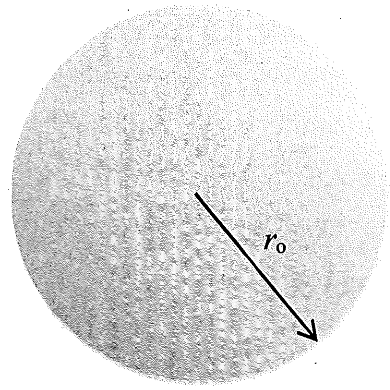
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| e | c | b | b | b | c | d | b | c | d |

## Part I. Lecture Multiple Choice (10 Questions, 43 points)

***The next three questions pertain to the situation described below.***

A solid insulating sphere of radius $r_o = 10$ cm, has a uniform charge density, $\rho = +30\ \mu C/m^3$, throughout its volume.




1. [5pts] What is the magnitude of the electric field E at $r = 2r_o$?

a) $1.13\times10^5$ N/C
b) $6.74\times10^6$ N/C
c) $3.39\times10^5$ N/C
d) $1.69\times10^6$ N/C
e) $2.82\times10^4$ N/C

Gauss's Law:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

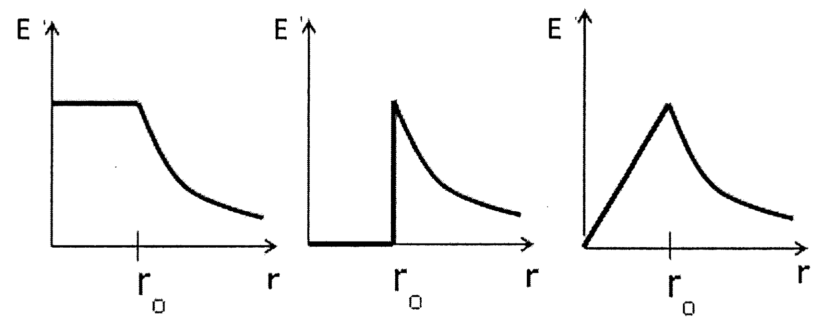
$$4\pi r^2 E = \frac{4\pi r_o^3 \rho}{3\varepsilon_o}$$

$$E = \frac{\rho}{3\varepsilon_o}\frac{r_o^3}{r^2} = \frac{30\ \mu C/m^3}{3\times 8.85\times10^{-12}\ \frac{C^2}{Nm^2}}\frac{(0.1\,m)^3}{(0.2\,m)^2} = 2.82\times10^4\ \frac{N}{C}$$

2. [3pts] Which one of these plots best represents the radial dependence of the magnitude of the electric field?

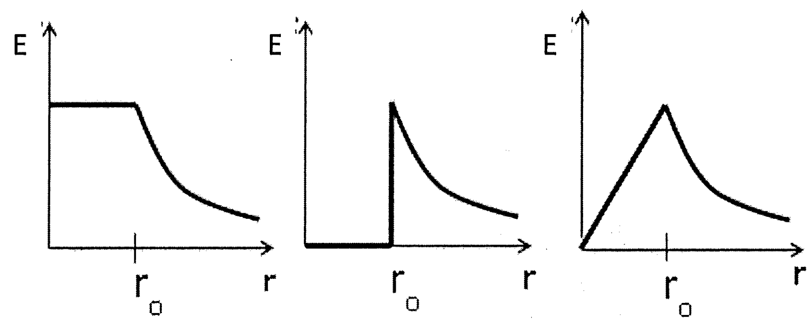
Field inside $\sim r$

Field outside $\sim \frac{1}{r^2}$




a) b) c)

3. [3pts] Suppose the sphere in question 1 is replaced with an identically sized conducting sphere with the same net charge. Which of the plots now best represents the radial dependence of the magnitude of the electric field?

