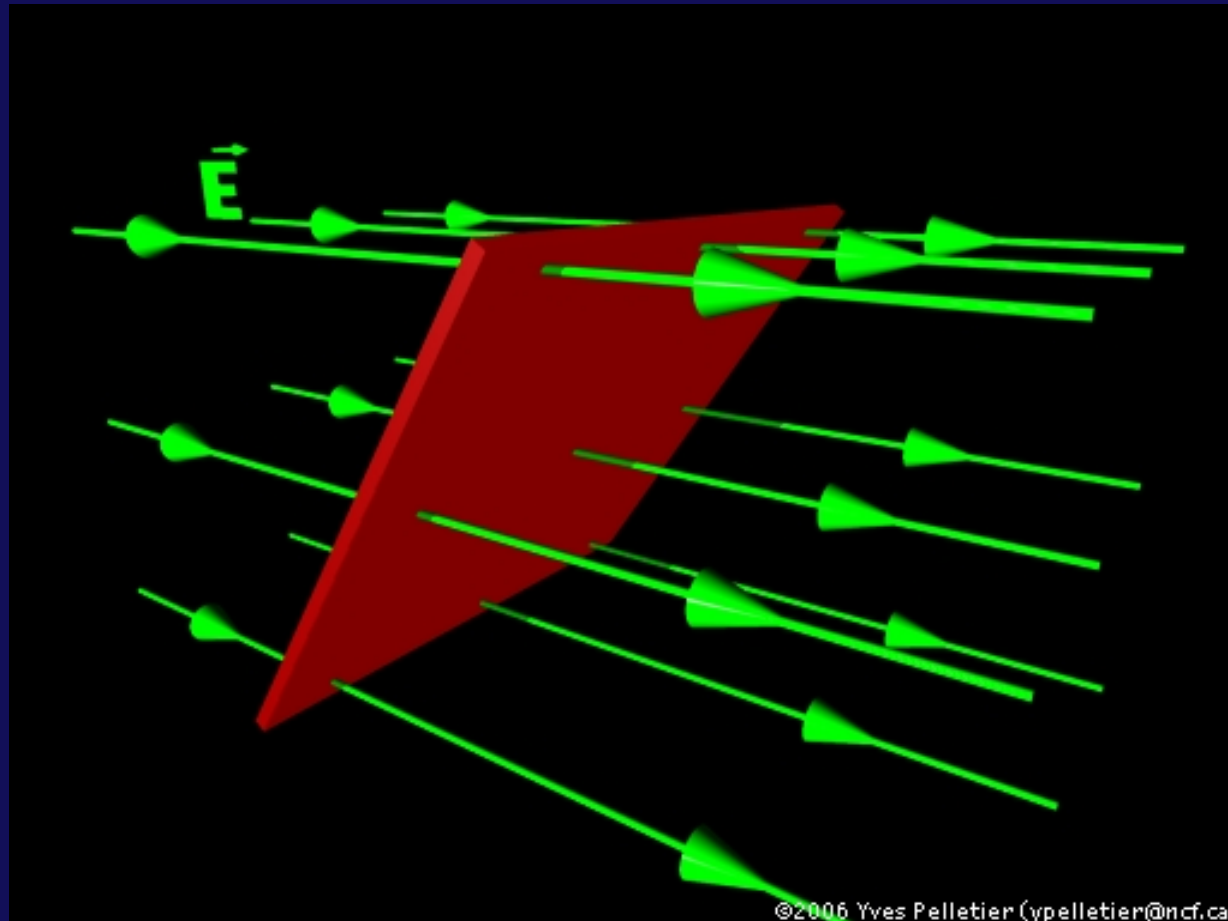


# Electric Flux

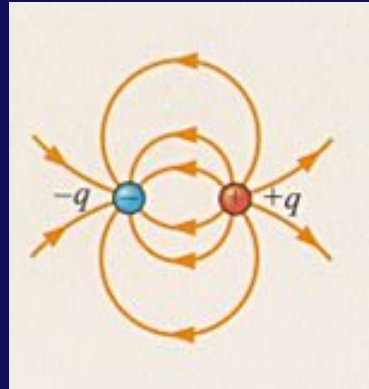


Phys 122 Lecture 4

G. Rybka

# Electric Field Distribution Summary

Dipole



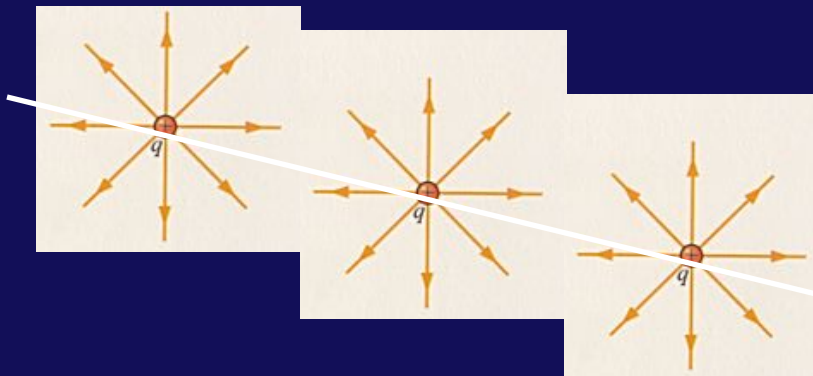
$$\sim 1 / R^3$$

Point Charge



$$\sim 1 / R^2$$

Infinite  
Line of Charge



$$\sim 1 / R$$

# Outline

- Electric field acting on charges
- Defining Electric Flux
  - *A measure of flow of the electric field through a given area.*
- A brief introduction to Gauss' Law

## Some of your Concerns

**What is the significance of electric flux? It seems like we are just defining various terms right now, with little application. And are we actually going to be calculating integrals in this class? We did not do so in Phys 121 (just focused on conceptual understanding), and these integrals look rather complex...**

**I would love a review of the prelecture question concerning the two charges within one circle. The charge is moved closer to one edge, but how does that affect the flux of points on opposite sides of the circle?**

**How to apply the formulas?**

# New Stuff: A moving charge ...

Recall from mechanics ...

