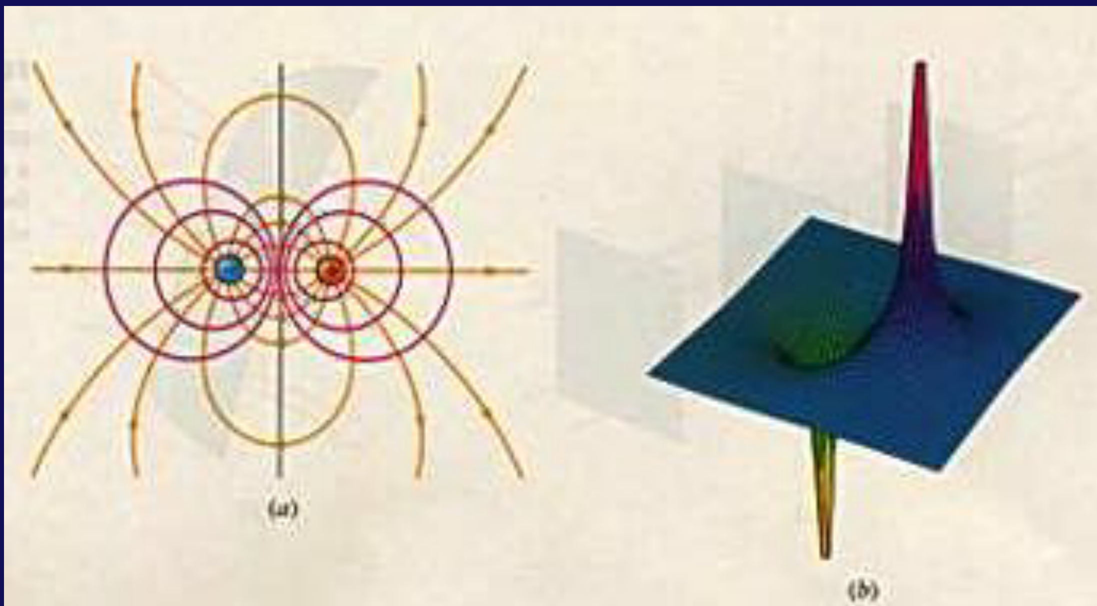
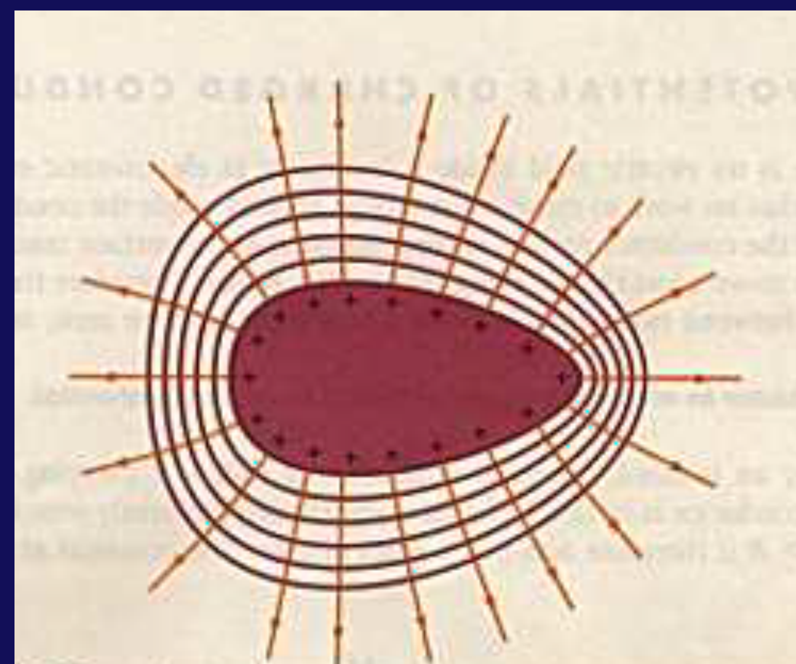
Electric Potential

Equipotentials and Energy





Phys 122 Lecture 8
G. Rybka

Your Thoughts

- **Nervousness about the midterm**

  I would like to have a final review of all the concepts that we should know for the midterm. I'm having a lot of trouble with the homework and do not think those questions are very similar to what we do in class, so if that's what the midterm is like I think we need to do more mathematical examples in class.

- **Confusion about Potential**

  I am really lost on all of this to be honest

  I'm having trouble understanding what Electric Potential actually is conceptually.

- **Some like the material!**

  These are yummy.

  I love potential energy. I think it is fascinating. The world is amazing and physics is everything.

A few answers

Why does this even matter? Please go over in detail, it kinda didn't resonate with practical value.

This is the first pre-lecture that really confused me. What was the deal with the hill thing?

don't feel confident in my understanding of the relationship of E and V. Particularly calculating these. If we go into gradients of V, I will not be a happy camper...

The Big Idea

Electric potential ENERGY of charge q in an electric field:

$$\Delta U_{a\to b} = -W_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

New quantity: Electric potential (<u>property of the space</u>) is the Potential ENERGY *per unit of charge*

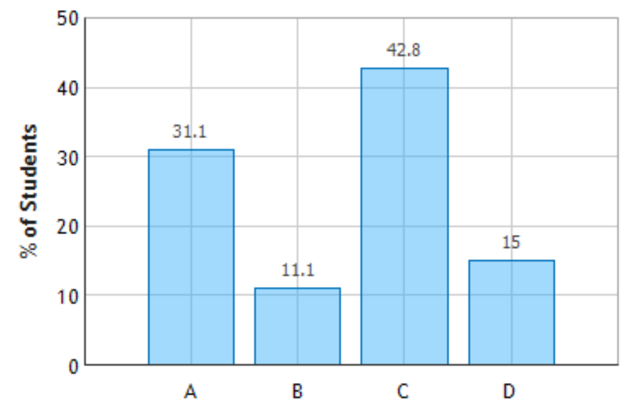
$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = \frac{-W_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

If we know E, we can get V

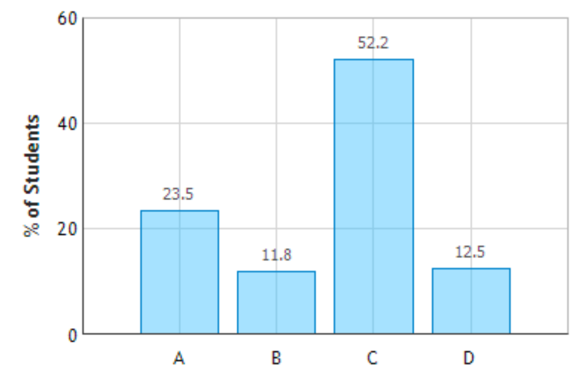
The CheckPoint had an "easy" E field

Suppose the electric field is zero in a certain region of space. Which of the following statements best describes the electric potential in this region?

