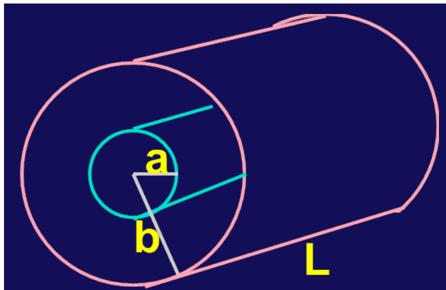
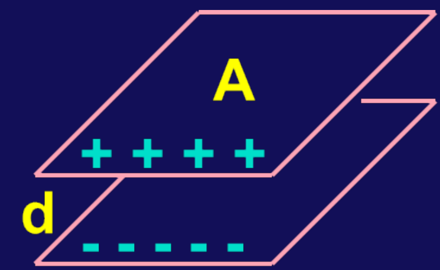
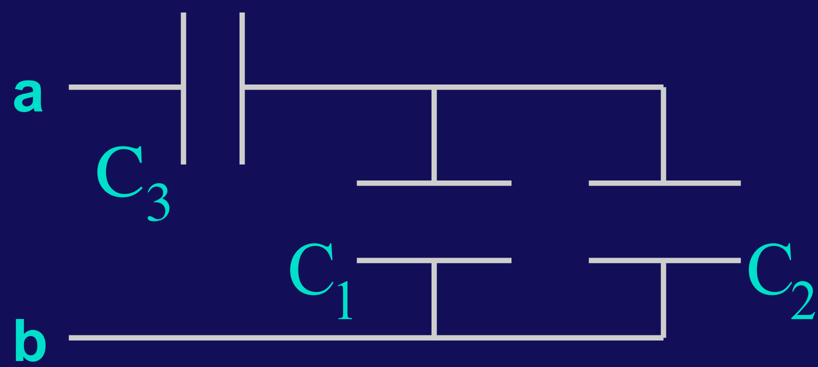


Capacitors: Part 1



$$C \equiv \frac{Q}{V}$$

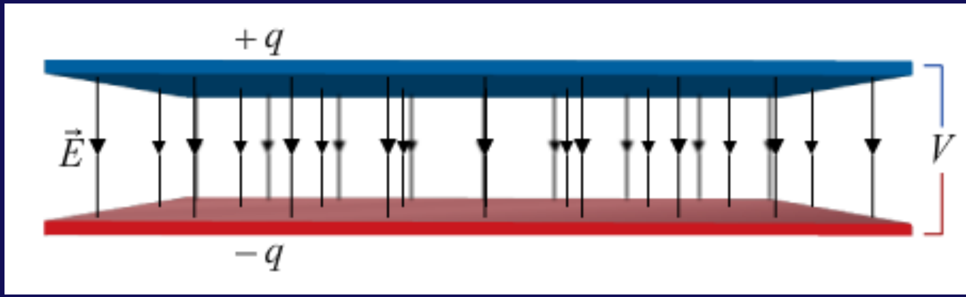


o

- **Long-answer REGRADEs DUE today by 4 pm; turn in directly to Alexis Olsho; no special form required**
- **Dielectrics and more Capacitor stuff on Monday**
  - **no new Smart Physics due**
- **Go over tough significant figure problem on homework**

Phys 122 Lecture 11
G. Rybka

Energy in Capacitors



- The work done (usually by a battery) to separate the charges on a capacitor is stored (and can later be retrieved) as ***Electric Potential Energy***

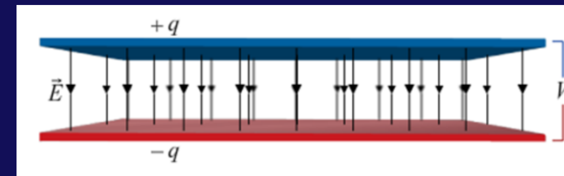
$$\Delta U = q\Delta V$$

- We charge a capacitor by moving *dq* through an existing *V*(*q*), and integrate to get energy stored:

$$\Delta U = \int dU = \int V(q)\,dq \quad \text{where} \quad V(q) = q/C$$

$$\Delta U = \int_0^{Q_{\max}} \frac{q}{C} dq = \frac{Q_{\max}^2}{2C} = \frac{Q_{\max} V_{\max}}{2} = \frac{1}{2} C V_{\max}^2$$

