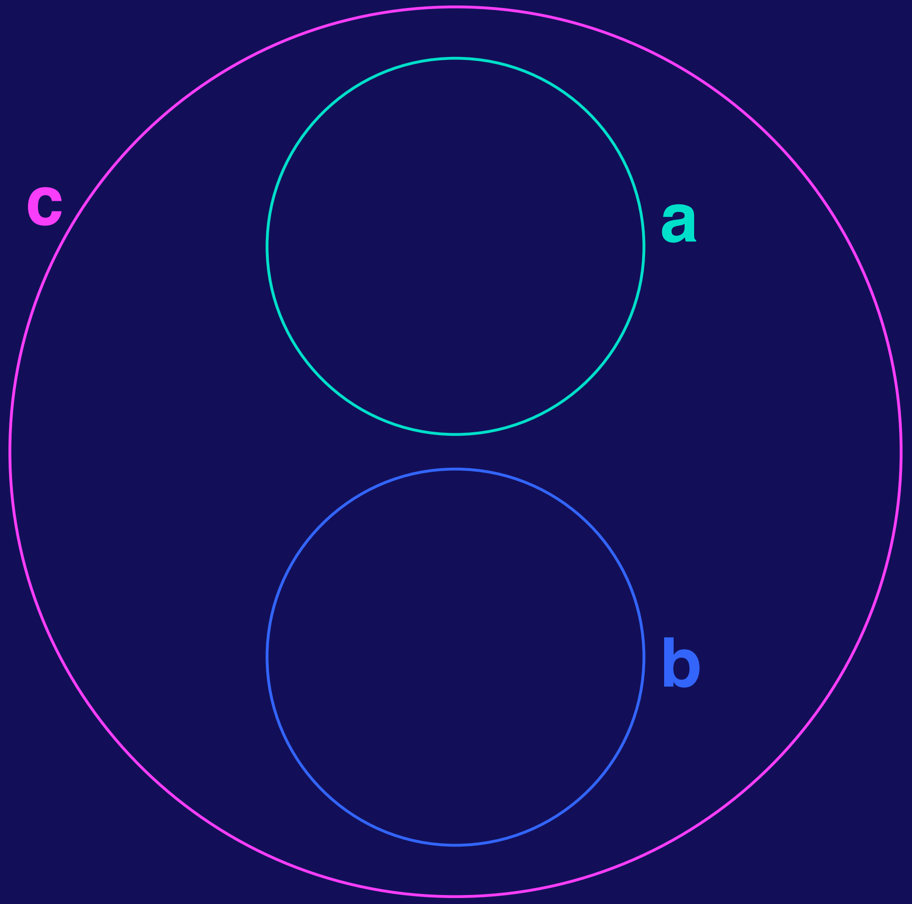
Gauss' Law



$$\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = q$$

Phys 122 Lecture 5

G. Rybka

Announcements

- Office hours:
  - Rybka – Friday 11:30-12:30 C501
  - Hicks (TA) – Thursday 11:30-1:30 Study Center
  - Garcia (Ph122B instructor) Tuesday 5:00-6:00 C521
  - Study Center Mon-Fri 9:30-4:30, downstairs

- First Midterm Exam
  - April 14
  - Practice exam posted on website under “exams”
- Clicker Registration
  - Follow link from website

Gauss' Law

- Gauss' Law (a FUNDAMENTAL Law):

  The net electric flux through any closed surface is proportional to the charge enclosed by that surface.

  $$\boxed{\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = \varepsilon_0 \Phi = q_{enclosed}}$$

– Always TRUE but not always easy to use

– Useful when situation exhibits SYMMETRY

– Choose a closed surface such that the <u>integral is trivial</u>

  – Direction: chose a surface so that E is parallel or perpendicular to each piece of the surface

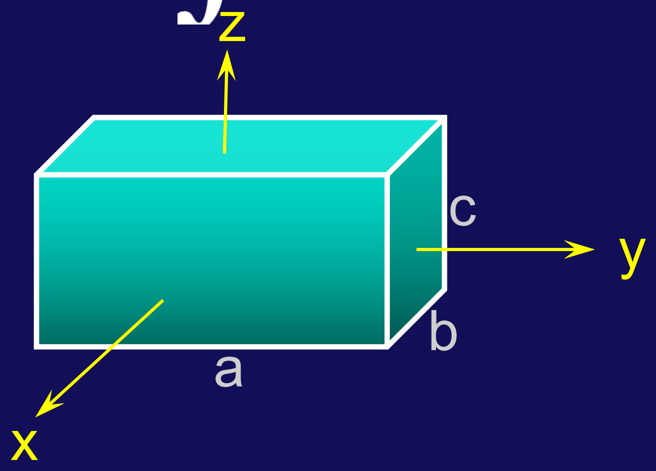
  – Magnitude: chose surface where E has the same value at all points on the surface when E is perpendicular to the surface.