A) The electric potential is zero everywhere in this region.
B) The electric potential is zero at at least one point in this region.
C) The electric potential is constant everywhere in this region.
D) There is not enough information given to distingush which of the above



Zero Electric Field: Question 1 (N = 136)



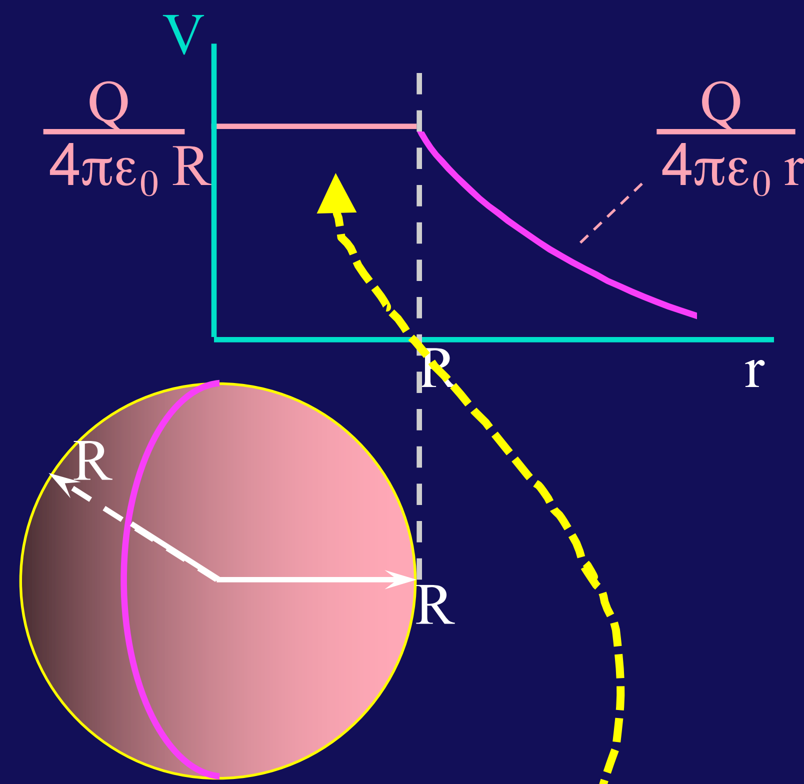
We just learned that

$$\Delta V_{A\to B} = -\int_A^B \vec{E}\cdot d\vec{l}$$

$\vec{E} = 0$ → $\Delta V_{A\to B} = 0$ → V **is constant!**

The Change in V is 0, the actual value is a constant

Potential from charged spherical conducting shell



- E Fields (from Gauss' Law)
  - **r < R:** $E = 0$
  - **r > R:** $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$
- Potentials
  - r > R:

$$V(r) = -\int_{\infty}^{r} \vec{E} \bullet d\vec{l} = -\int_{\infty}^{r} E(dr') = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

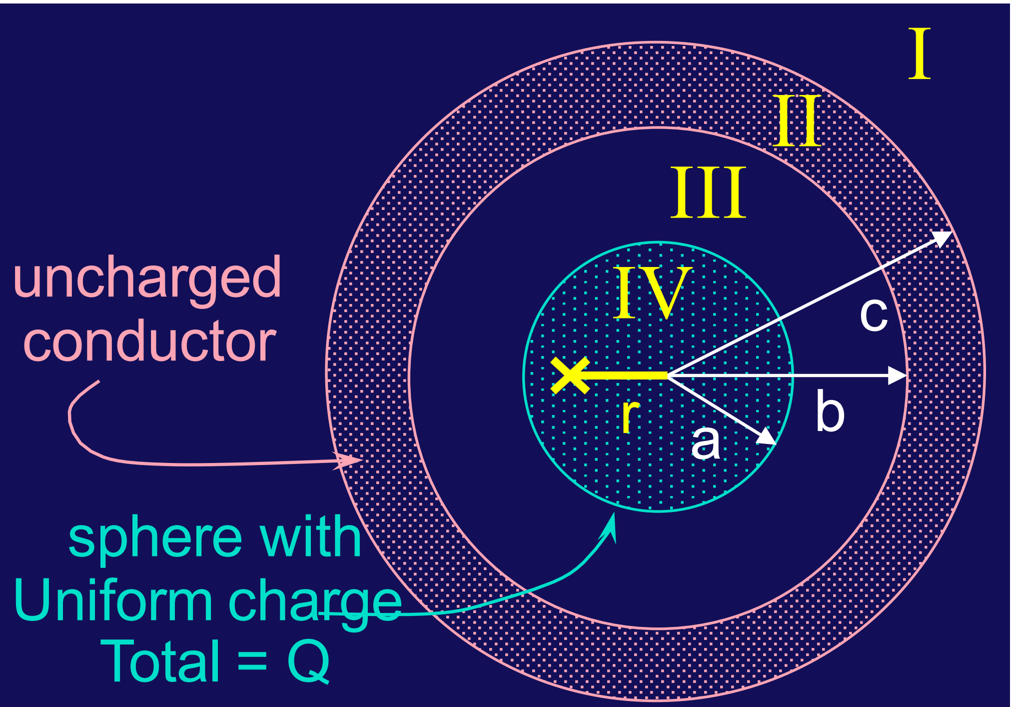
  - r < R:

$$V(r) = -\int_{\infty}^{R} E(dr) - \int_{R}^{r} E(dr') = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + 0$$

We just learned V = constant when E = 0

More challenging …

Calculate the potential V(r) at x



- Work from outside in …

  V=0 at **r=** ...

- Determine **E(r)** everywhere

$$\vec{E}_I(r) = \frac{kQ}{r^2}\,\hat{r} \qquad \vec{E}_{III}(r) = \frac{kQ}{r^2}\,\hat{r}$$

$$\vec{E}_{II}(r) = 0 \qquad \vec{E}_{IV}(r) = \frac{kQr}{a^3}\,\hat{r}$$

- Determine **ΔV** for each region by integration

$$V(r) = V_\infty + \Delta V_{\infty\to c} + \Delta V_{c\to b} + \Delta V_{b\to a} + \Delta V_{a\to r}$$

$$\Delta V_{\infty\to c} = -\int_\infty^c E_I\,dr = -\int_\infty^c k\frac{Q}{r^2}\,dr = \frac{kQ}{c} \qquad \Delta V_{\infty\to b} = \frac{kQ}{c} - \int_c^b E_{II}\,dr = \frac{kQ}{c} + 0$$

$$\Delta V_{\infty\to a} = \frac{kQ}{c} + 0 - \int_b^a E_{III}\,dr = \frac{kQ}{c} + 0 - \int_b^a \frac{kQ}{r^2}\,dr = \frac{kQ}{c} + 0 + kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Delta V_{\infty\to r<a} = kQ\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right) - \int_r^a \frac{kQr'}{a^3}\,dr' = kQ\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b} - \left(\frac{1}{2a} - \frac{r^2}{2a^3}\right)\right)$$

Yes, yes, a mess !

