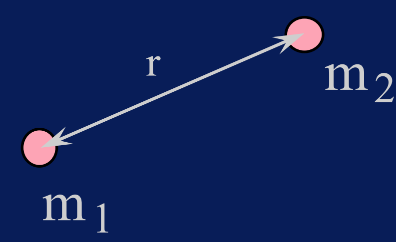
The World According to Physics 121

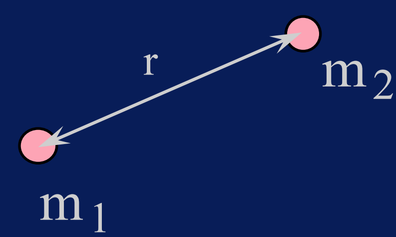
- **Objects**
  - Specified by geometry and mass
- **Forces**
  - Gravity: $F = G\frac{m_1 m_2}{r^2}$



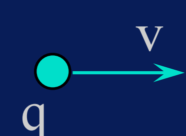
  - Others: Tension, Normal, Friction
- **Space and Time**
  - Euclidean with Galilean Invariance
    - “ordinary” 3D space; “slow” velocities

The World According to Physics 122

- Things -- Bodies and Fields (E,B)
  - Specified by geometry and mass and charge
- Forces
  - Gravity: $\vec{F} = -G\frac{m_1 m_2}{r^2}\hat{r}$



  - Electromagnetic: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$



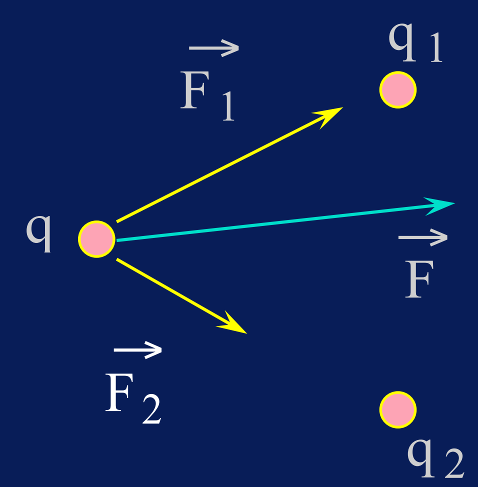
- Space and Time
  - Euclidean with Lorentz Invariance
    - “ordinary space” but can be really really fast...

More on Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$\frac{F_{elec}}{F_{grav}} = \frac{q_1 q_2}{m_1 m_2}\,\frac{\frac{1}{4\pi\varepsilon_0}}{G}$$

$$\vec{E} \equiv \frac{\vec{F}}{q}$$



Superposition!!

Your Lecture Thoughts

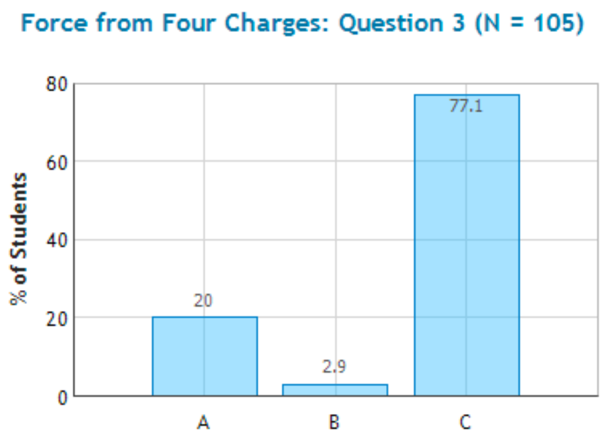
- Nervous about forces from multiple charges
- Questions about continuous line of charge
- We will continue on FRIDAY
  - Electric Field Lines
  - Example Calculations:
    - Discrete: Electric Dipole
    - Continuous: Infinite Line of Charge
- No new SmartPhysics due on Friday

Last time … CheckPoint

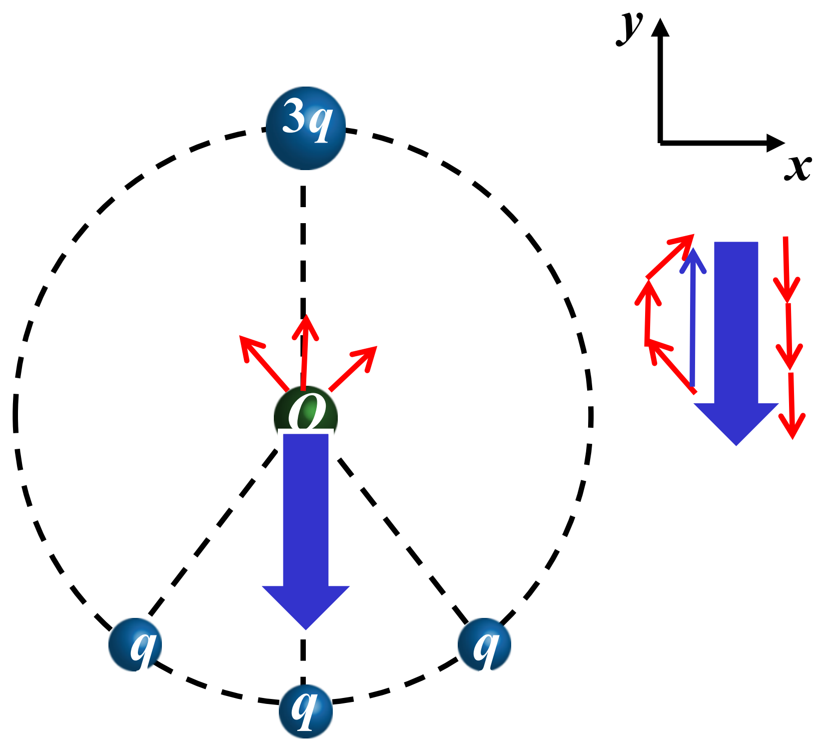
Four charged particles are placed on a circular ring with radius 3 m as shown below. A particle with charge Q is placed in the center of the ring

What is vertical force on Q?