$$F = ma$$

... and force on  $q$  in  $E$

$$F = qE$$

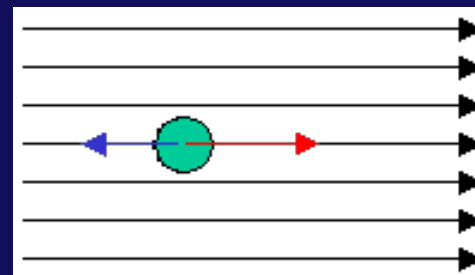
Acceleration of a charge is then:

$$a = (q/m)E$$

Tipler Problem: A particle beam of protons, each with a  $KE = 3.25 \times 10^{-15} \text{ J}$ , is stopped by a uniform electric field in 2 m. A proton has:

$$m_p = 1.673 \times 10^{-27} \text{ kg}$$

$$q_p = 1.602 \times 10^{-19} \text{ C}$$



What is  $E$  ?

1. Given the kinetic energy
2. 1-D kinematics, uniform  $a$ ; stops in distance  $x$ :
3. Substitute:
4. Recall from above that
5. Therefore
6. Or,
7. You know all these, so only now calculate:

$$KE = \frac{1}{2} mv^2$$

$$2ax = v^2$$

$$ax = KE/m$$

$$a = (q/m)E$$

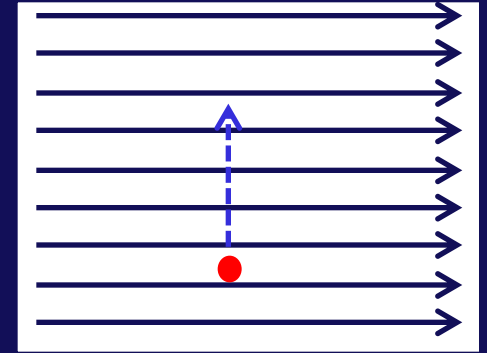
$$qEx = KE$$

$$E = KE/(qx)$$

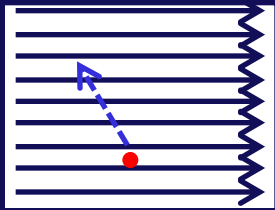
$$E = 10143 \text{ N/C}$$

# Clicker

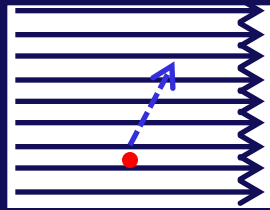
An electron is fired upward at speed  $v_0$  toward the top of the page as shown. A uniform electric field points to the right.