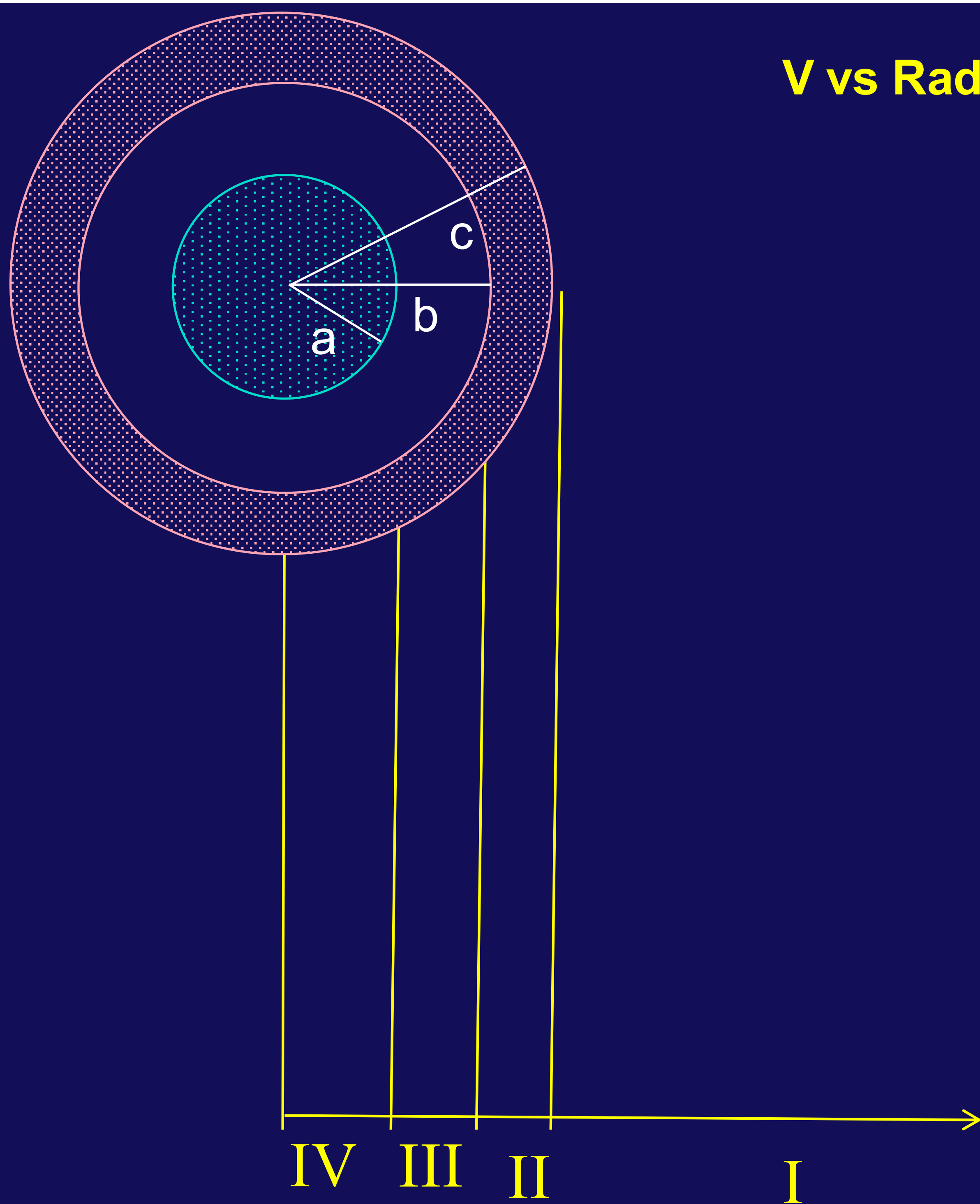
V vs Radius

$$\Delta V_{\infty \to c} = \frac{kQ}{r}$$

$$\Delta V_{c->b} = 0$$

$$\Delta V_{b \to a} = \frac{kQ}{r}$$

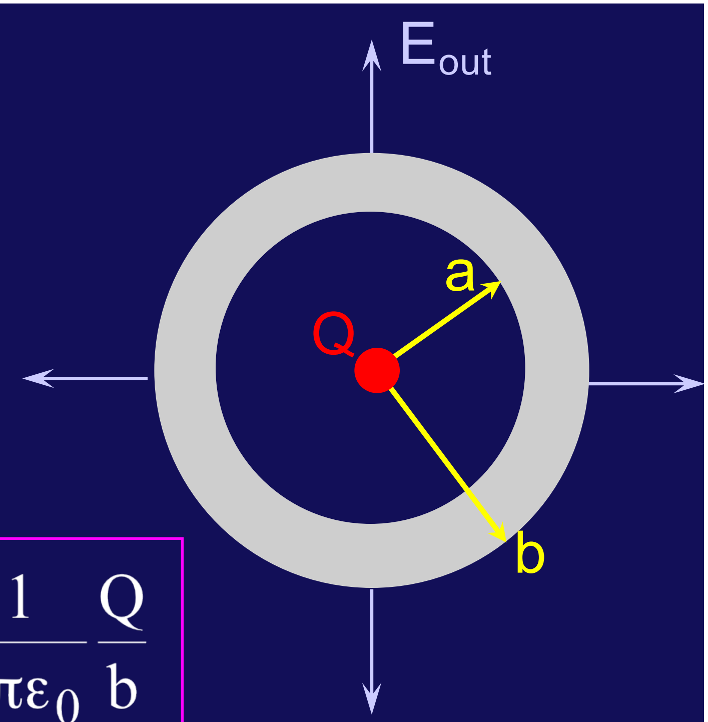
$$\Delta V_{a \to r<a} = kQ\left(\frac{1}{2a} - \frac{r^2}{2a^3}\right)$$



Clicker

1A A point charge Q is fixed at the center of an uncharged conducting spherical shell of inner radius a and outer radius b.

- What is the value of the potential V_a at the inner surface of the spherical shell?



