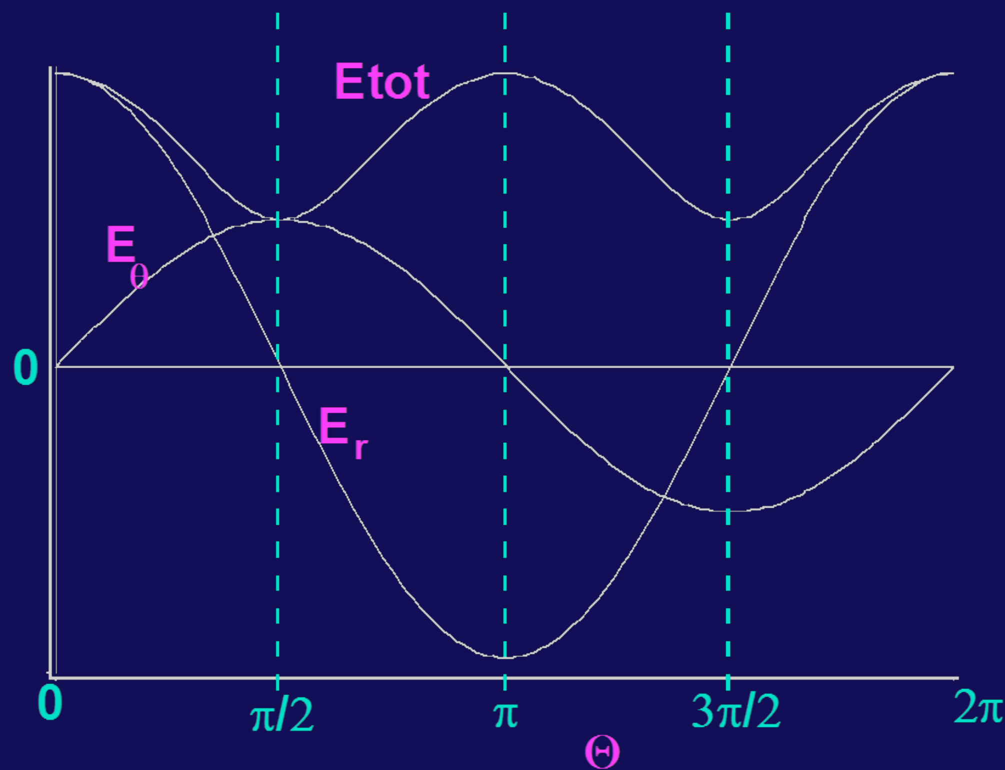
More Potential & "E from V"

What is this ?



- **Exam grading still in progress. Expect them on Wednesday**
- **SmartPhysics "Conductors and Capacitance" due Wed.**

Phys 122 Lecture 9
G. Rybka

First, some Demos on E = 0 in conductor

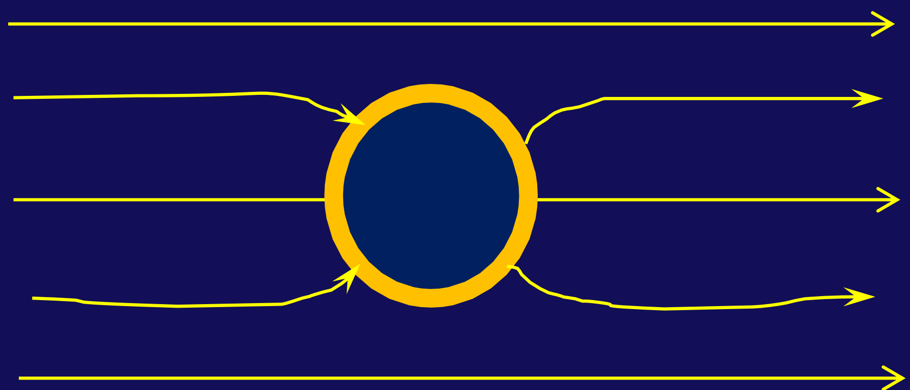
Consider an E field | **Move a conductor into the field**