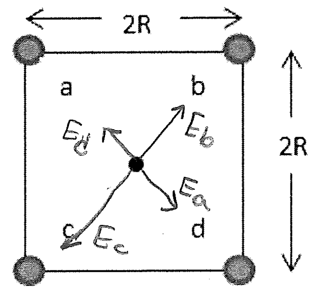



a) b) c)

Field inside = 0 (conductor)

Field outside unchanged $(\sim \frac{1}{r^2})$

4. [5pts] Four point charges are placed at the four corners of a square with side length $2R$. Points a, b, and d have charges of $Q$, $-2Q$, and $Q$, respectively. The electric field at the center of the square has magnitude $\frac{kQ}{2R^2}$ and is directed towards point c. What is the charge on point c?



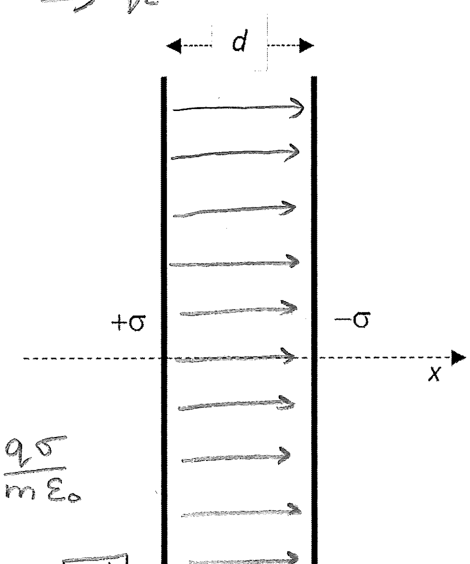

a) -2Q
b) -3Q
c) -Q
d) +Q
e) +3Q

Field components from a and d cancel by symmetry.

$$E_b = \frac{K(2Q)}{(\sqrt{2}R)^2} = \frac{KQ}{R^2} \quad \text{(directed up-right)}$$

$$\text{So } E_c \text{ is } \frac{3kQ}{2R^2} \quad \text{(directed down-left)} \Rightarrow q_c = -3Q$$

5. [5pts] Consider two surfaces with equal and opposite charge density σ separated by a distance $d$ as shown in the figure. A particle with positive charge $q$ and mass $m$ is released from being at rest on the plate on the left. How long does it take the particle to reach the plate on the right?




a) It never collides
b) $\sqrt{\frac{2dm\epsilon_0}{q\sigma}}$
c) $\frac{dm\epsilon_0}{q\sigma}$
d) $\sqrt{\frac{4dm\epsilon_0}{q\sigma}}$
e) Insufficient information is given to determine this

Field between plates:

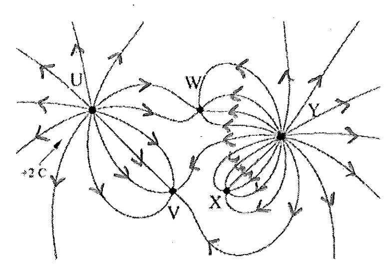
$$E = E_{left} + E_{right} = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

$$F = qE = ma \Rightarrow a = \frac{qE}{m} = \frac{q\sigma}{m\varepsilon_0}$$

Kinematics: $\Delta x = d = \frac{1}{2}at^2$

$$\Rightarrow t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dm\varepsilon_0}{q\sigma}}$$

6. [4pts] Five charges, labeled U through Y, and the electric field lines they create are shown in the figure. Given that the charge on U is +2C, the electric charges on the others are



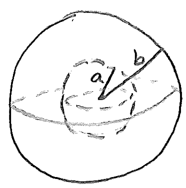

a) V = +4/3C  W = +2/3C  X = -1C  Y = -2C
b) V = -4C  W = -2C  X = +6C  Y = -6C
c) V = -1C  W = -1C  X = -1C  Y = +3C
d) V = +1C  W = -1C  X = -1C  Y = +3C