Equivalent Expressions: Energy in Capacitors



$$U = \frac{1}{2} C V^2$$

C = Q/V →

$$U = \frac{1}{2} Q V$$

V = Q/C →

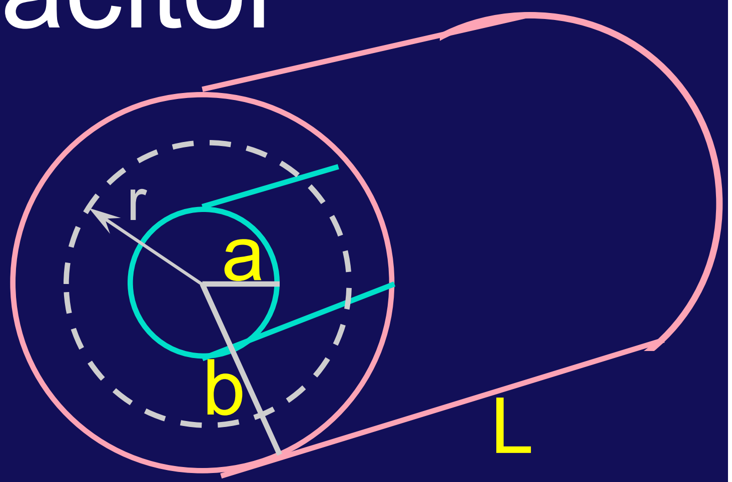
$$U = \frac{1}{2} \frac{Q^2}{C}$$

- **Energy Density:**
  - The <u>energy</u> <u>stored</u> (per volume) in ANY electric field
    - » Not just our parallel plate case
    - » Units: J / m³

$$u \equiv \frac{U}{Vol} = \frac{\frac{1}{2} C V^2}{Ad} = \frac{\frac{1}{2} \varepsilon_0 A V^2 / d}{Ad} = \frac{1}{2} \frac{\varepsilon_0 V^2}{d^2} \Rightarrow u = \frac{1}{2} \varepsilon_o E^2$$

Recall: (V/d = E)

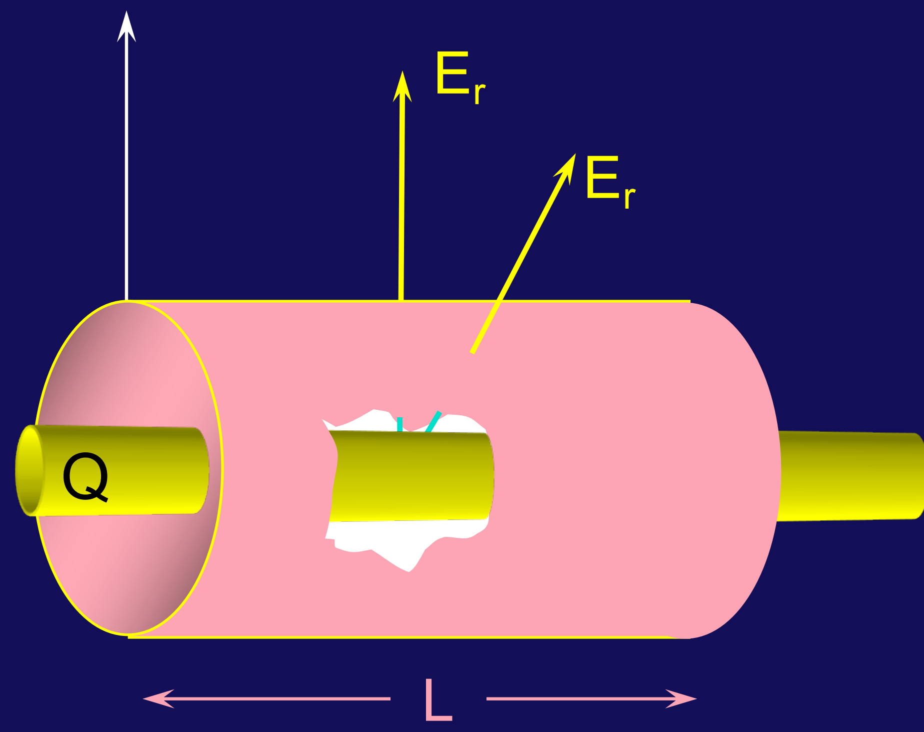
Example:
Cylindrical Capacitor



- Calculate the capacitance:
- Assume +Q, -Q on surface of cylinders with potential difference *V*.