Field Lines will adjust to make E = 0 inside

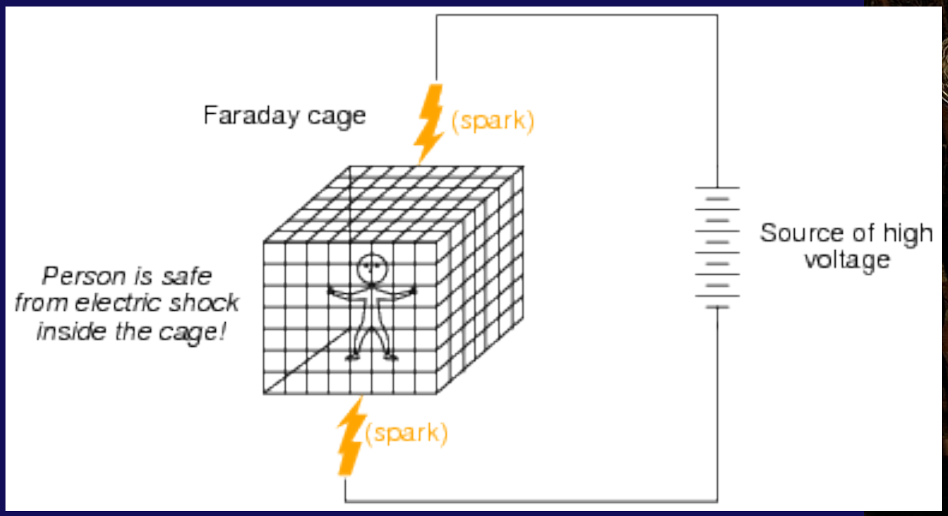
First, some Demos on E = 0 in conductor

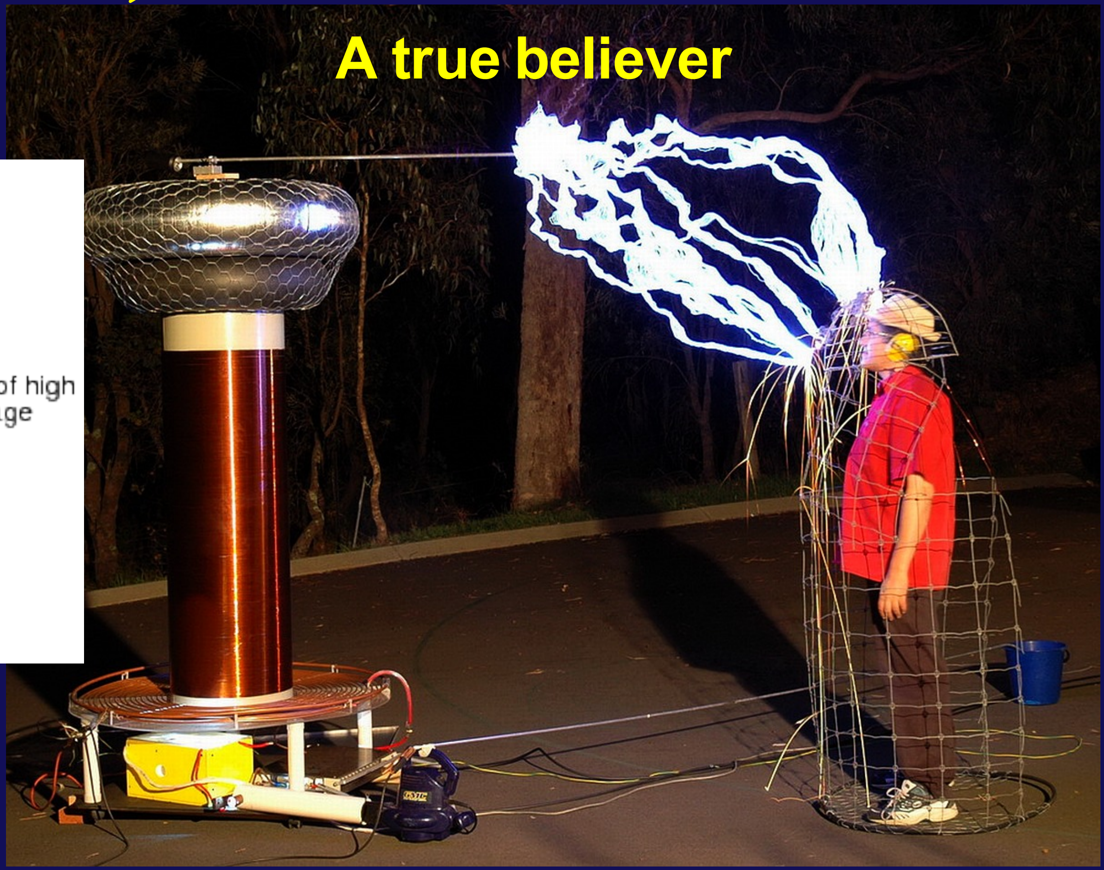
Consider an E field

Move a conductor into the field



Field No E in Conductor





Remember: V from E & E from V

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Electric potential , a property of the space

$$\vec{\mathrm{E}} = -\vec{\nabla}\mathrm{V}$$

Electric field , also a property of the space

Examples of Gradient

$$\vec{\nabla}\mathrm{V} = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

$$\vec{\nabla}\mathrm{V} = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

What is electric potential?

X_1 at ∞

- **Define the electric potential of a point in space as the potential difference between that point and a reference point, V_2-V_1.**
  - A good reference point is often* infinity … we typically set $\mathbf{V} \;\; = \mathbf{0}$
  - The electric potential is then defined as:

X_2 Q

$$V(r) \equiv V_r - V_\infty$$

  - The potential from a point charge is then:

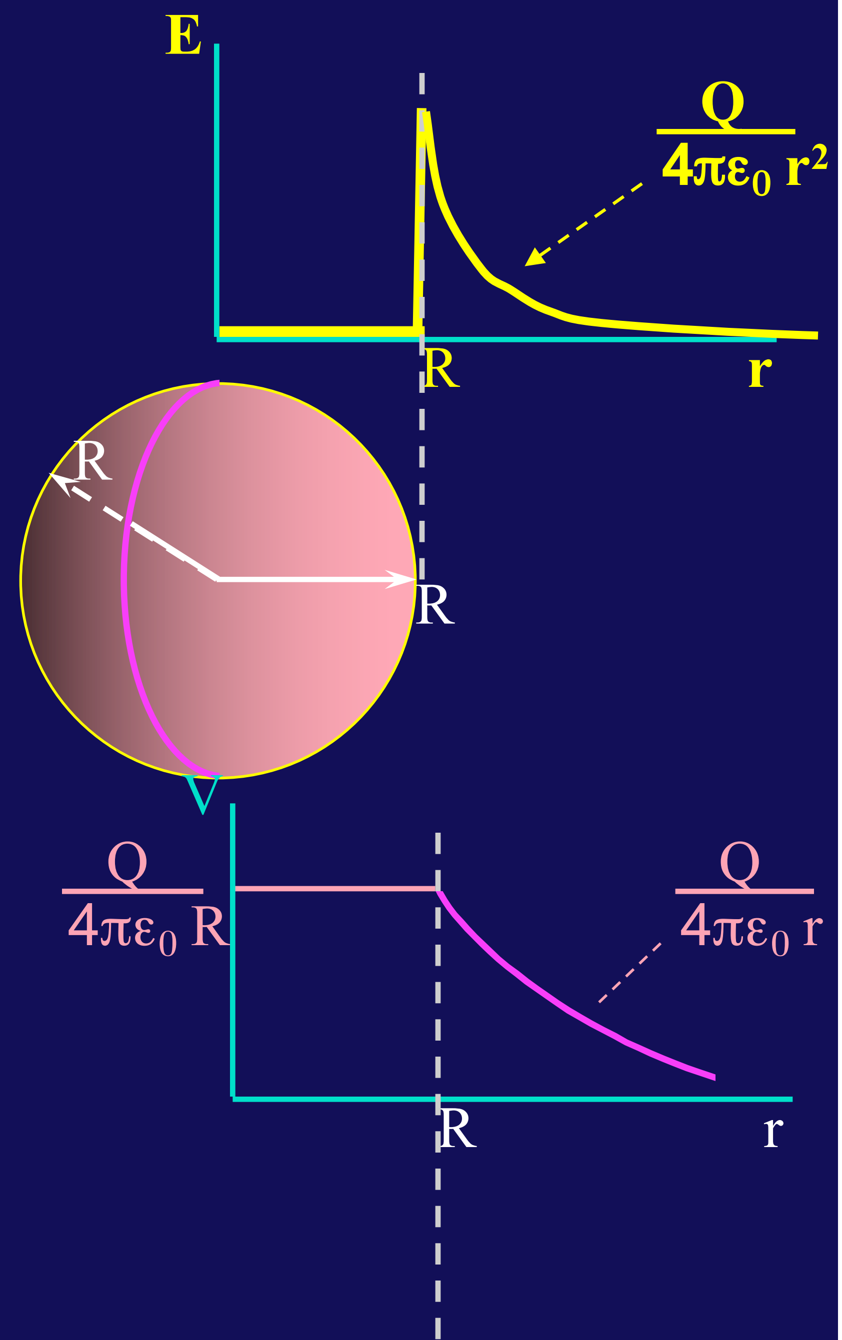
$$V(r) \equiv V_r - V_\infty = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

* Often → sets of point charges. Will not be true for “infinitly” large charged sheets; see later in lecture

Last time we sketched the Potential from charged a spherical conducting shell

- Given E from Gauss' Law

- We obtained V by integration



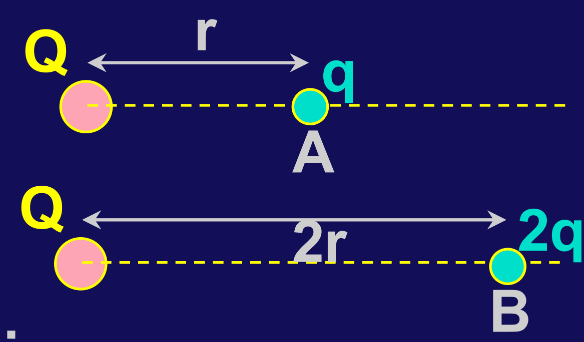
Notes:

1. E can be discontinuous at boundary
2. V must be continuous at boundary
3. Here, we set V = 0 at infinity
   - That will not always be possible
   - V plot is really "potential difference"

Clicker (think carefully)

Two test charges are brought separately to the vicinity of positive charge Q.

- **Charge +q is brought to A, a distance r from Q.**
- **Charge + 2q is brought to B, a distance 2r from Q.**
- **Compare the potential at A (V_A) to that at B:**



(a) $V_A < V_B$ (b) $V_A = V_B$ (c) $V_A > V_B$

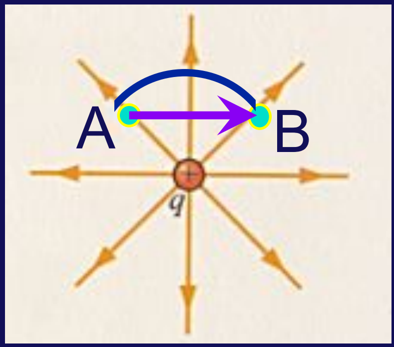
The Potential is a function of the space … not the test charge q or 2q

A positive "test charge" would move from point A towards point B. (repulsion from E field at A. Therefore, $V_A > V_B$

Since B is twice as far from Q as A, $V_A = 2\ V_B$

Clicker

A positive charge Q is moved from A to B along the path shown. What is the sign of the work done to move the charge from A to B?