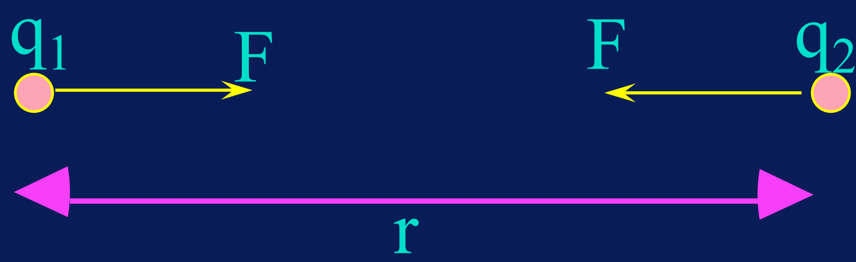
A) $F_y > 0$ B) $F_y = 0$ C) $F_y < 0$



3q is will have greater effect than the three q charges, because two of the three bottom q charges are at an angle to the middle charge. - Nathan



Reminder: Coulomb's Law



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

MKS Units:

- r in meters
- q in Coulombs
- F in Newtons

Þ $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N-m^2/C^2

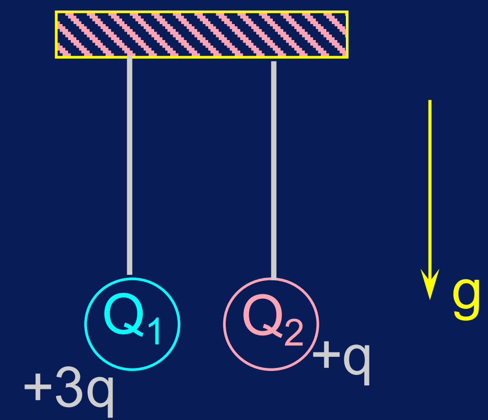
We call this group of constants “k” as in: $F = k\, q_1 q_2/r^2$

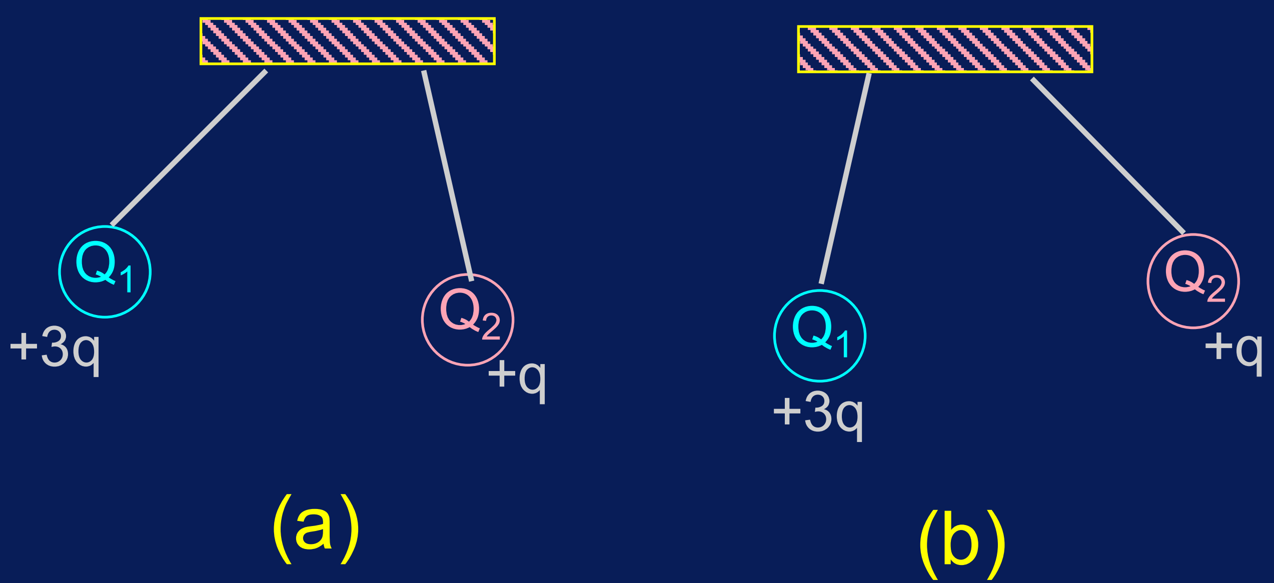
- Same spatial dependence as gravitational force,
  - NO mention of mass
- Strength determined by the charge of the two objects.

Clicker

Two equal mass balls are suspended from the ceiling with equal length nonconducting threads as shown. Ball 1 has charge Q_1 = +3q and Ball 2 has charge Q_2 = +q.