(a) $V_a = 0$ (b) $V_a = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$ (c) $V_a = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$

The potential is given by:

$$V_a = -\int_{\infty}^{a} \vec{E} \bullet d\vec{l}$$

E outside the spherical shell $\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$

E inside the spherical shell: **E = 0**

$$V_a = -\int_{\infty}^{b} \vec{E} \bullet d\vec{l} - \int_{b}^{a} \vec{E} \bullet d\vec{l} \Rightarrow V_a = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} + 0 \Rightarrow V_a = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

E from V

Since we can get V from integrating E

$$\Delta V_{a \to b} = -\int_a^b \vec{E} \cdot d\vec{l}$$

We should get E by differentiating V

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

- **Expressed as a vector, E is μ gradient of V**

$$\vec{\mathrm{E}} = -\vec{\nabla}\,\mathrm{V}$$

- **Cartesian coords:**

$$\vec{\nabla}\,\mathrm{V} = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

- **Spherical coords:**

$$\vec{\nabla}\,\mathrm{V} = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

E from V: a simple Example

- **Consider the following electric potential:**

$$V(x, y, z) = 3x^2 + 2xy - z^2$$

- **What electric field does this describe?**

$$E_x = -\frac{\partial V}{\partial x} = -6x - 2y \qquad E_y = -\frac{\partial V}{\partial y} = -2x \qquad E_z = -\frac{\partial V}{\partial z} = 2z$$

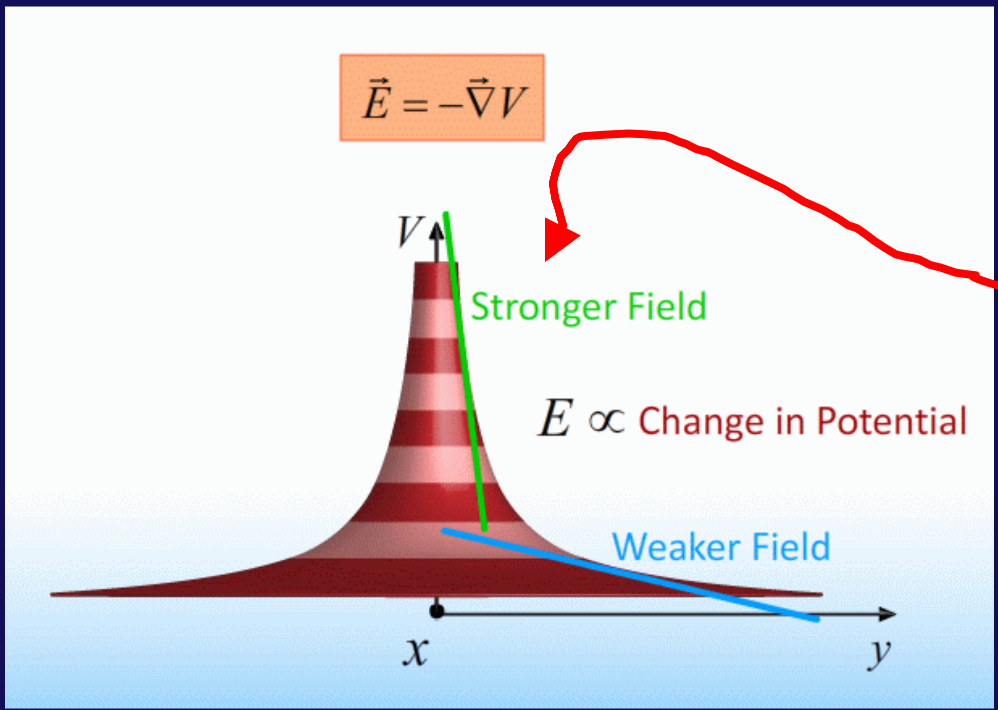
... expressing this as a vector: $\vec{E} = (-6x - 2y)\,\hat{\mathbf{x}} - 2x\,\hat{\mathbf{y}} + 2z\,\hat{\mathbf{z}}$

"Can we please go over the "gradient" more?"

What is going on ?

$$\vec{E} = -\vec{\nabla} V$$

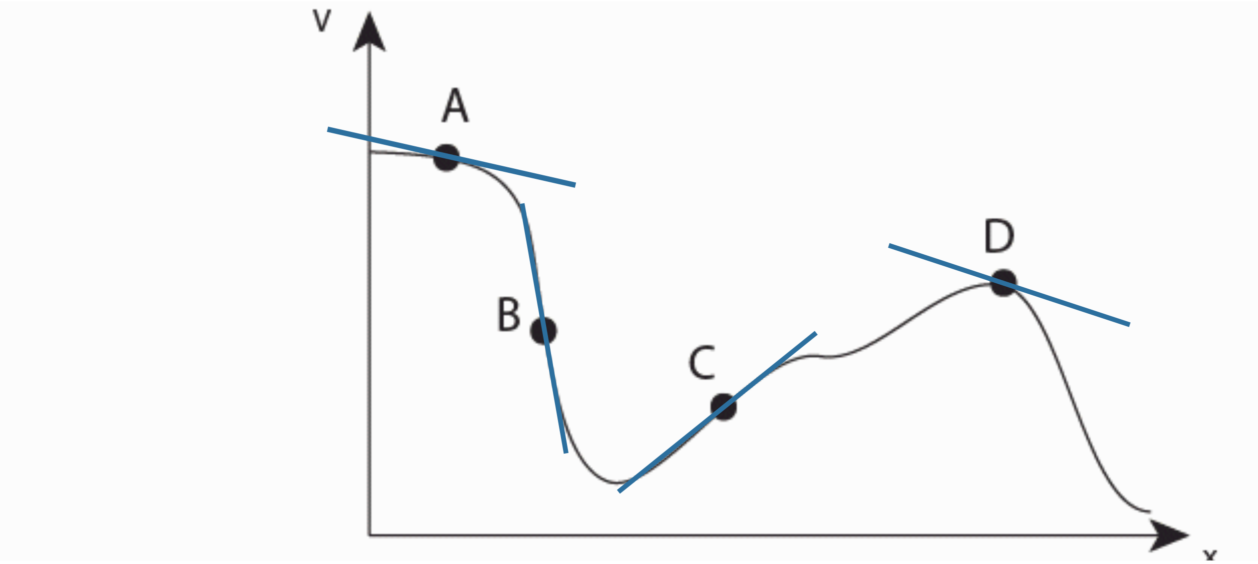
- We are finding the SLOPE in the potential function
- The sign is telling us which way E increases

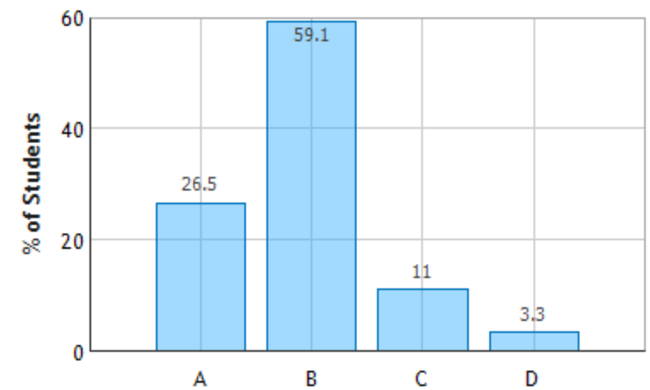


The SP folks like this picture of a potential

CheckPoint Review

The electric potential in a certain region is plotted in the following graph





At which point is the magnitude of the E-field greatest?

