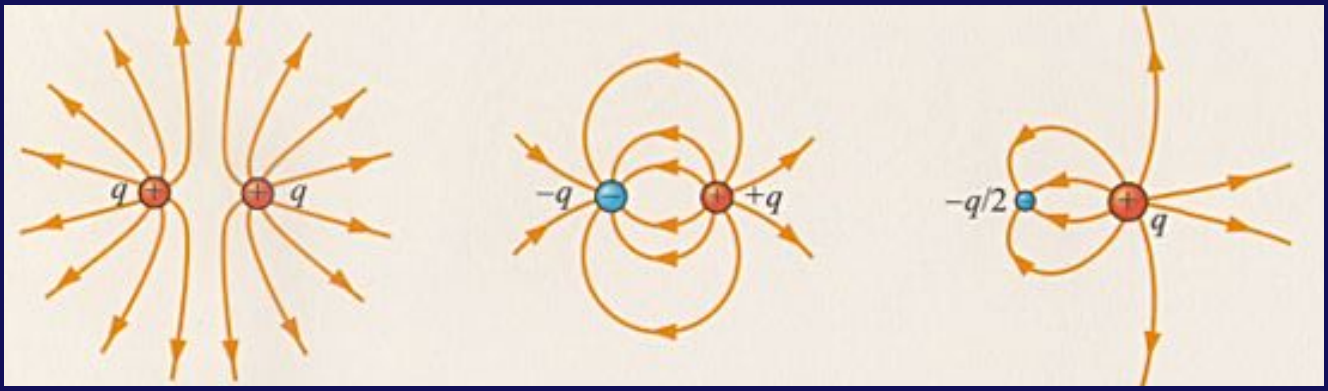
Electric Fields



Phys 122 Lecture 3
G. Rybka

Overview

- A few more Demos
- Electric Field Lines
- Example Calculations:
  - Discrete: Electric Dipole
  - Continuous: Infinite Line of Charge

Next week Labs and Tutorials begin

Electric Fields

The force, F, on any charge q due to some collection of charges is always proportional to q:

$$\vec{F} = \frac{q}{4\pi\varepsilon_0} \sum \frac{q_i \hat{r}_i}{r_i^2}$$

Last time, we introduced the Electric Field:

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

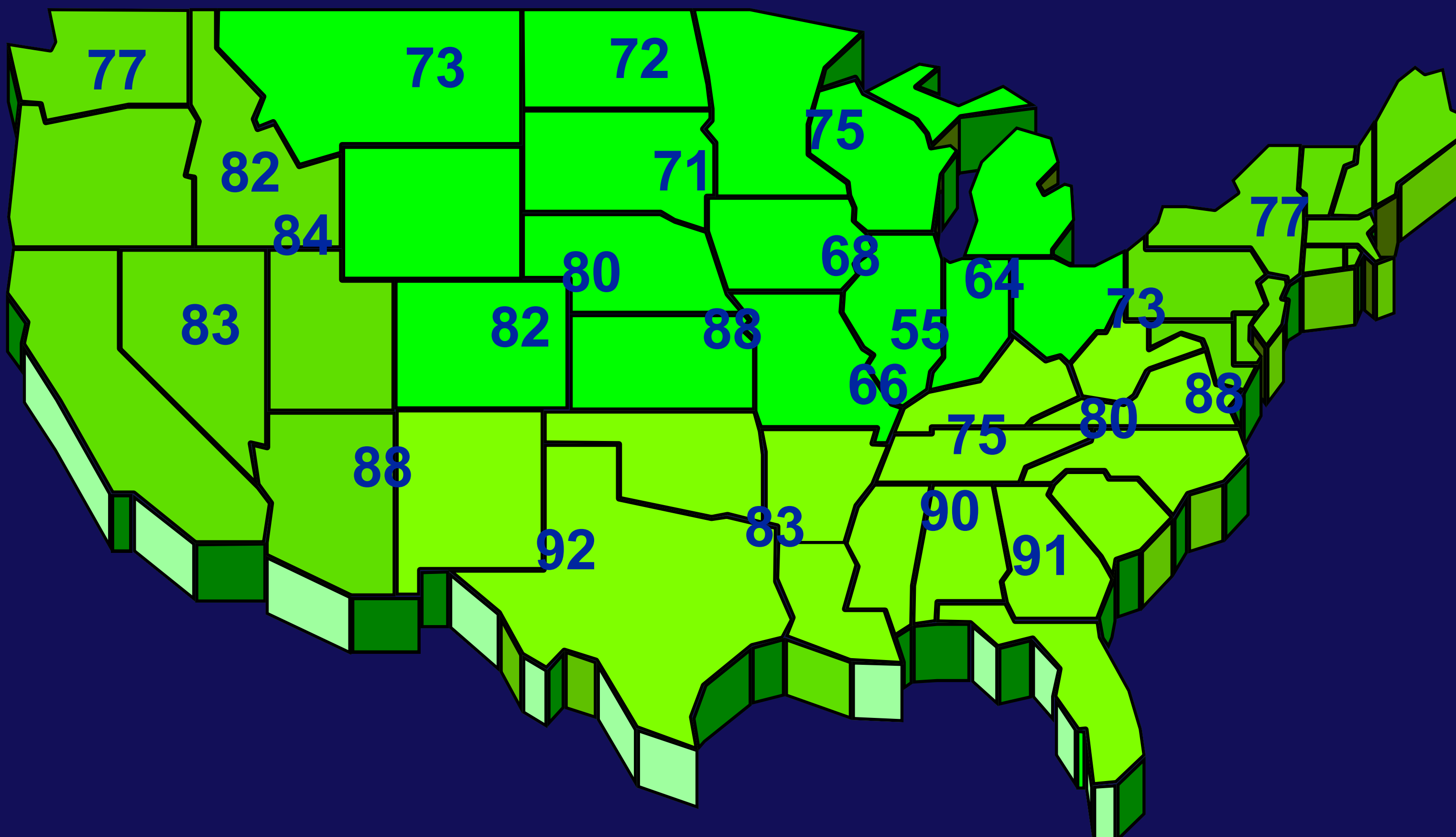
a quantity, which is independent of that charge q, and depends only upon its position relative to the collection of charges.

A FIELD is something that can be defined anywhere in space

it can be a scalar field (e.g., a Temperature Field)

it can be a vector field (as we have for the Electric Field)

Fields of all kinds...

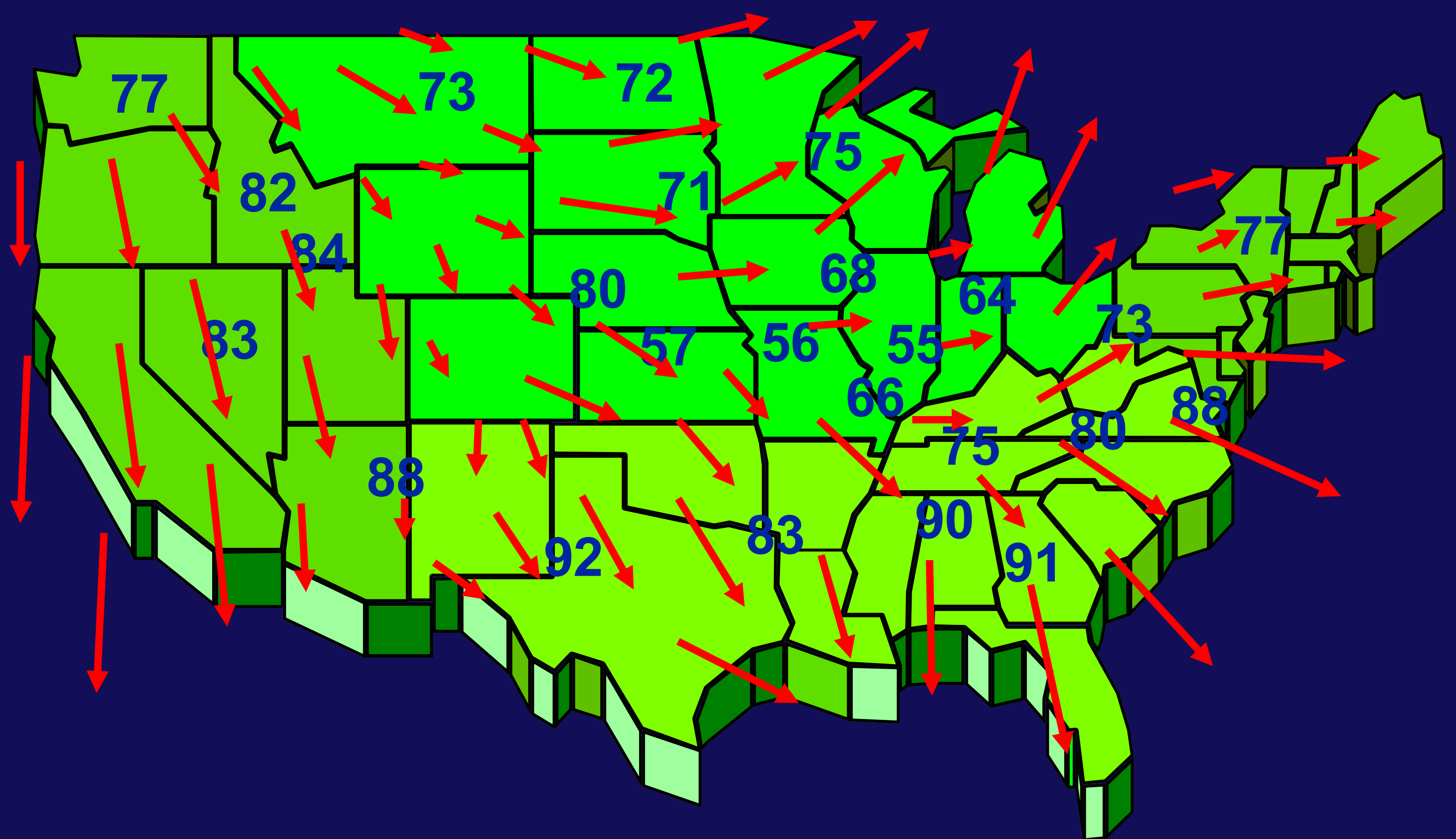


These isolated Temperatures make up a Scalar Field

(you learn only the temperature at a place you choose)

Fields of all kinds...