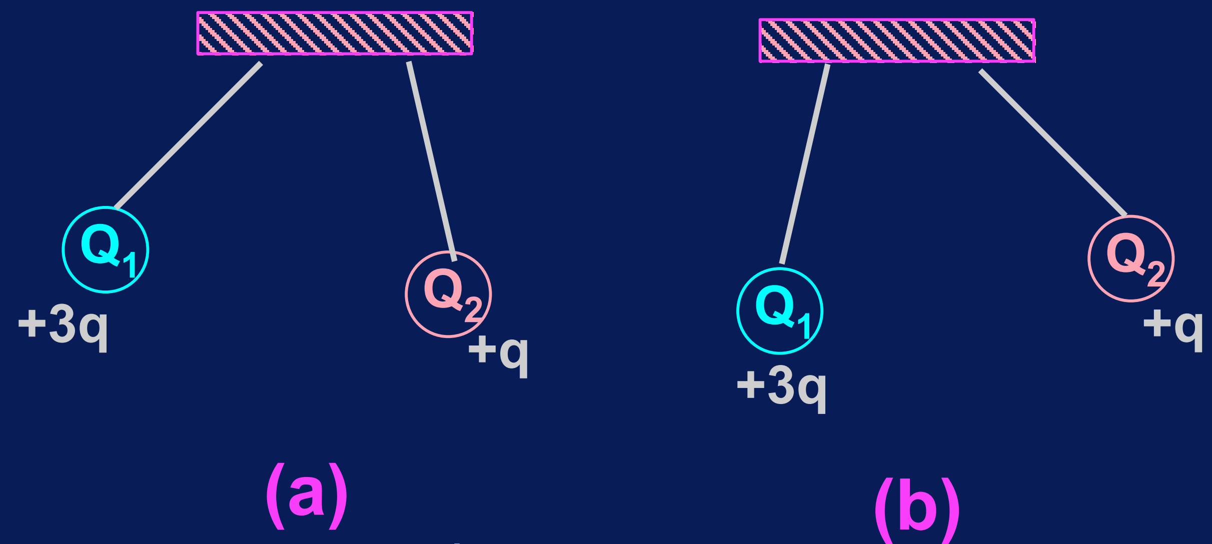
- Which of the following pictures best represents the equilibrium position?

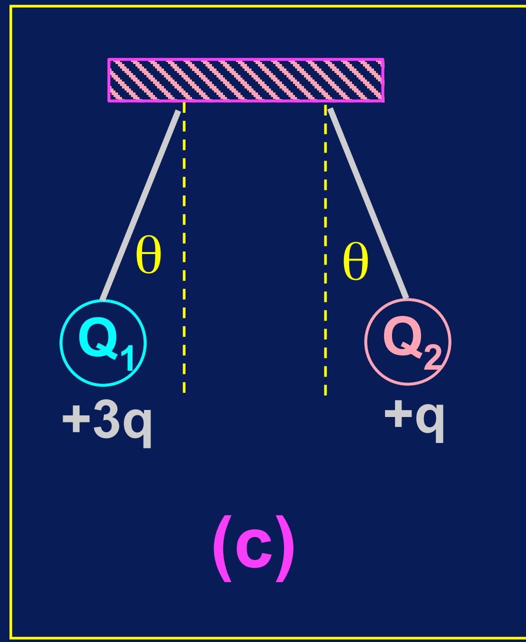




Clicker

- Which of the following pictures best represents the equilibrium position?





- Remember Newton's Third Law!

The electrical force Q_1 exerts on Q_2 must be equal and opposite to the electrical force that Q_2 exerts on Q_1.

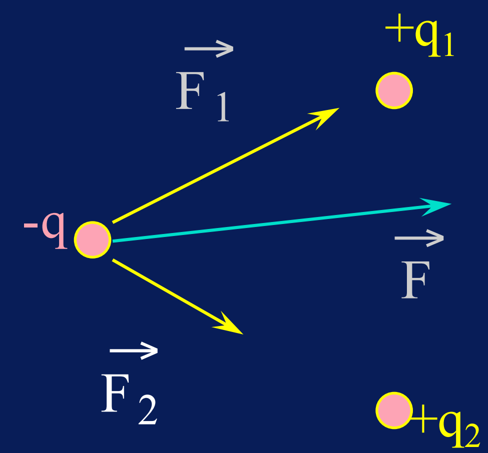
The amount of charge each ball has determines the magnitude of the force, but each ball experiences the same force.

Therefore, the symmetry demands (c) !!

Consider more than two charges now

If q_1 were the only other charge, we would know the force on q due to q_1 .

If q_2 were the only other charge, we would know the force on q due to q_2 .



- What is the force on q when both q_1 and q_2 are present??
  - As in mechanics, we have the Law of Superposition:
    - The TOTAL FORCE on the object is just the VECTOR SUM of the individual forces.

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

Clicker

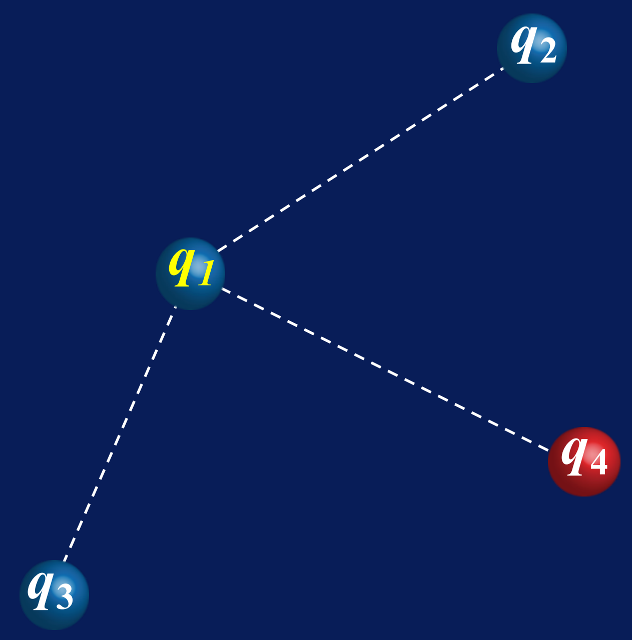
What happens to the Force on q_1 if its sign is changed?