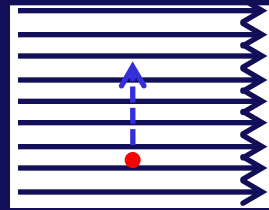
Which trajectory best represents its motion:



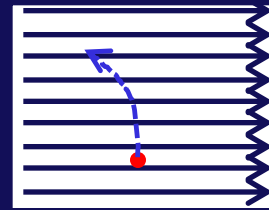
(A)



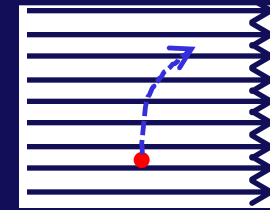
(B)



(C)



(D)



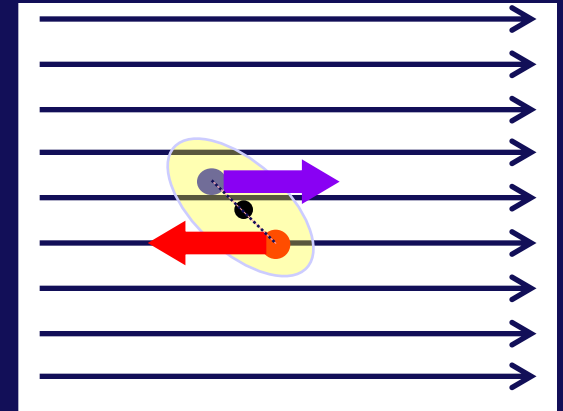
(E)

- **Vertical speed unchanged; no vertical force**
- **Horizontal force:**  $F_x = qE_x = ma$ 
  - **Recall constant acceleration  $\rightarrow$  parabolic trajectory in  $x$**
- **Which way ? Electron is NEGATIVE charge so sign of  $F_x$  is opposite to sign of  $E$**

# Clicker: Torque on a dipole

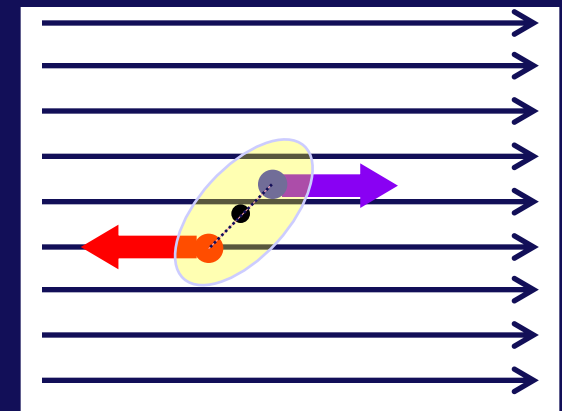
An electric dipole consists of two equal and opposite charges, fixed a distance  $2a$  apart. It is placed in a uniform electric field as shown. It will

- A. Not translate in any direction
- B. Translate horizontally
- C. Translate vertically
- D. Start to rotate clockwise**
- E. Start to rotate counter-clockwise



Consider the Forces on the charges:

- equal in magnitude
- opposite in direction
  - Dipole will not translate
- Displaced compared to center of mass
  - non-zero torque