Recall: Cylindrical Symmetry

- Gaussian surface is cylinder of radius r and length L
- Cylinder has charge Q



- Apply Gauss' Law:

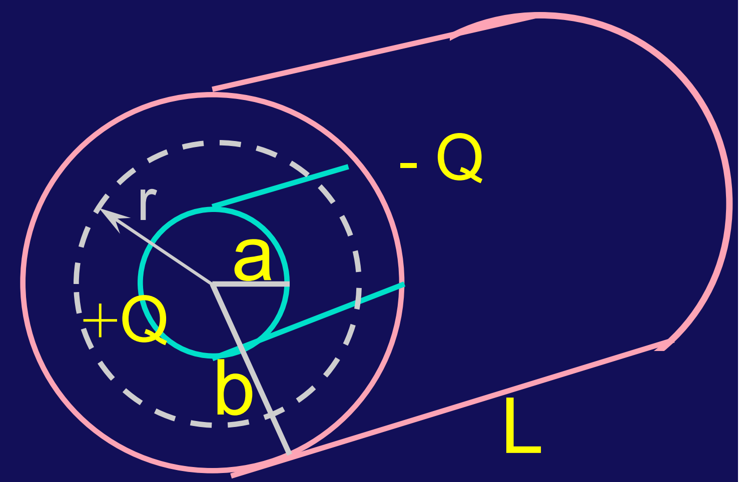
$$\oint \vec{E} \bullet d\vec{S} = 2\pi r L E = \frac{Q}{\varepsilon_0}$$

þ

$$E = \frac{Q}{2\pi\varepsilon_0 L r}$$

This is not new

Example: Cylindrical Capacitor



- Calculate the capacitance:
- Assume +Q, -Q on surface of cylinders with potential difference *V*.
- From Gauss' Law:

$$E = \frac{Q}{2\pi\varepsilon_0 r L}$$

If inner cylinder has +Q, then the potential V is positive if we take the zero of potential to be defined at r = b:

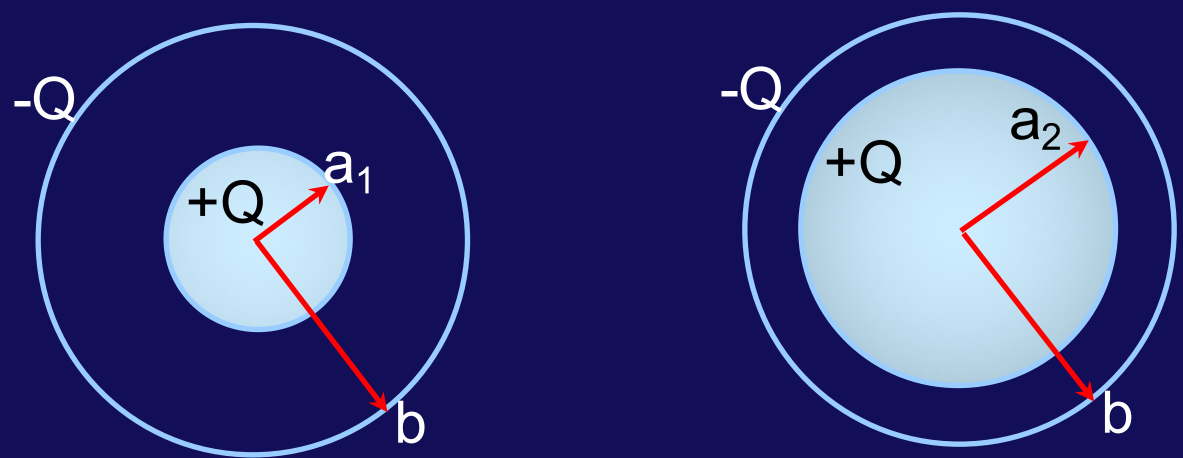
$$V = -\int_b^a \vec{E} \bullet d\vec{l} = -\int_b^a E dr = \int_a^b \frac{Q}{2\pi\varepsilon_0 r L} dr = \frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right)$$

Þ

$$C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

Clicker

- In each case below, a charge of +Q is placed on a solid spherical conductor and a charge of -Q is placed on a concentric conducting spherical shell.
  - Let V_1 be the potential difference between the spheres with (a_1, b).
  - Let V_2 be the potential difference between the spheres with (a_2, b).
  - What is the relationship between the absolute value of V_1 and V_2?