(a) $W_{AB} < 0$ (b) $W_{AB} = 0$ (c) $W_{AB} > 0$

A direct calculation of the work along arrow is not easy

Magnitude and Direction of the E field are changing along that straight path from A to B,

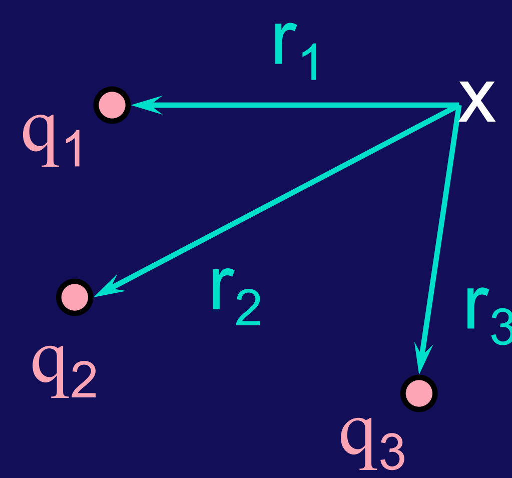
i.e., the integrand $\vec{E} \cdot d\vec{l}$ is messy

Remember: potential difference is independent of the path, so, take any path you wish. Here we move along an equipotential

Choose a path along the arc of a circle centered at the charge. Along this path $\vec{E} \bullet d\vec{l} = 0$ at every point!!

Potential from N charges …

At point X is just the algebraic sum of the potential due to each charge separately

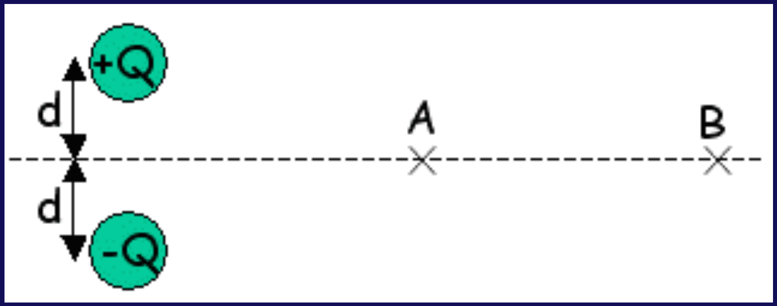


$$V_X = -\int_{\infty}^{r} \sum_{n=1}^{N} \vec{E}_n \bullet d\vec{l}$$

Þ $$V_X = \sum_{n=1}^{N} V_n(r) = \frac{1}{4\pi\varepsilon_0} \sum_{n=1}^{N} \frac{q_n}{r_n}$$

Where r_n is distance from charge to point X

Clicker (70% missed this on first attempt in PreLecture last week)



An electric dipole with charge magnitude Q and separation 2d is shown. Compare V_A, the electric potential at point A, with V_B, the electric potential at point B.

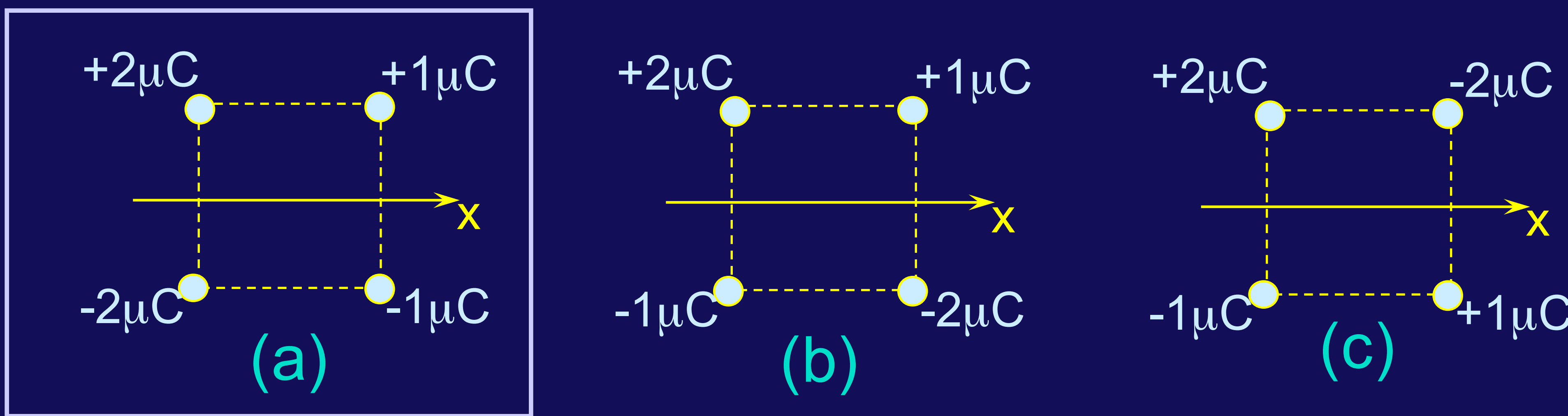
(a) $V_A < V_B$ (b) $V_A = V_B$ (c) $V_A > V_B$

Both +Q and –Q are same distances from A or B and have opposite sign

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r_A} + \frac{-Q}{r_A}\right) = 0 = V_B$$