Positive: field lines outwards
Negative: field lines inwards

# of field lines starting/ending @ a point is directly proportional to the point's charge.

7. [5pts] A hollow spherical non-conducting shell of inner radius $a$ and outer radius $b$ carries charge density $\frac{C}{r^2}$ in the region $a<r<b$. Which integral best describes the total charge within a spherical Gaussian surface drawn at radius r, where $a<r<b$?

a) $\int_a^r 4\pi r^2 C\, dr$
b) $\int_r^b 4\pi r^2 C\, dr$
c) $\int_a^b 4\pi C\, dr$
d) $\int_a^r 4\pi C\, dr$
e) $\int_a^r \frac{4}{3}\pi r C\, dr$




$Q_{enc} = \int \rho\, dV$

$dV = A\,dr = 4\pi r^2 dr$

volume of a thin spherical shell

Want only the charge within radius r sphere

$Q_{enc} = \int_a^r \rho(4\pi r^2)\, dr = \int_a^r 4\pi C\, dr$

8. [4pts] A dipole with dipole moment p=2 C-m is placed at an angle of 30 degrees to a uniform electric field of magnitude E=3 N/C. What is the torque on the dipole?

a) 0 N-m
b) 3 N-m
c) 5.2 N-m
d) 6 N-m
e) 12 N-m

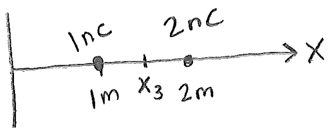
$\vec{\tau} = \vec{p} \times \vec{E}$

$|\vec{\tau}| = |\vec{p}|\,|\vec{E}| \sin\theta$

$= (2\text{ C}\cdot\text{m})(3\tfrac{N}{C})(\tfrac{1}{2}) = 3\text{ N}\cdot\text{m}$

9. [4pts] Point charges $q_1$ and $q_2$ are located on the x-axis at positions $x_1$ and $x_2$ respectively. Charge $q_3$ is then positioned on the x-axis at $x_3$ such that each charge feels no net electric force. If $x_1 = 1.00$ m, $x_2$=2.00 m, $q_1$= 1.00 nC, $q_2$=2.00 nC, then the value of $x_3$ is:

a) 0.75 m
b) 1.28 m
c) 1.41 m
d) 1.78 m
e) 2.33 m




$X_3$ must be between $X_1$ & $X_2$.

Field @ $X_3$:

$E = \frac{K(1\,nC)}{(X_3 - 1m)^2} - \frac{K(2\,nC)}{(X_3 - 2m)^2} = 0$

$(X_3 - 2m)^2 = 2(X_3 - 1m)^2$

$\Rightarrow X_3^2 = 2\,m^2$

$X_3 = \sqrt{2}\,m$

$= 1.41\,m$

10. [5pts] A dipole in a uniform electric field is placed so that it feels a torque. Which of the following will increase the torque the most?

a) Double both charges on the dipole
b) Double the separation distance between the charges of the dipole
c) Double the electric field
d) Answers a,b, and c will have exactly the same effect
e) None of these will change the torque on the dipole

$\vec{p} = q\vec{L},\ \vec{\tau} = \vec{p} \times \vec{E} \Rightarrow |\vec{\tau}| = qLE\sin\theta$

→ Torque directly proportional to q, L, and E.

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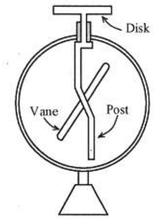
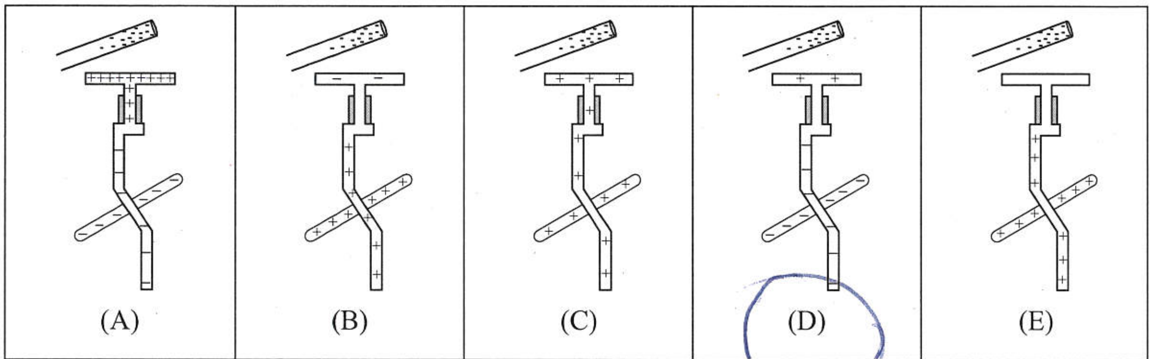
**II.** Lab questions [12 pts] ANSWER THESE ON YOUR SCANTRON SHEET.

