# More Gauss, Less Potential

- **Today:**
  - **Gauss' Law examples**
- **Monday:**
  - **Electrical Potential Energy (Guest Lecturer)**
    - **new SmartPhysics material**
- **Wednesday:**
  - **Electric Potential**
    - **new SmartPhysics material**
- **Thursday:**
  - **Midterm 1 → SEE OUR COURSE WEBSITE !!!!!**
    - **Equation sheet**
    - **Practice exam**
    - **Will *not include* Electrical Potential Energy**
- **Friday: No Class**

**Phys 122 Lecture 6**

**G. Rybka**

# Gauss' Law Reminder

The net electric flux through any closed surface is proportional to the charge enclosed by that surface.

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = \varepsilon_0 \Phi = q_{enclosed}$$

- TRUE always, USEFUL only when symmetries exist in physical situation

  - SPHERES (hollow or solid)
  - CYLINDERS (thick, thin, hollow, solid), and LINES
  - PLANES (thick or thin)

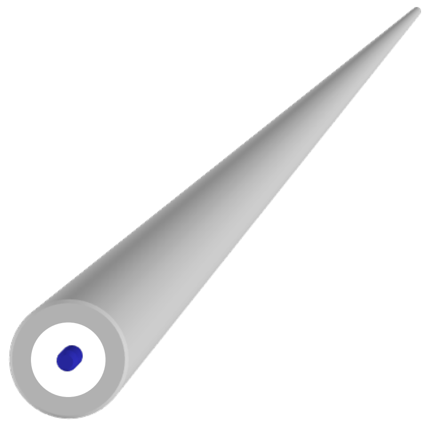
- These items can be
  - Conducting
  - Non-conducting
  - Charged
  - Uncharged

- Gaussian Surfaces selected so E is uniform or Zero when it penetrates a surface.

# We didn’t get to this last class

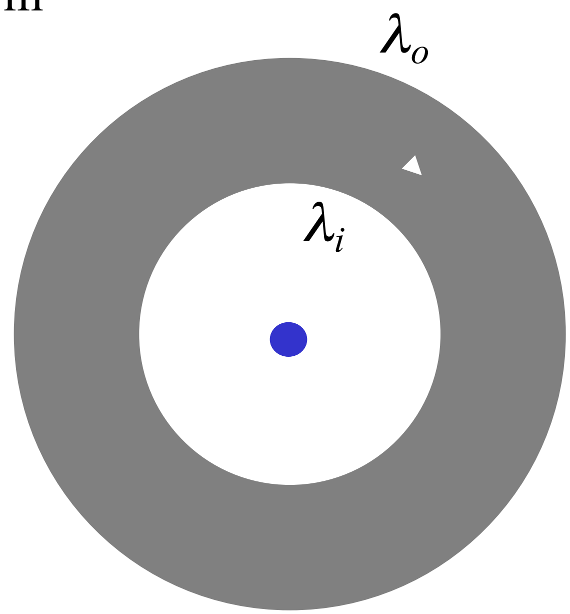
A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of –4 C/m.

What is the linear charge density of the induced charge on the inner surface of the conducting cylinder ($\lambda_i$) and on the outer surface ($\lambda_o$)?



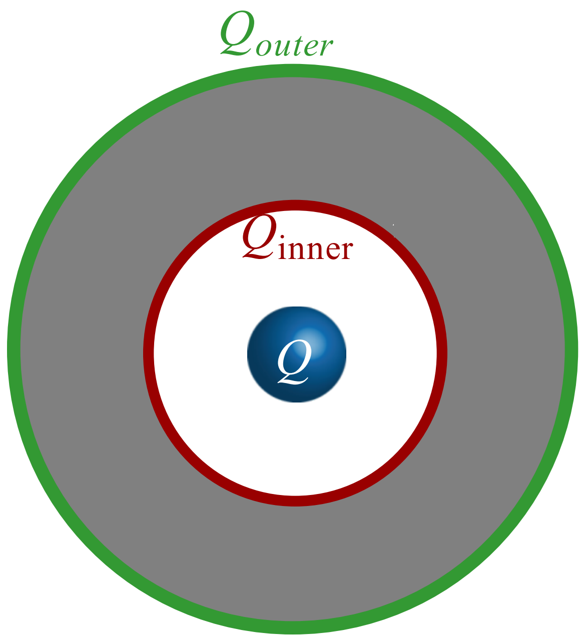
| | | | | | |
|---|---|---|---|---|---|
| $\lambda_i$: | +2.5 C/m | –4 C/m | –2.5 C/m | –2.5 C/m | 0 |
| $\lambda_o$: | –6.5 C/m | 0 | +2.5 C/m | –1.5 C/m | –4 C/m |
| | A | B | C | D | E |