  – Therefore: then, bring E outside of the integral

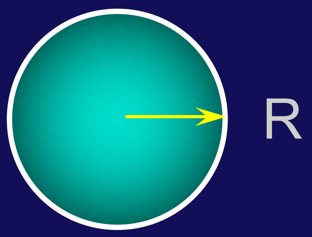
Geometry and Surface Integrals

If E is constant over a surface, and normal to it everywhere, we can take E outside the integral, leaving only a surface area

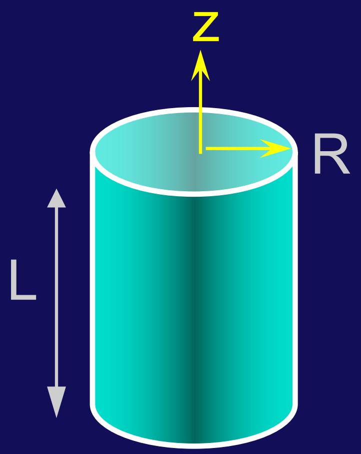
$$\oint \vec{E} \bullet d\vec{S} = E\oint dS$$



$$\oint dS = 2ac + 2bc + 2ab$$



$$\oint dS = 4\pi R^2$$



$$\oint dS = 2\pi R^2 + 2\pi RL$$

PreLecture:

How does Gauss's law differ from Coloumb's Law? How will we be expected to apply them?

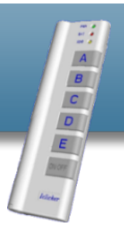
Very difficult concept, determining how to solve a conductor versus an insulator was confusing, please go over the differences.

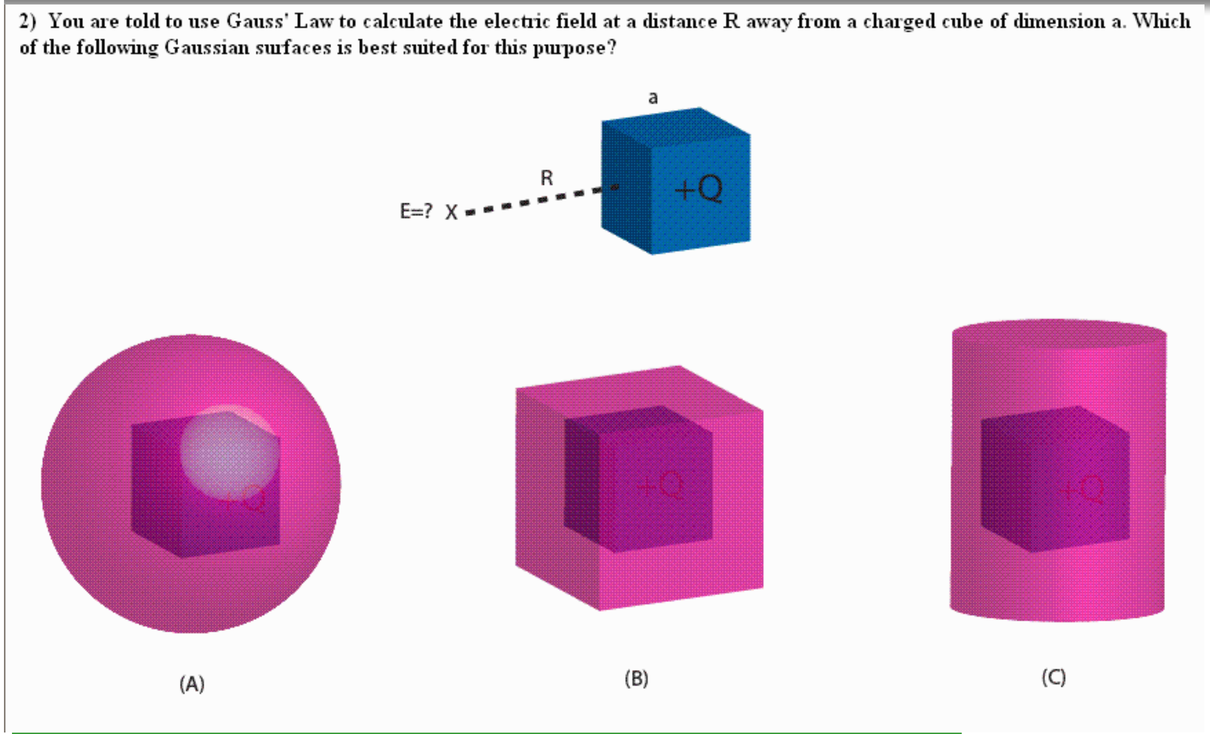
I can get the right answers, but I don't understand any of this conceptually.

VS

I mostly understood the conductor concepts, the formulas were hard to follow.