It may be more interesting to know which way the wind is blowing …

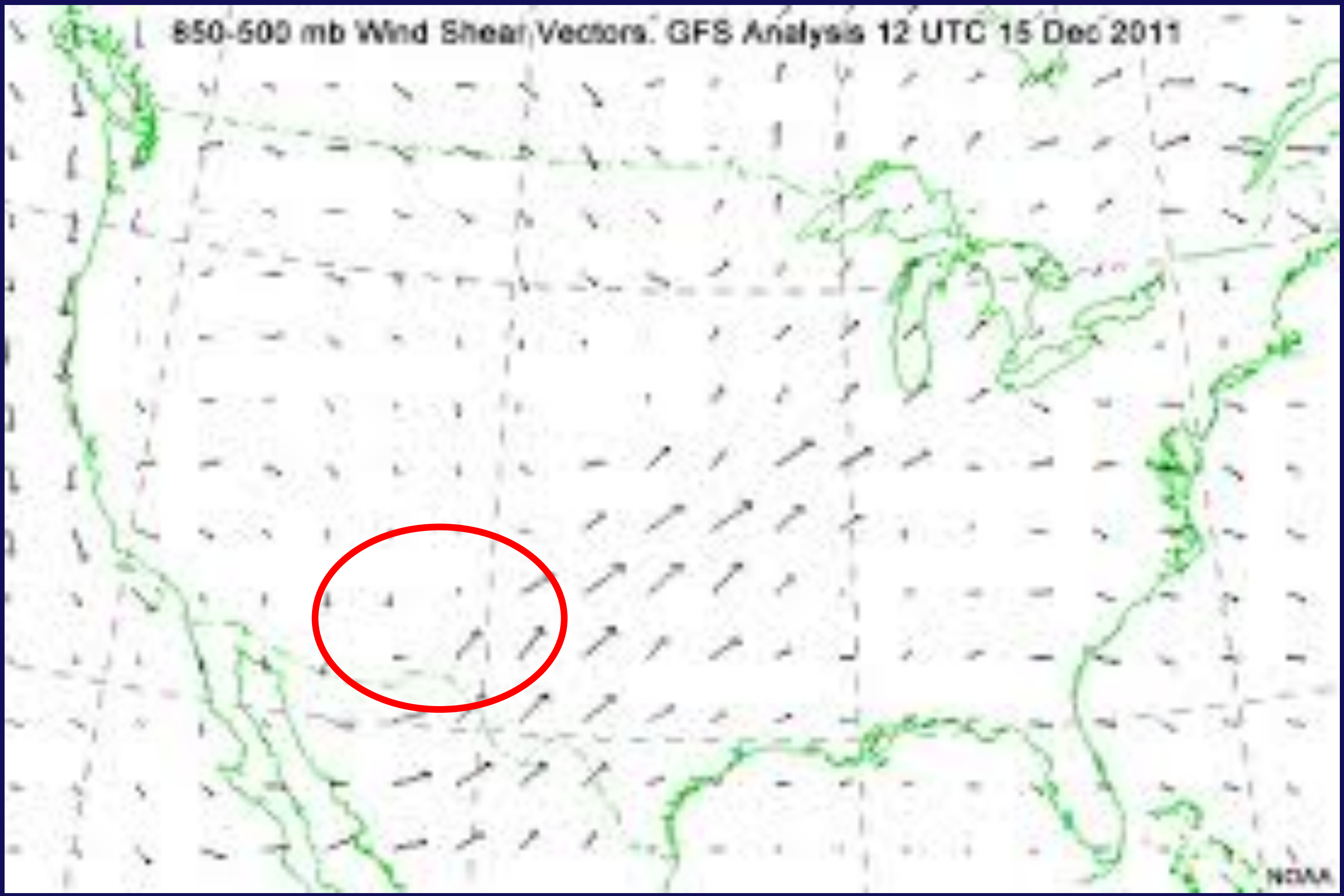


That would require a VECTOR field.

(you learn how fast the wind is blowing, AND in what direction)

Fields of all kinds...

I didn't just make this up. Here's a real one

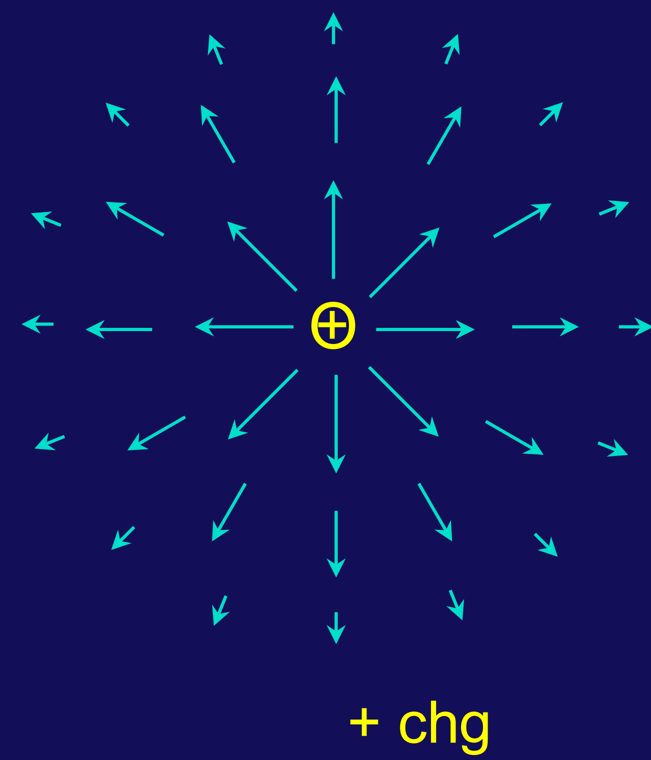


Notice: Anywhere you look, the lines do NOT cross each other

How Can We Visualize the E Field?

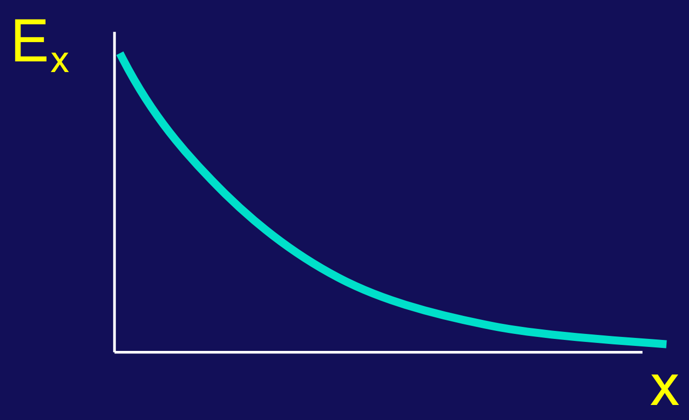
- Vector Maps:

  arrow length indicates vector magnitude

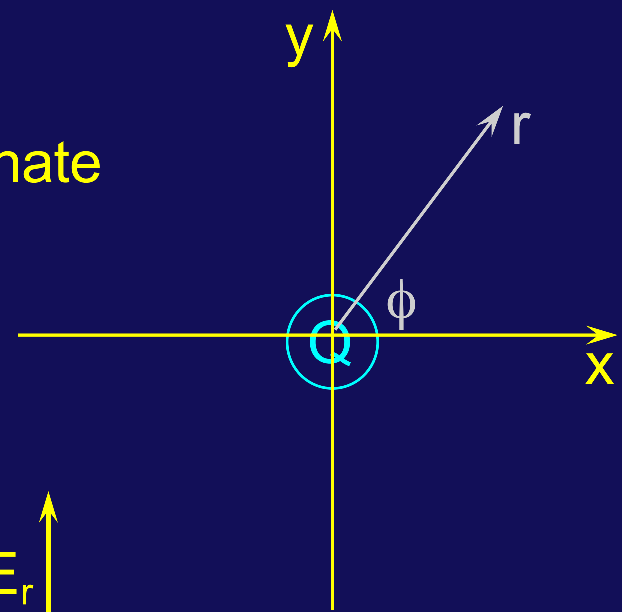


- Graphs:

  E_x, E_y, E_z as a function of (x, y, z)
  E_r, E_θ, E_Φ as a function of (r, θ, Φ)

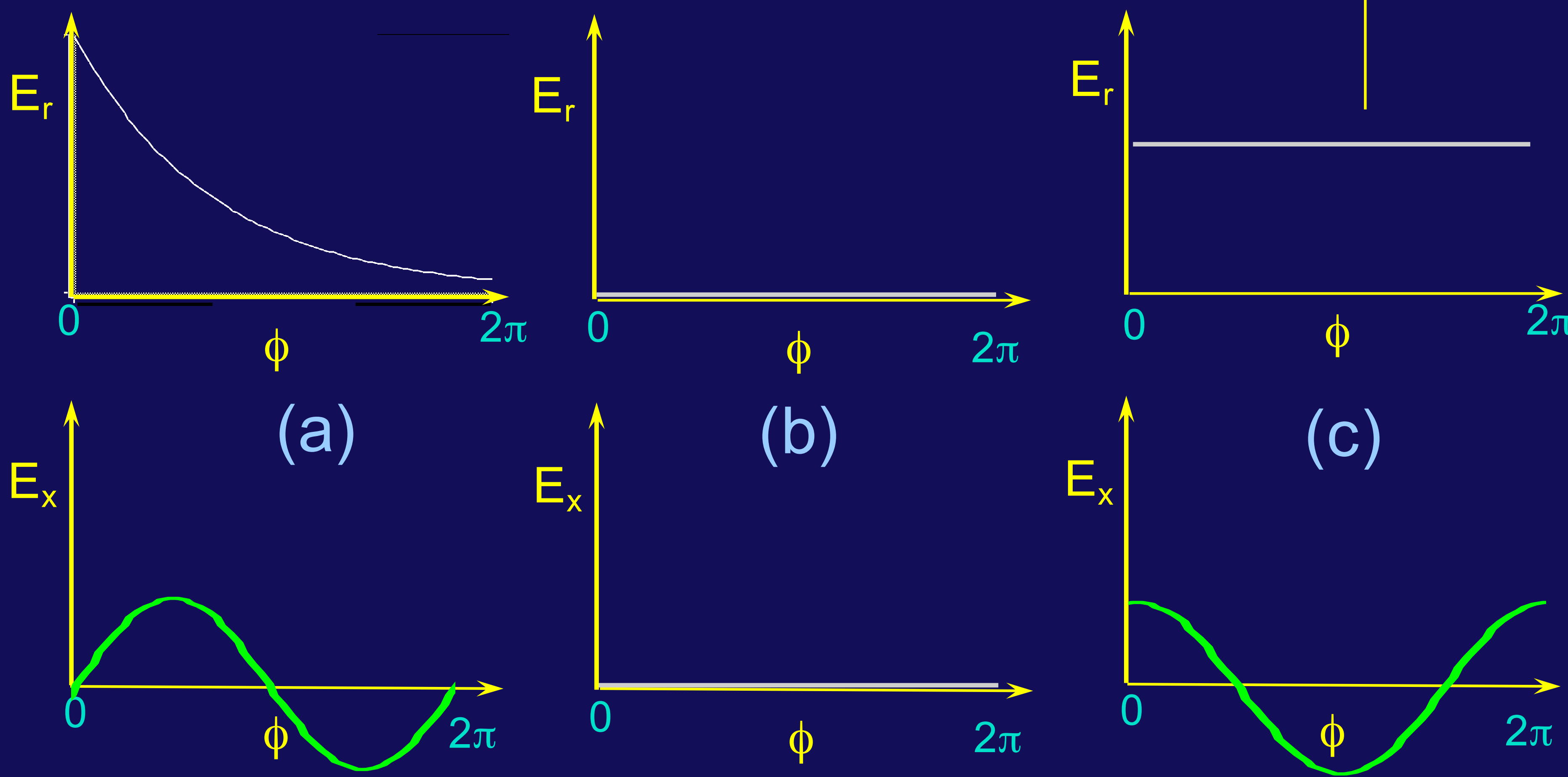


Clickers



Consider a point charge fixed at the origin of a co-ordinate system as shown.

Which of the following graphs best represents the functional dependence of the Electric Field?



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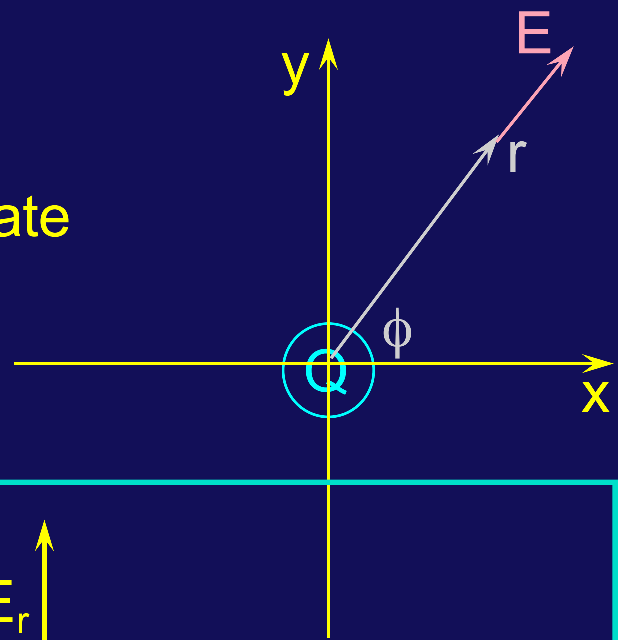

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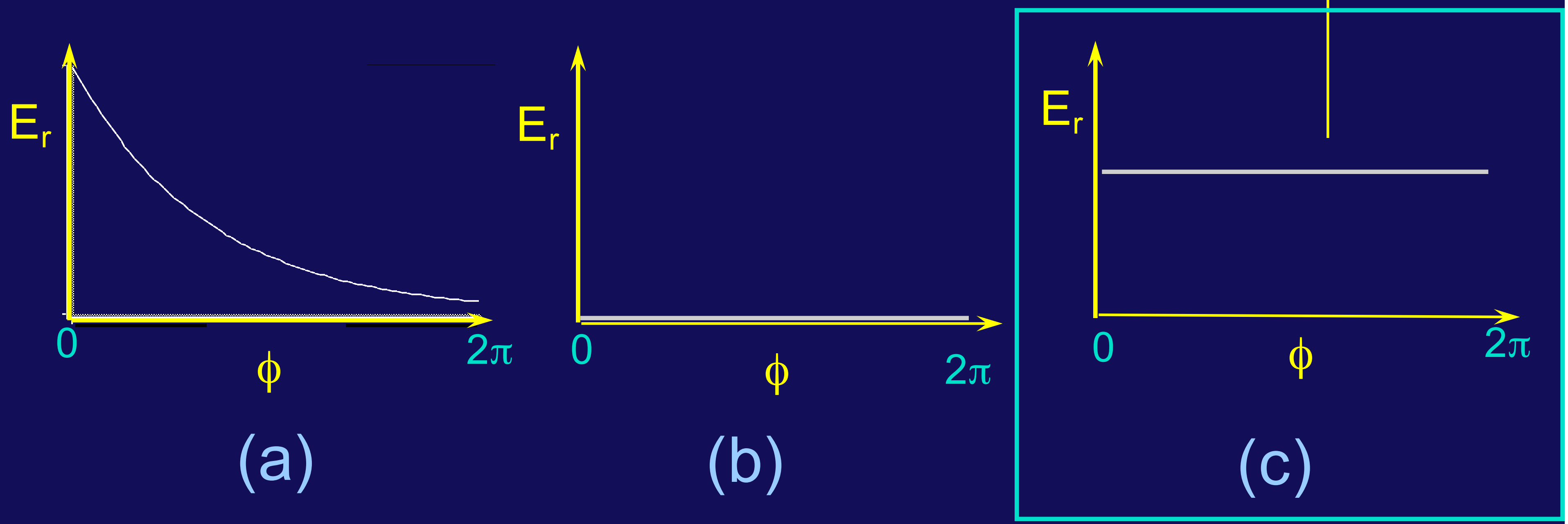
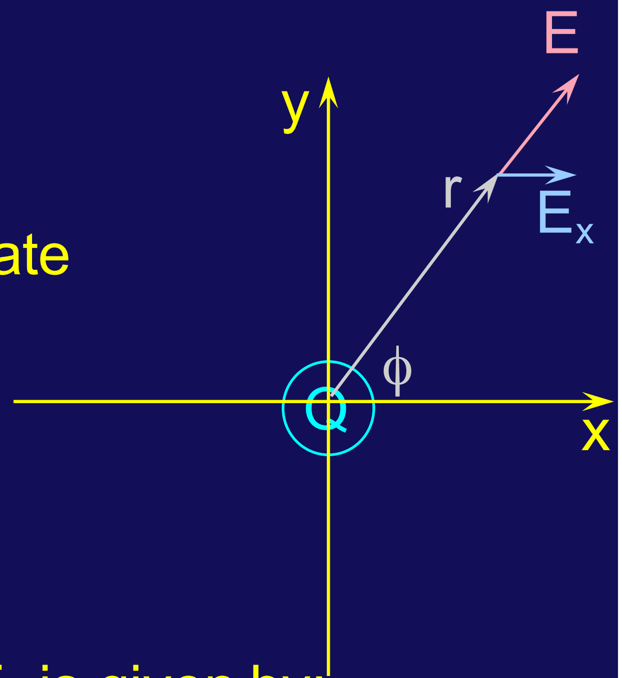


At a fixed r, the radial component of the field is a constant, independent of ϕ!!