Dipole will initially rotate Clockwise

# Torque on a dipole

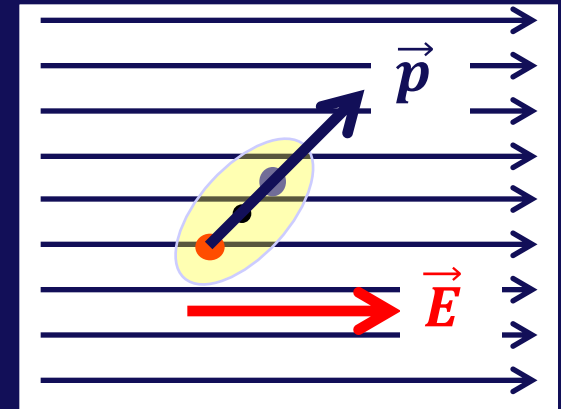
- **Electric dipole moment:**

$$\vec{p} = q\vec{L}$$

q = charge

$\vec{L}$  = vector from - to + charge

Units: C-m



- **Torque on dipole in uniform field**

$$\vec{\tau} = \vec{p} \times \vec{E}$$

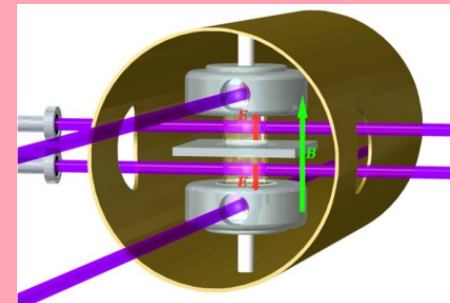
Aligns dipole with field

- **Potential energy of dipole**

$$U = -\vec{p} \cdot \vec{E} + U_0$$

$U_0$  is usually 0

Aside: UW Hg-199 EDM experiment



A test of new physics;  
most sensitive in world

# Fundamental Law of Electrostatics

- **Coulomb's Law**

**Force between two point charges**

**OR**

- **Gauss' Law**

**Relationship between Electric Fields  
and charges**



# Electric Dipole Field Lines

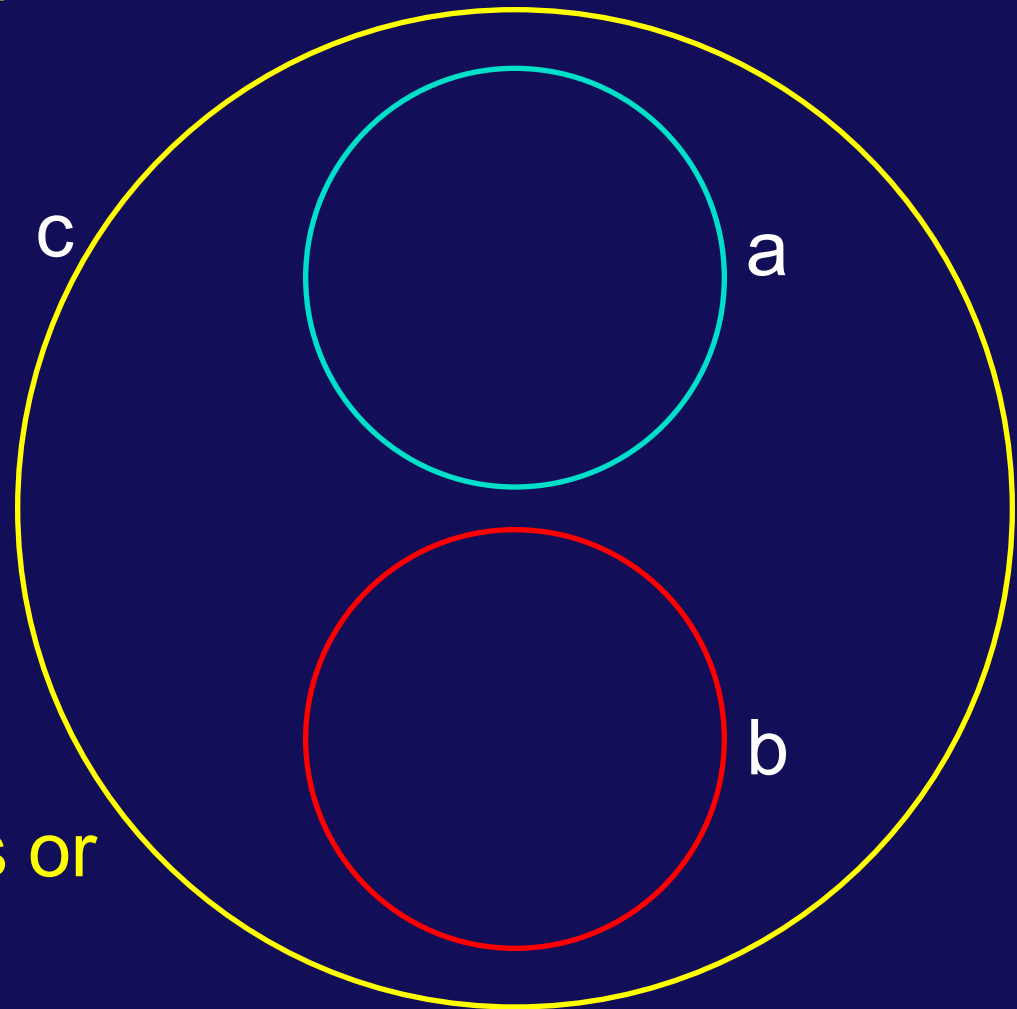
Consider imaginary  
spheres centered on :

a)  $+q$  (green)

b)  $-q$  (red)

c) midpoint (yellow)

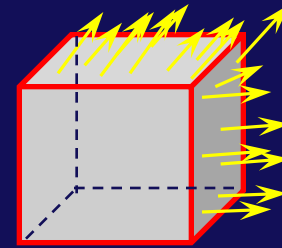
- All lines leave a
- All lines enter b
- Equal number leaves or enters sphere c



# Quantify Electric Flux

Define: electric flux  $\Phi_E$  through the closed surface  $S$

$$\Phi_E \equiv \oint \vec{E} \cdot d\vec{S}$$



- The integral is over the entire “CLOSED SURFACE”
- The result (the net electric flux) is a SCALAR quantity

$\vec{E} \cdot d\vec{S}$  is the component of  $E$  which is NORMAL to the SURFACE  
(normal means it points OUT, “dS is positive”)

- Net Electric flux is the sum of the normal components of the electric field all over the closed surface.
- The direction of the normal component of  $E$  as it penetrates the surface can point IN or it can point OUT.
- IN is NEGATIVE                      OUT is POSITIVE