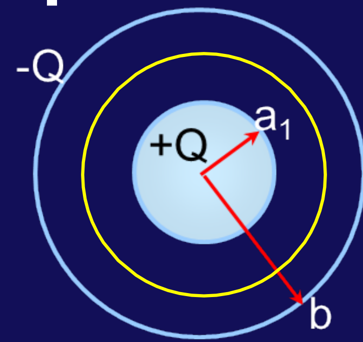


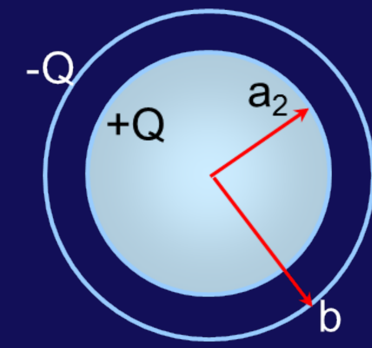
(a) $V_1 < V_2$ (b) $V_1 = V_2$ (c) $V_1 > V_2$

- Two spherical capacitors.
- Intuition: for parallel plate capacitors: $V = (Q/C) = (Qd)/(A\varepsilon_0)$.
  Therefore you might expect that $V_1 > V_2$ since $(b-a_1) > (b-a_2)$.
- In fact this is the case as we can show directly from the definition of V!

Follow up

Compute V_1 from E (let $V_b = 0$)





$$V_1 = -\int_b^{a_1} E(r)dr = \int_{a_1}^{a_2} E(r)dr + \int_{a_2}^{b} E(r)dr$$

Since E(r) has the same form for r > a_2,

$$V_1 = \int_{a_1}^{a_2} E(r)dr + V_2$$

(c) $V_1 > V_2$

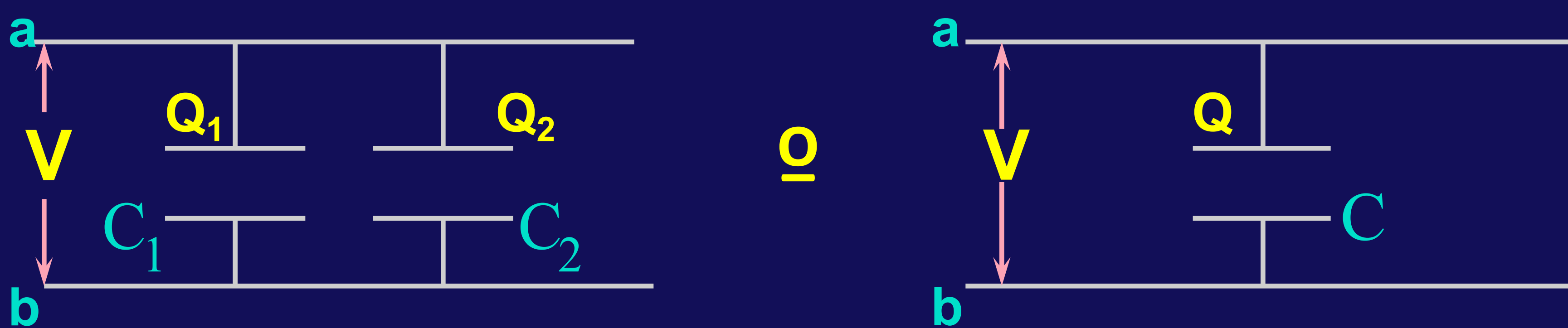
(Abs. value)

The actual functional form for the spherical capacitor:

$$V = -\int_b^a E(r)dr = -\frac{Q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_b^a = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C \equiv \frac{Q}{V} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Capacitors in Parallel