○A ○B ○C ○D

B: “ The slope of the electric potential is the magnitude of the electric “

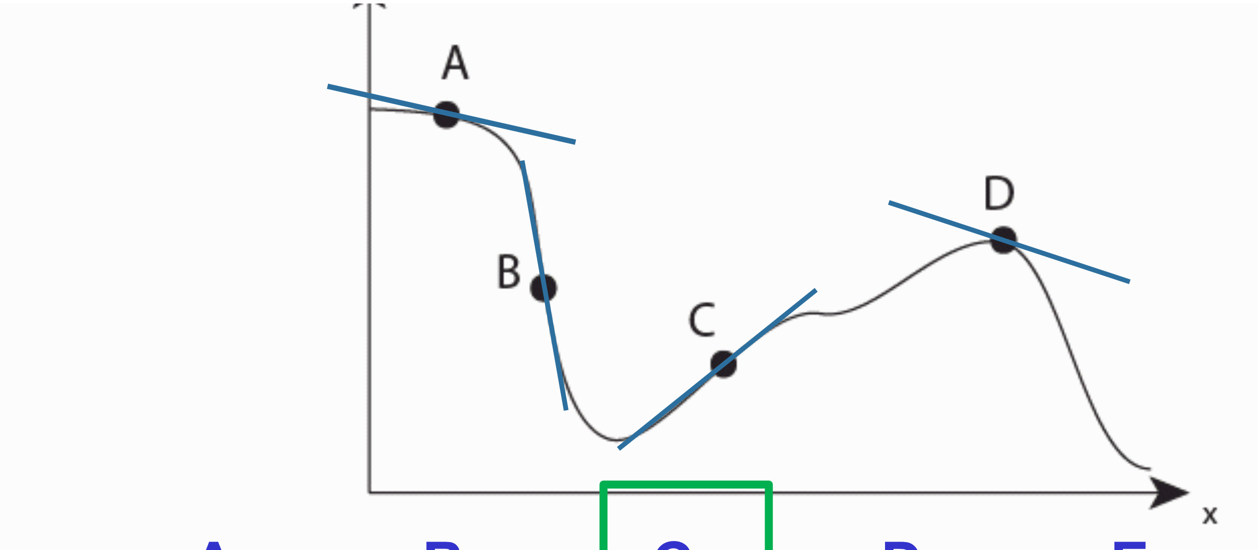
How do we get E from V?

$$\vec{E} = -\vec{\nabla} V \quad \rightarrow \quad E_x = -\frac{\partial V}{dx} \quad \rightarrow$$

Look at slopes!

Clicker of a Checkpoint

At which point is the direction of the E field along the negative x axis ?



A **B** **C** **D** **E = none of these**

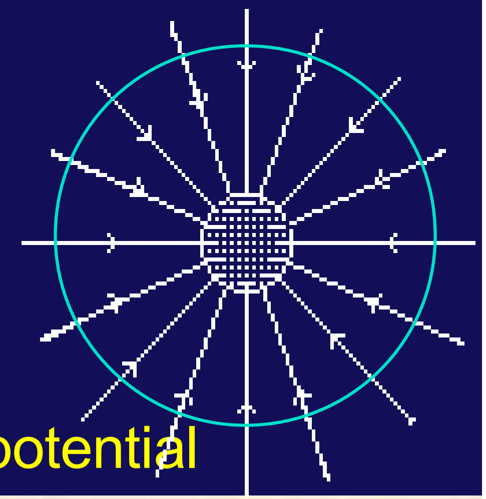
How do we get E from V?

$$\vec{E} = -\vec{\nabla} V \quad \rightarrow \quad E_x = -\frac{\partial V}{dx} \quad \rightarrow$$ **Look at slopes!**

Equipotentials

Defined as: The locus of points with the same potential.

- Example: for a point charge, the equipotentials are spheres centered on the charge.

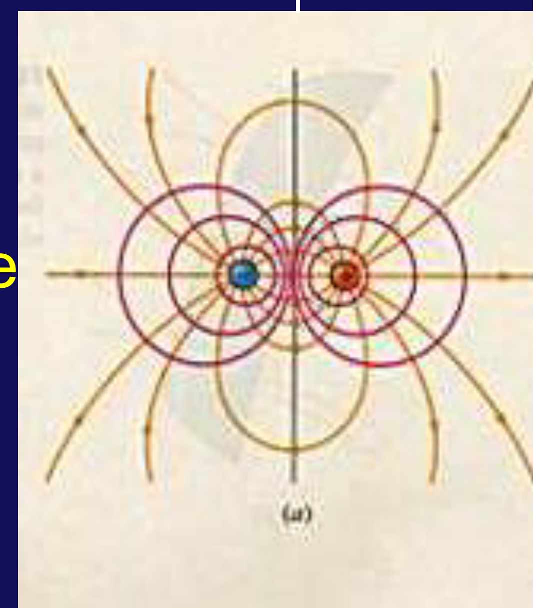


- GENERAL PROPERTY:
  - The Electric Field is always perpendicular to an Equipotential Surface.
- Why?? $\vec{E} = -\vec{\nabla} V$

The gradient ($\vec{\nabla}$) says E is in the direction of max rate of change.