Clicker

Which of the following charge distributions produces V(x) = 0 for all points on the x axis? (we are defining $V(x) \equiv 0$ at $x = \infty$)



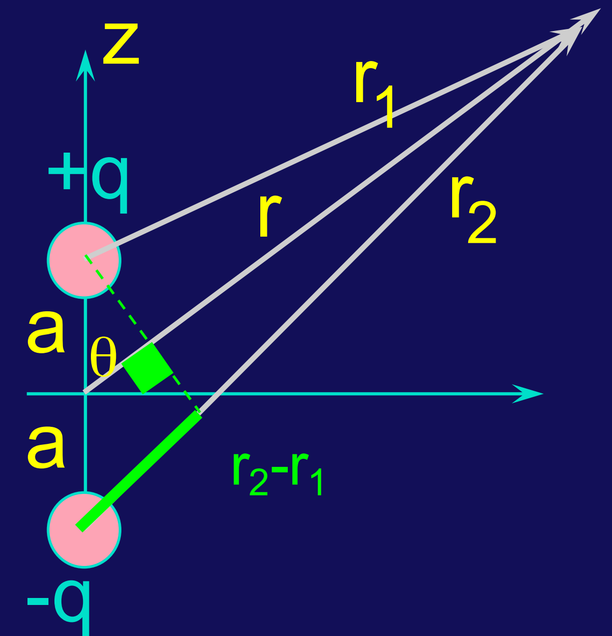
Need the ALGEBRAIC sum of the individual contributions

To make V(x) = 0 for all x, we must have the +Q and -Q contributions cancel, which means that any point on the x-axis must be equidistant from +2μC and -2μC and also from +1μC and -1μC.

This condition is met only in case (a)!

Electric Dipole: $V(r,\theta)$

The potential is much easier to calculate than the field since it is an algebraic sum of 2 scalar terms.



$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_1 r_2}$$

- Rewrite this for special case $r >> a$:

$$r_2 - r_1 \approx 2a\cos\theta$$

$$r_1 r_2 \approx r^2$$

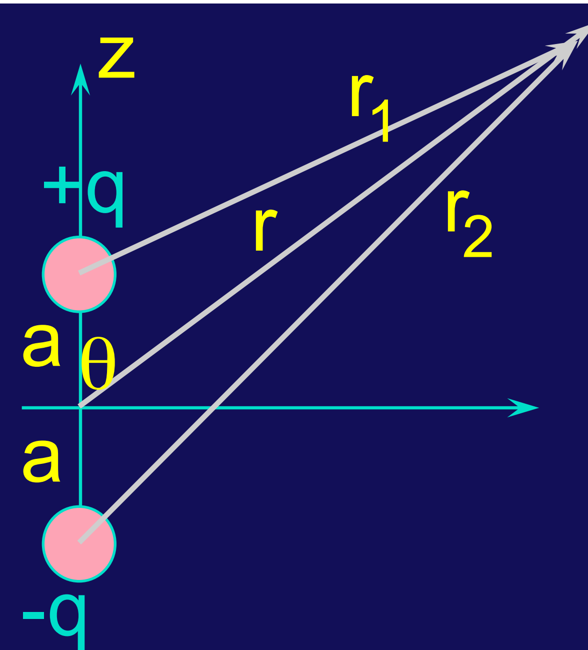
$\Rightarrow$

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2}$$

Can we use this potential somehow to calculate the E field of a dipole?

(the direct calculation was messy)

Electric Dipole: $\vec{E}(r,\theta)$



Start with $V(r,\theta)$ for r >> a:

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2}$$

To calculate E, need gradient in SPC.

(but no ϕ dependence)
$$\vec{\nabla}V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{2aq}{4\pi\varepsilon_0}\left(\frac{-2\cos\theta}{r^3}\right)$$

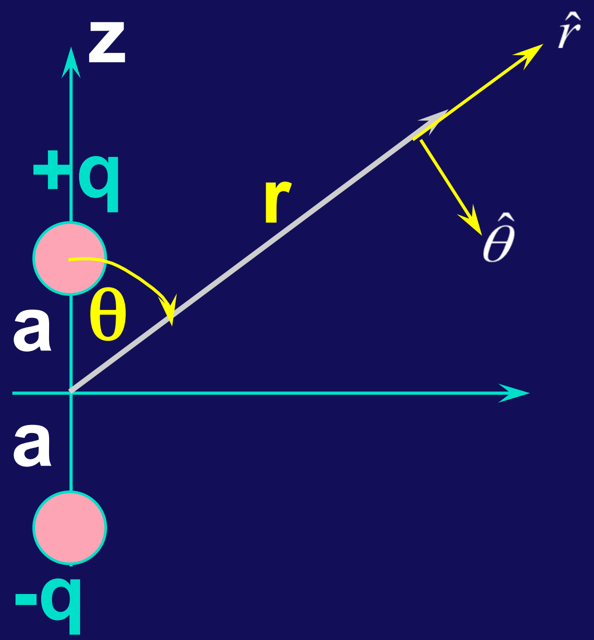
$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\frac{2aq}{4\pi\varepsilon_0}\left(\frac{-\sin\theta}{r^3}\right)$$