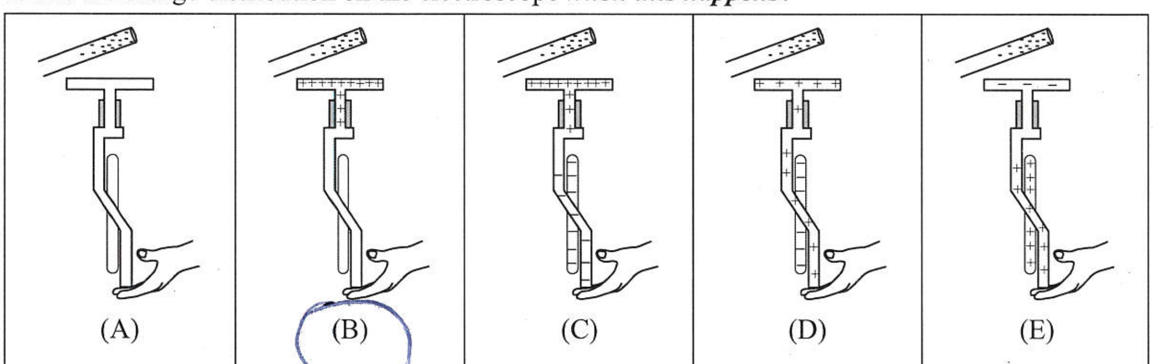



Initially, an electroscope's vane is **open**, as shown at right, and no charged objects are nearby. Then a **Teflon** rod is rubbed with a wool cloth, giving it a **negative charge**. The rod is held *near* the electroscope disk (but does not touch and no sparks jump), and it is observed that the vane **opens further**.

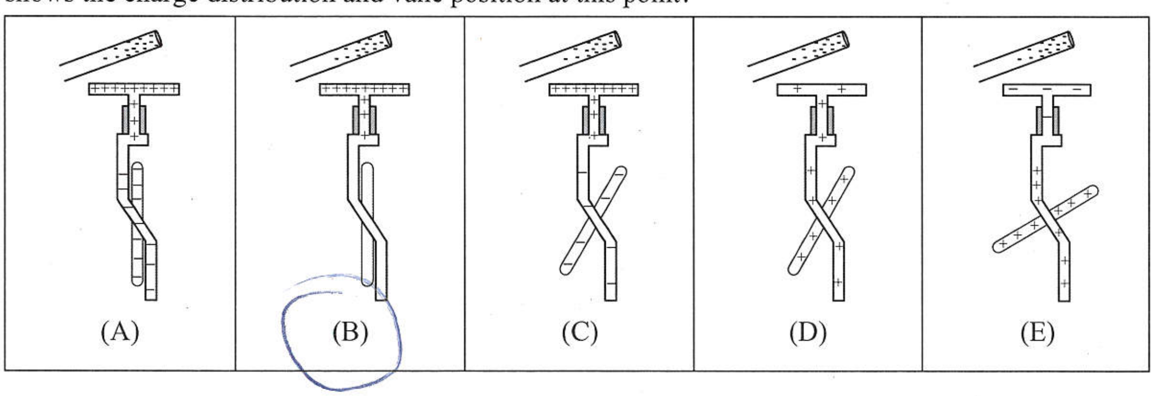
11. [4 pts] Which picture below shows the charge distribution on the electroscope ***while*** the Teflon rod is held near the disk? Look carefully, the differences are subtle.