CheckPoint

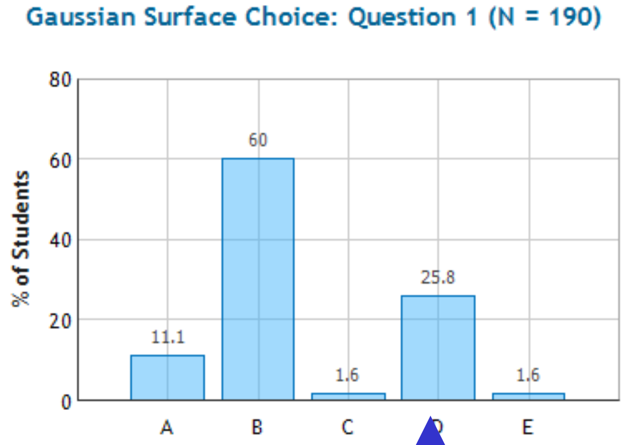




2) You are told to use Gauss' Law to calculate the electric field at a distance R away from a charged cube of dimension a. Which of the following Gaussian surfaces is best suited for this purpose?

D) The field cannot be calculated using Gauss' Law

E) None of the above



Gaussian Surface Choice: Question 1 (N = 190)

THE CUBE HAS NO GLOBAL SYMMETRY!

THE FIELD AT THE FACE OF THE CUBE

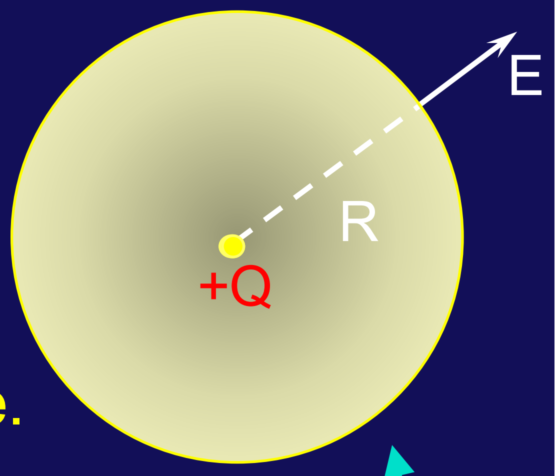
IS NOT

PERPENDICULAR OR PARALLEL

3D	POINT	SPHERICAL
2D	LINE	CYLINDRICAL
1D	PLANE	PLANAR

Example: Rediscovering Coulomb's Law

What is E field around a point charge?



Symmetry Þ E field of point charge is radial and spherically symmetric

Draw a sphere of radius R centered on the charge.

E is normal to every point on that surface

Þ $$\vec{E} \bullet d\vec{S} = EdS$$

E has same value at every point on surface

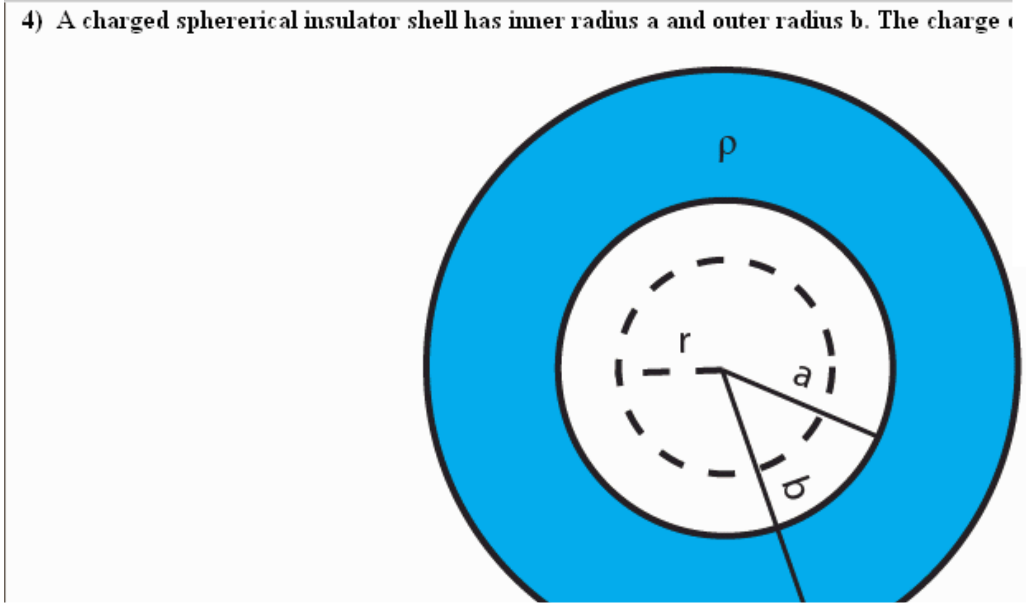
Þ can take E outside of the integral!

$$\oint \vec{E} \bullet d\vec{S} = \oint EdS = E\oint dS = 4\pi R^2 E$$

Gauss' Law Þ $\varepsilon_0 4\pi R^2 E = Q$ ⇒ $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}}$

We are free to choose the surface in such problems…we call this a "Gaussian" surface

