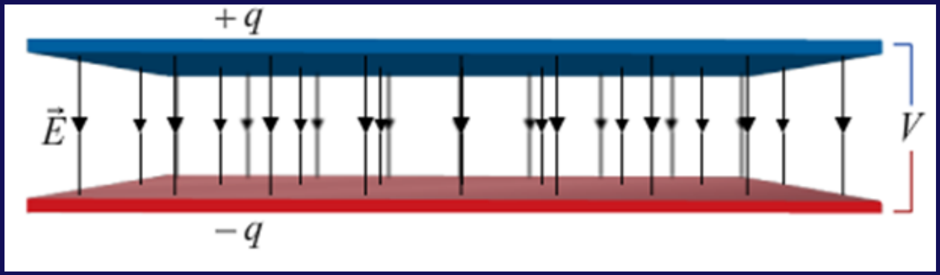
Conductors & Capacitance



- **PICK UP YOUR EXAM; Average of the three classes is *approximately 51*. Standard deviation is 18. It may go up (or down) by a point or two once all grading is finished.**

- **Exam KEY is posted on the home page**

- **REGRADE requests are DUE Friday at 4 PM in Alexis Olsho's office**
  - **Put your name on paper explaining your request reason.**
  - **Take it to Alexis Olsho's office yourself (1st floor of PAB, near h bar)**
  - **Same office as Susan Miller**

Phys 122 Lecture 10
Gray Rybka

Your thoughts

I am really confused by capacitance, especially with how/why it relates to potential.

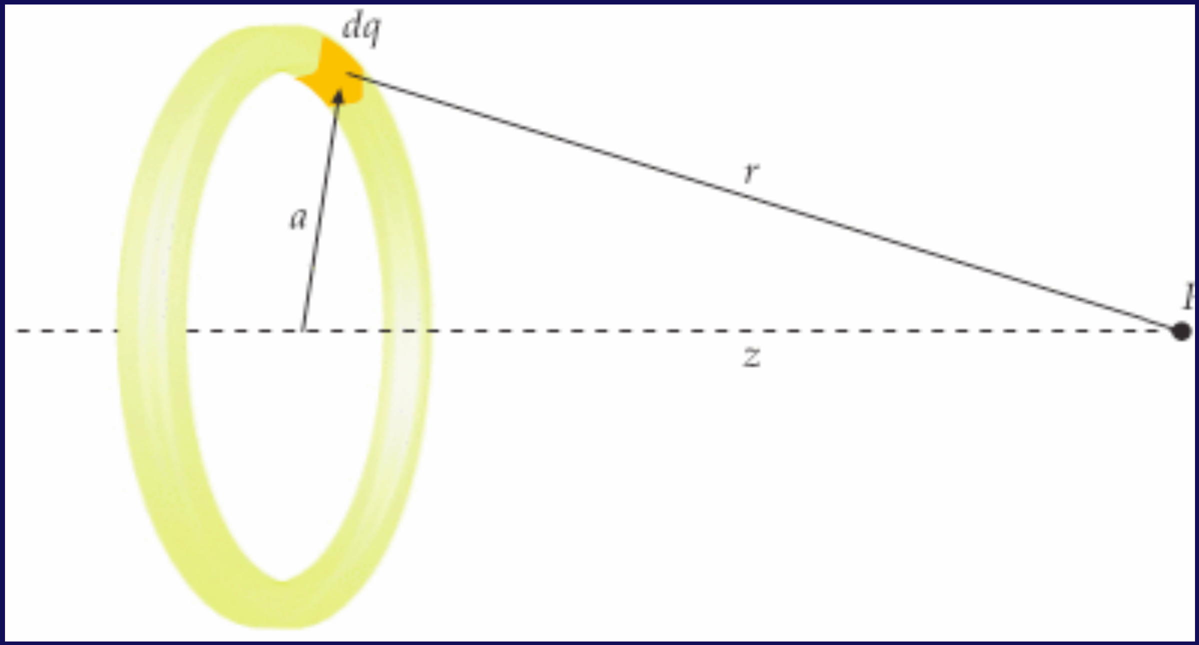
what are some practical applications of capacitors? What is the benefit of having a capacitor?

This is a little confusing to me mostly because of residual confusion from Electric potential, which I still don't understand that well conceptually. If you could talk thoroughly about the link between voltage and capacitance that would be nice.

We left off with Potential from a ring of charge

- Consider a uniformly charged ring with total charge of Q

$$V = \int \frac{k\,dq}{r} = \frac{k}{\sqrt{a^2 + z^2}} \int dq = \frac{kQ}{\sqrt{a^2 + z^2}}$$