For r>0, this constant is > 0.

(note: the azimuthal component E_ϕ is zero)

Clicker

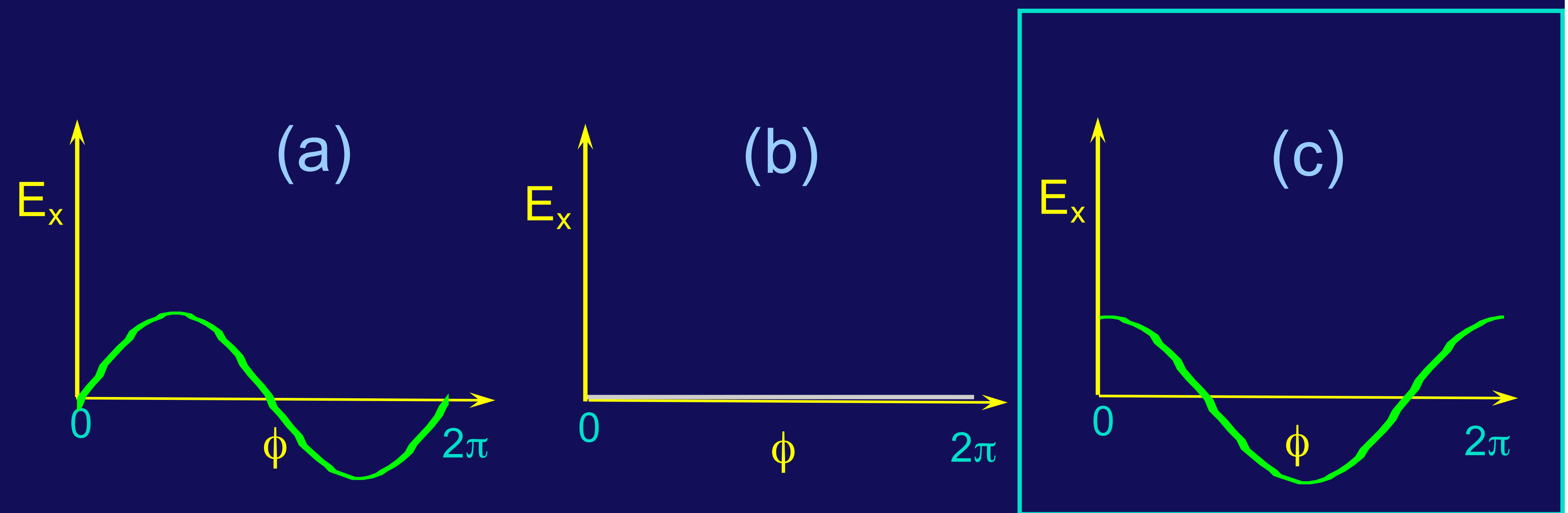


Consider a point charge fixed at the origin of a co-ordinate system as shown.

Which of the following graphs best represents the functional dependence of the Electric Field?

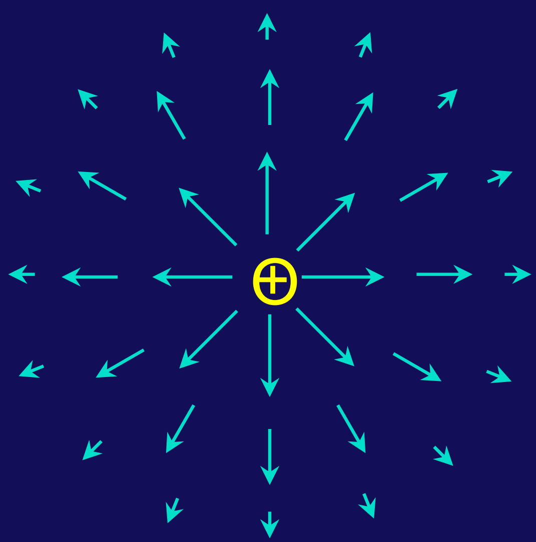
- At fixed r, the horizontal component of the field E_x is given by:

$$E_x = E_r \cos\phi$$

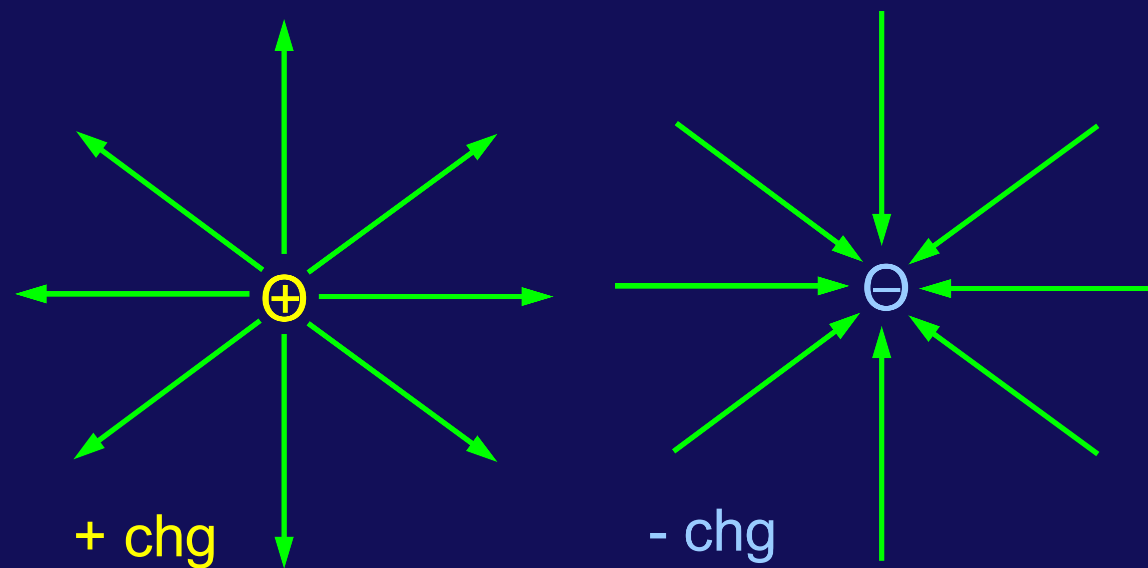


Another Way to Visualize E ...

- The Old Way:
  Vector Maps



- A New Way:
  Electric Field Lines



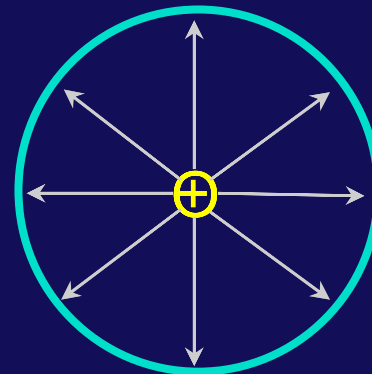
- Lines leave positive charges and return to negative charges
- Number of lines leaving/entering charge = amount of charge
- Tangent of line = direction of E
- Density of lines = magnitude of E

"Density of Field Lines   E"

Electric field from a point charge:

– is spherically symmetric

– and has magnitude: $$E = k\frac{Q}{R^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$$



+ chg

Suppose *N* field lines penetrate the surface of sphere of radius R,

- the density of lines has the same dependence of R^2:

$$\text{Density} = \frac{N}{A} = \frac{N}{4\pi R^2}$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} \propto \frac{N}{4\pi R^2} \quad \rightarrow N \propto Q$$

$$N \equiv \frac{Q}{\varepsilon_0}$$

$$k \equiv \frac{1}{4\pi\varepsilon_o}$$

$k = 9 \text{ x } 10^9 \text{ N m}^2 / \text{C}^2$

$\varepsilon_o = 8.85 \, x \, 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2$

Constants

Flux lines … coming

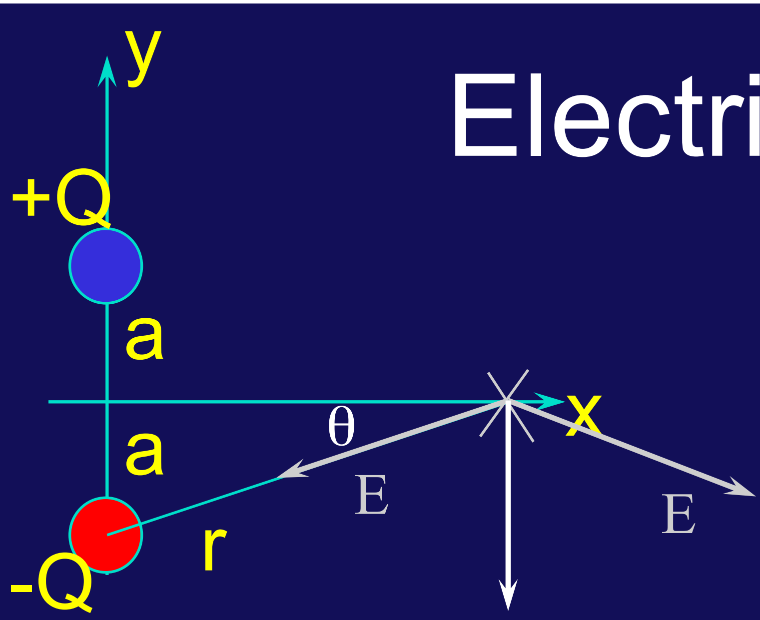
vacuum permittivity, or electric constant

Calculations for today

- Point Charges:
  - Electric Dipole
- Continuous Charge Distribution:
  - Infinite line of charge

Note: We will see these configurations again when we discuss Gauss' Law and Electric Potential.