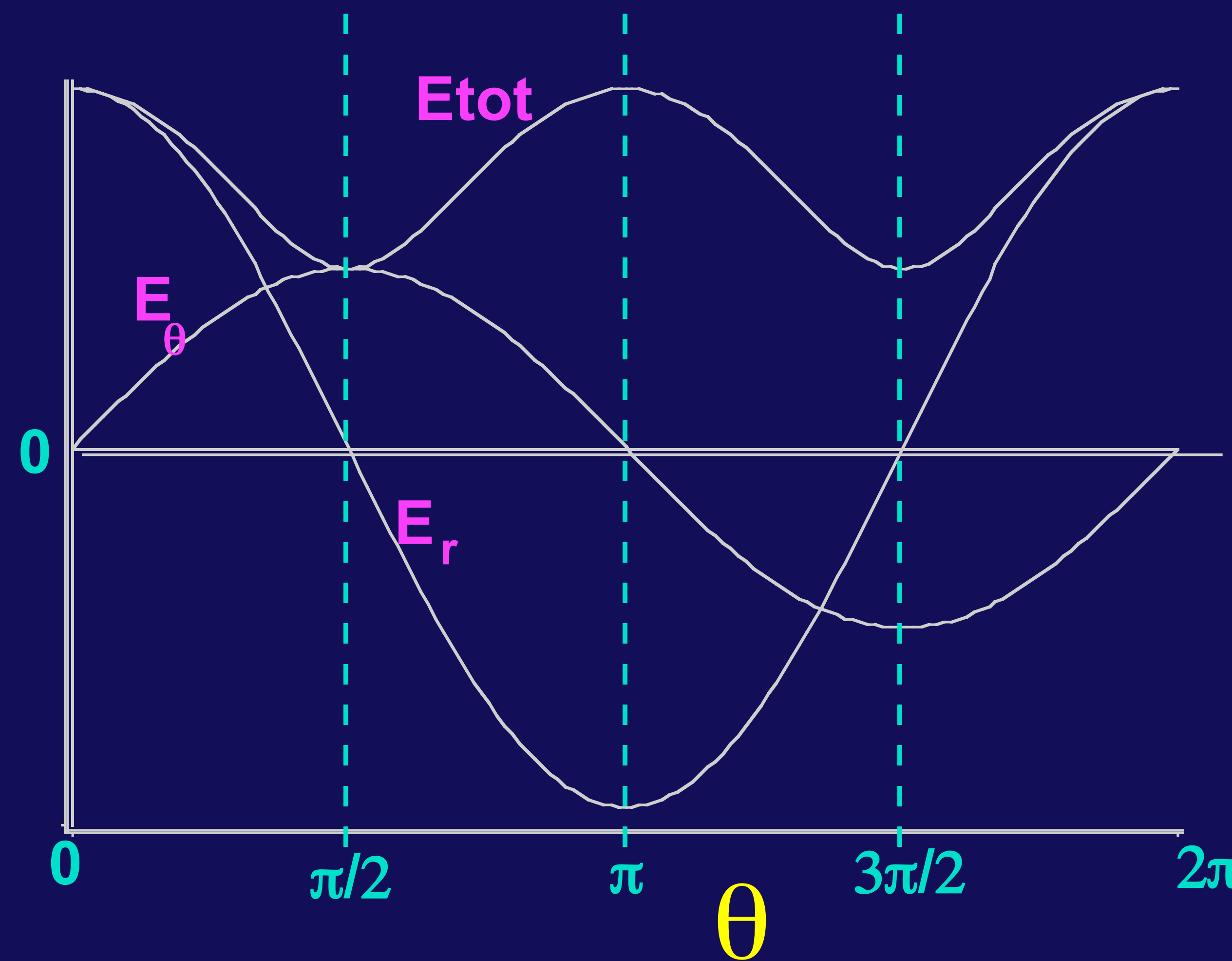
the dipole moment !

Þ

$$\vec{E} = \frac{2aq}{4\pi\varepsilon_0 r^3}\left((2\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}\right)$$

Dipole Field vs θ for a fixed r





$$\vec{E} = \frac{2\,aq}{4\pi\varepsilon_0 r^3}\left(\left(2\cos\theta\right)\hat{r} + (\sin\theta)\hat{\theta}\right)$$