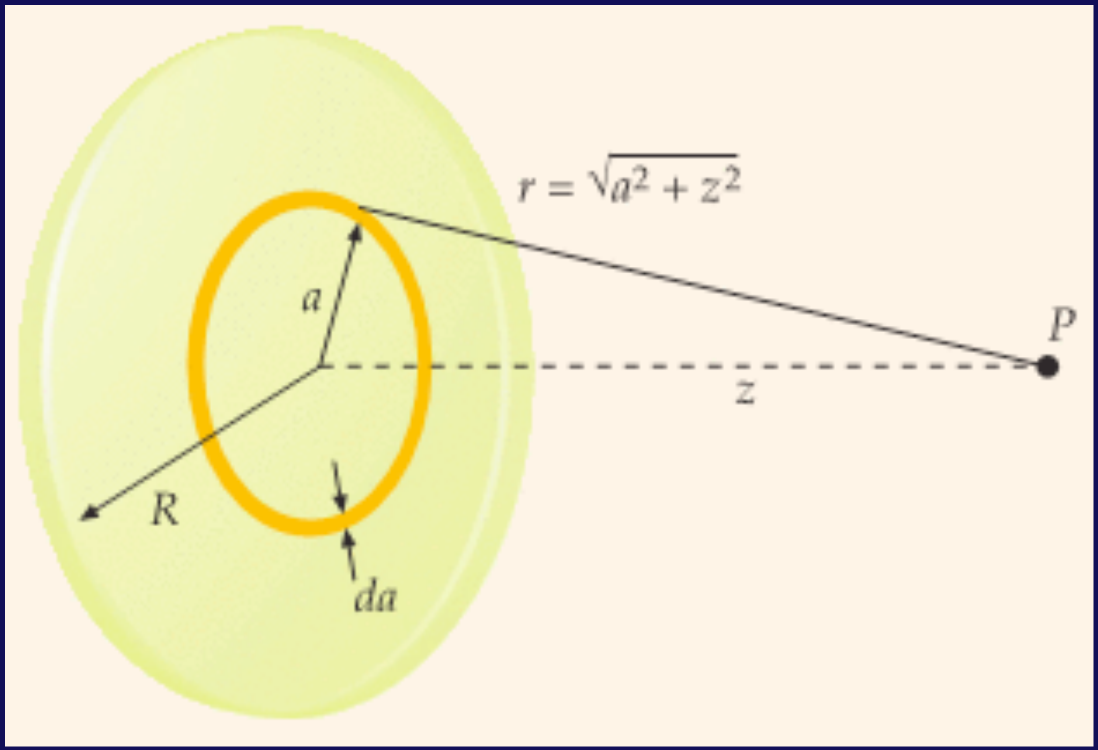


This can be defined anywhere with V = 0 assumed at infinity

Potential from a disk of charge

Uniformly charged disk with area charge density σ

Can we solve it as a series of charged rings?



$$V = \int_0^R \frac{k\sigma 2\pi a\,da}{\sqrt{a^2 + z^2}}$$

$$= k\sigma 2\pi \int_0^R \frac{a\,da}{\sqrt{a^2 + z^2}}$$

$$= 2\pi k\sigma \left[\sqrt{a^2 + z^2}\right]_0^R$$

$$= 2\pi k\sigma \left[\sqrt{R^2 + z^2} - |z|\right]$$

A short aside: Energy Units

MKS: U = QV þ 1 coul-volt = 1 joule

for particles (e, p, ...) 1 eV = 1.6×10^{-19} joules

Accelerators

- Electrostatic: VandeGraaff

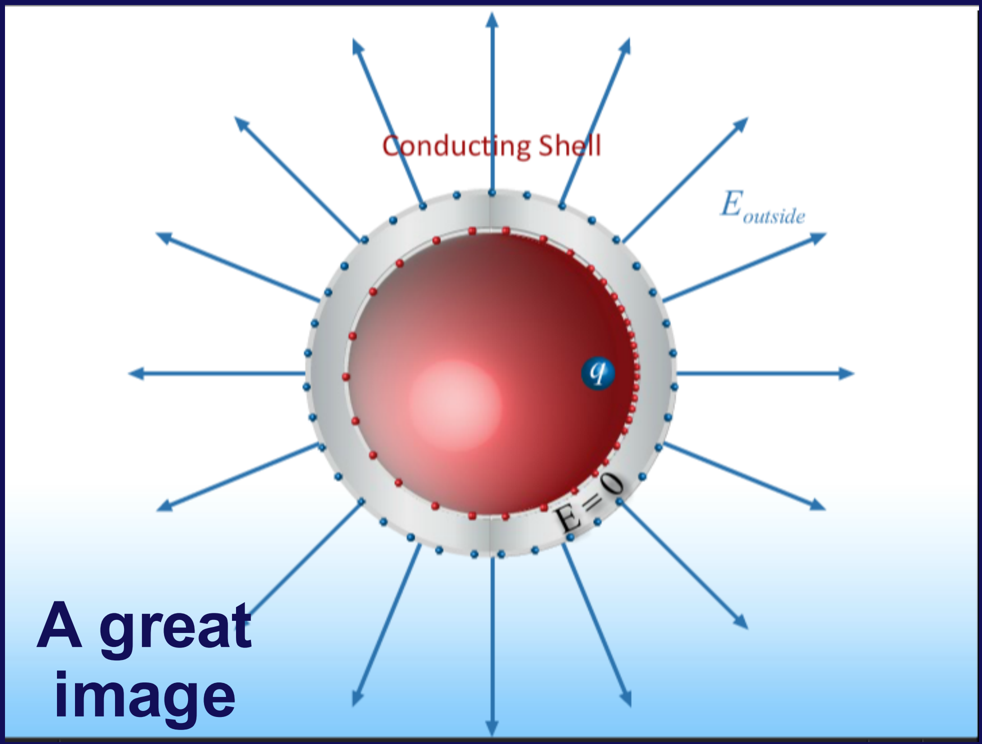
electrons → 100 keV (10^{5} eV)

- Electromagnetic: Large Hadron Collider

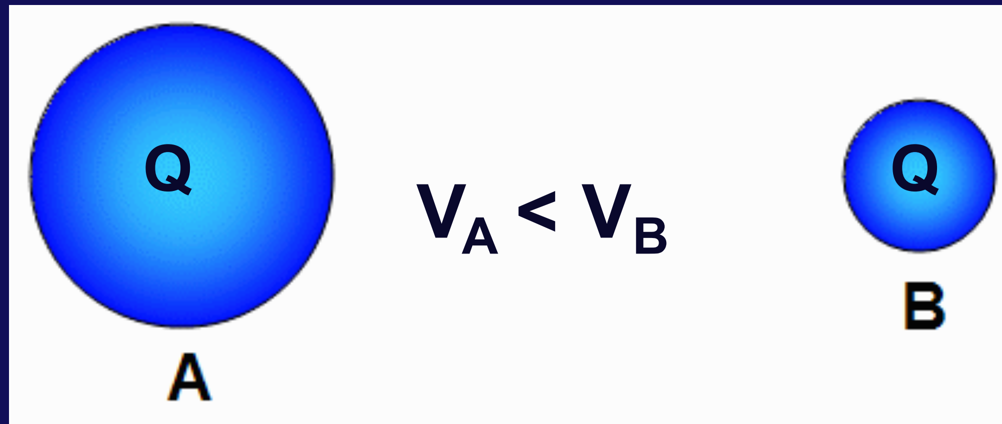
protons → 7 TeV (7×10^{12} eV)