Electric Dipole



What is the Electric Field generated by this charge arrangement?

Calculate for a point along x-axis: (x,0)

$E_x = ?$

Symmetry

$$E_x(x,0) = 0$$

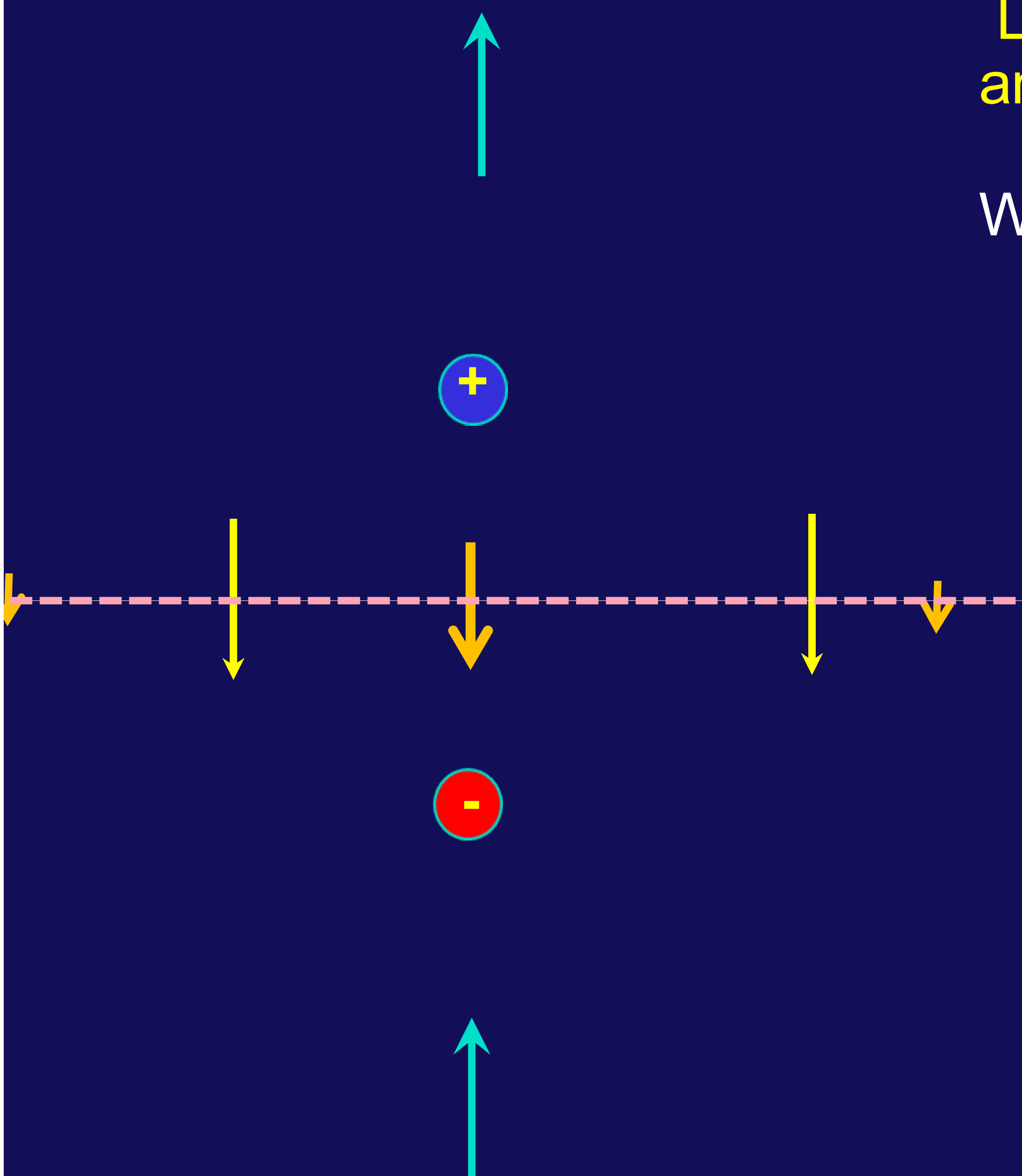
$E_y = ?$

$$E_y(x,0) = -2\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\sin\theta$$

$$\sin\theta = \frac{a}{r} \qquad r^2 = x^2 + a^2$$

$$E_y(x,0) = -2\frac{1}{4\pi\varepsilon_0}\frac{Qa}{\left(x^2 + a^2\right)^{3/2}}$$

Electric Dipole: Field Lines



Lines leave positive charge and return to negative charge

What can we observe about E?

- $E_x(x,0) = 0$
- $E_x(0,y) = 0$

Field largest in space between the two charges

- We derived:

$$E_y(x,0) = -2\frac{1}{4\pi\varepsilon_0}\frac{Qa}{\left(x^2+a^2\right)^{3/2}}$$

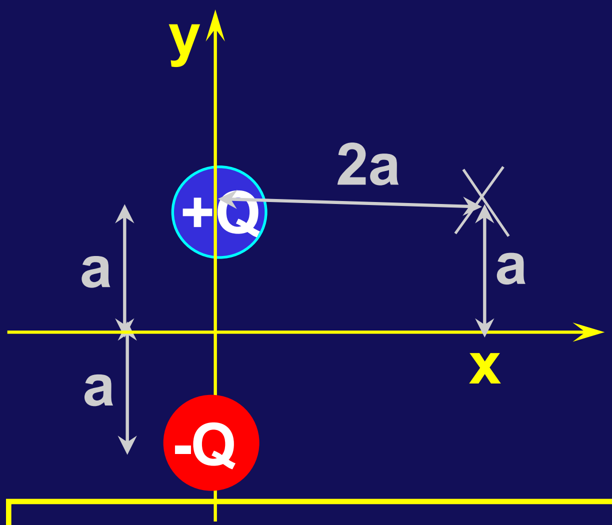
... for r >> a,

$$E_y(x,0) \propto \frac{1}{x^3}$$

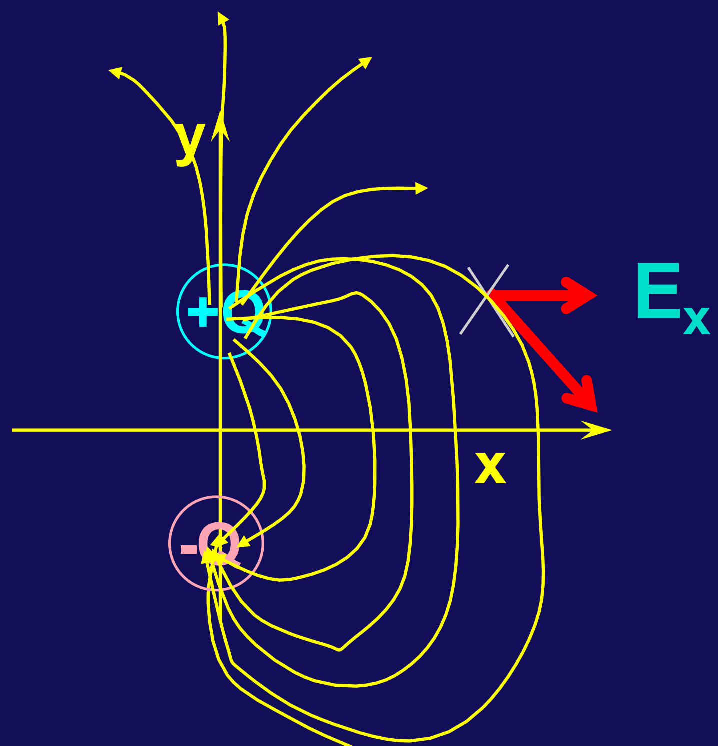
Clicker

Consider a dipole aligned with the y-axis as shown.

Which of the following statements about $E_x(2a,a)$ is true?