A) $|F_1|$ increases

B) $|F_1|$ remains the same

C) $|F_1|$ decreases

D) Need more information to determine



Clicker

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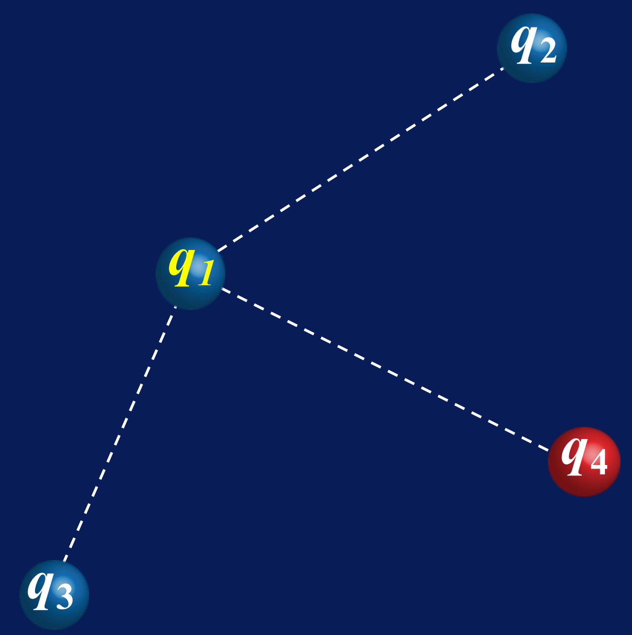
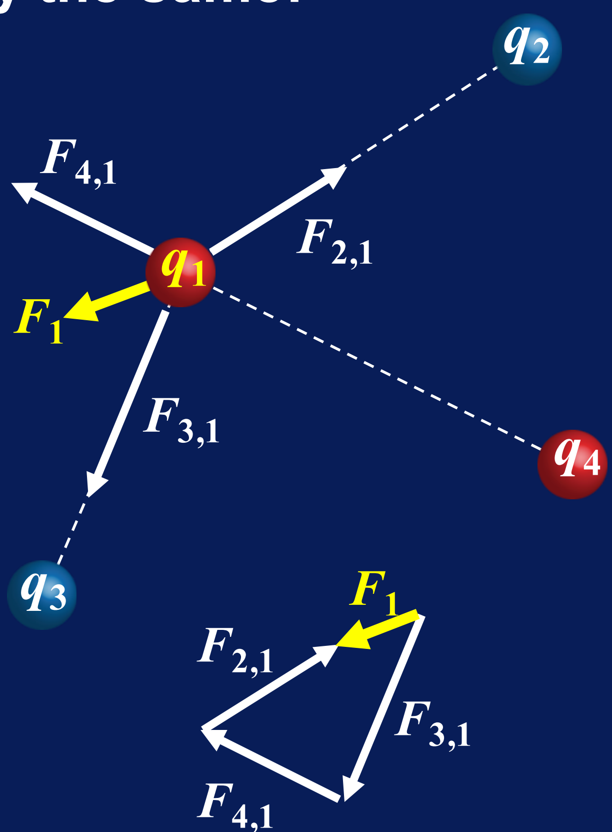
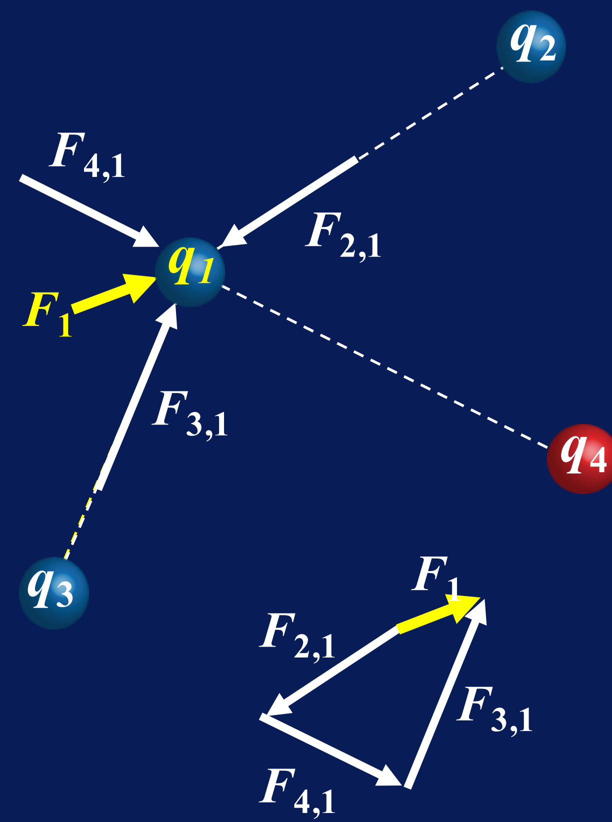


The direction of all forces changes by 180° – the magnitudes stay the same:



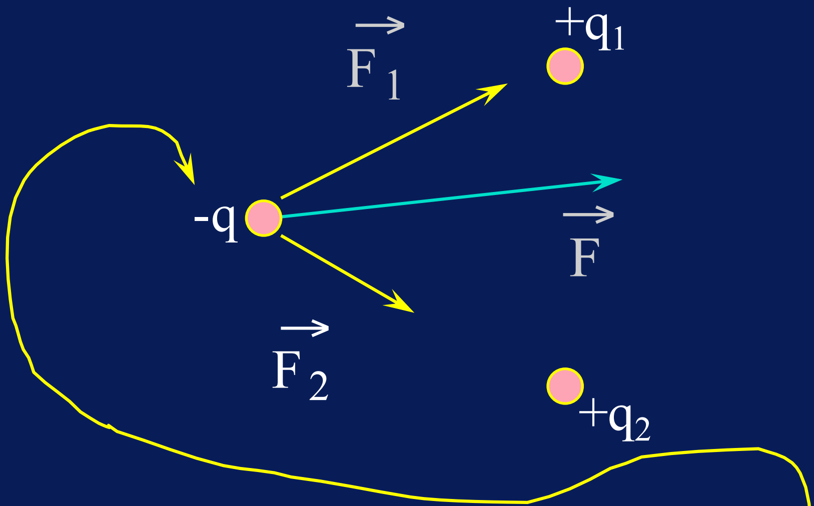


$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1} + \ldots$$

$$-\vec{F}_1 = -\vec{F}_{2,1} - \vec{F}_{3,1} - \vec{F}_{4,1} - \ldots$$

The Electric Field

- A simple but profound observation
  - The net Coulomb force on a given charge is always proportional to the strength of that charge.



$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F} = \frac{q}{4\pi\varepsilon_0}\left(\frac{q_1\hat{r}_1}{r_1^2} + \frac{q_2\hat{r}_2}{r_2^2}\right)$$

– We can therefore define a quantity, the electric field, which is independent of the "test" charge q and depends only on the position in space:

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

This is one important definition!!!

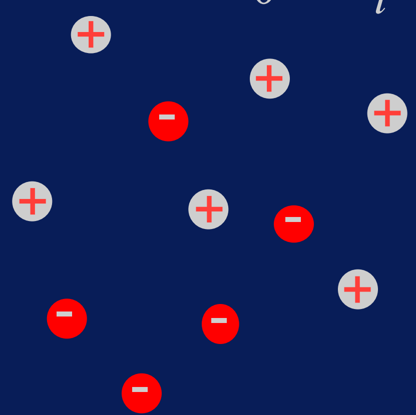
The Electric Field

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

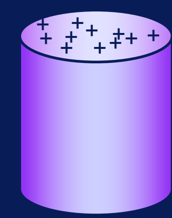
With this concept, we can "map" the electric field anywhere in space, produced by any arbitrary

Collection of charges or Charge distribution

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{r_i^2} \hat{r}_i$$

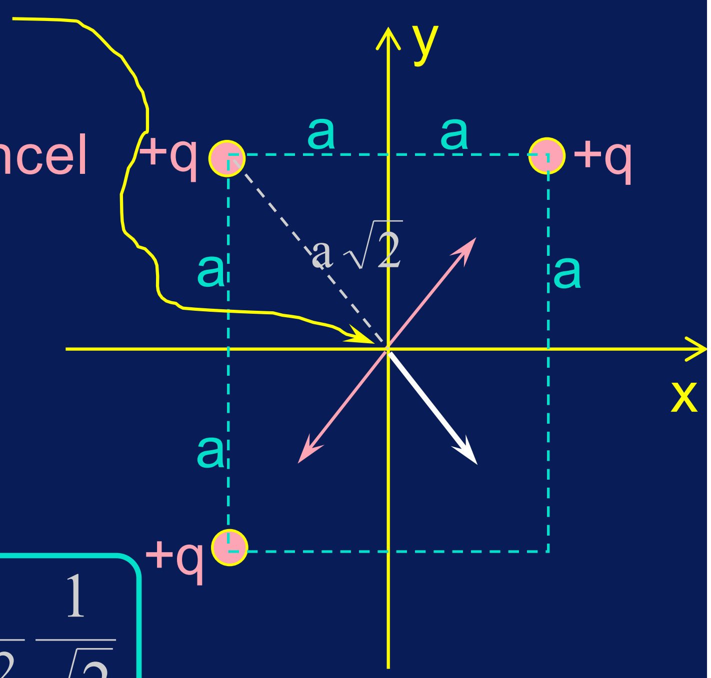


$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \hat{r}$$



Example

What is the electric field at the origin?



The fields from the top right and bottom left cancel

The total field is from the top left charge only

– The components are:

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{q}{2a^2}\frac{1}{\sqrt{2}}$$

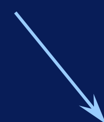
$$E_y = -\frac{1}{4\pi\varepsilon_0}\frac{q}{2a^2}\frac{1}{\sqrt{2}}$$

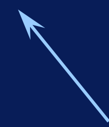
If a charge Q were placed at the origin, the force on this charge would be:

$$F_x = QE_x = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{2\sqrt{2}a^2}$$

$$F_y = QE_y = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{2\sqrt{2}a^2}$$

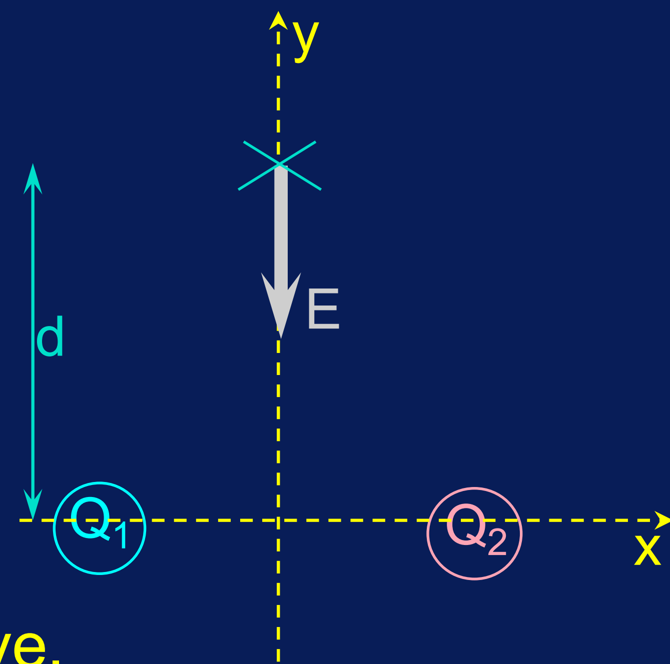
Note:

if Q>0, F= 

if Q<0, F= 

Clicker

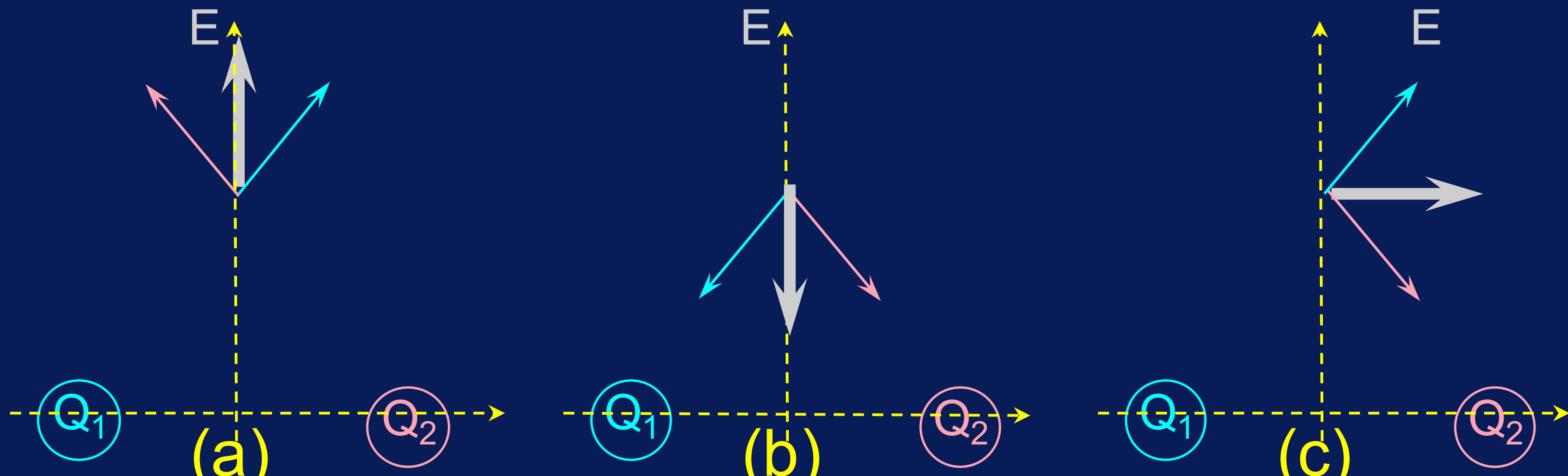
- Two charges, Q_1 and Q_2 , fixed along the x-axis as shown, produce an electric field E at a point (x,y) = (0,d) which is directed along the negative y-axis.
  - Which of the following statements is true?



(a) Both charges Q_1 and Q_2 must be positive.

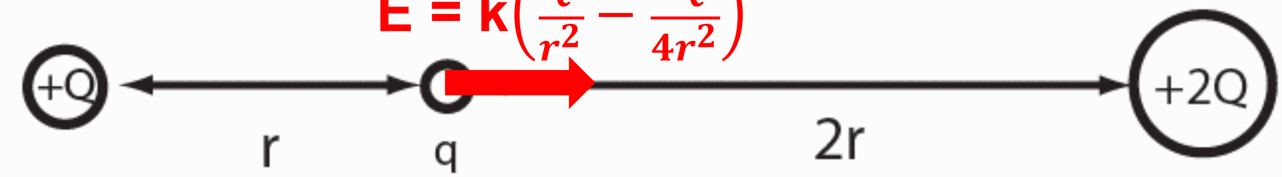
(b) Both charges Q_1 and Q_2 must be negative.

(c) The charges Q_1 and Q_2 must have opposite signs.



CheckPoint

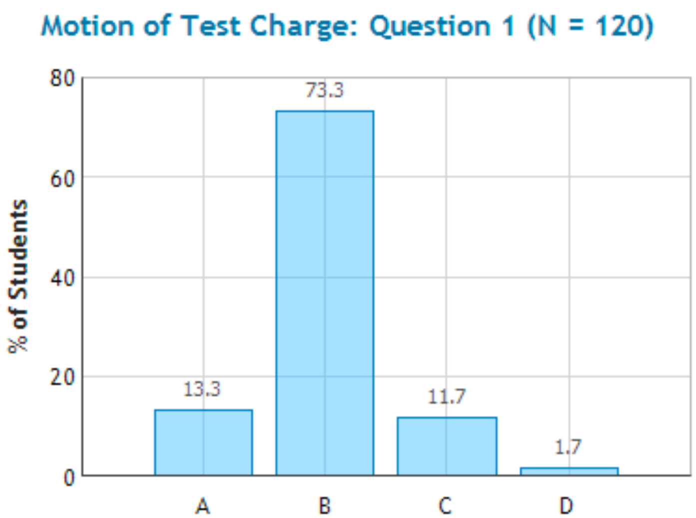
$$E = k\left(\frac{Q}{r^2} - \frac{2Q}{4r^2}\right)$$



A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q. How will the test charge move immediately after being released?

8) How will the test charge move immediately after being released?