Along an equipotential surface there is NO change in V so E *along this surface* does not change

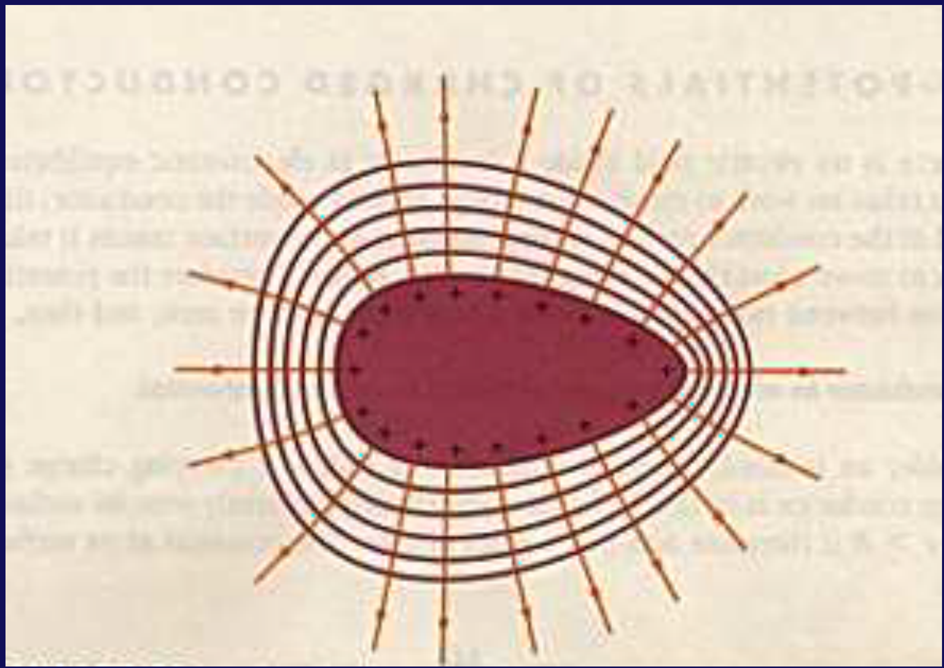
→ **E must be normal to the equipotential surface**



Dipole Equipotentials

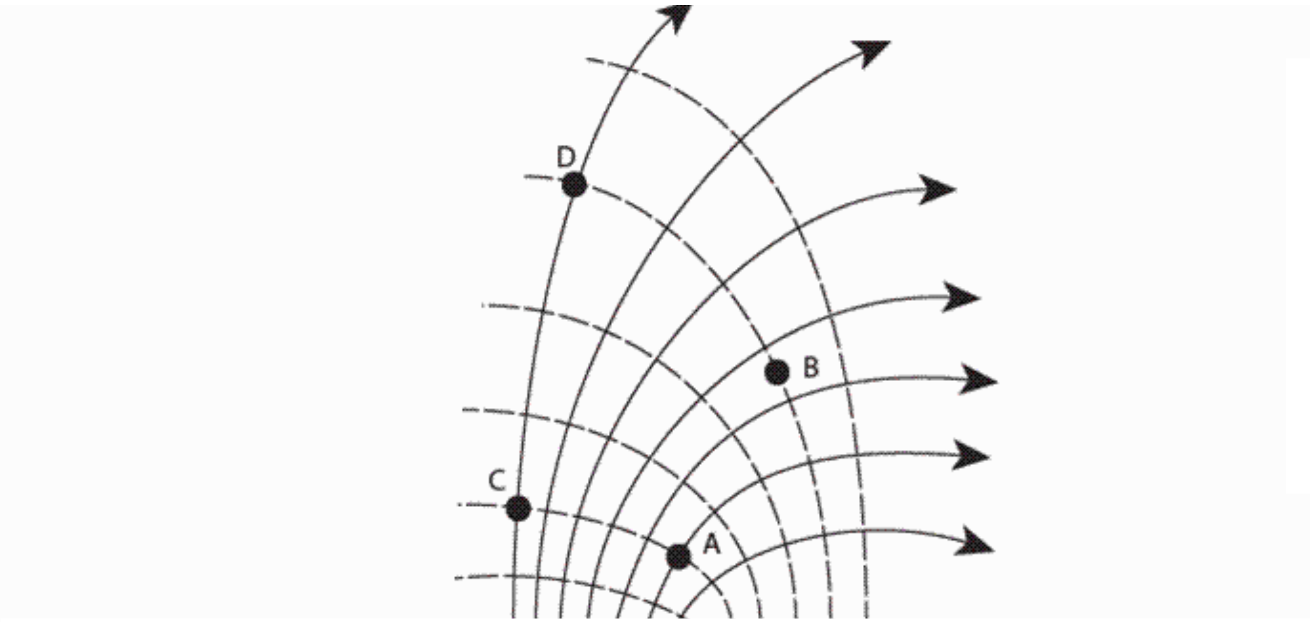
Equipotential Example

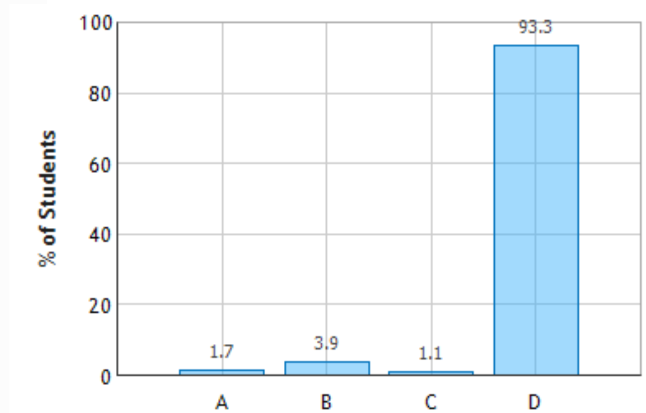
- Field lines more closely spaced near end with most curvature .
- Field lines ⊥ to surface near the surface (since surface is equipotential).
- Equipotentials have similar shape as surface near the surface.
- Equipotentials will look more circular (spherical) at large r.



Let's look at this series of checkpoints

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.





At which point in space is the E-field the weakest?

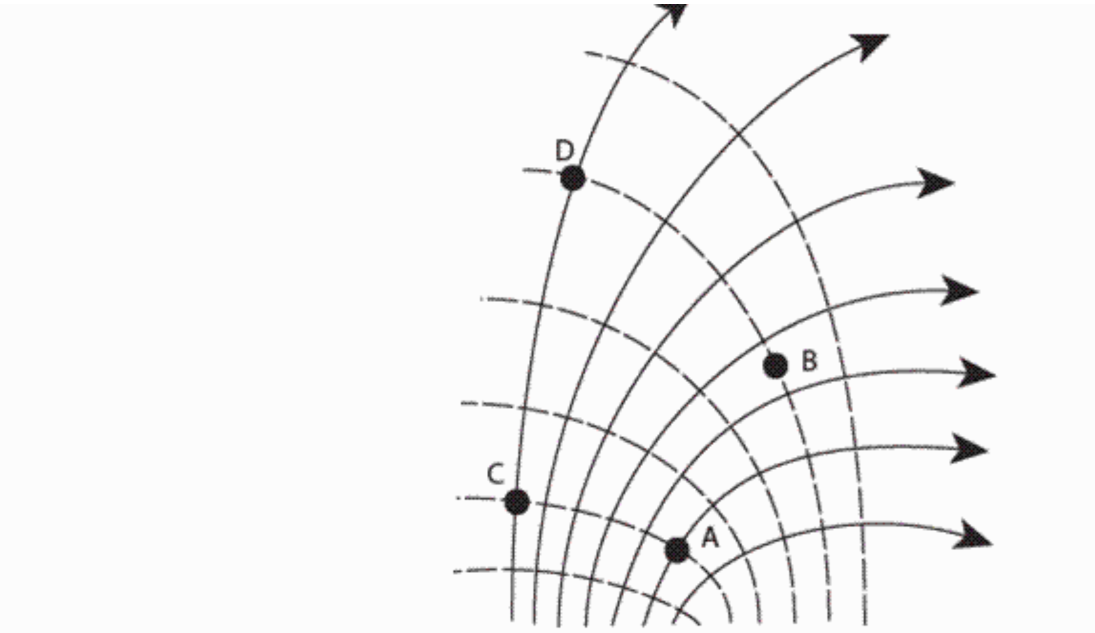
○A ○B ○C ○D

"The electric field lines are the least dense at D "