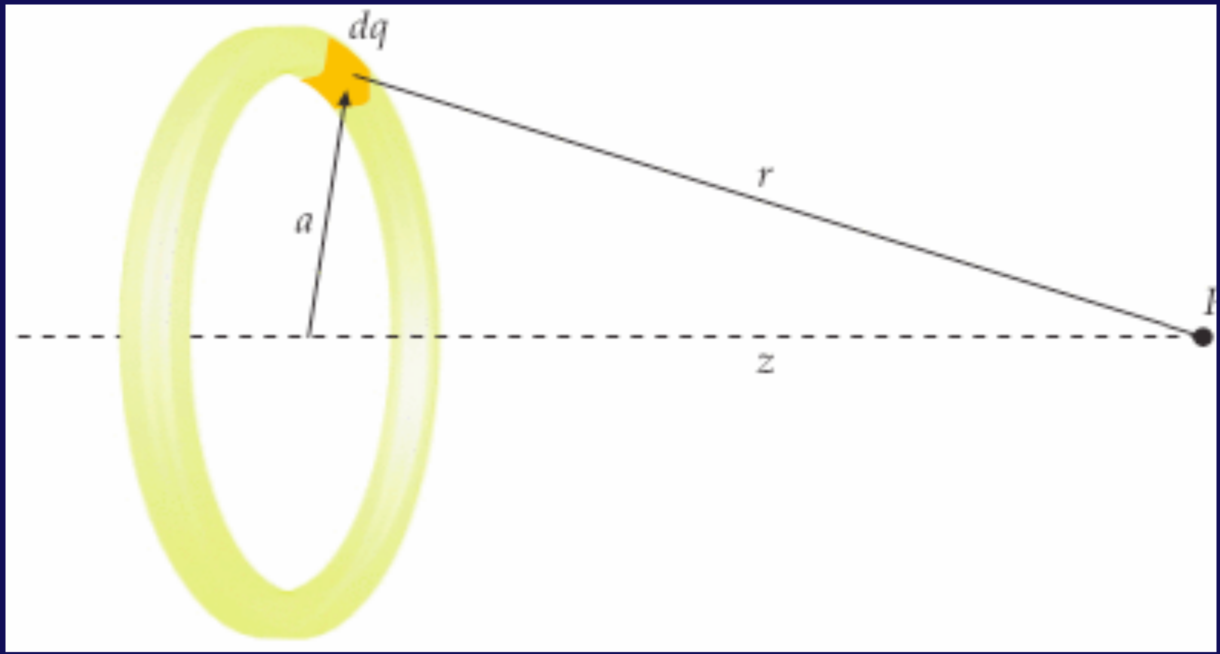
Potential from a ring of charge

- Consider a uniformly charged ring with total charge of Q

$$V = \int \frac{kdq}{r}$$

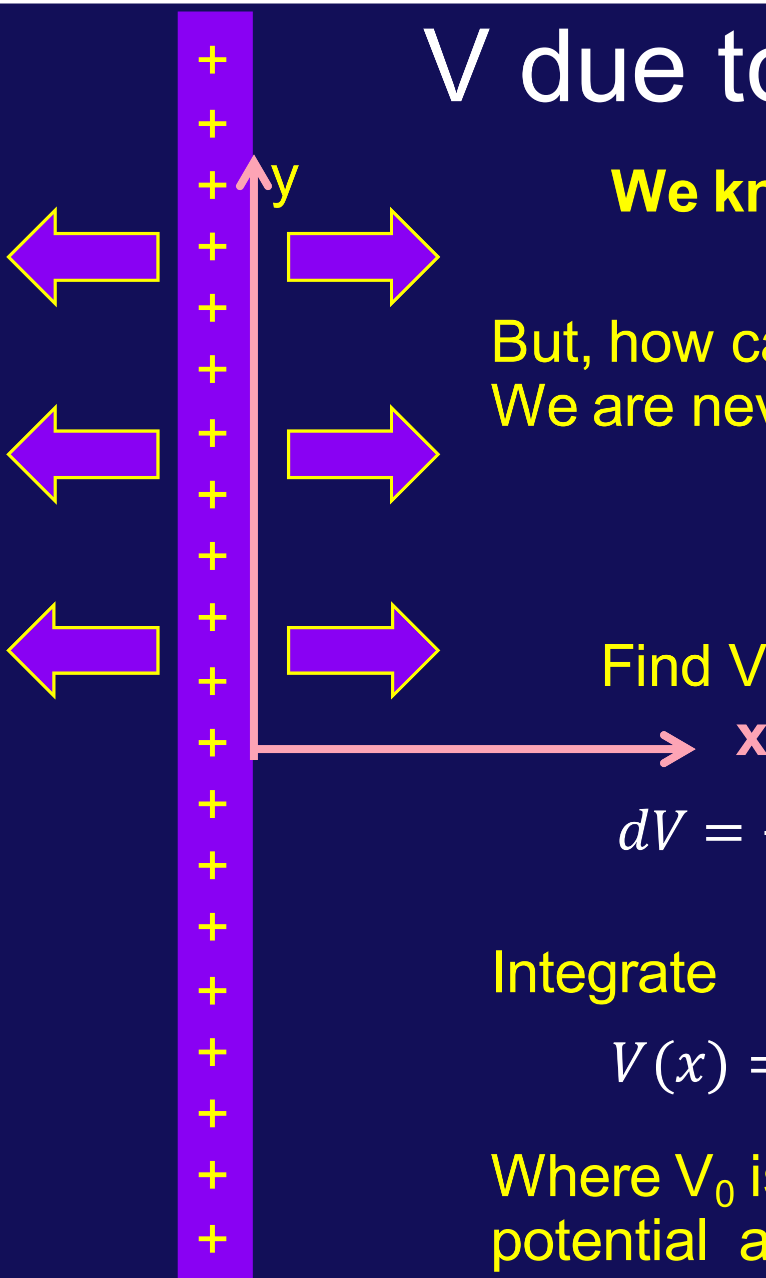
$$= \frac{k}{\sqrt{a^2 + z^2}} \int dq$$

$$= \frac{kQ}{\sqrt{a^2 + z^2}}$$



This can be defined anywhere with V = 0 assumed at infinity

V due to infinite Plane of Charge



We know E = $\sigma/2\varepsilon_0 = 2\pi k\sigma$ for x >0

But, how can V = 0 at "infinity" since
We are never infinitely far from the charges??