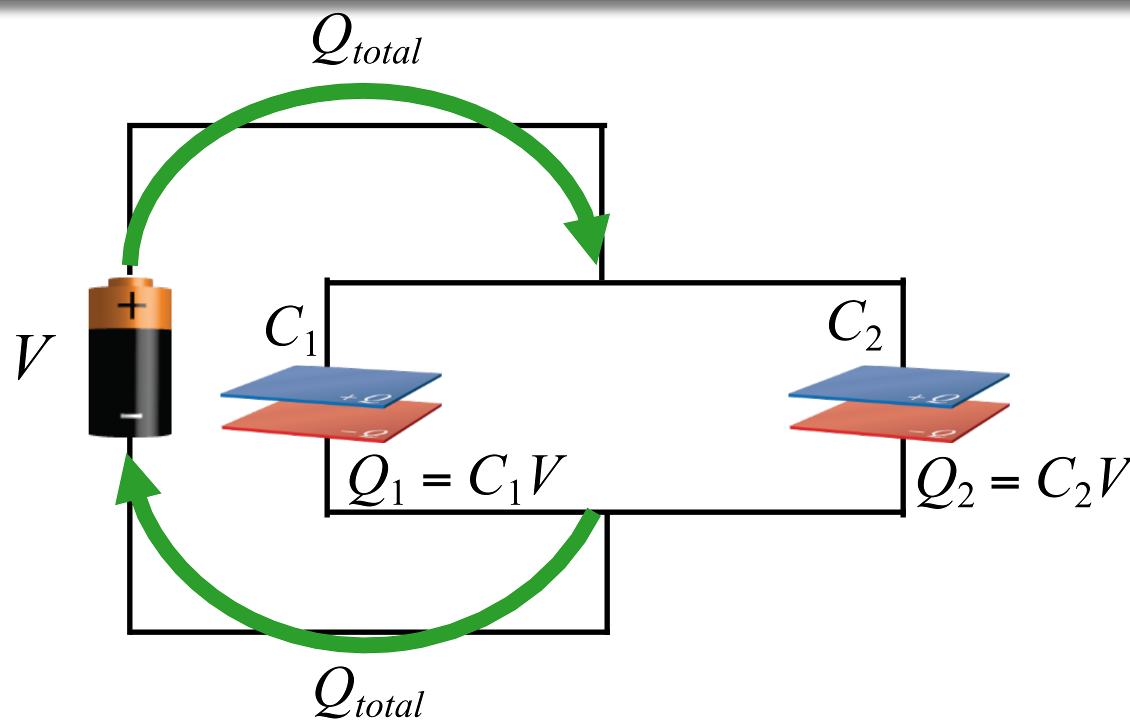
- Find "equivalent" capacitance C

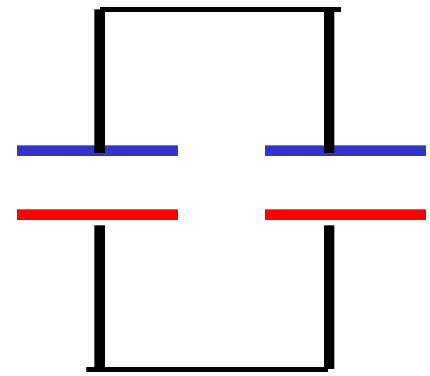
Parallel Combination: $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

Equivalent Capacitor: Þ $C \equiv \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{C_1 V + C_2 V}{V}$

