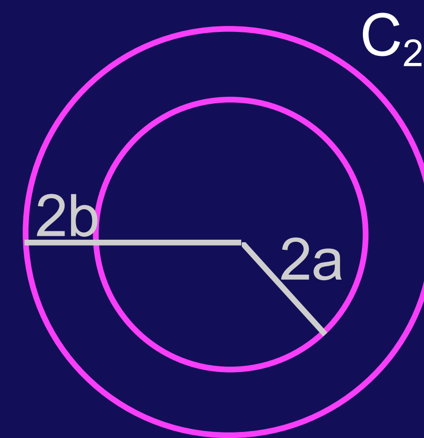
$$u = \frac{U}{volume} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Units: $\frac{\mathrm{J}}{\mathrm{m}^3}$

Clicker



- Consider two cylindrical capacitors, each of length L.
  - C_1 has inner radius a and outer radius b.
  - C_2 has inner radius 2a and outer radius 2b.



If both capacitors are given the same amount of charge, what is the relation between U_1, the energy stored in C_1, and U_2, the energy stored in C_2?

(a) $U_2 < U_1$

(b) $U_2 = U_1$

(c) $U_2 > U_1$

$$U_1 = \frac{1}{2}\frac{Q^2}{C_1}$$

$$U_2 = \frac{1}{2}\frac{Q^2}{C_2}$$

$$\frac{U_1}{U_2} = \frac{C_2}{C_1} \propto \frac{\ln(b/a)}{\ln(2b/2a)} = 1$$

$$C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

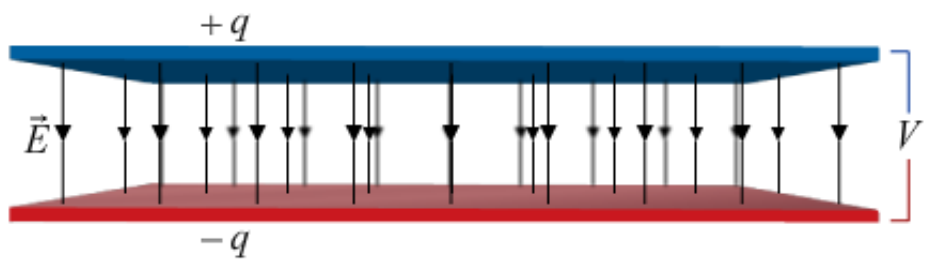
Energy expressions in Capacitors (various forms)

Energy Stored in Capacitors

$$U = \frac{1}{2}QV \quad \text{or} \quad U = \frac{1}{2}\frac{Q^2}{C} \quad \text{or} \quad U = \frac{1}{2}CV^2$$

Energy Density

$$u = \frac{1}{2}\varepsilon_o E^2$$



From: **C = Q/V**
CV = Q
V = Q/C