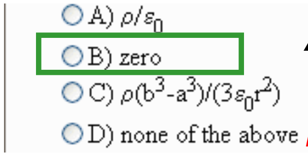
CheckPoint



4) A charged sphererical insulator shell has inner radius a and outer radius b. The charge

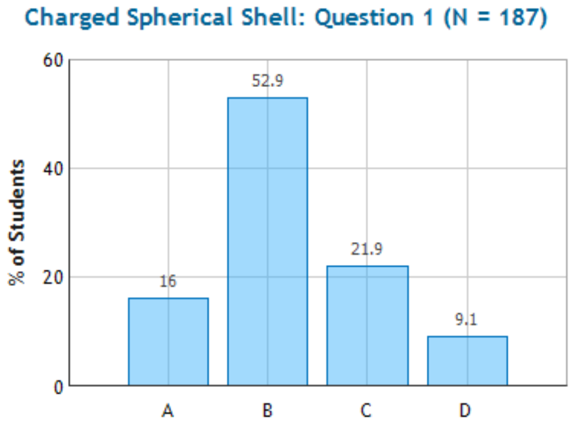
A charged spherical insulating shell has inner radius a and outer radius b. The charge density on the shell is ρ.

What is the magnitude of the E-field at a distance r away from the center of the shell where r < a?



○ A) ρ/ε_0
○ B) zero
○ C) $\rho(b^3-a^3)/(3\varepsilon_0 r^2)$
○ D) none of the above

A. when r is inside a, Electric field changes linearly
B. There is no charge inside an insulated shell
C. Proportional to volume
D. electric field using Gauss' rule.
E. When r<a, then the charge should e=pr/3e0,



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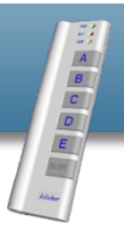
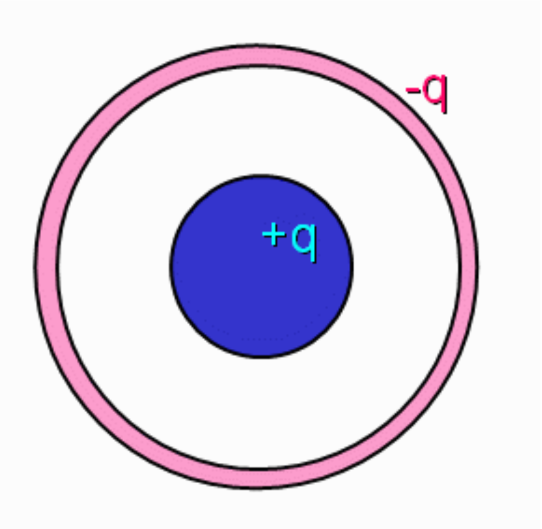
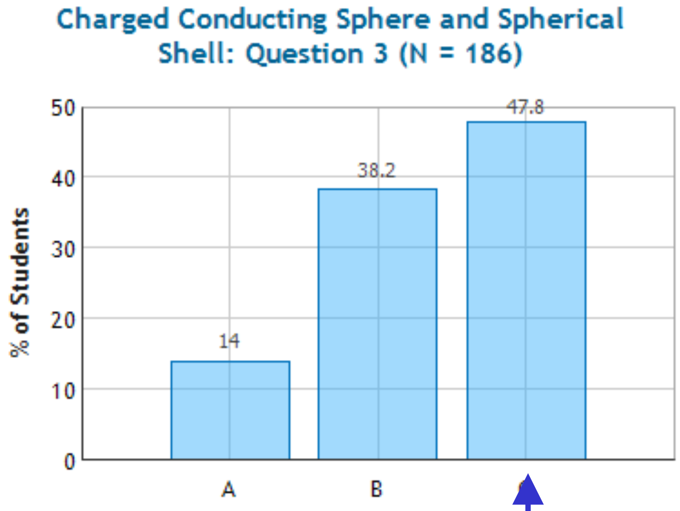


4) A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.



What is direction of field **OUTSIDE** the red sphere?

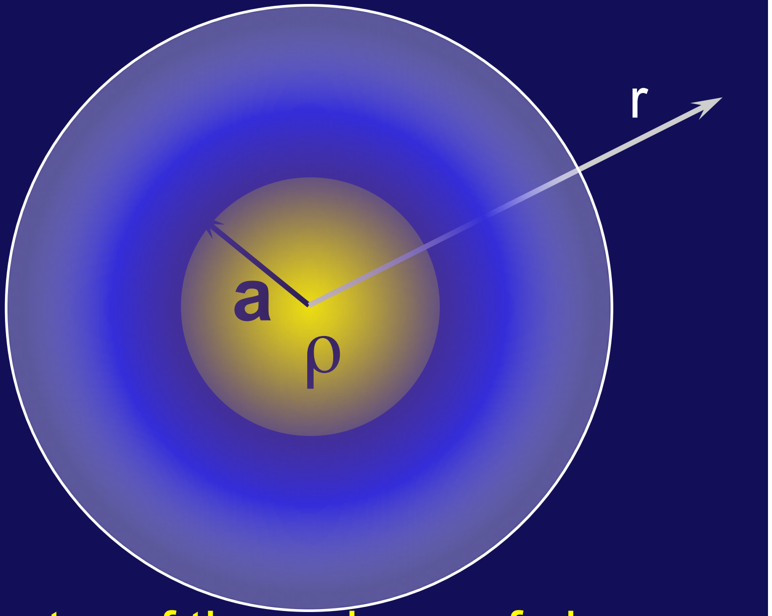
○ The field point radially outward ○ The field point radially inward ○ The field is zero