Þ $C = C_1 + C_2$

Parallel Capacitor Circuit





$C_1 + C_2$

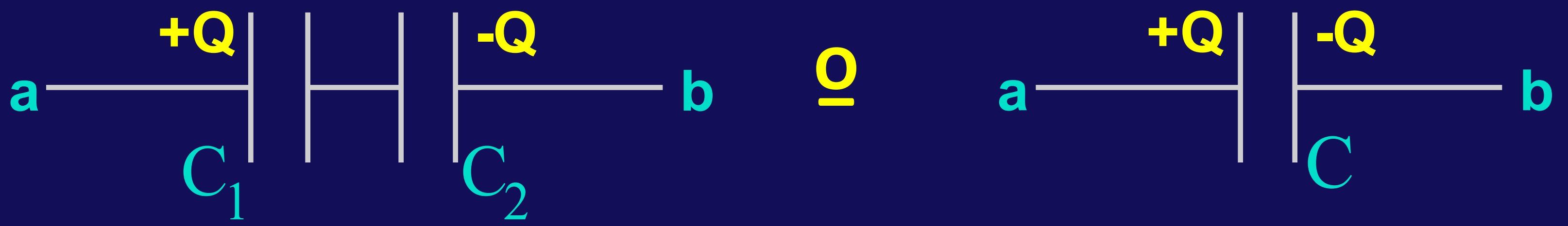
$= C_{eq}$

Key point: V is the same for both capacitors

Key Point: $Q_{total} = Q_1 + Q_2 = VC_1 + VC_2 = V(C_1 + C_2)$

$C_{total} = C_1 + C_2$

Capacitors in Series



- Find "equivalent" capacitance C

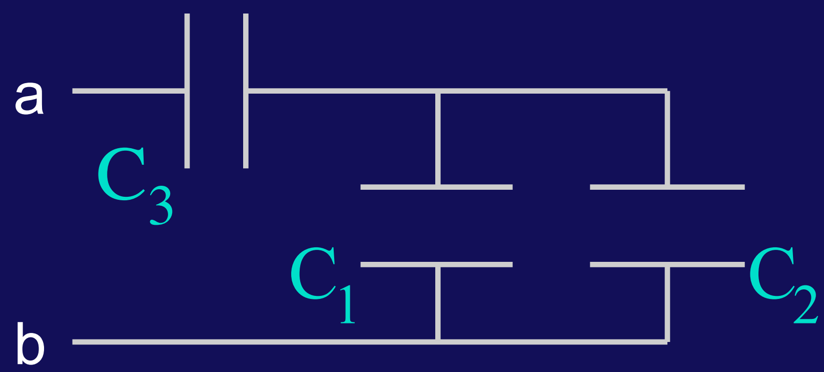
- Charge on C_1 same as charge on C_2
  - since applying a potential difference across ab cannot produce a net charge on the inner plates of C_1 and C_2 .

RHS: $V_{ab} = \frac{Q}{C}$

LHS: $V_{ab} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$

Þ $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Examples:
Combinations of Capacitors



o

Note: C_3 is in series with the parallel combination on C_1 and C_2. *i.e.*

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2}$$

Þ

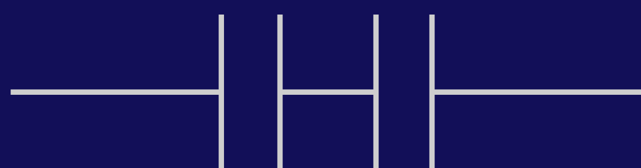
$$C = \frac{C_3(C_1 + C_2)}{C_1 + C_2 + C_3}$$

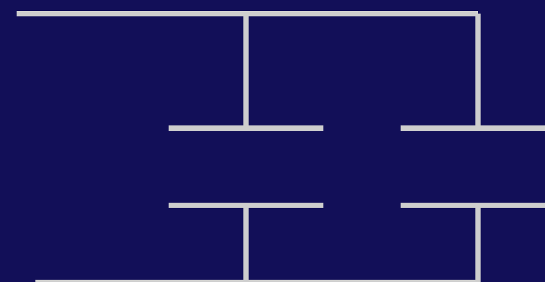
PreLecture Trouble … (click)

Suppose you have two identical capacitors, each having capacitance C. C_{max} is the biggest possible equivalent capacitance that can be made by combining these two, and C_{min} is the smallest.

How does C_{max} compare to C_{min}?

A. $C_{max} = 4C_{min}$
B. $C_{max} = 3C_{min}$
C. $C_{max} = 2C_{min}$
D. $C_{max} = (3/2)C_{min}$
E. $C_{max} = C_{min}$





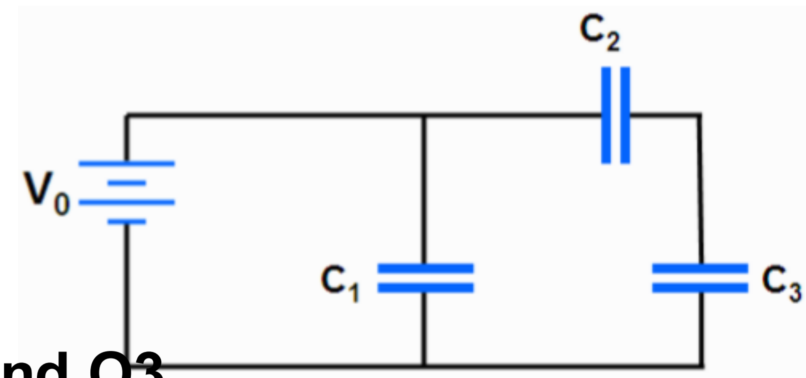
$$\frac{1}{C_{min}} = \frac{1}{C} + \frac{1}{C} \Rightarrow C/2$$

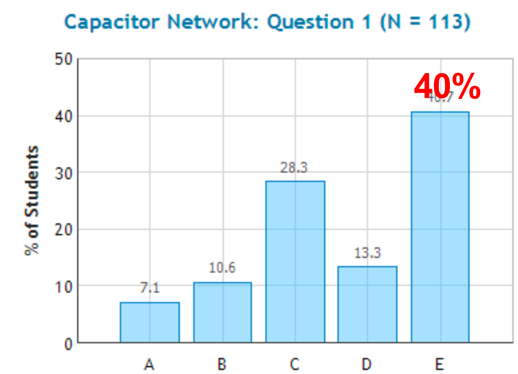
$$C_{max} = C_1 + C_2 = 2C$$

$$C_{max} / C_{min} = 4$$

Checkpoint difficulty

A circuit consists of three unequal capacitors C1, C2, and C3 which are connected to a battery of voltage V0. The capacitance of C2 is twice that of C1. The capacitance of C3 is three times that of C1. The capacitors obtain charges Q1, Q2, and Q3.