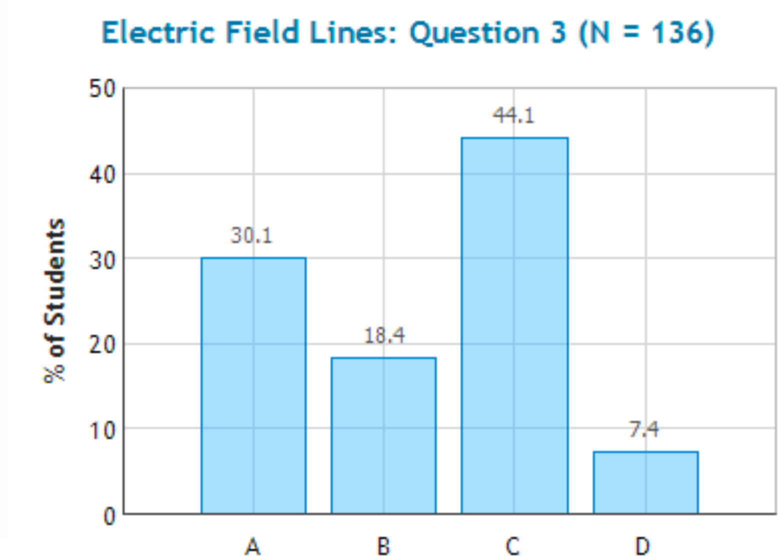
Okay, so far, so good ☺

What ?

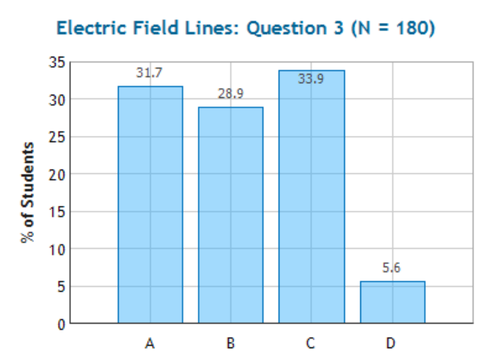
Compare the work done moving a negative charge from A to B and from C to D. Which one requires more work?



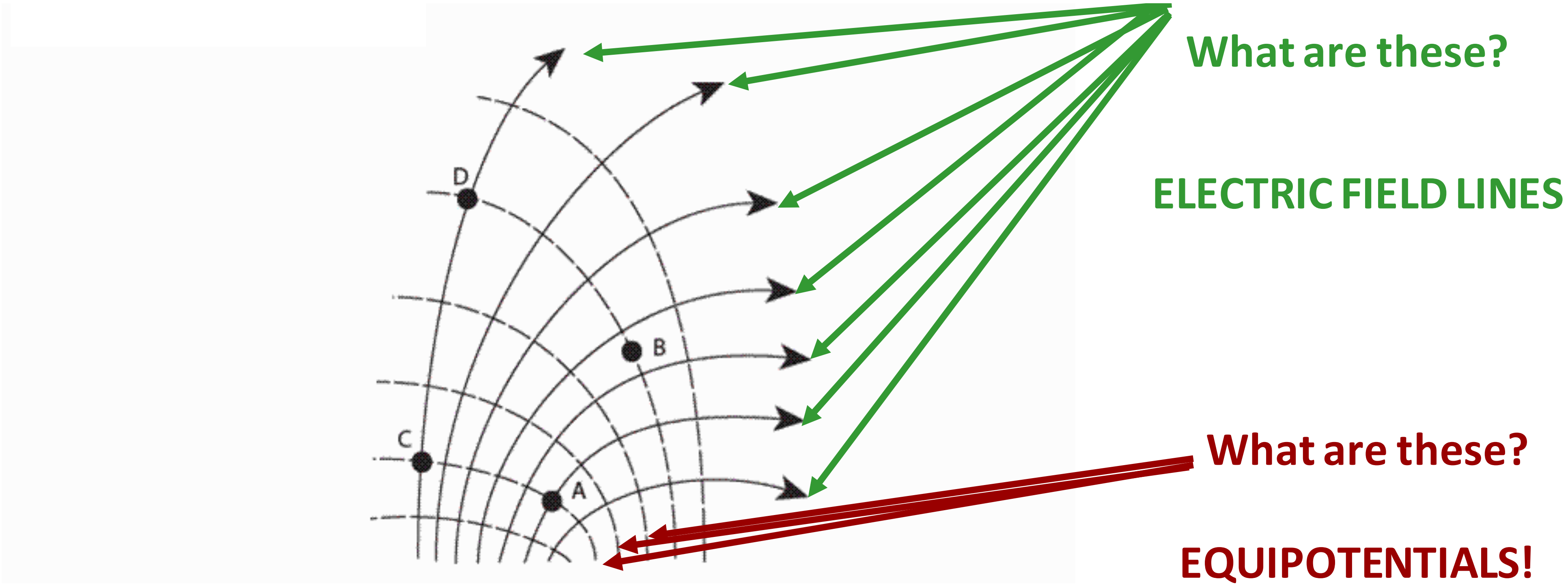
Problem !



A) from A to B
B) from C to D
C) the same
D) cannot determine without performing calculation



First a Hint
Now a Clicker



What is the sign of W_{AC} = work done by **E field** to move negative charge from **A** to **C** ?