Conductors: Main Points

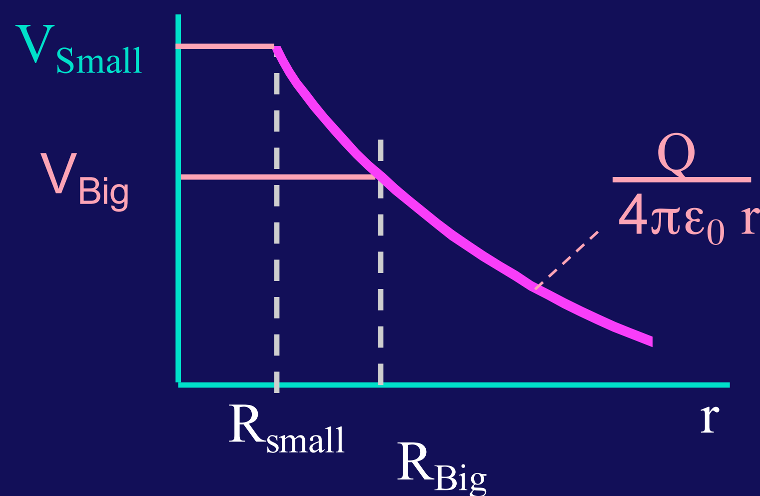
- Charges free to move
- *E* = 0 in a conductor
- Surface = Equipotential
- *E* at surface perpendicular to surface



A few minutes on Conductors

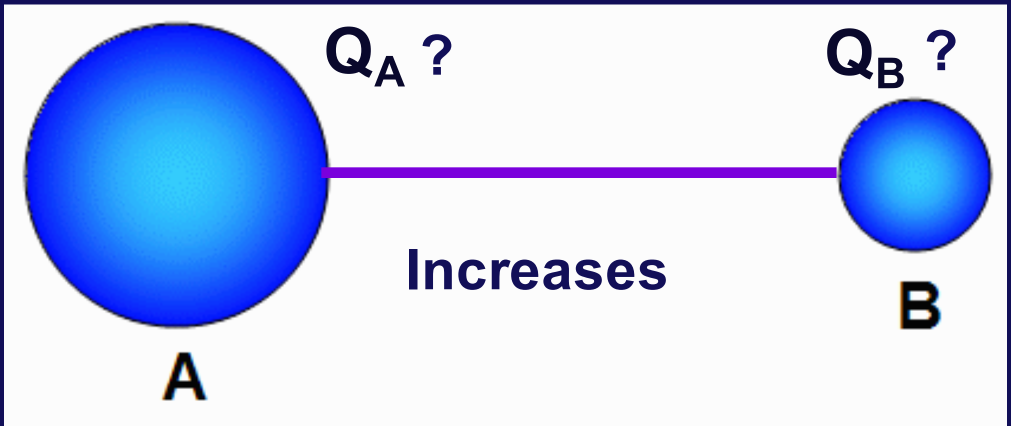


- Same total Q
- Different radii
- V at edge different



Checkpoints 1, 3, 5 … pretty good job here overall

A few minutes on Conductors



- Now the entire thing is a single conductor
  - E = 0 in a conductor (implies ΔV = 0 between any 2 points in a conductor
- If there was a potential difference, charges would move

What happens to the charge on conductor A after it is connected to conductor B by the wire?

The charges <u>do move</u> initially … enough to make $V_A = V_B$

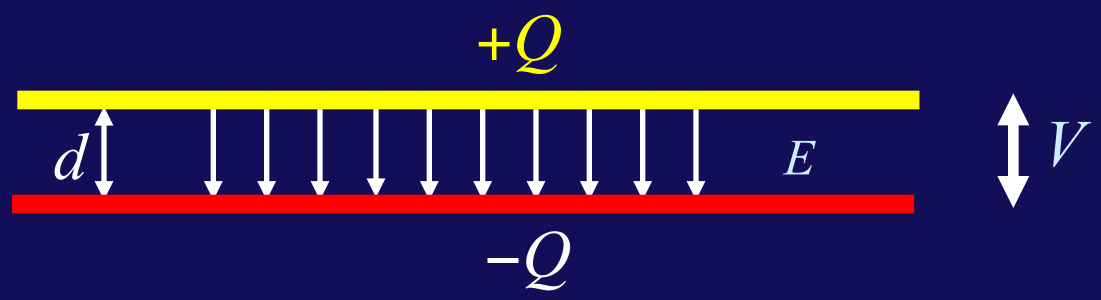
Potentials: $\frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}$ ⇨ $Q_A = \frac{R_A}{R_B} Q_B > 1$

Toward Capacitance

Capacitance is **defined** for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

Two conductors: one with excess charge = $+Q$
the other with excess charge = $-Q$

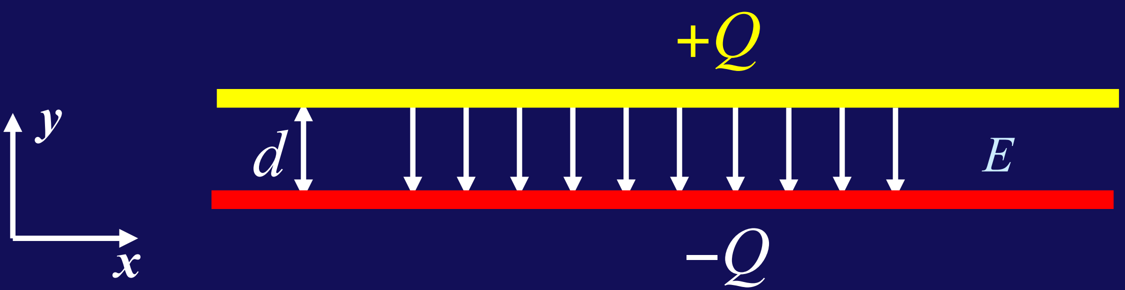


Charges create an electric field in the space between

We can integrate the electric field between them to find the potential difference between the conductor

This potential difference WILL be proportional to Q !
The ratio of Q to the potential difference is the capacitance
and it only depends on the geometry of the conductors