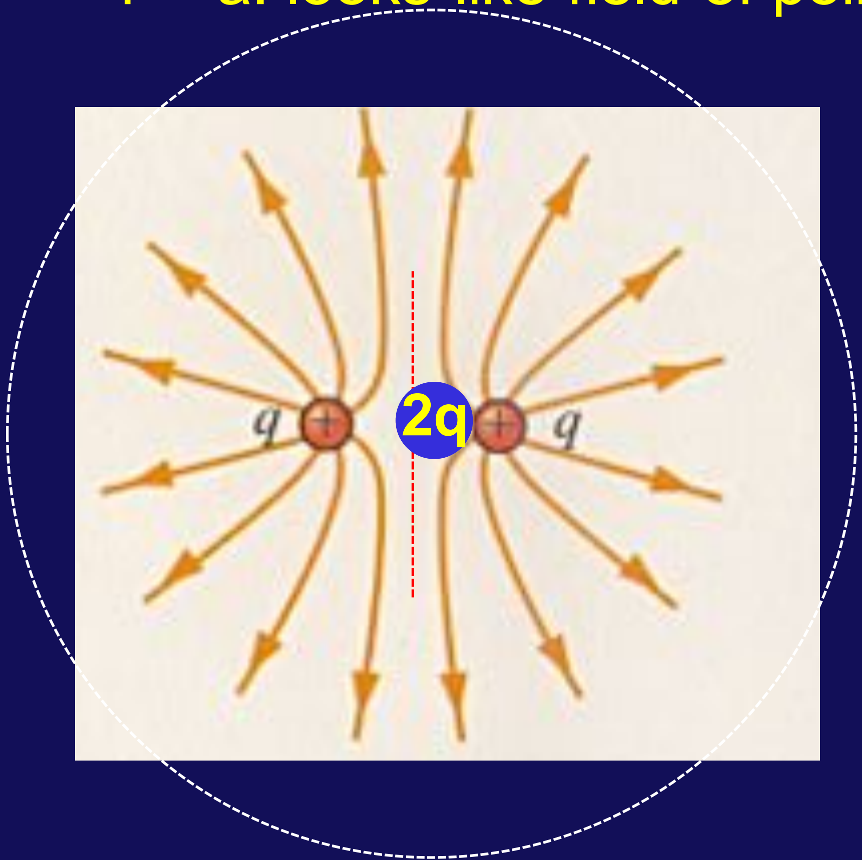
(a) $E_x(2a,a) < 0$ (b) $E_x(2a,a) = 0$ (c) $E_x(2a,a) > 0$



Solution: Draw some field lines according to our rules.

Field Lines from 2 SAME-SIGN Charges

- The field lines are quite different compared to the dipole
  - There is a zero halfway between the charges
  - r>>a: looks like field of point charge (+2q) at origin.

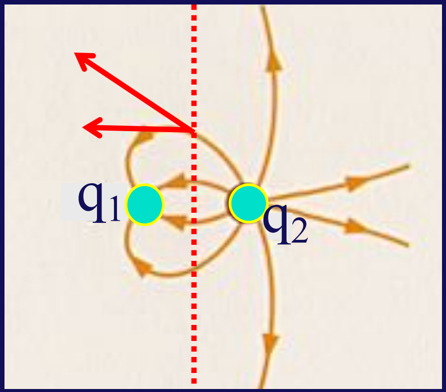


Dipole

Clicker

Examine the electric field lines produced by the charges in this figure.

Which statement is true?



(a) q_1 and q_2 have the same sign
(b) q_1 and q_2 have the opposite signs and $|q_1| > |q_2|$
(c) q_1 and q_2 have the opposite signs and $|q_1| < |q_2|$

Field lines start from q_2 and terminate on q_1. This means q_2 is positive; q_1 is negative; so… not (a)

Now, which one is bigger?

Notice along a line of symmetry between the two, that the E field still has a positive y component. If they were equal, it would be zero; This indicates that q_2 is greater than q_1

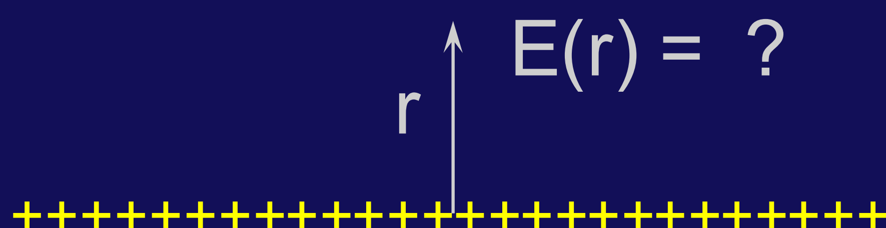
Electric Fields from Continuous Charge Distributions

- Principles (Coulomb's Law + Law of Superposition) remain the same.

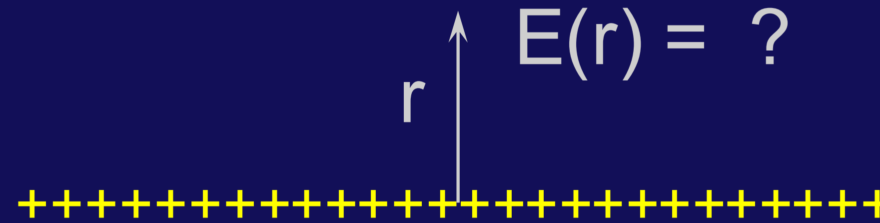
Only change:

$$\Sigma \rightarrow \int$$

Example: Infinite line of charge



How do we approach this calculation?

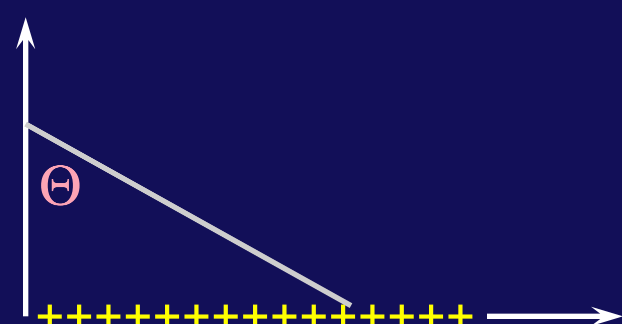


In words:

"add up the electric field contribution from each bit of charge, using superposition of the results to get the final field"

In practice:

- Use Coulomb's Law to find the E field per segment of charge
- Plan to integrate along the line…
  - x: from −¥ to +¥ OR θ: from $-\pi/2$ to $+\pi/2$



- Any symmetries ? This may help for easy cancellations.

Charge Densities

- How do we represent the charge "Q" on an extended object?

total charge Q → small pieces of charge dq

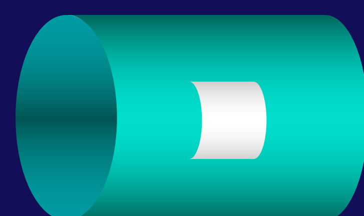
- Line of charge:

λ = charge per unit length

$$dq = \lambda\, dx$$

- Surface of charge:

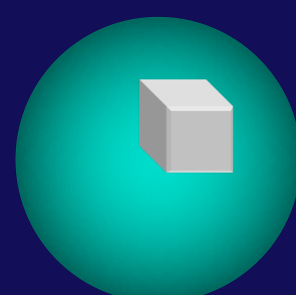
σ = charge per unit area



$$dq = \sigma\, dA$$

- Volume of charge:

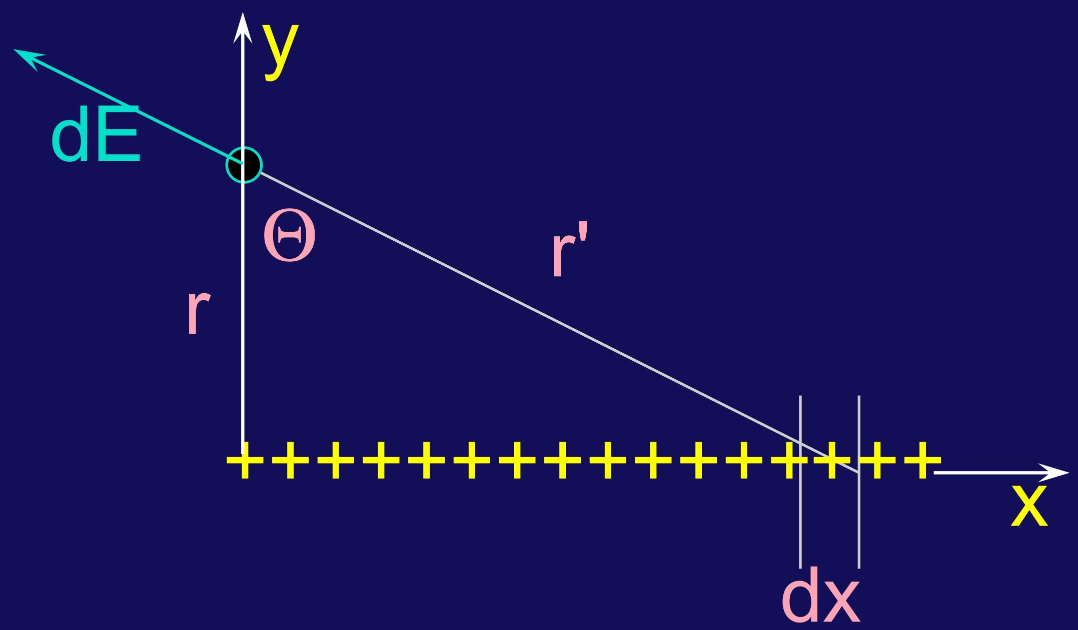
ρ = charge per unit volume



$$dq = \rho\, dV$$

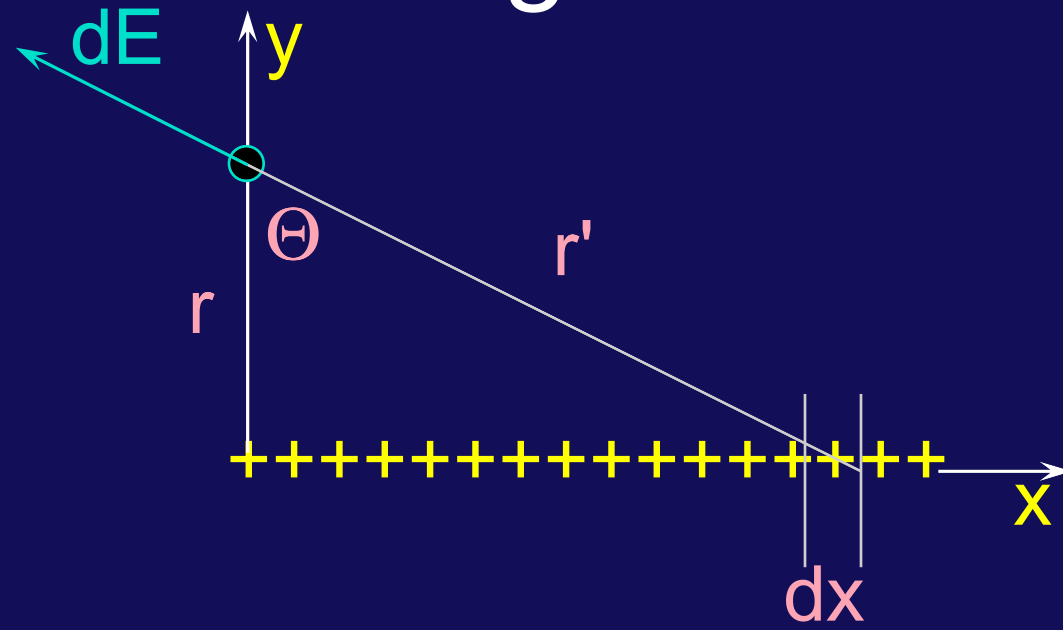
Infinite Line of Charge

Charge density = λ



We need to add up the E field contributions from all segments dx along the line.

Infinite Line of Charge



We use Coulomb's Law to find dE:

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'^2}$$

We need dq in terms of dx:

$$dq = \lambda dx$$

And r' in terms of r:

$$r' = \frac{r}{\cos\theta}$$

Therefore,

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(r/\cos\theta)^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cos^2\theta dx}{r^2}$$

But x and θ are not independent!

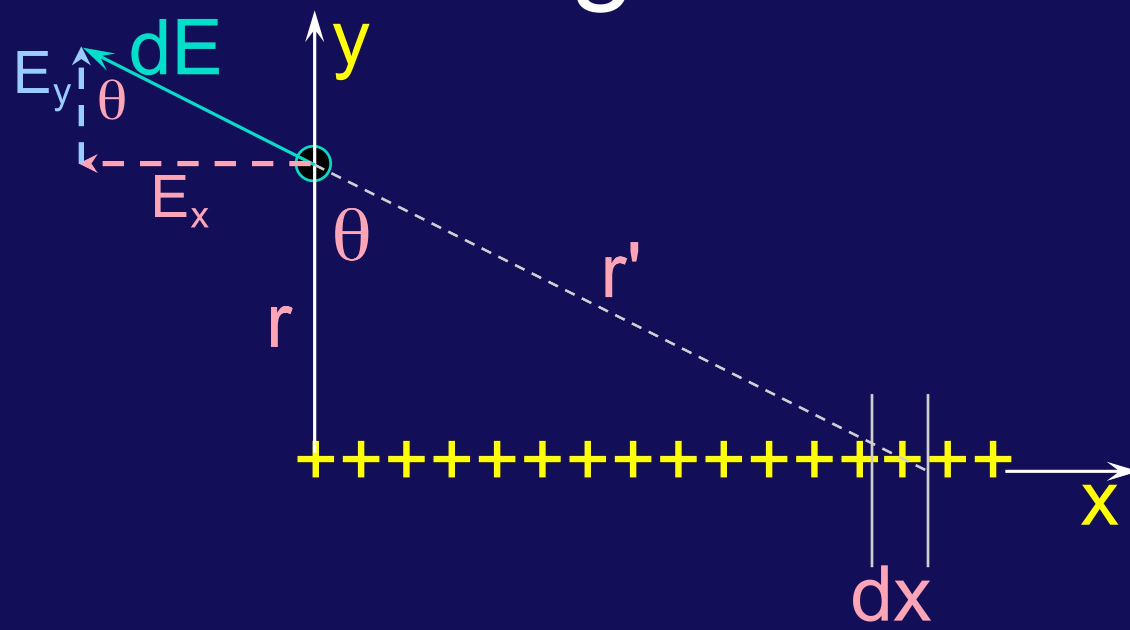
$$x/r = \tan\theta$$

$$dx = r\sec^2\theta\, d\theta$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r}$$

Infinite Line of Charge

- Components:



$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r}\sin\theta$$

$$dE_y = +\frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r}\cos\theta$$

- Integrate:

$$E_x = \int dE_x = -\int_{-\pi/2}^{+\pi/2}\frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\int_{-\pi/2}^{+\pi/2}\sin\theta d\theta$$

$$E_y = \int dE_y = \int_{-\pi/2}^{+\pi/2}\frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\int_{-\pi/2}^{+\pi/2}\cos\theta d\theta$$

Infinite Line of Charge

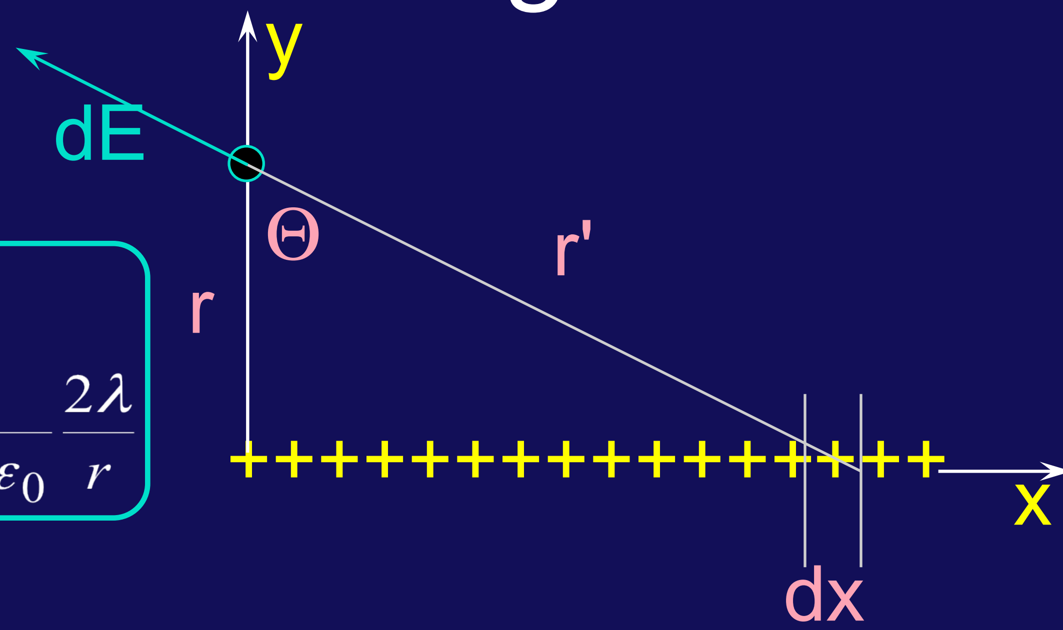
- Solution:

$$\int_{-\pi/2}^{+\pi/2} \sin\theta \, d\theta = 0$$

$$\int_{-\pi/2}^{+\pi/2} \cos\theta \, d\theta = 2$$

$$E_x = 0$$

$$E_y = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r}$$



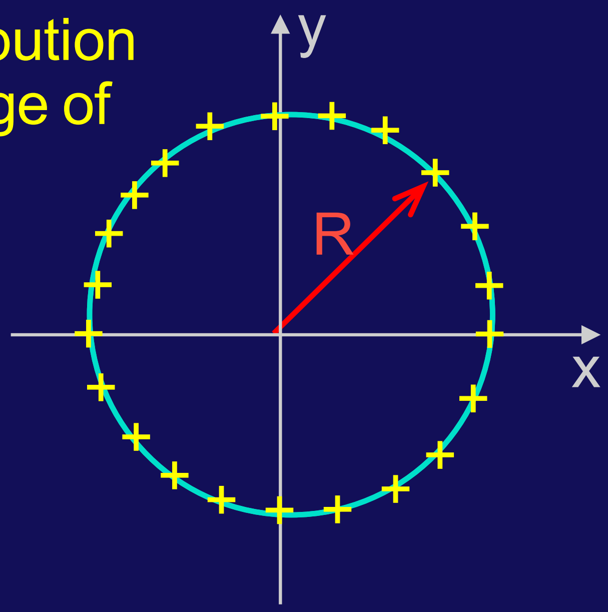
- Conclusion:

The Electric Field produced by an infinite line of charge is:

- everywhere perpendicular to the line
- is proportional to the charge density
- decreases as 1/r.