Compare Q1, Q2, and Q3.

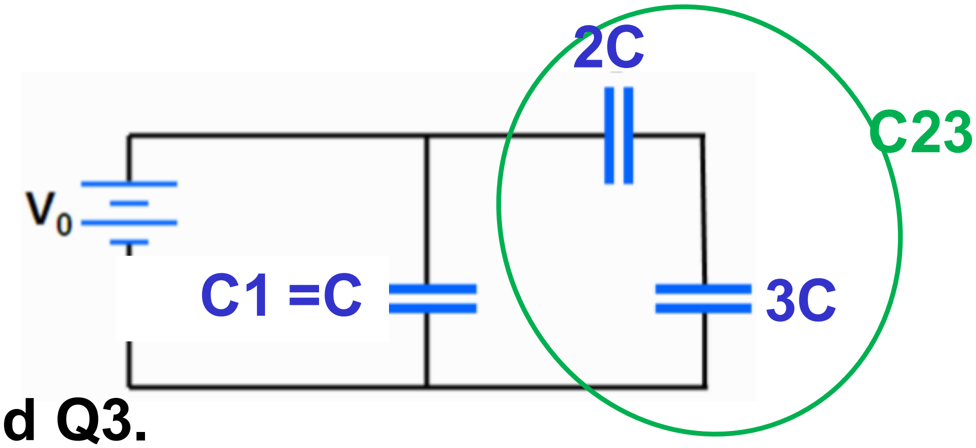
Q1 > Q3 > Q2
Q1 > Q2 > Q3
Q1 > Q2 = Q3
Q1 = Q2 = Q3
Q1 < Q2 = Q3

Great response

Q2 and Q3 will be the same since C2 and C3 are in series. The capacitance of the series C2 and C3 is 1.2 times the capacitance of C1. V0 voltage will be running through the two parallel branches, so C1 will have less charge than C2 and C3 because C2 and C3 have a combined capacitance of 1.2C, where C is the capacitance of C1.

Checkpoint difficulty

A circuit consists of three unequal capacitors C1, C2, and C3 which are connected to a battery of voltage V0. The capacitance of C2 is twice that of C1. The capacitance of C3 is three times that of C1. The capacitors obtain charges Q1, Q2, and Q3.



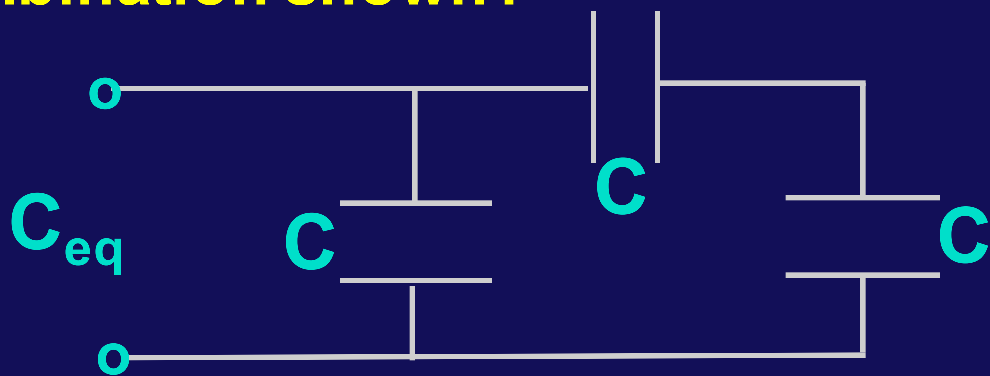
Compare Q1, Q2, and Q3.

Q1 > Q3 > Q2
Q1 > Q2 > Q3
Q1 > Q2 = Q3
Q1 = Q2 = Q3
Q1 < Q2 = Q3

- $V_0 = V_{C1} = V_{C23}$
- Which is larger? C1 or C23 ?
- $C_{23} = \left(\frac{1}{2C} + \frac{1}{3C}\right)^{-1} = \frac{6}{5}C$ SO, C23 > C1
- Recall Q2 = Q3 for series capacitors = Q23
- Q = CV
- V's the same, so Q23 > Q1 since C23 > C1

Clicker

What is the equivalent capacitance, C_{eq}, of the combination shown?