Example from Prelecture



$$E = \frac{\sigma}{\varepsilon_o}$$

What is σ ?

$$\sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

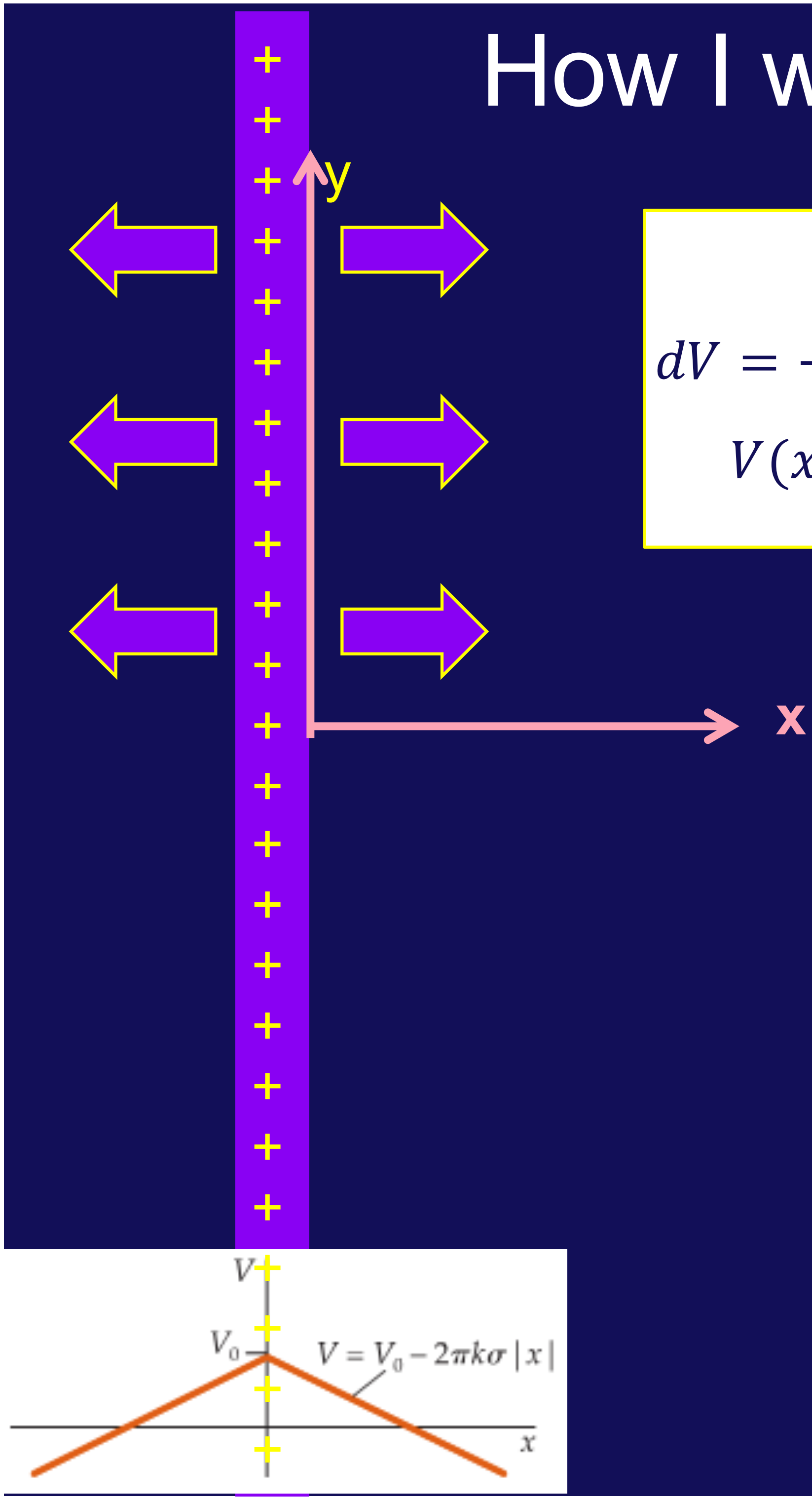
$$V = -\int_0^d \vec{E} \cdot d\vec{y} \quad \rightarrow \quad V = -\int_0^d (-E dy) = E \int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

As promised, V is proportional to Q !

$$C \equiv \frac{Q}{V} = \frac{\cancel{Q}}{\cancel{Q}d / \varepsilon_o A} \quad \rightarrow \quad C = \frac{\varepsilon_0 A}{d}$$