- **Key point: The inside + outside linear charge densities on cylinder surfaces must ADD up to the total charge of the cylinder, i.e., -4 C/m**
- **Key point: E=0 inside conductor means inner surface charge must be opposite to charge on the enclosed wire since E is to charge enclosed.**



# Similar Clicker:


A particle with charge $+Q$ is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?




A) inner = $-Q$, outer = $+Q$

B) inner = $-Q/2$ , outer = $+Q/2$

C) inner = $0$, outer = $0$

D) inner = $+Q/2$, outer = $-Q/2$

E) inner = $+Q$, outer = $-Q$

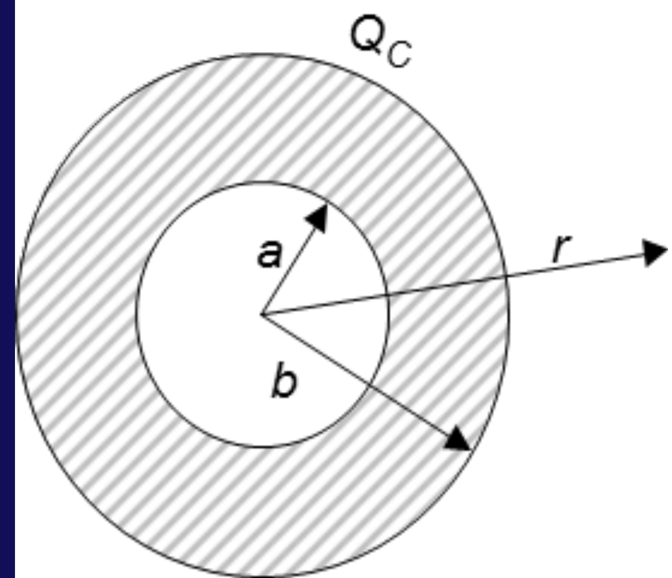
Since $E = 0$ in conductor

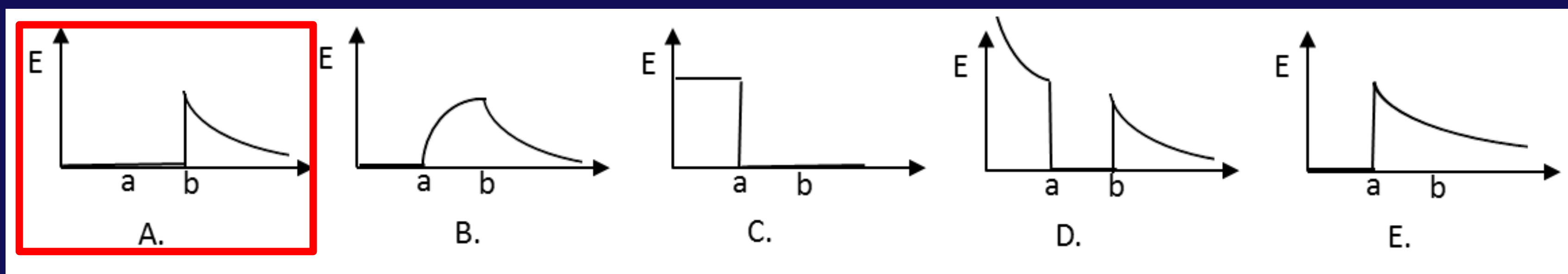
- Gauss' Law: $\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$ → $Q_{enc} = 0$

# Clicker

A conducting spherical shell of inner radius a and outer radius b has a net charge Q = +6 C.