Gauss' Law: No net enclosed charge

Let’s try another: Uniform charged sphere

What is the magnitude of the electric field due to a solid sphere of radius a with uniform charge density ρ (C/m^3)?



- Outside sphere: (r > a)
    - Spherical symmetry centered on the center of the sphere of charge
    - Choose Gaussian surface to be hollow sphere of radius r

$$\oint \vec{E} \bullet d\vec{S} = 4\pi r^2 E = \frac{q}{\varepsilon_0}$$

$$q = \frac{4}{3}\pi a^3 \rho$$

Þ

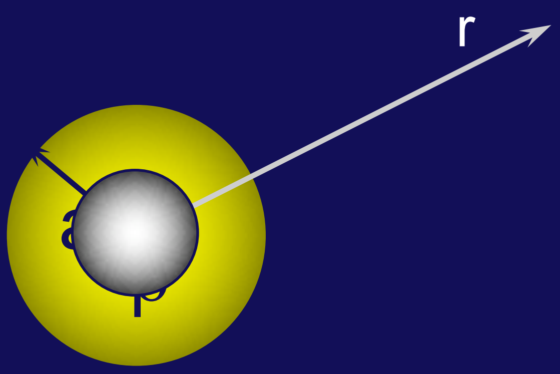
Gauss’ Law

$$\boxed{E = \frac{\rho a^3}{3\varepsilon_0 r^2}}$$

Uniform charged sphere

- Outside sphere: (r > a)

$$\boxed{E = \frac{\rho a^3}{3\varepsilon_0 r^2}}$$

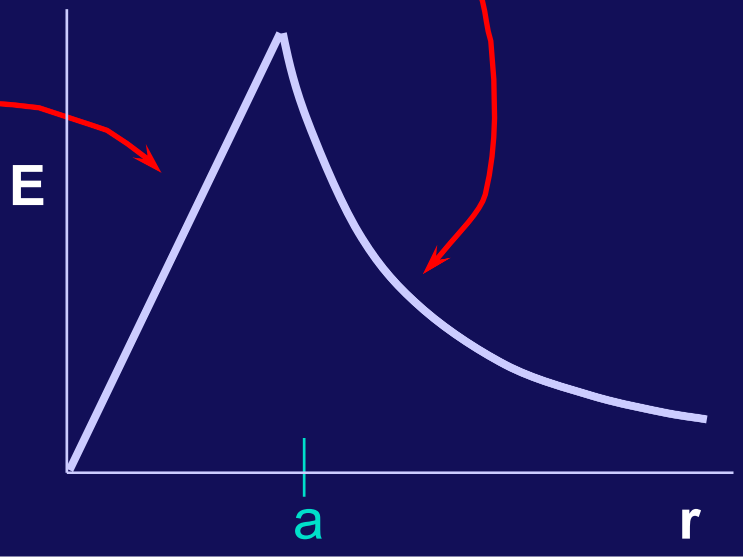


- Inside sphere: (r < a)
  - Spherical symmetry, centered on the center of the sphere of charge.
  - Choose Gaussian surface to be sphere of radius r.

Gauss' Law $$\oint \vec{E} \bullet d\vec{S} = 4\pi r^2 E = \frac{q}{\varepsilon_0}$$

But, $$q = \frac{4}{3}\pi r^3 \rho$$

Thus: $$E = \frac{\rho}{3\varepsilon_0} r$$



How is charge carried on macroscopic objects?

Assume (for now) that only two kinds of objects in the world:

Insulators.. Once charged, the charges CANNOT MOVE.

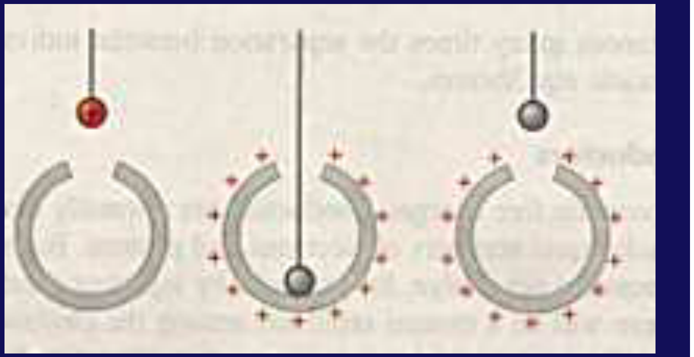
Plastics, glass, and other “bad conductors of electricity” are good examples of insulators.