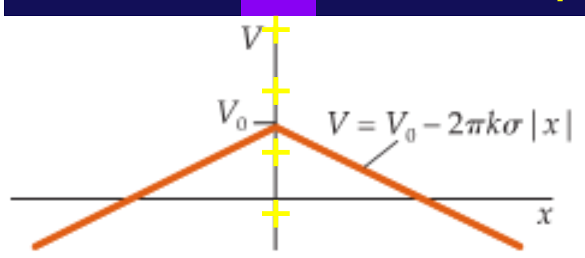
Find V from its defining relation:

$$dV = -\vec{E}\cdot\vec{dl} = -2\pi k\sigma\, dx \qquad x > 0$$

Integrate

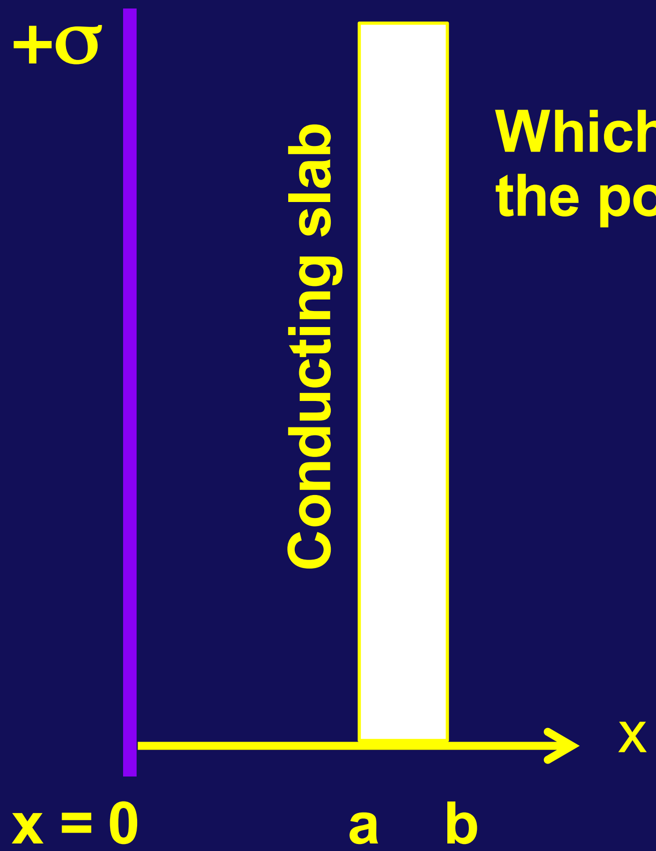
$$V(x) = -2\pi k\sigma\, x + V_0 \qquad x > 0$$

Where V_0 is constant of integration defined as the potential at x = 0

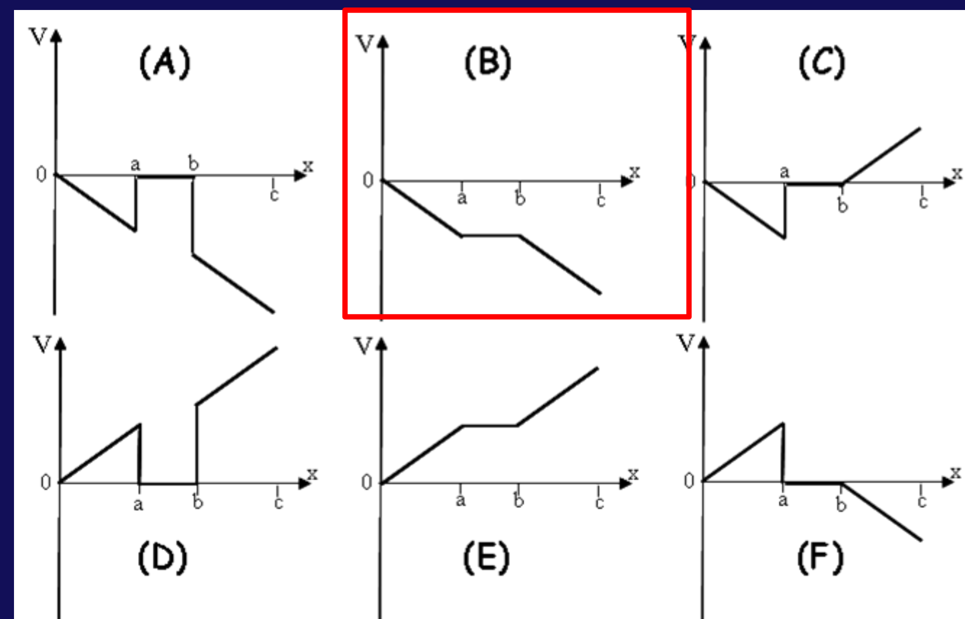


It would look like this

Clicker



Which curve best represents the Potential in the positive x direction?



- E is constant between x = 0 and x = a
  - V must be a linear function of x, downward since **sigma is positive**
- E = 0 in conductor
  - V = constant
- E has same value to right of conductor
  - V continues downward with same slope