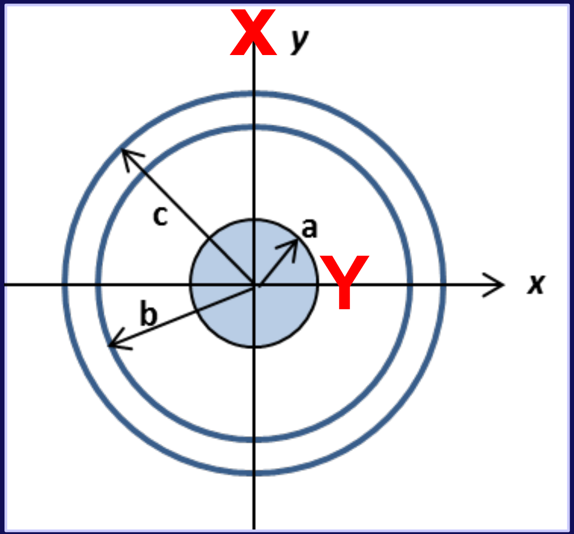
Which of the plots below best describes the radial component of the electric field as a function of distance from the center of symmetry for the system?




- **From 0 < r < a: Gauss' Law; net charge enclosed = 0 → E = 0**
- **From a < r < b: Inside a conductor. E = 0 !! (always)**
- **From r > b: Gauss' Law: E $1/r^2$**

# Clicker




An insulating cylinder, concentric with the $z$ axis, has a radius a and a linear charge density $\lambda$.

It is surrounded by a concentric, cylindrical conducting shell with inner radius b, outer radius c and linear charge density $-\lambda$.

The cylinders will be treated as infinitely long.

Which way would a proton move if it was placed at position X in the figure?

A) Right B) Left C) Up D) Down **E) It will not move**

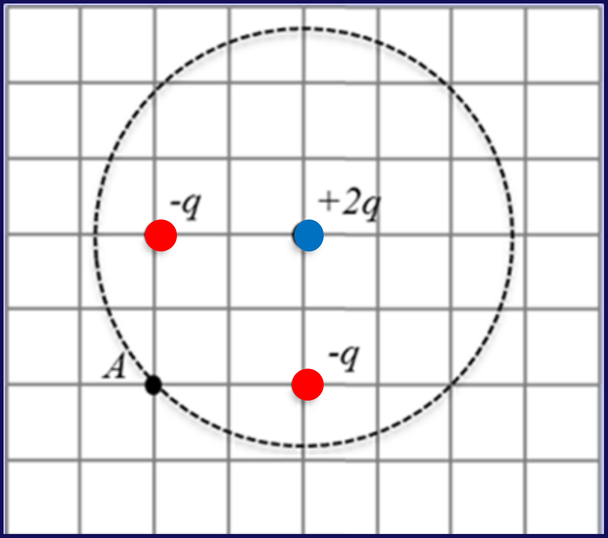
- Gauss' Law: Field at X is zero; Net charge enclosed is 0

Clicker Followup (oral) Which way would an electron move if it was placed at position Y in the figure?

A) Right **B) Left** C) Up D) Down E) It will not move

# Clickers

A spherical Gaussian surface (dashed line) that passes through point A encloses the three point charges as shown




**What is the net flux through the Gaussian surface?**

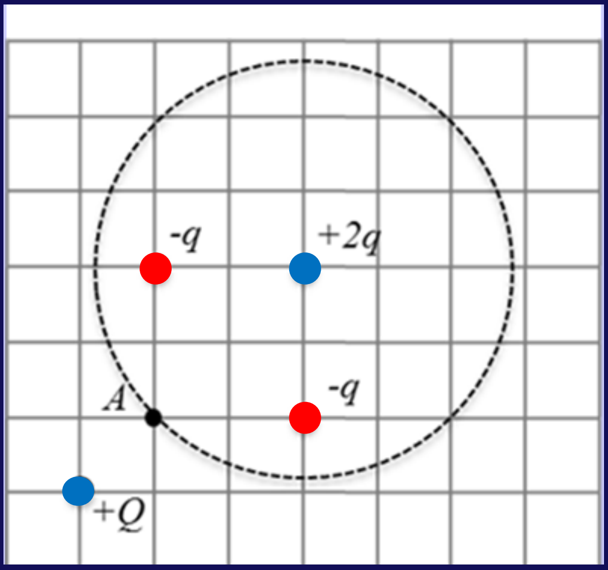
A. Zero

**Net charge enclosed is zero**

B. $4q/\varepsilon_0$

C. You can't determine from this geometry

**A new point charge +Q, of unknown magnitude, is added as shown. The magnitude of the flux through the Gaussian surface**