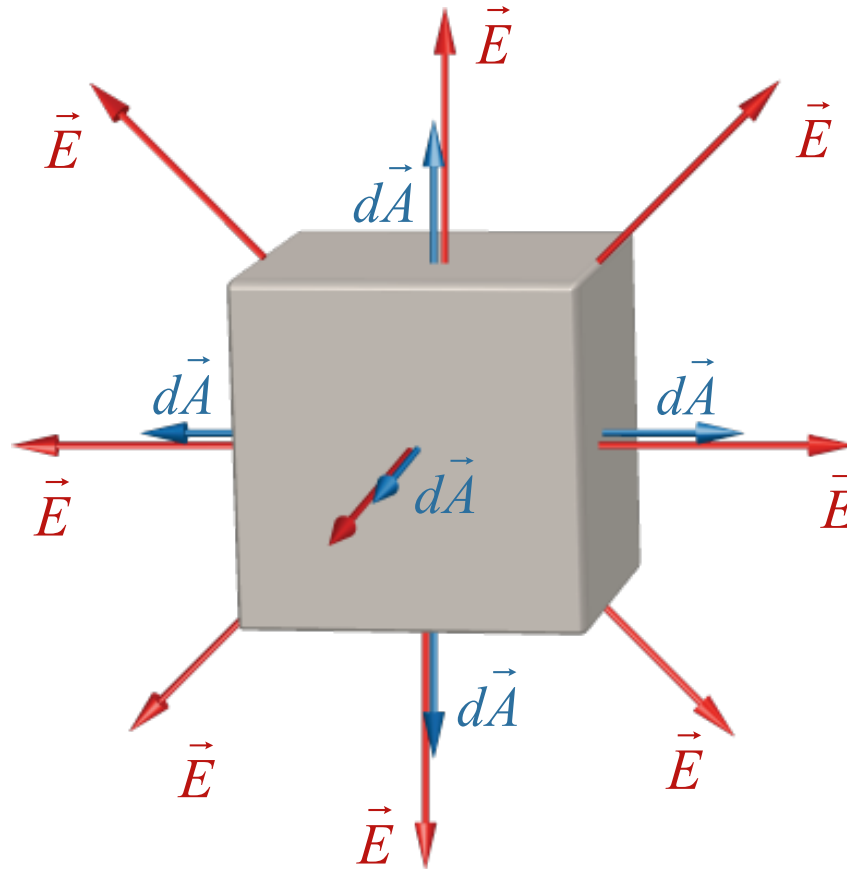
# Other Kinds of Flux

- In general, flux is a measure of something that is flowing. “How much is going how fast through a given area?”
- Examples: Water, Light
- Electric field isn't really flowing, but if tiny charges were present, they would.
- This will make more sense when we get to the electric current

# Direction Matters:



What might be in this box ????



For a closed surface,  $d\vec{A}$  points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