- A: to the left
- B: to the right
- C: stay still
- D: other

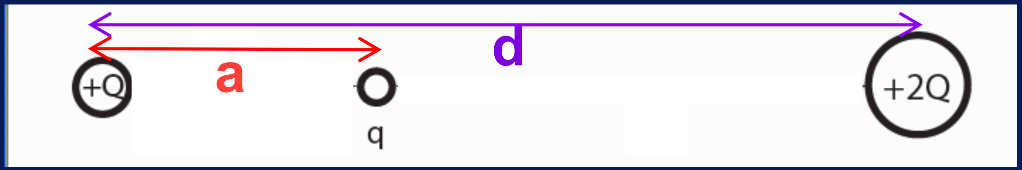


And for the motion, recall

$$\mathbf{F} = q\mathbf{E} = m\mathbf{a}$$

(F, E and a are vectors)

Clicker followup



Same setup:

- Q_1 has charge +Q
- Q_2 has charge +2Q
- They are separated by d.
- Charge q is a distance a away from Q_1

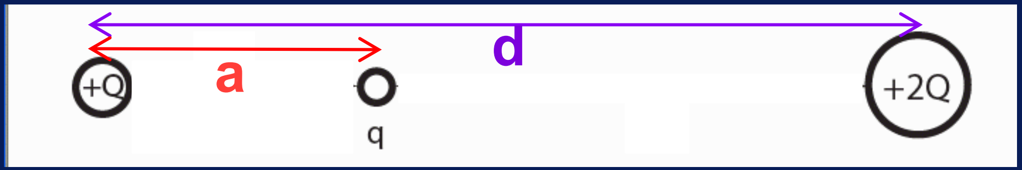
Is there a place – the value for a -- between Q_1 and Q_2 where the force on ANY charge (positive or negative) is zero?

(a) NO

(b) Yes, but I can't find it with all this time pressure.

(c) Yes and my answer is ________ from Q_1. I will volunteer to specify if you ask me

Clicker followup



Is there a place – the value for a -- between Q_1 and Q_2 where the force on ANY charge (positive or negative) is zero?

The force on charge q a distance a away from Q_1 is

$$F = \frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{a^2} - \frac{2Q}{(d-a)^2}\right) = 0?$$

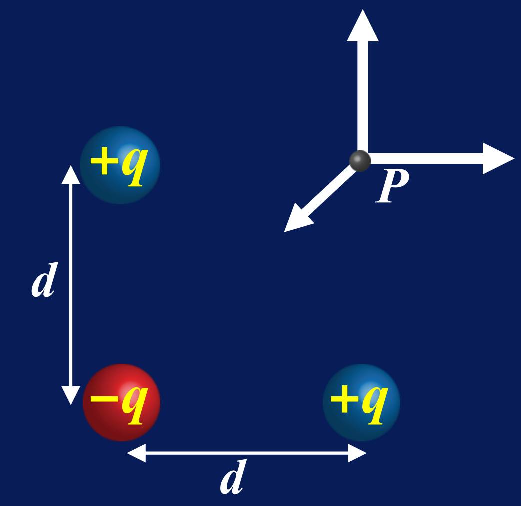
True if: $$\frac{1}{a^2} = \frac{2}{(d-a)^2}$$ for some value of a

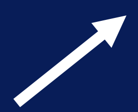
(c) Yes and my answer is $a = d\left(\sqrt{2}-1\right)$ from Q_1.

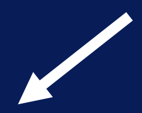
(you can check if that's right)

Clicker

What is the direction of the electric field at point P, the unoccupied corner of the square?



A) 

B) 

C) $E = 0$

D) Need to know d

E) Need to know d & q

Calculate E at point P.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

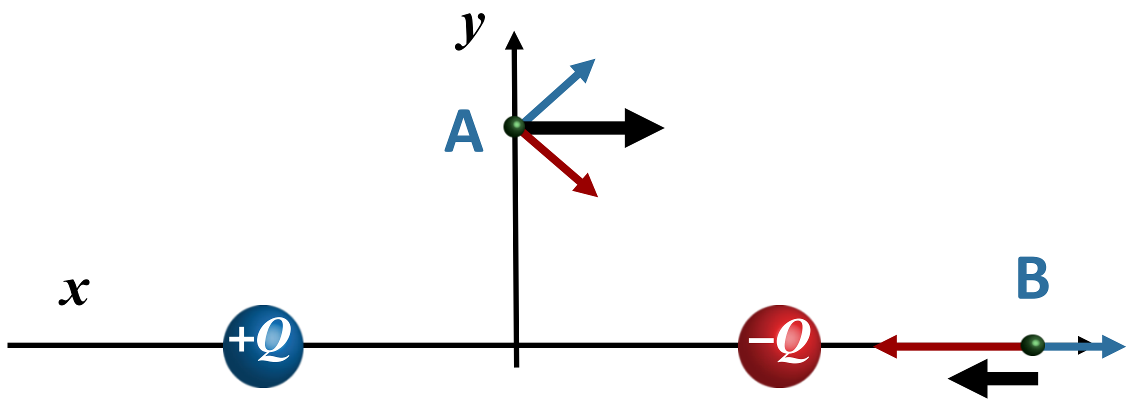
$$E_x = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\cos\frac{\pi}{4}\right)$$

> 0

$$E_y = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\sin\frac{\pi}{4}\right)$$

> 0

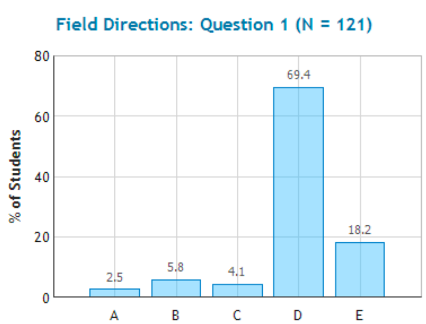
CheckPoints



Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

2) What is the direction of the electric field at point **A?**
○ Up ○ Down ○ Left ○ Right ○ Zero