A. increases

B. decreases

C. remains the same

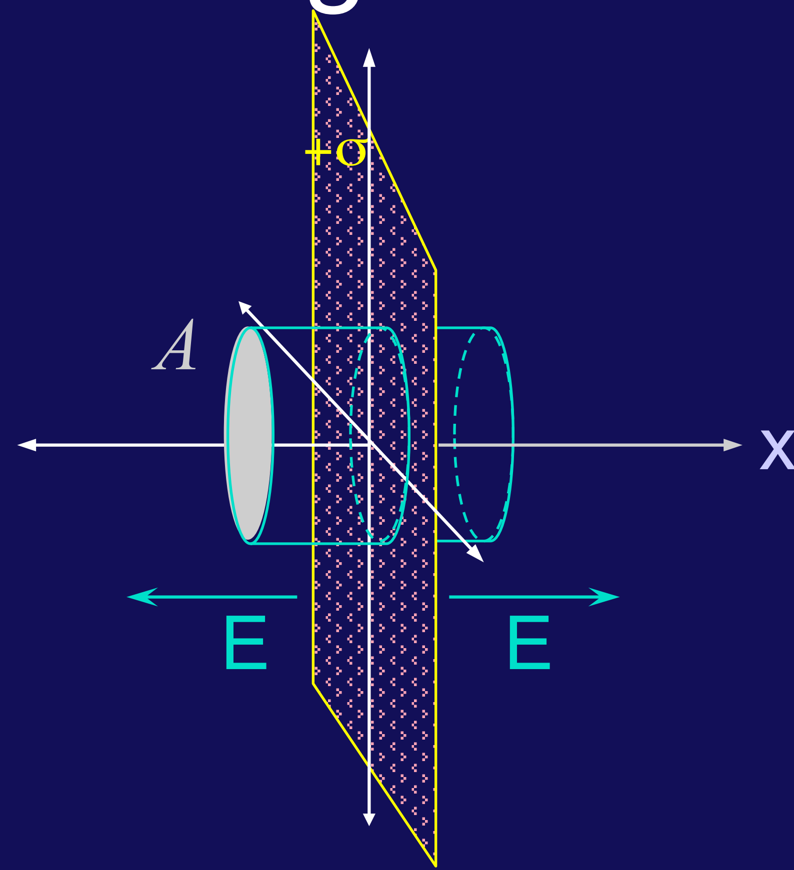
**New charge is outside Gaussian surface**

# Infinite sheet of charge

## Symmetry:

direction of E = x-axis

CHOOSE Gaussian surface to be a cylinder aligned with the x-axis.




## Apply Gauss' Law:

Integrate the barrel, $\vec{E} \bullet d\vec{S} = 0$

Now the ends, $\oint \vec{E} \bullet d\vec{S} = 2AE$

The charge enclosed = $\sigma A$

Therefore, Gauss' Law Þ $\varepsilon_0(2EA) = \sigma A$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Conclusion: An infinite plane sheet of charge creates a CONSTANT electric field .

# Two Infinite Sheets

(into screen)

Field outside the sheets must be zero. Two ways to see:

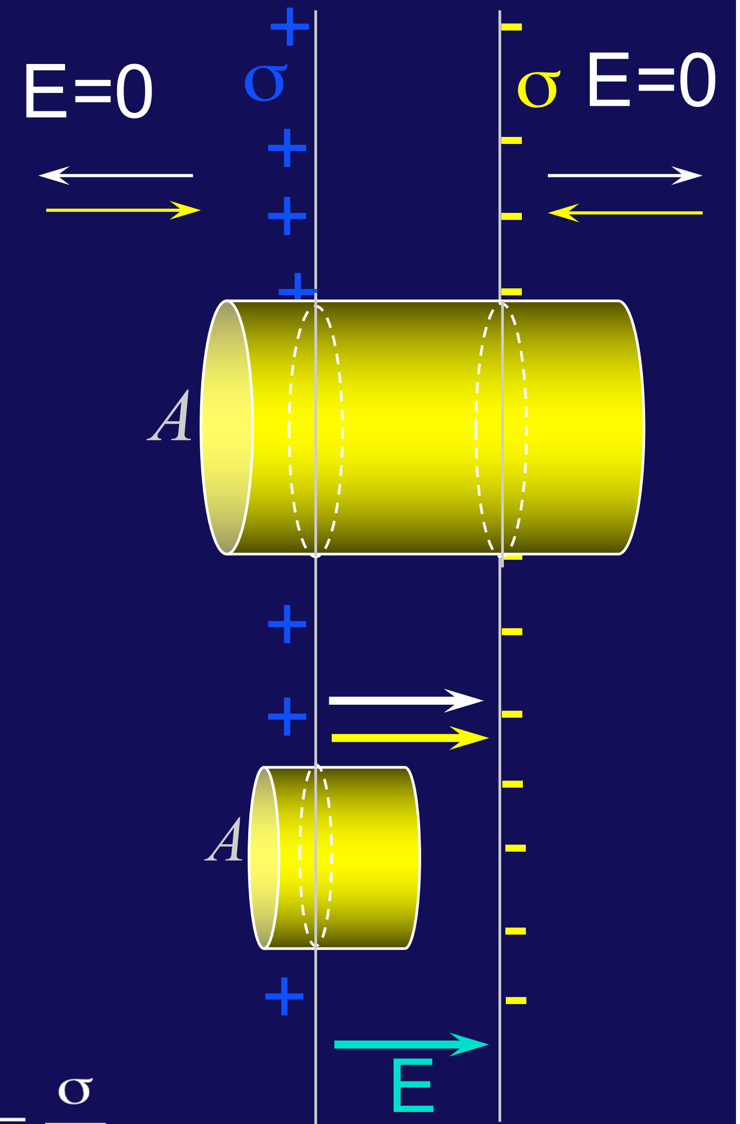
Superposition

Gaussian surface encloses zero charge

Field inside sheets is NOT zero:

Superposition

Gaussian surface encloses non-zero chg




$$Q = \sigma A$$

$$\oint \vec{E} \bullet d\vec{S} = AE_{outside} + AE_{inside}$$

(with $AE_{outside}$ crossed out and marked 0)

$$E = \frac{\sigma}{\varepsilon_0}$$

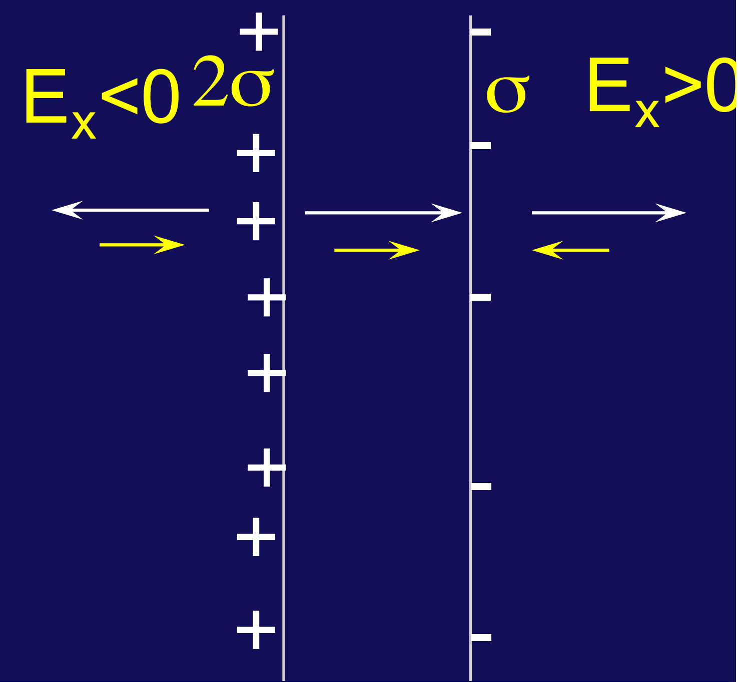
# More Sheets

(into screen)

Now, asymmetrically charged

Superposition

Show fields in all locations from each sheet




$$E_x = \frac{-2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E_x = \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E_x = \frac{2\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