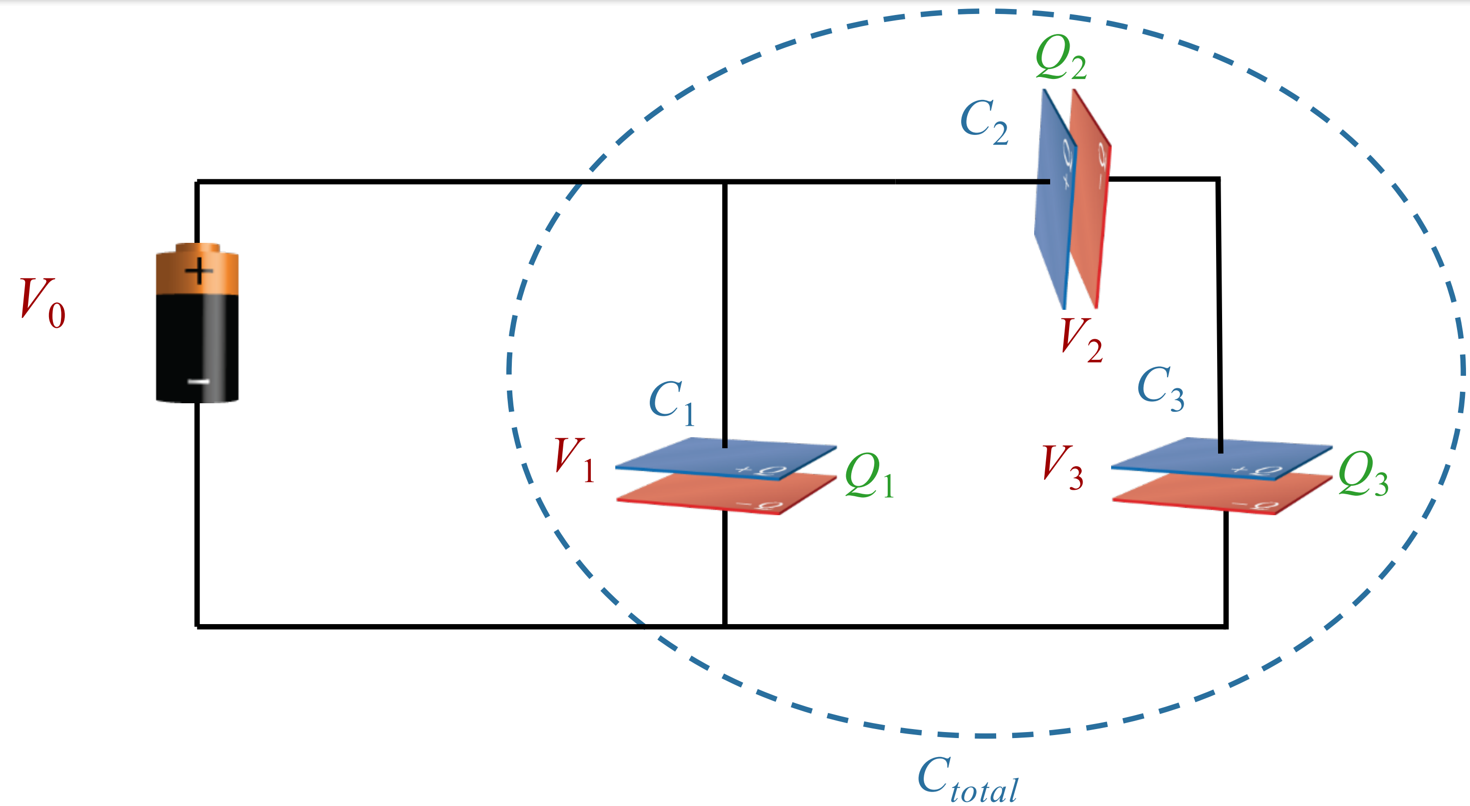
(a) $C_{eq} = (3/2)C$

(b) $C_{eq} = (2/3)C$

(c) $C_{eq} = 3C$

$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_1 = \frac{C}{2} \Rightarrow C_{eq} = C + \frac{C}{2} = \frac{3}{2}C$$

Similar to CheckPoint



Which of the following is NOT necessarily true:

A) $V_0 = V_1$
B) $C_{total} > C_1$
C) $V_2 = V_3$
D) $Q_2 = Q_3$
E) $V_1 = V_2 + V_3$