C determined by geometry! Not Q

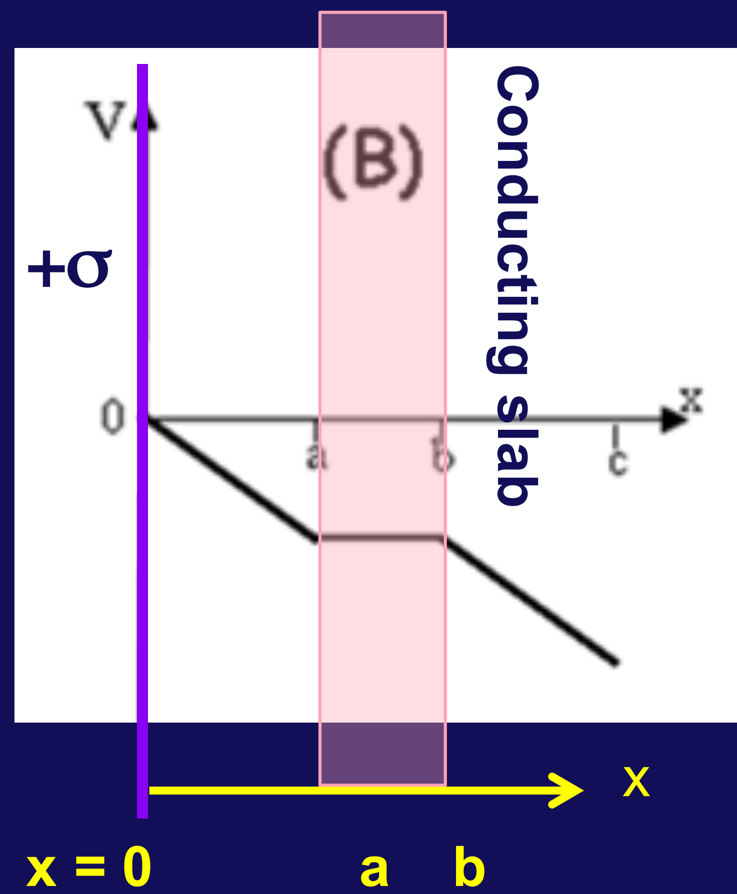
How I was getting you ready …



E = $\sigma/2\varepsilon_0$ **for x >0**

$$dV = -\vec{E}\cdot\overrightarrow{dl} = -2\pi k\sigma\,dx \qquad x > 0$$

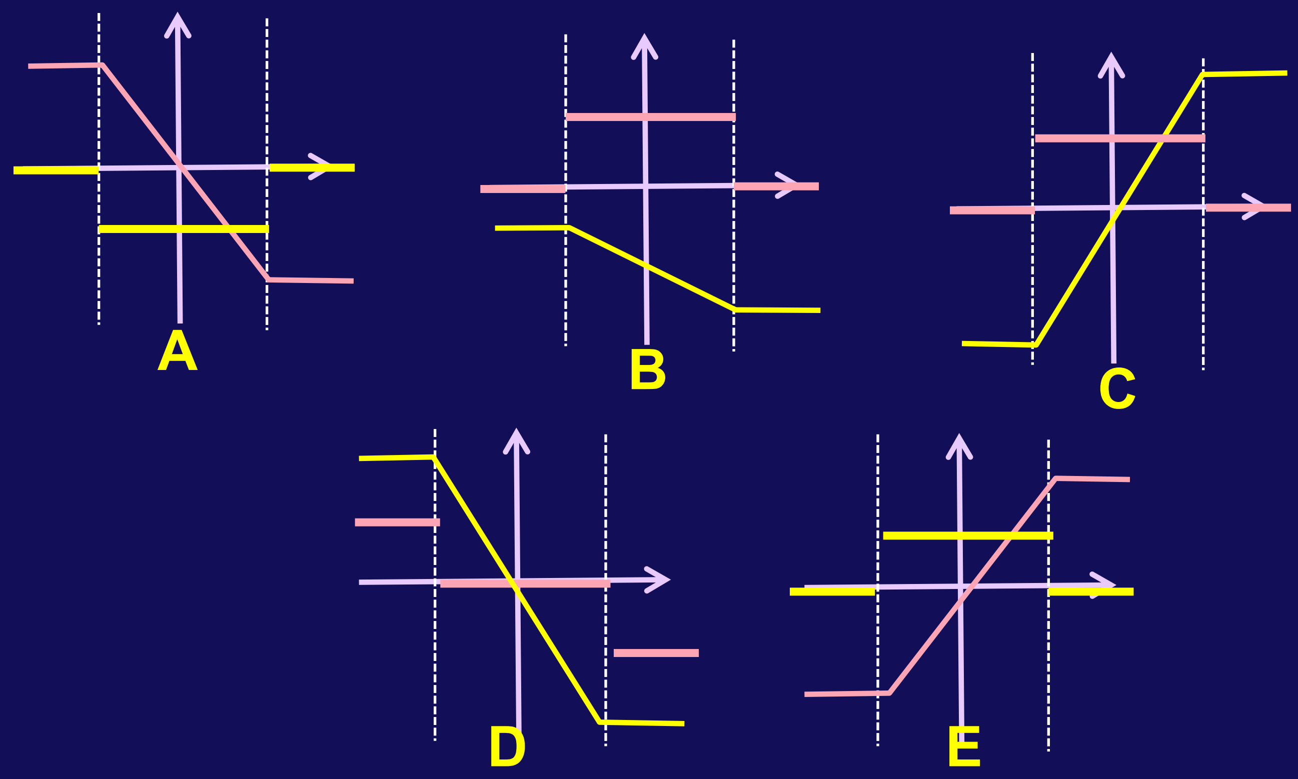
$$V(x) = \color{red}{-2\pi k\sigma\,x} + V_0 \qquad x > 0$$



Clicker: 2 planes

$+\sigma$ $-\sigma$

Two "infinite" oppositely charged parallel plates are located at –d and +d on the x axis. Which graphs best represent the Electric Field and the Potential Difference vs x ?



Clicker

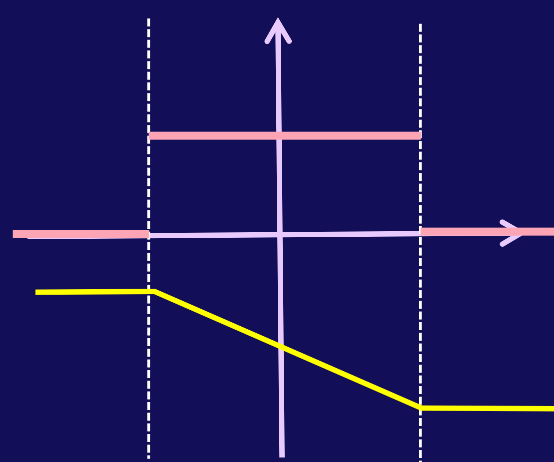
$+\sigma$ E=0

$-\sigma$ E=0

Two "infinite" oppositely charged parallel plates are located at –d and +d on the x axis. Which graphs best represent the Electric Field and the Potential Difference vs x ?

E constant

$$V(x) = -2\pi k\sigma\, x + V_0$$



B

Going green …

Please explain the green thing and why it does the things it does. :/

The charged plate questions were the most difficult, especially with the green conductor plates in the middle, I don't know how to account for that.