# One more question here …

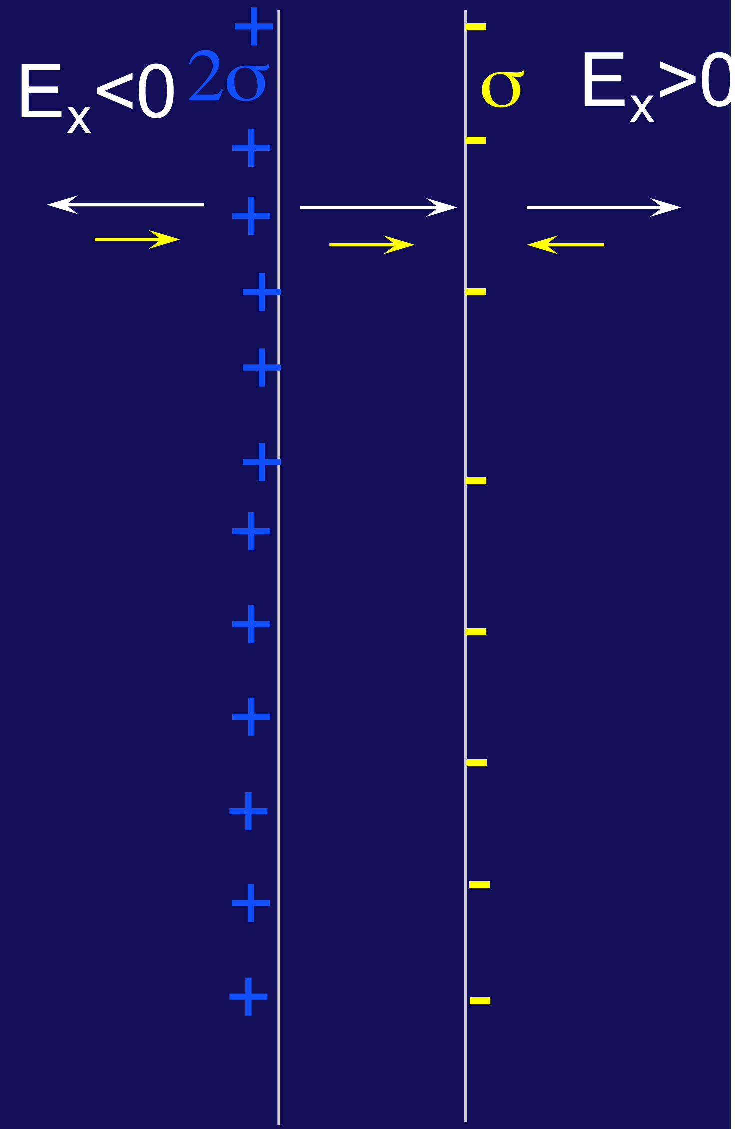
(into screen)

Now, asymmetrically charged

Superposition

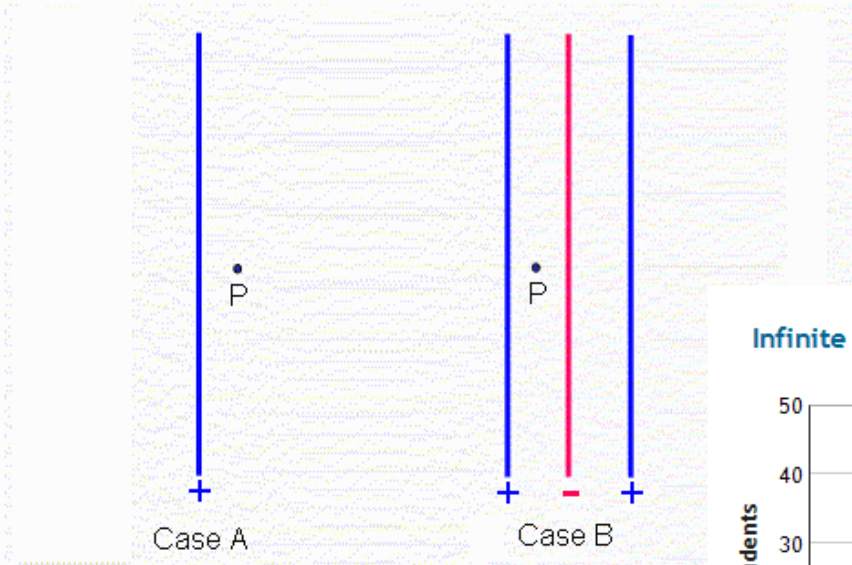
Show fields in all locations from each sheet