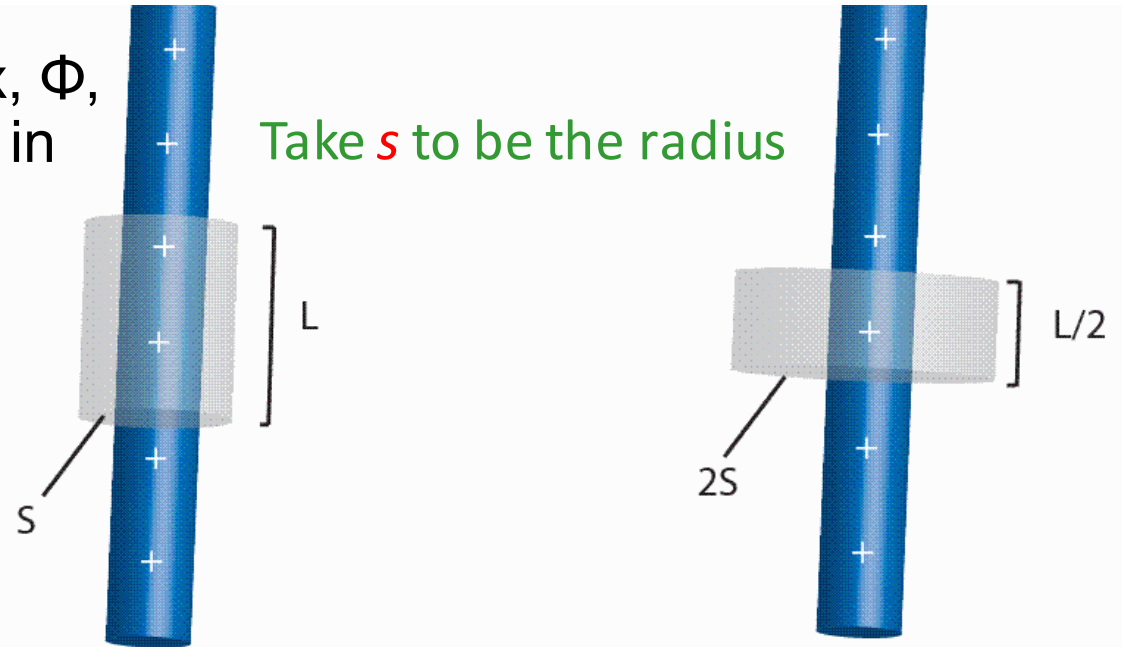
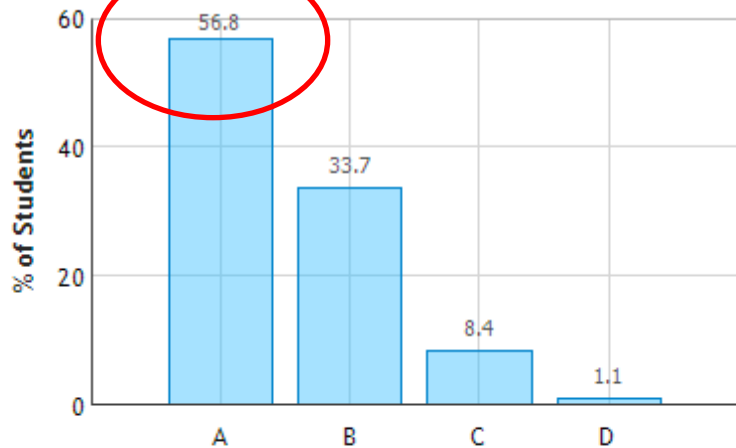
# CheckPoint: Electric Flux and Field Lines



An infinitely long charged rod has uniform charge density of  $\lambda$ , and passes through a cylinder (gray). The cylinder in case 2 has twice the radius and half the length compared to the cylinder in case 1.

Compare the magnitude of the flux,  $\Phi$ , through the surface of the cylinder in both cases.

Flux from Uniformly Charged Rod: Question 1  
(N = 95)



Case 1	Case 2
$\Phi_1 = 2\Phi_2$ (A)	$\Phi_1 = \Phi_2$ (B)
	$\Phi_1 = 1/2\Phi_2$ (C)
	none (D)

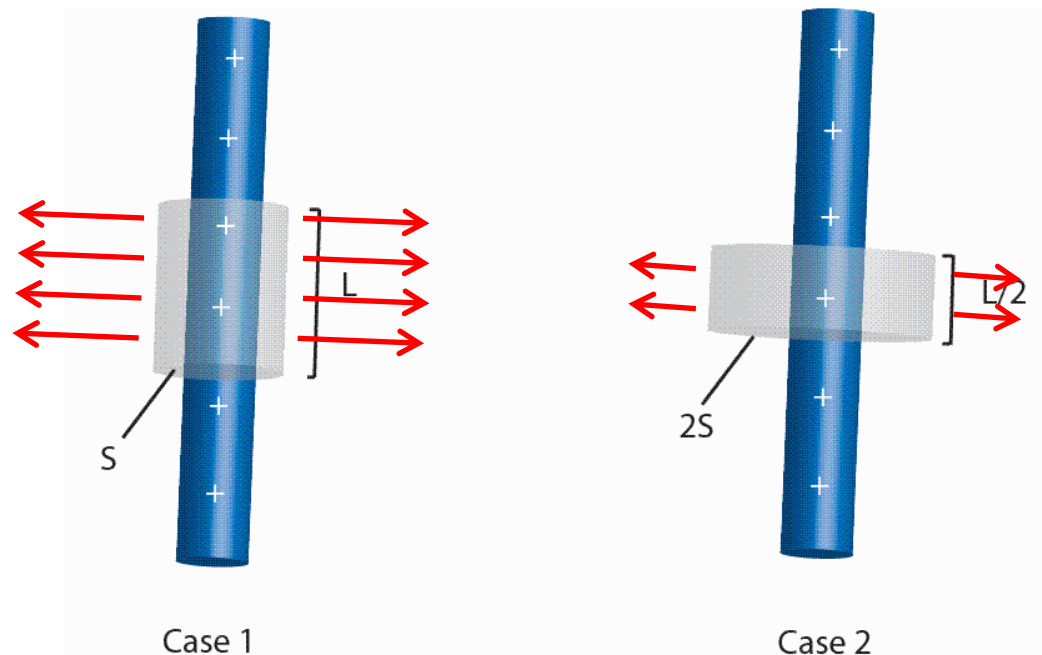
# CheckPoint: Electric Flux and Field Lines