12. [4 pts] Next, the experimenter continues to hold the rod in one hand **near** the electroscope's disk, and **touches the bottom end of the post with the other hand**, and the **vane closes.** Which picture below shows the charge distribution on the electroscope ***when this happens***?

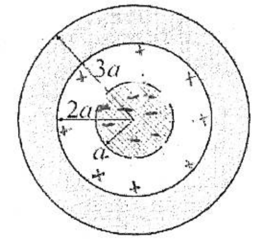



13. [4 pts] Finally, the hand is **removed**, but the rod is **still held near the disk**. Which picture below shows the charge distribution and vane position at this point?



Last Name, First Name Answer Key student id ____________ score ________

**III-A [18 points]** A cylindrical insulator of infinite length and radius $a$ has charge uniformly distributed through its volume with negative charge density $\rho_1$ ($\rho_1 < 0$). The insulator is surrounded by a conducting shell of inner radius $2a$ and outer radius $3a$ containing net charge per unit length $\lambda_2 = 3\pi a^2 \rho_1$.




i. [4 points] Use Gauss' Law to get an expression for the magnitude of the electric field, $E = |\vec{E}|$, at $r$ such that $a<r<2a$.

For a Gaussian cylinder of length L + radius r: $\oint \vec{E}\cdot d\vec{A} = EA = 2\pi r L E$

magnitude of enclosed charge is $|\rho_1| \times$ enclosed wire volume $= \pi a^2 L |\rho_1|$

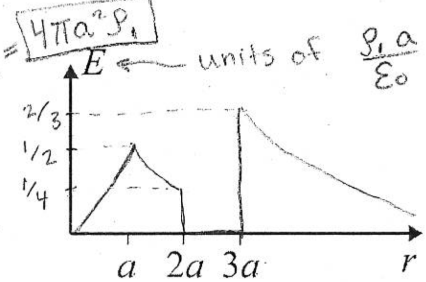
$2\pi r L E = \pi a^2 L \rho_1/\varepsilon_0 \Rightarrow \boxed{E = \dfrac{|\rho_1| a^2}{2\varepsilon_0 r}}$

ii. [4 points] Use Gauss' Law to calculate the charge density at the outer surface of the conductor. For a cylindrical Gaussian surface located at $2a<r<3a$, net flux is zero, since it's inside a conductor $\Rightarrow$ No net charge is enclosed.

So $\lambda_{inside} = -\lambda_{wire} = -\pi a^2 \rho_1$.

This leaves $\lambda_{outside} = \lambda_{conductor} - \lambda_{inside} = \boxed{4\pi a^2 \rho_1}$

iii. [6 points] Use Gauss' Law to make a graph of $E$ versus $r$ using the axes. Make sure your graph shows the correct ratio between $E(a)$, $E(2a)$, $E(3a)$. Show your work.

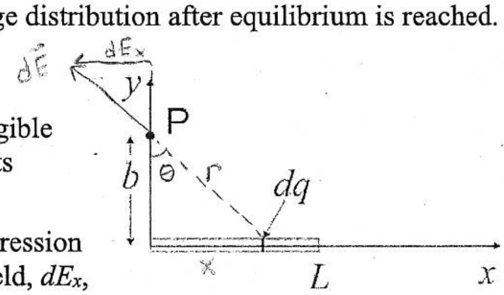



$0<r<a$: $E = \dfrac{|\rho_1| r}{2\varepsilon_0}$  $a<r<2a$: $E = \dfrac{|\rho_1| a^2}{2\pi\varepsilon_0 r}$

$2a<r<3a$: $E = 0$ (conductor)  $3a<r$: $E = \dfrac{2|\rho_1| a^2}{\varepsilon_0 r}$

iv. [4 points] A wire is used to connect the outside of the shell to ground. Using + and – signs on the figure of part i, make a sketch of the charge distribution after equilibrium is reached.