**What if an uncharged conducting cylinder was introduced here ?**



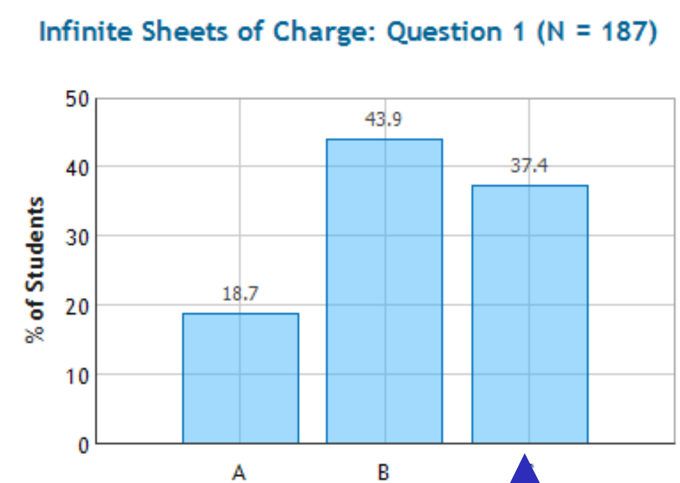
# CheckPoint

10) In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same.

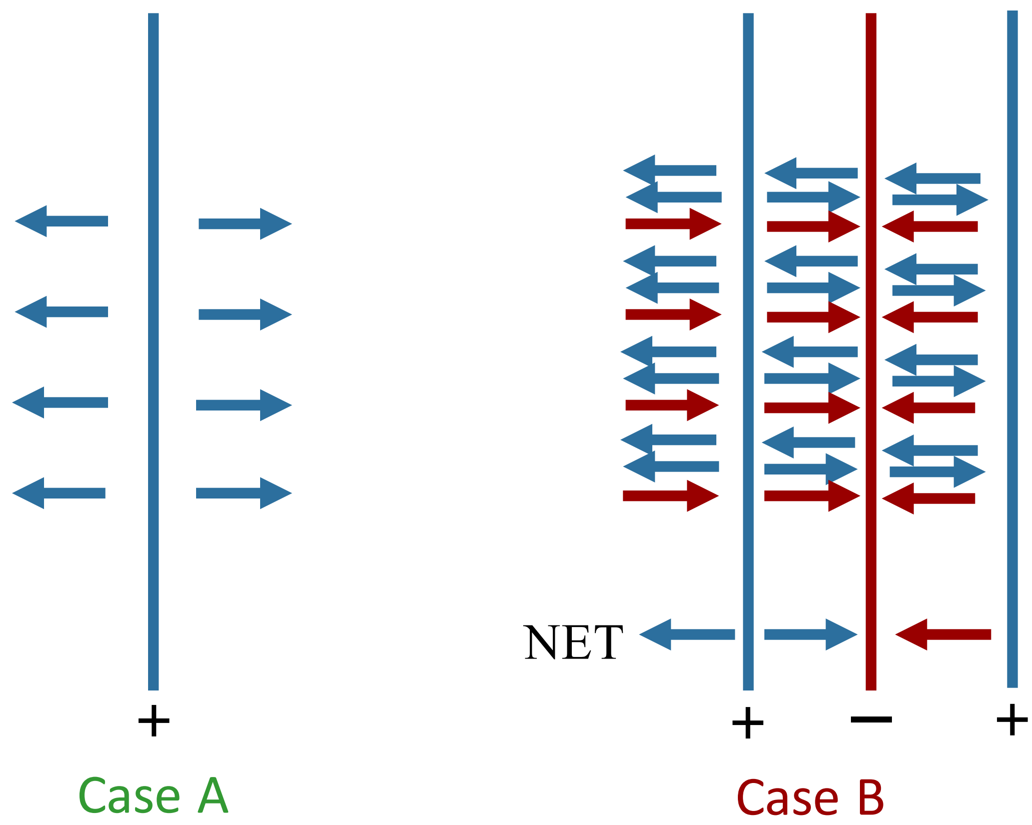



In which case is $E$ at point $P$ the biggest?