Clicker

Consider a circular ring with a uniform charge distribution (λ charge per unit length) as shown. The total charge of this ring is +Q.



The electric field at the origin is

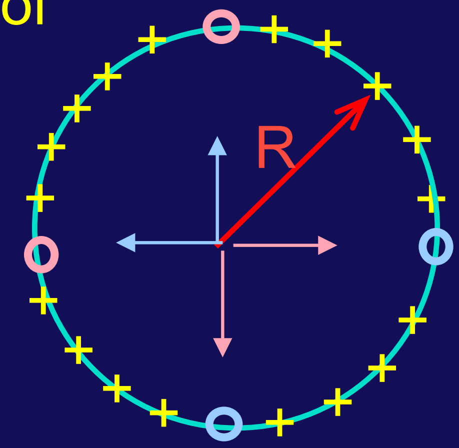
(a) zero

(b) $\frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda}{R}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{\pi R\lambda}{R^2}$

Clicker

Consider a circular ring with a uniform charge distribution (λ charge per unit length) as shown. The total charge of this ring is +Q.



The electric field at the origin is

(a) zero

(b) $\frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda}{R}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{\pi R\lambda}{R^2}$

The total field at the origin is given by the VECTOR SUM of the contributions from all bits of charge.

If the total field were given by the ALGEBRAIC SUM, then (b) would be correct. (exercise for the student).

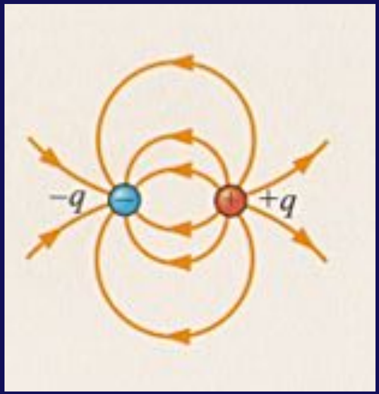
Note that the electric field at the origin produced by one bit of charge is exactly cancelled by that produced by the bit of charge diametrically opposite!!

Therefore, the VECTOR SUM of all these contributions is ZERO!!

Summary

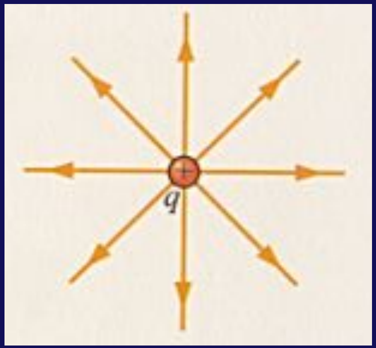
Electric Field Distibutions

Dipole



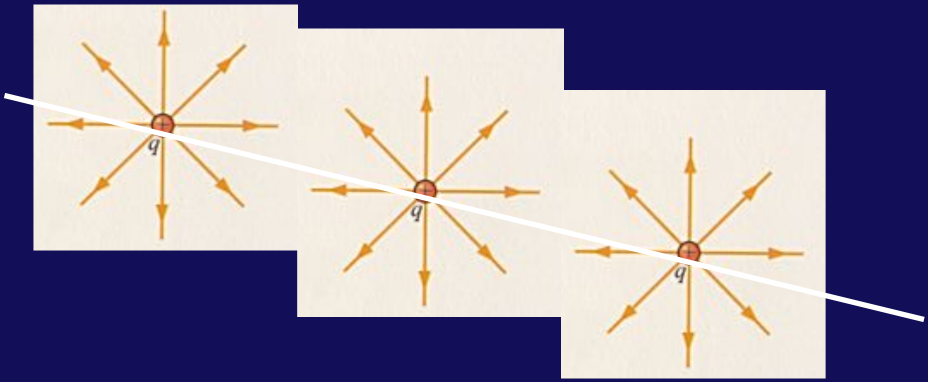
$\sim 1 / R^3$

Point Charge



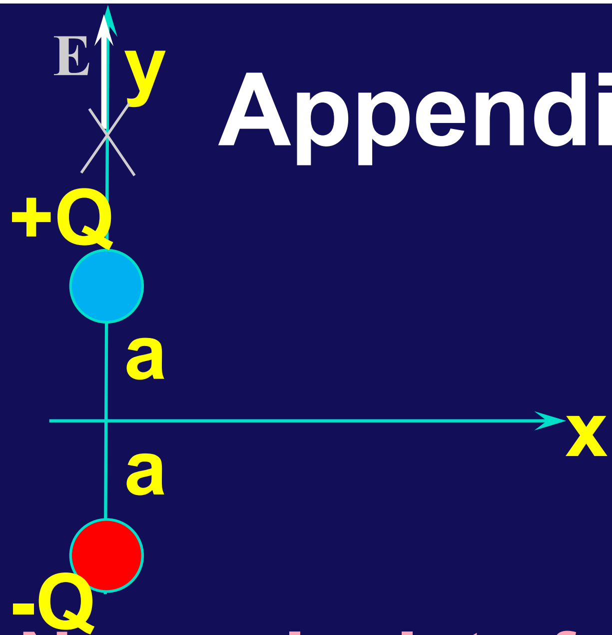
$\sim 1 / R^2$

Infinite
Line of Charge



$\sim 1 / R$

Appendix: Electric Dipole



What is the Electric Field generated by this charge arrangement?

Now calculate for a pt along y-axis: (0,y)

E_x = ?

Coulomb Force
Radial

$$E_x(0,y) = 0$$

E_y =?

$$E_y(0,y) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{(y-a)^2} - \frac{1}{(y+a)^2}\right)$$

$$E_y(0,y) = \frac{Q}{4\pi\varepsilon_0}\frac{4ay}{y^4\left(1-\frac{a^2}{y^2}\right)^2}$$

Appendix: Electric Dipole

Case of special interest:

(antennas, molecules)

r >> a

For pts along x-axis:

$$E_x(r,0) = 0$$

$$E_y(r,0) = -2\frac{1}{4\pi\varepsilon_0}\frac{Qa}{\left(r^2 + a^2\right)^{3/2}}$$

For r >>a,

$$E_y(r,0) \approx -2\frac{1}{4\pi\varepsilon_0}\frac{Qa}{r^3}$$

For pts along y-axis:

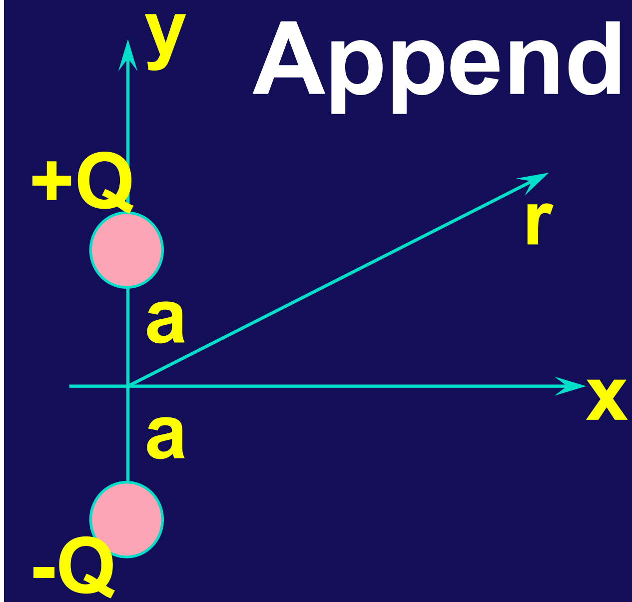
$$E_x(0,r) = 0$$

$$E_y(0,r) = \frac{Q}{4\pi\varepsilon_0}\frac{4ar}{r^4\left(1 - \frac{a^2}{r^2}\right)^2}$$

For r >>a,

$$E_y(0,r) \approx +4\frac{1}{4\pi\varepsilon_0}\frac{Qa}{r^3}$$

Appendix: Electric Dipole Summary



Case of special interest:

(antennas, molecules)

r >> a

- Along x-axis

$$E_x(r,0) = 0$$

$$E_y(r,0) \approx -2\frac{1}{4\pi\varepsilon_0}\frac{Qa}{r^3}$$

- Along y-axis

$$E_x(0,r) = 0$$

$$E_y(0,r) \approx +4\frac{1}{4\pi\varepsilon_0}\frac{Qa}{r^3}$$

- Along arbitrary angle Θ

$$E \propto Qa$$

$$E \propto \frac{1}{r^3}$$