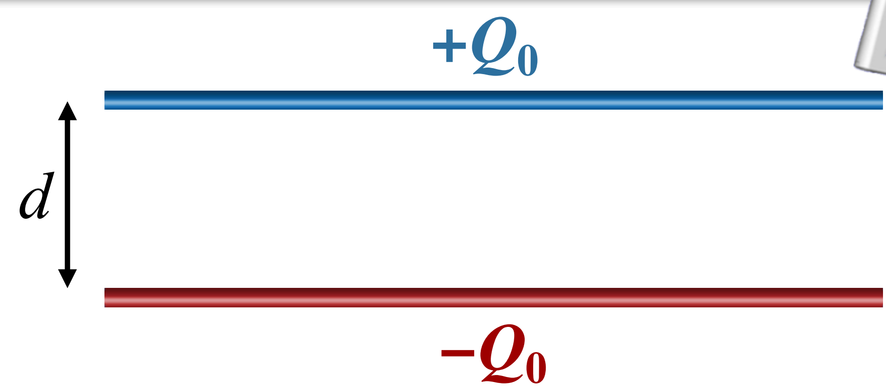
Can we discuss the second part of the second checkpoint question? C=Q/deltaV but C= epsilon_0*Area/distance. So what actually happens if you increase Q in such a way that keeps deltaV the same?? These formulas seem to contradict.

Warning: Lots of Green Thing Questions are next

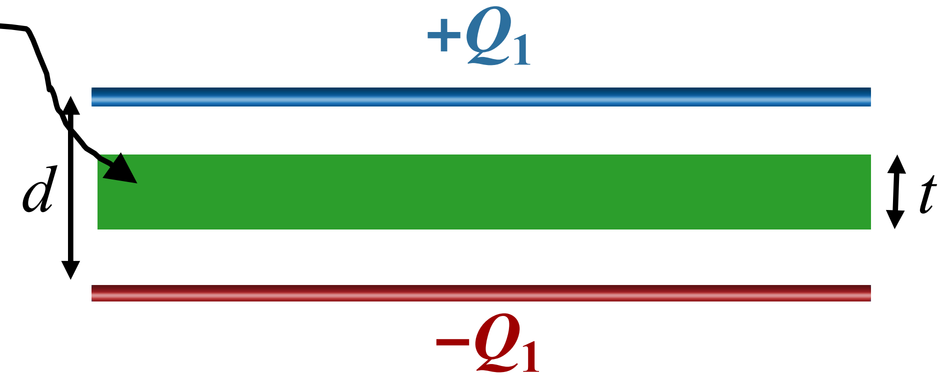
(these Checkpoint issues were more of a struggle)

Clicker: (not the Checkpoint)

Initial charge on capacitor $= Q_0$



Insert an uncharged conductor



Charge on capacitor now called Q_1

How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$

B) $Q_1 = Q_0$

C) $Q_1 > Q_0$

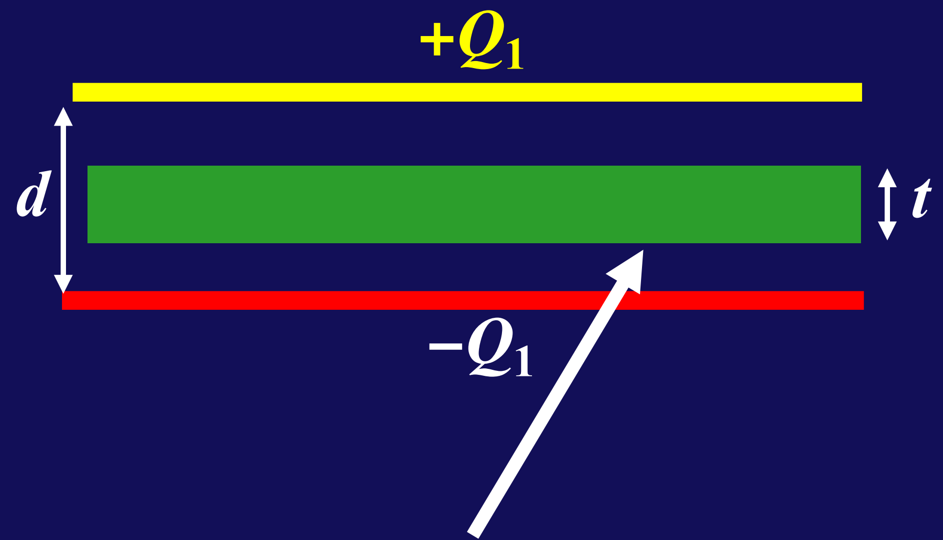
Plates not connected to anything

↓

CHARGE CANNOT CHANGE !