Conductors.. Here, the charges ARE FREE TO MOVE.

Metals are good examples of conductors.

How do the charges move in a conductor?

DEMO: Faraday Pail



Charge the inside, all of this charge moves to the outside.

Recall Conductors →Charges Free to Move

Claim: $E = 0$ inside any conductor at equilibrium

Charges in conductor move to make E field zero inside. (Induced charge distribution).

If $E \neq 0$, then charge would feel a force and move!

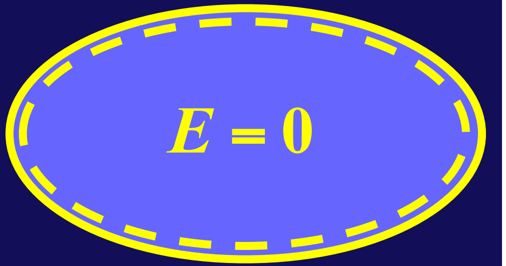
Claim: Excess charge on conductor is only on surface

To demonstrate this, we apply Gauss' Law

-- Take Gaussian surface to be just inside conductor surface

-- $E = 0$ everywhere inside conductor

$$\oint_{surface} \vec{E} \cdot d\vec{A} = 0$$

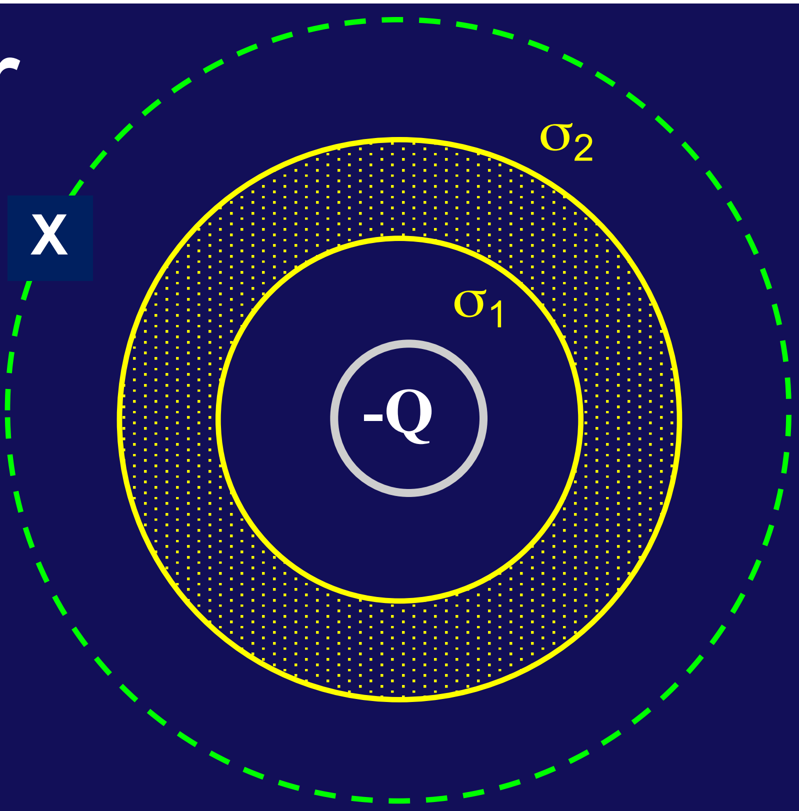


-- Gauss' Law:

$$\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o} \Rightarrow Q_{enc} = 0$$

Clicker

A **Q** = **-3 μC** charge is surrounded by an uncharged conducting spherical shell (in yellow)



Compare the electric field at point **X** to the one you would find if the conducting shell was removed.

(a) $E_{shell} < E_{NoShell}$

(b) $E_{shell} = E_{NoShell}$

(c) $E_{shell} > E_{NoShell}$

Select a sphere passing through the point X as the Gaussian surface.

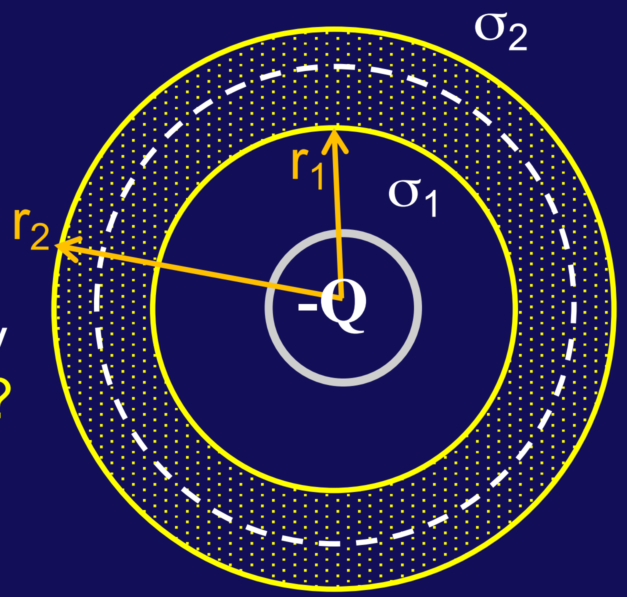
How much charge does it enclose?