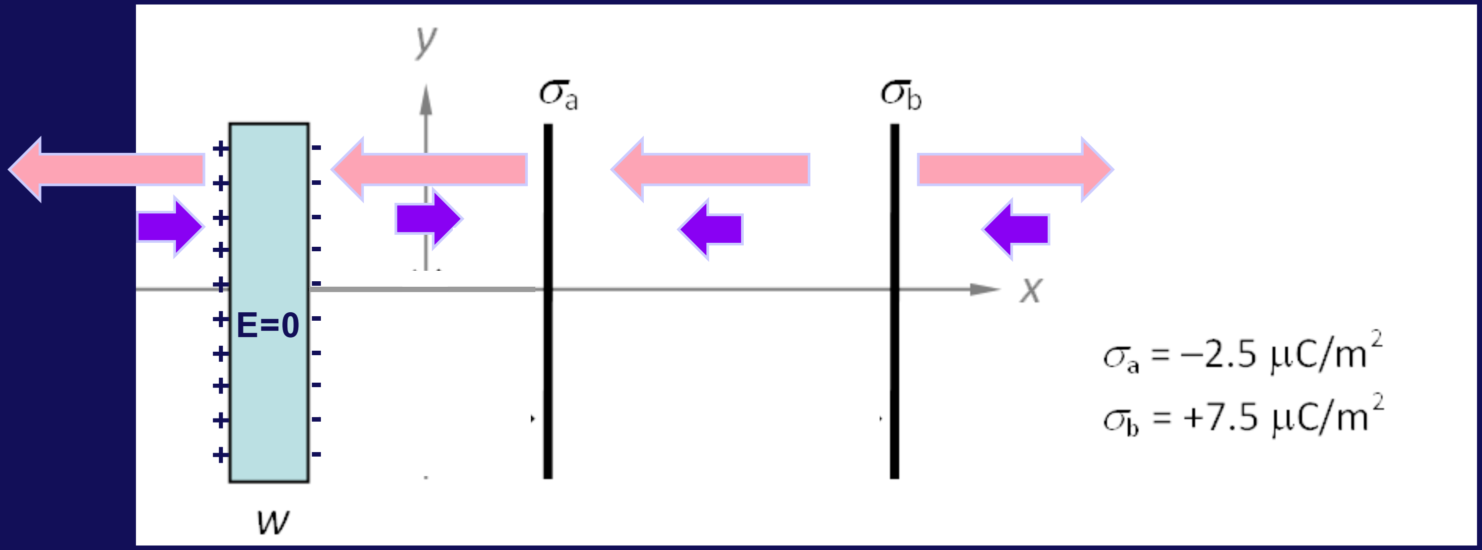
A) A B) B C) the same



# Superposition:



# Clicker   3 Infinite sheets, into page




**The two thin plates are made of insulating material and carry uniformly-distributed surface charge densities of $\sigma_a$ and $\sigma_b$ respectively. The thick metal plate has width *w*, and is uncharged.**

**What is the sign of the surface charge density $\sigma_{LHS}$ on the left-hand side of the thick metal plate?**

***A.*** $\sigma_{LHS} < 0$

***B.*** $\sigma_{LHS} = 0$

***C.*** $\sigma_{LHS} > 0$

- **Fields between planes**
- **E = 0 inside conductor**
- **Surface charge on conductor induced to cancel E inside**

# Gauss' Law tips

- Gauss' Law is ALWAYS VALID!! $\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = q$

  If you have (a) spherical, (b) cylindrical, or (c) planar symmetry AND:

  If you know the charge (RHS), you can calculate the electric field (LHS)

  If you know the field (LHS, usually because E=0 inside conductor), you can calculate the charge (RHS).

  - Spherical Symmetry: Gaussian surface = Sphere of radius r

    LHS: $\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = 4\pi\varepsilon_0 r^2 E$

    RHS: $q = \text{ALL charge inside radius r}$

    ⇒ $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$

  - Cylindrical Symmetry: Gaussian surface = Cylinder of radius r

    LHS: $\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = \varepsilon_0 2\pi r L E$

    RHS: $q = \text{ALL charge inside radius r, length L}$

    ⇒ $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

  - Planar Symmetry: Gaussian surface = Cylinder of area A

    LHS: $\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = \varepsilon_0 2AE$

    RHS: $q = \text{ALL charge inside cylinder} = \sigma A$

    ⇒ $E = \frac{\sigma}{2\varepsilon_0}$