Clicker to reinforce …

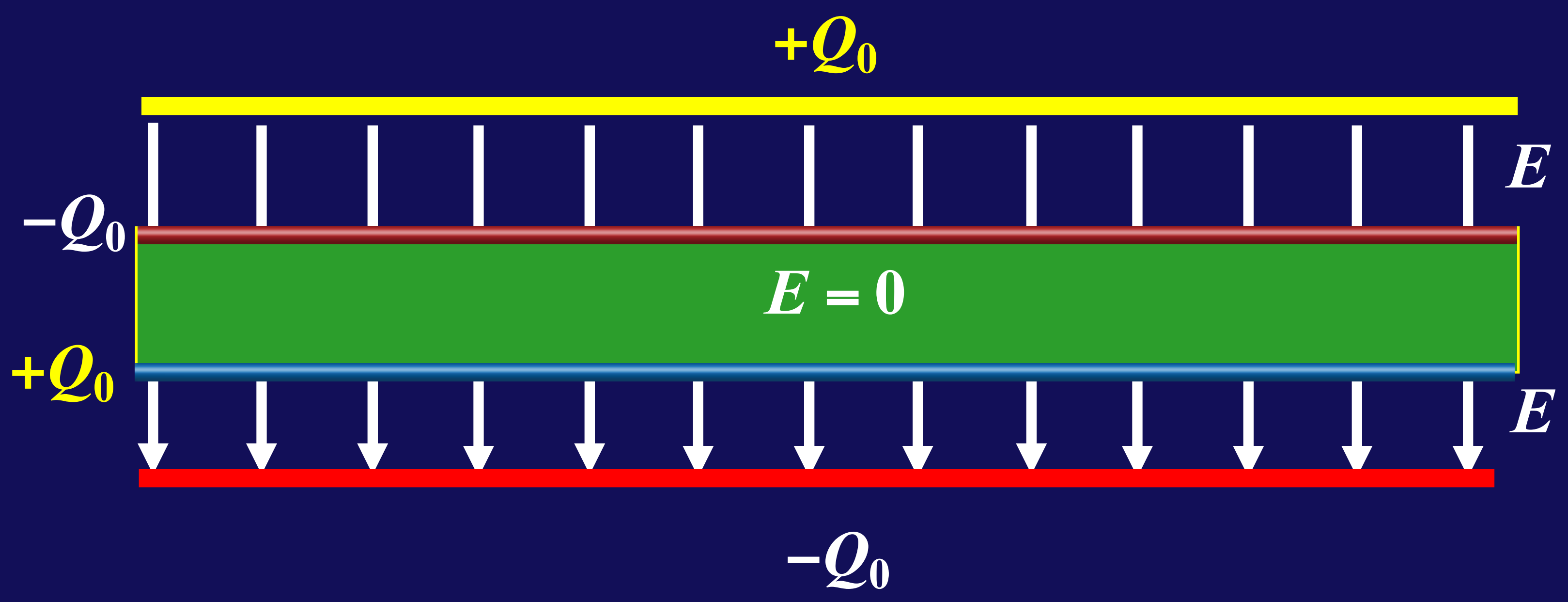
Initial charge on capacitor $= Q_0$



What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$
B) $-Q_0$
C) 0
D) **Positive but the magnitude unknown**
E) **Negative but the magnitude unknown**

Why ?



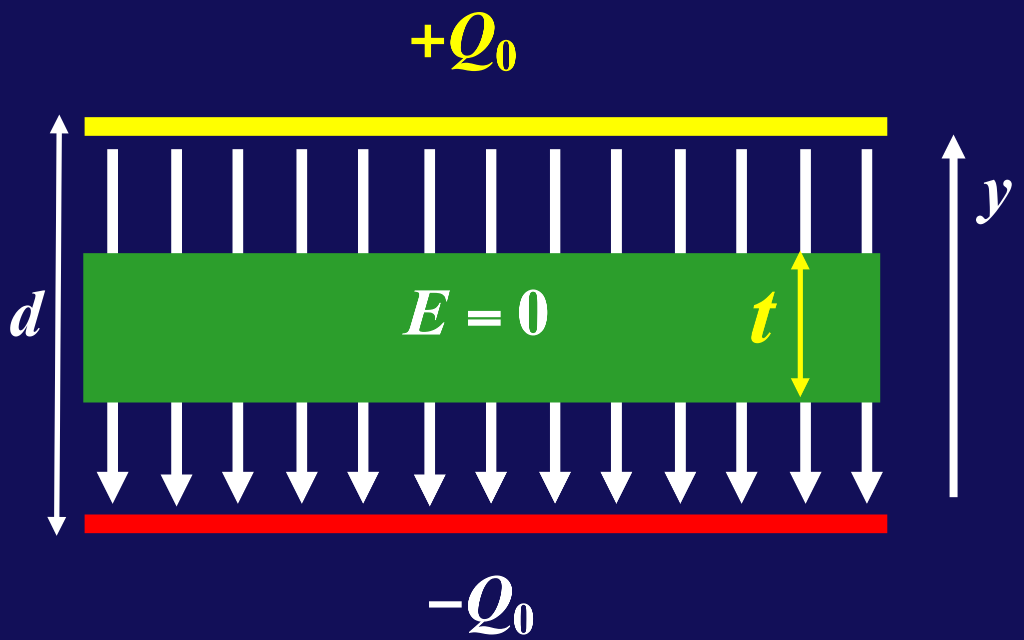
***E* must be = 0 in conductor !**

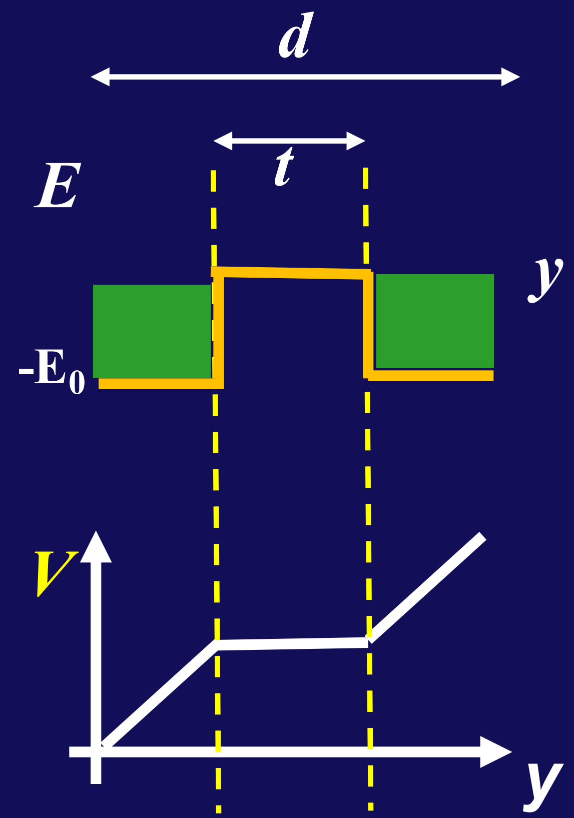
Charges inside conductor move to cancel *E* field from top & bottom plates.