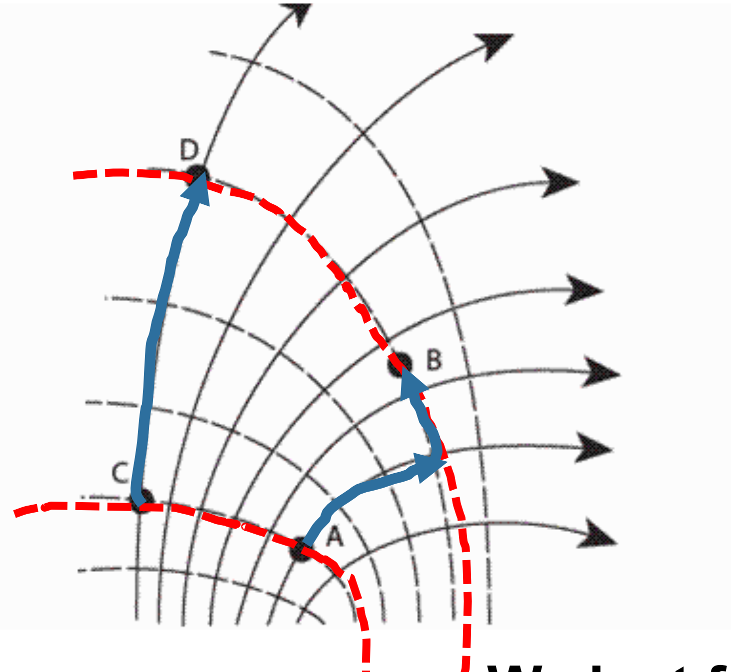
A) $W_{AC} < 0$ B) $W_{AC} = 0$ C) $W_{AC} > 0$

A and **C** are on the same equipotential → $W_{AC} = 0$

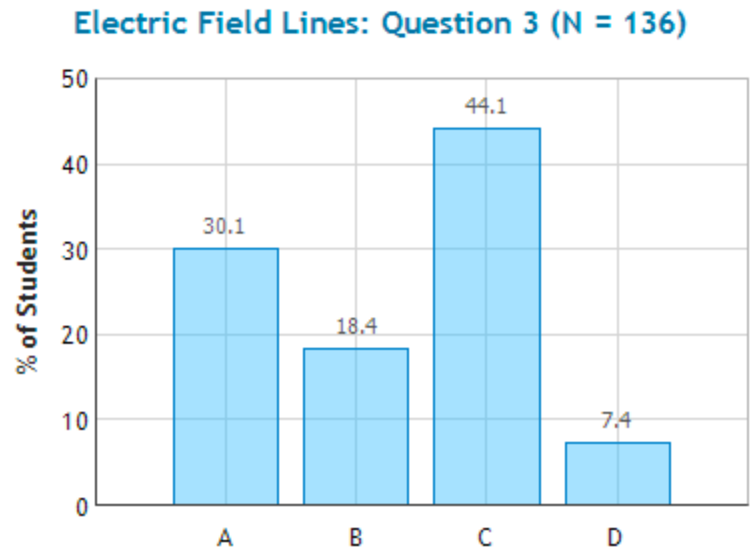
Equipotentials are perpendicular to the E **field**: No work is done along an equipotential

Back to the Checkpoint …

Compare the work done moving a negative charge from A to B and from C to D. <u>Which one requires more work</u>?



Problem !



work?

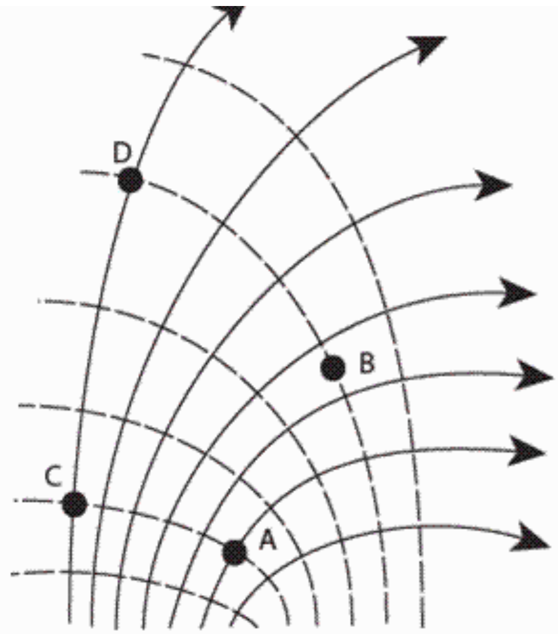
A) from A to B
B) from C to D
C) the same
D) cannot determine without performing calculation

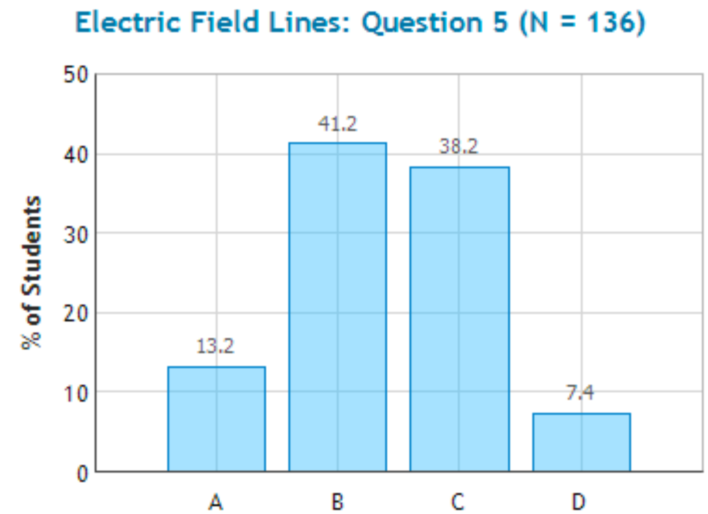
- We just found: $W_{AC} = 0;$
  - → A&C at same potential
  - Similarly: B&D at same potential

- Look at path from A to B and consider change in potential
- Look at path from C to D and consider change in potential
  - THEY ARE THE SAME so <u>Work done is the same.</u>

Another one … now, A to B or D

Compare the work done moving a negative charge from A to B and from A to D. Which one requires more work?





A) from A to B
B) from A to D
C) the same
D) cannot determine without performing calculation

A answer: "E field weak at d"

B answer: "Moving the charge from A to D crosses more equipotential lines, so it requires more work.

C answer: "Since B and D are on the same equipotential line, the change in potential energy (and therefore the work required) between A and either point is the same."

The Bottom Line

If we know the electric field E everywhere,

$$V_B - V_A \equiv \frac{W_{AB}}{q_0} \quad Þ \quad V_B - V_A = -\int_A^B \vec{E} \bullet d\vec{l}$$

allows us to calculate the potential function V everywhere
(keep in mind, we often define $V_A = 0$ at some convenient place)

If we know the potential function V everywhere,

$$\vec{E} = -\vec{\nabla} V$$

allows us to calculate the electric field E everywhere

- **Units for Potential! 1 Joule/Coul = 1 VOLT**