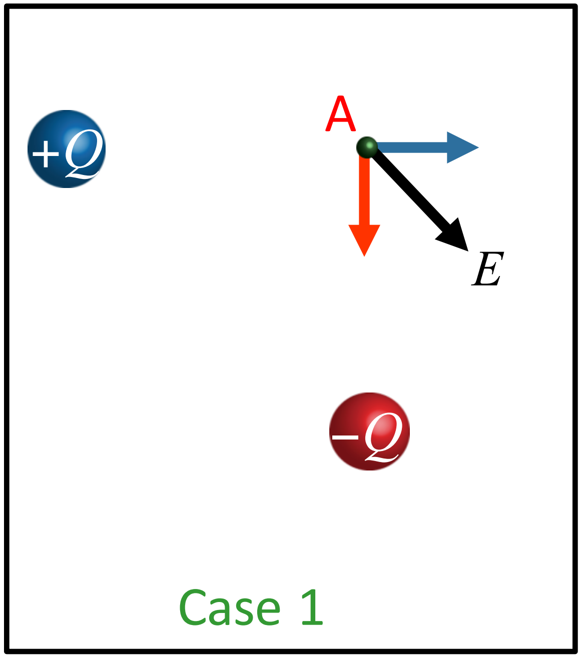
What is the direction of the electric field at point **B?**
○ Up ○ Down ○ Left ○ Right ○ Zero

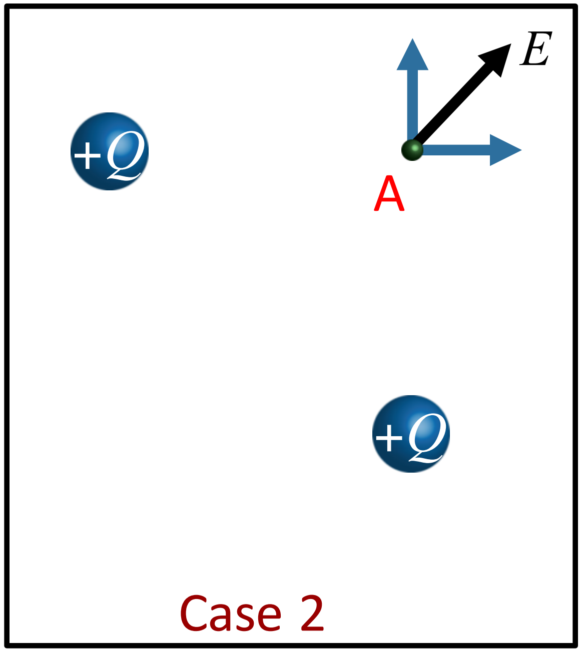


(For point A) The electric field due to left charge is up and to the right (repulsive because both points are positive). The electric field due to right charge is down and to the right because points have different signs. The two electric fields have a net electric field to the right. - *Andrea*

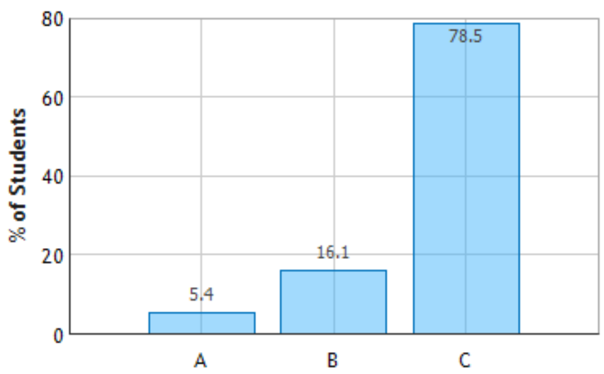
CheckPoint

In which case is the magnitude of the electric field at A the largest?





SAME



Reality of Electric Field

The electric field has been introduced as a mathematical convenience, just as the gravitational field in Phys 121.

Spoiler Alert:

There is MUCH MORE to the electric field than this!

IMPORTANT: E field propagates at speed of light.

NO *instantaneous* action at a distance!!

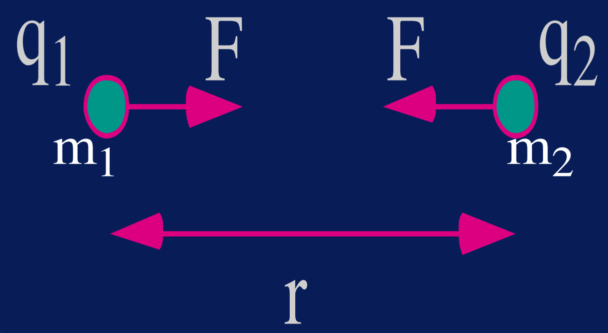
(we will explore this aspect when we get to electromagnetic waves)

As a charge moves, resultant E field at time t depends on where the charge was at time t-dt.

We avoid these complications for now by restricting ourselves to situations in which the SOURCE of the E field is AT REST!

(*i.e.* electrostatics)

Force Comparison
Electrical vs Gravitational



$$F_{elec} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$F_{grav} = G\frac{m_1 m_2}{r^2}$$

$$\frac{F_{elec}}{F_{grav}} = \frac{q_1 q_2}{m_1 m_2}\frac{\frac{1}{4\pi\varepsilon_0}}{G}$$

For a proton,

* $q = 1.6 \times 10^{-19}$ C

$m = 1.67 \times 10^{-27}$ kg

$$\frac{F_{elec}}{F_{grav}} = 1.23 \times 10^{+36}$$

* Note: smallest charge seen in nature *!*

See you Friday

- We will do the Dipole
- We will do the line of charge

- No new SP on Friday

- Start your Webassign HW soon please