Calculate V

Now calculate V as a function of distance from the bottom conductor
(define V=0 there).

$$V(y) = -\int_0^y \vec{E} \cdot d\vec{y}$$





What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

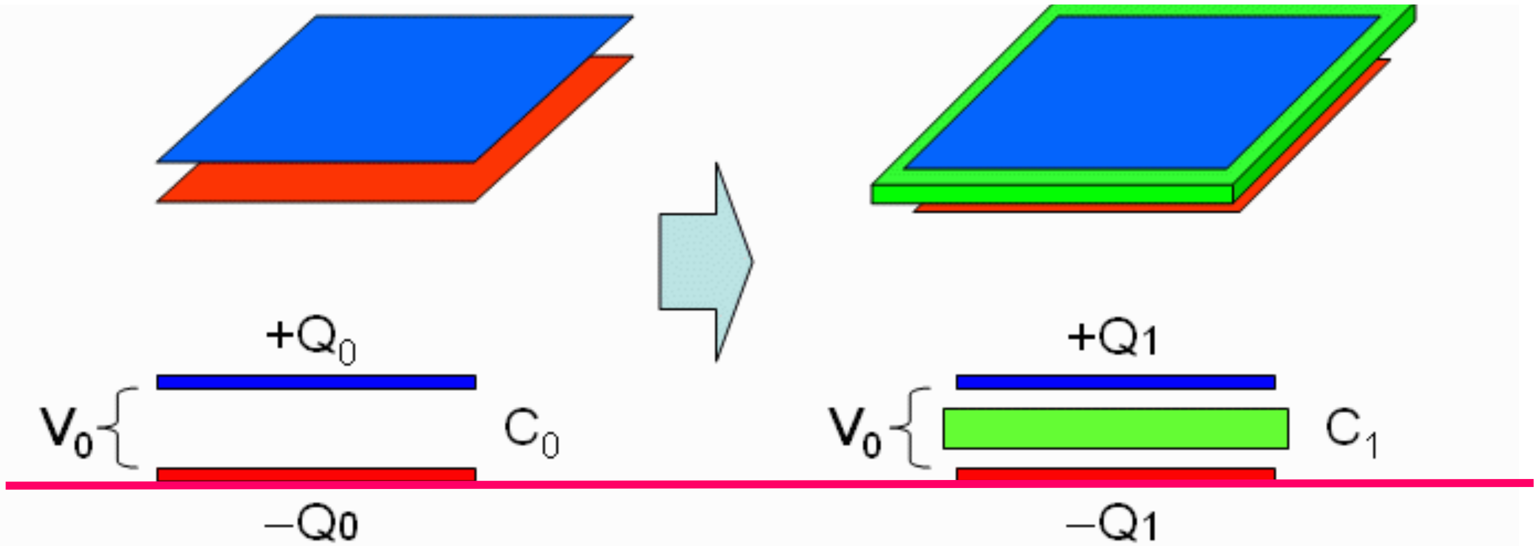
B) $\Delta V = E_0(d - t)$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

Now, Click on CheckPoint 8 again

Two parallel plates with charge Q_0 on each plate.
Potential difference is V_0.
Put in uncharged conductor (green)
ADJUST the potential difference to be the SAME as before: i.e, V_0.



How is charge Q_1 related to the original Q_0 ?

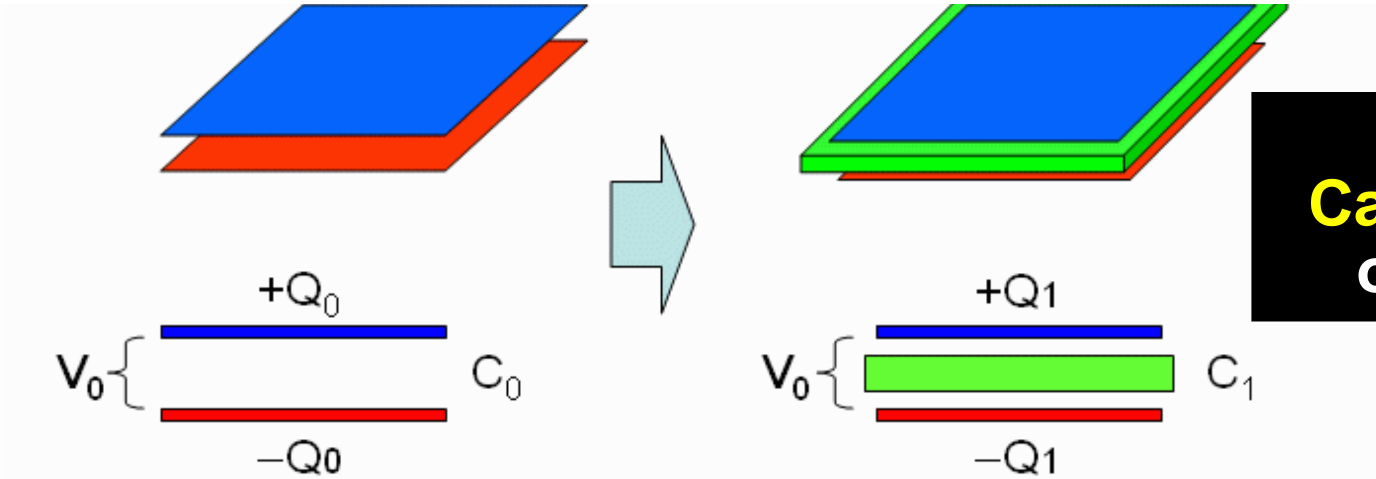
A) $Q_1 < Q_o$ B) $Q_1 = Q_o$ C) $Q_1 > Q_o$

We just found $\Delta V = E_2(d - t)$
In the 1st case $\Delta V = E_1(d)$
For **same ΔV** we need $E_2 > E_1$ implies $Q_1 > Q_o$

This charge came from some connected circuit that made V_0 the same

Let's click this one too …

Same setup. Potential difference is V_0. Insert conductor (green)
CHARGE Q_1 is ADJUSTED to make Potential Difference equal to V_0.
(see previous Clicker question where Q_1 had to increase)



How does the **Capacitance** of the object change?

A) $C_1 > C_0$ B) $C_1 = C_0$ C) $C_1 < C_0$

Same V_0 → $C_0 = Q_0 / V_0$ *and* $C_1 = Q_1 / V_0$
Therefore, since $Q_1 > Q_0$ *it means* $C_1 > C_0$

Alternate explanation using Previous situation

Same Q_0 → $V_0 = E_0 d$ *and* $V_1 = E_0(d - t)$
$C_0 = Q_0 / E_0 d$ *and* $C_1 = Q_0 / (E_0(d - t))$
→ $C_0 = \varepsilon_0 A / d$ *and* $C_1 = \varepsilon_0 A / (d - t)$

PROPERTY OF GEOMETRY ONLY