## Definition of Flux:

$$\Phi \equiv \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

$E$  constant on barrel of cylinder  
 $E$  perpendicular to barrel surface  
 ( $E$  parallel to  $dA$ )

$$\Phi = E \int_{\text{barrel}} d\vec{A} = EA_{\text{barrel}}$$



$\Phi_1 = 2\Phi_2$ (A)	$\Phi_1 = \Phi_2$ (B)	$\Phi_1 = 1/2\Phi_2$ (C)	none (D)
---------------------------	--------------------------	-----------------------------	-------------

**RESULT: GAUSS' LAW**

$\Phi$  proportional to charge enclosed !

**Case 1**

$$E_1 = \frac{\lambda}{2\pi\epsilon_0 s} \rightarrow \Phi_1 = \frac{\lambda L}{\epsilon_0}$$

$A_1 = 2\pi s L$

**Case 2**

$$E_2 = \frac{\lambda}{2\pi\epsilon_0 (2s)} \rightarrow \Phi_2 = \frac{\lambda(L/2)}{\epsilon_0}$$

$A_2 = (2\pi(2s))L/2 = 2\pi s L$

# Trapezoid in Constant Field



This is a “shout-out” clicker

$$\vec{E} = E_0 \hat{x}$$

Label faces:

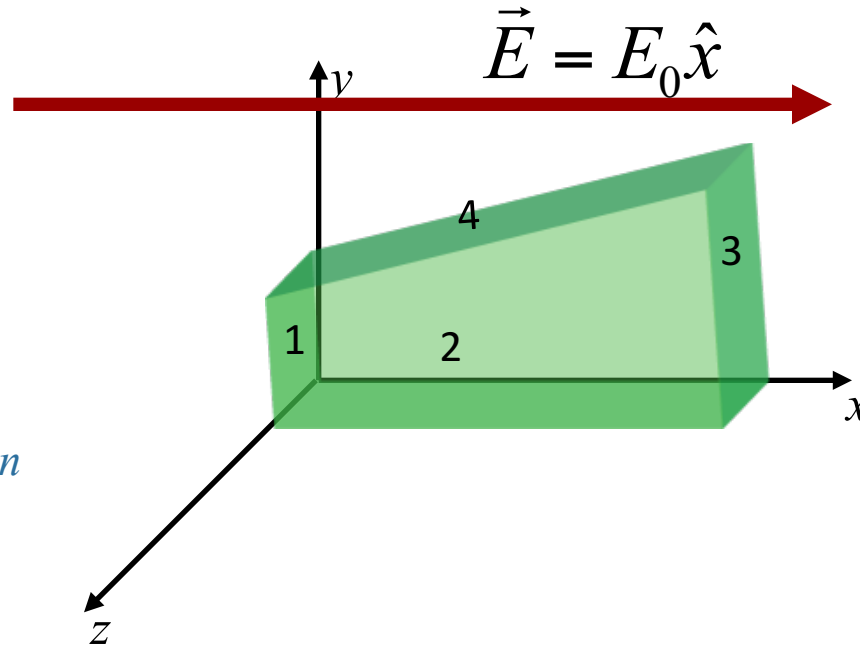
1:  $x = 0$

2:  $z = +a$

3:  $x = +a$

4: slanted

Define  $\Phi_n =$  Flux through Face  $n$



A)  $\Phi_1 < 0$

B)  $\Phi_1 = 0$

C)  $\Phi_1 > 0$

A)  $\Phi_2 < 0$

B)  $\Phi_2 = 0$

C)  $\Phi_2 > 0$

A)  $\Phi_3 < 0$

B)  $\Phi_3 = 0$

C)  $\Phi_3 > 0$

A)  $\Phi_4 < 0$

B)  $\Phi_4 = 0$

C)  $\Phi_4 > 0$