Answer: **-Q**, whether or not the uncharged shell is present.

(The field at point **X** is determined only by the objects with NET CHARGE.)

Clicker

A $Q = -3\ \mu C$ charge is surrounded by an uncharged conducting spherical shell (in yellow)



What is the value of the surface charge density σ_1 on the inner surface of the conducting shell?

(a) $\sigma_1 = -Q$ (b) $\sigma_1 = +Q$ (c) $\sigma_1 = 0$ (d) $\sigma_1 = \frac{-Q}{4\pi r_1^2}$ (e) $\boxed{\sigma_1 = \frac{+Q}{4\pi r_1^2}}$

Inside the conductor, we know the field: E = 0

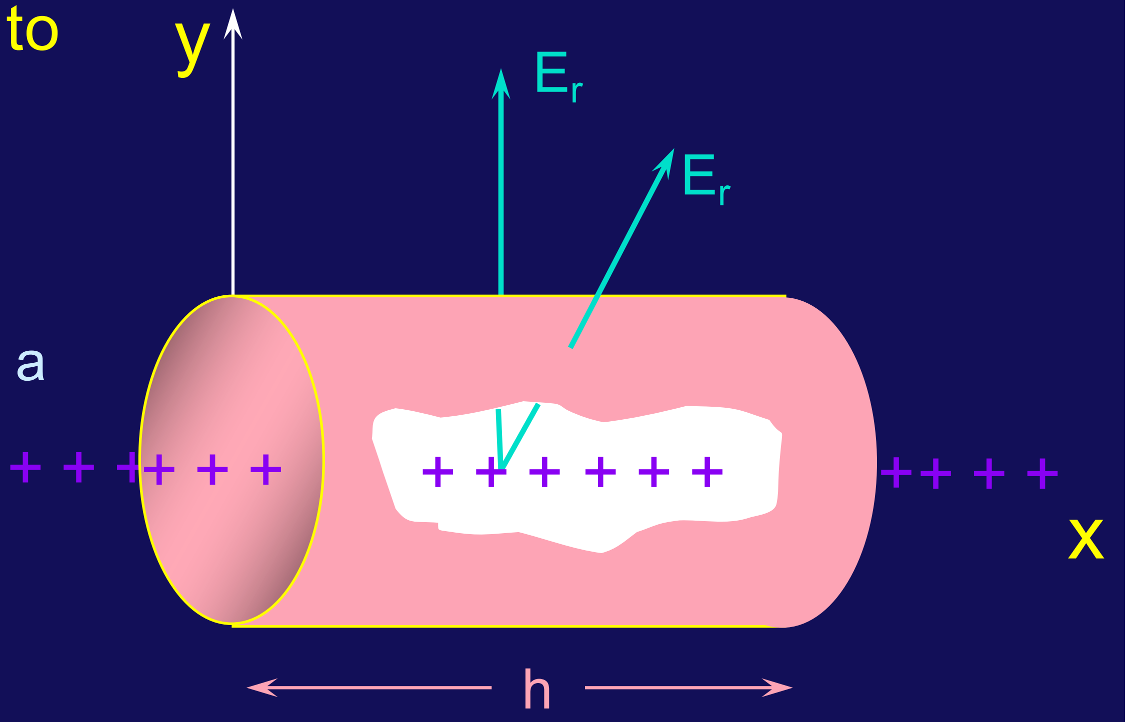
Select a Gaussian surface inside the conductor

Since E = 0 on this Gaussian surface, the total enclosed charge must be 0

Therefore σ_1 must be positive, to cancel the charge **-Q**

Infinite Line of Charge

Symmetry Þ E field must be ⊥ to line and can only depend on distance from line



CHOOSE Gaussian surface to be a cylinder of radius r and length h aligned with the x-axis.

Apply Gauss' Law:

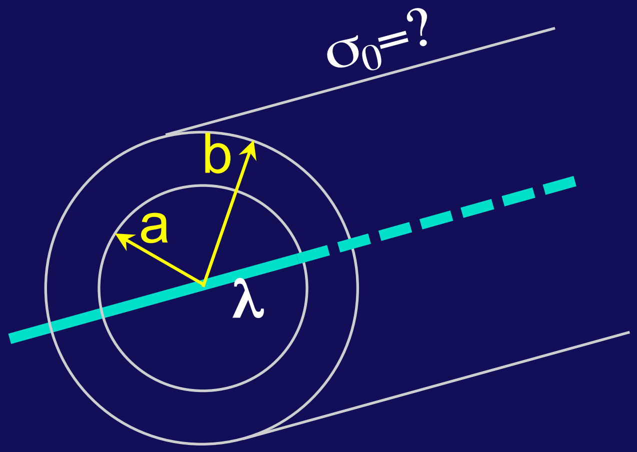
On the ends, $\vec{E} \bullet d\vec{S} = 0$