# Trapezoid in Constant Field + $Q$



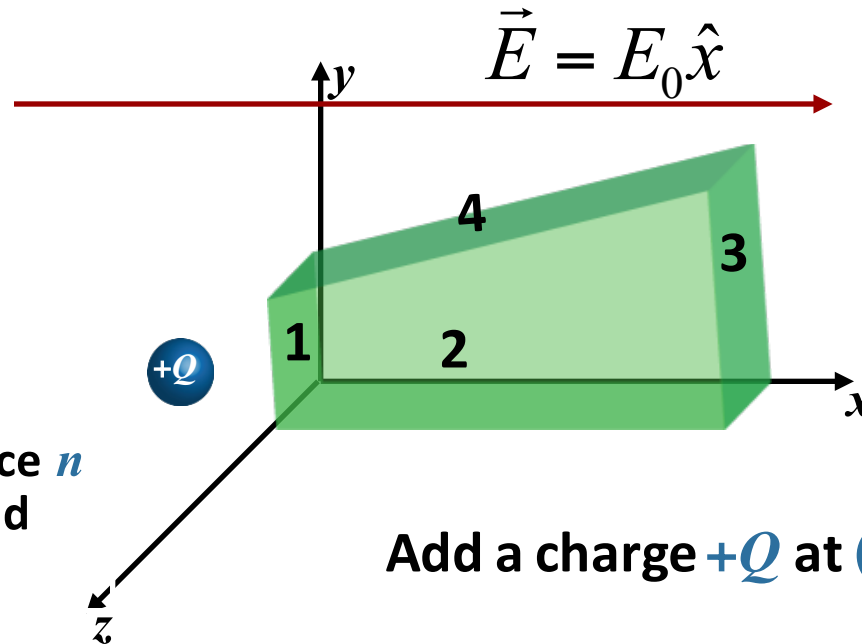
This is a “shout-out” clicker

$$\vec{E} = E_0 \hat{x}$$

Label faces:

- 1:  $x = 0$
- 2:  $z = +a$
- 3:  $x = +a$
- 4: slanted

Define  $\Phi_n =$  Flux through Face  $n$   
 $\Phi =$  Flux through Trapezoid



Add a charge  $+Q$  at  $(-a, a/2, a/2)$

How does Flux change?

A)  $\Phi_1$  increases

B)  $\Phi_1$  decreases

C)  $\Phi_1$  remains same

A)  $\Phi_3$  increases

B)  $\Phi_3$  decreases

C)  $\Phi_3$  remains same

A)  $\Phi$  increases

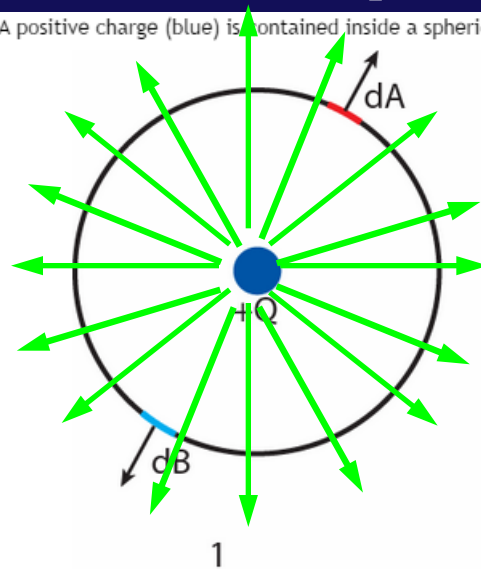
B)  $\Phi$  decreases

C)  $\Phi$  remains same

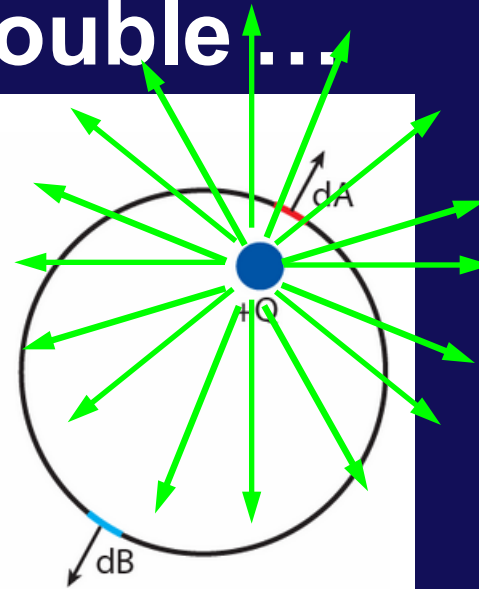


# Checkpoint trouble...

A positive charge (blue) is contained inside a spherical shell (black).



1



2

Consider two small surface elements on the shell surface  $dA$  and  $dB$ , located on opposite side of the shell.

1) How does the electric flux through the two surface elements,  $d\Phi_A$  and  $d\Phi_B$  change when the charge is moved from position 1 to position 2?

- $d\Phi_A$  increases and  $d\Phi_B$  decreases
- $d\Phi_A$  decreases and  $d\Phi_B$  increases
- Both  $d\Phi_A$  and  $d\Phi_B$  do not change

**63%**

Flux from Point Charge Through Surfaces of Sphere: Question 1 (N = 94)