On the barrel, $\oint \vec{E} \bullet d\vec{S} = 2\pi r h E$ and $q_{enclosed} = \lambda h$ Þ

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Clicker

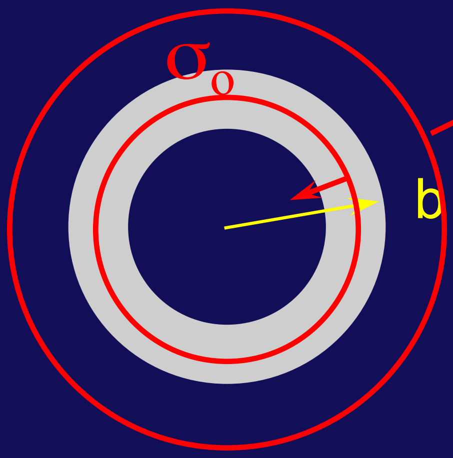
A line charge λ C/m is placed along the axis of an uncharged conducting cylinder of inner radius $r_i = a$, and outer radius $r_o = b$ as shown.



What is the value of the charge density σ_o **(C/m²)** on the outer surface of the cylinder?

(a) $\sigma_o = -\frac{\lambda}{2\pi b}$ (b) $\sigma_o = 0$ $\boxed{\text{(c) } \sigma_o = +\frac{\lambda}{2\pi b}}$

View end on:
Draw Gaussian tube which surrounds only the outer edge

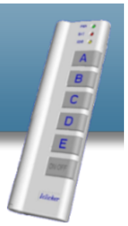


$$E_{outside} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\oint EdS = (2\pi r_{inside} L)\overset{0}{\cancel{E_{conductor}}} + (\cancel{2\pi} r_{outside} L)E_{outside} = \frac{q}{\varepsilon_0} = \frac{\sigma_o \cancel{2\pi} bL}{\varepsilon_0}$$

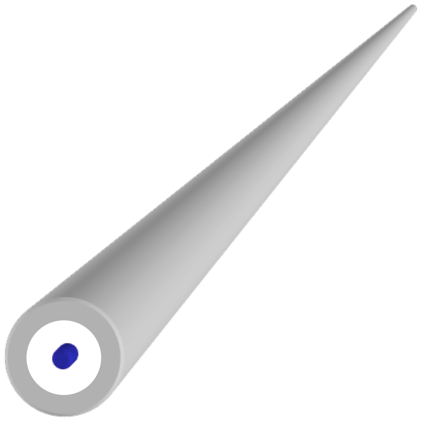
$$E_{outside} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma_o b}{\varepsilon_0 r} \Rightarrow \sigma_o = \frac{\lambda}{2\pi b}$$

Smart Clicker: Infinite Cylinders

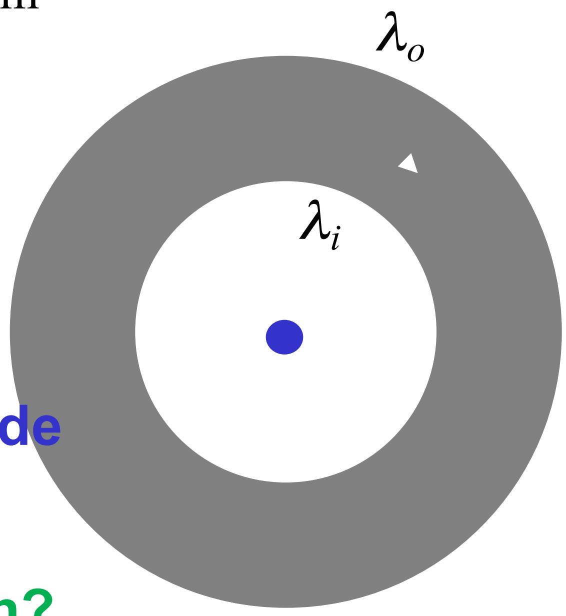


A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of –4 C/m.

What is the linear charge density of the induced charge on the inner surface of the conducting cylinder (λ_i) and on the outer surface (λ_o)?



λ_i:	+2.5 C/m	–4 C/m	–2.5 C/m	–2.5 C/m	0
λ_o:	–6.5 C/m	0	+2.5 C/m	–1.5 C/m	–4 C/m
	A	B	C	[✓ D]	E



- **E = 0 in conducting cylinder means net charge enclosed = 0**
  - **Thus, λ_ι = -2.5 C/m to cancel wire**
- **If NET is -4 C/m and inside is -2.5 C/m, then outside has to be -1.5 C/m to add up**
- **Follow up for you: What is field outside based on?**