**III-B [7 points]** Consider now a different situation consisting of an insulator of finite length $L$ and negligible radius with charge $Q$ uniformly distributed through its volume.




i. [2 points] Use Coulomb's Law to write an expression for the contribution to the $x$ component of the field, $dE_x$, at point P ($x = 0$, $y = b$) by the element of charge $dq$ as shown.

$dE_x = -\dfrac{K\,dq}{r^2}\sin\theta = \boxed{\dfrac{-K\,dq\,x}{(b^2+x^2)^{3/2}}}$  $\left(\sin\theta = \dfrac{x}{r}\right)$

ii. [5 points] Integrate the previous to obtain the $x$ component of the field at point P.

$dq = Q\dfrac{dx}{L} \longrightarrow E_x = -\displaystyle\int_0^L dx\, \frac{KQ}{L}\frac{x}{(x^2+b^2)^{3/2}}$

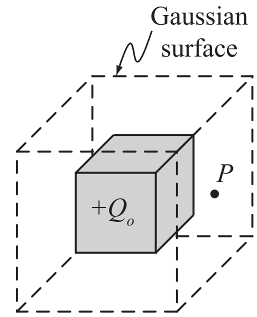
u-substitution: $u = x^2 + b^2$, $du = 2x\,dx$ $\Rightarrow E_x = -\dfrac{KQ}{2L}\displaystyle\int_{b^2}^{L^2+b^2} \frac{du}{u^{3/2}} = -\frac{KQ}{2L}\left(-2u^{-1/2}\right)\Big|_{b^2}^{L^2+b^2}$

$= \boxed{\dfrac{KQ}{L}\left(\dfrac{1}{\sqrt{L^2+b^2}} - \dfrac{1}{b}\right)}$

**Name** ________________________________________ **Student ID** ________________ **Score**________

*last* *first*

IV. [20 points total] A cube has a charge $+Q_o$ spread uniformly throughout its volume. A cube-shaped Gaussian surface encloses the cube of charge as shown. The centers of the cube of charge and the Gaussian surface are at the same point, and point $P$ is at the center of the right face of the Gaussian surface.



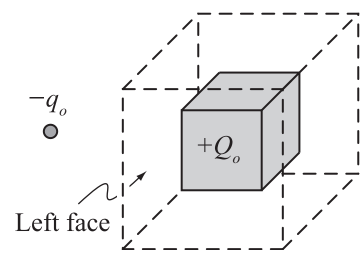

A. [6 pts] Is the magnitude of the electric field from the cube constant over any face of the Gaussian surface? Explain how you can tell.

*No, the magnitude of the electric field is not constant over any face. The cube is not a point charge, so we need to break it up into small pieces and take the vector sum of the field from each piece. Different portions of the charged cube are at different distances to each point in a way that's not symmetric in the same way as a sphere or line charge. For example, point P is the closest point to the center of the cube. A corner of the right face is farther from the center, but might be nearer to the vertex of the cube of charge. There's no reason to expect the vector sum to have the same magnitude at different points.*

B. [7 pts] Can this Gaussian surface be used to find the magnitude of the electric field at point $P$ due to the cube of charge? Explain why or why not.

*No, this surface can't be used to find $E_P$. In order to find $E_P$, we need to be able to isolate it in an equation. Since it starts out in a dot product inside a flux integral in Gauss' law, we need to be able to reduce the dot product and the integral. To get $E_P$ out of the integral, we need $\vec{E}$ to be parallel to $d\vec{A}$ everywhere on the surface* and *$E_P$ must be constant at every point over a surface. We determined above that the electric field is not constant over any part of the Gaussian surface, so we can't use this surface to solve for the electric field.*

A point charge $-q_o$ is brought near the center of the left face of the Gaussian surface as shown at right.




C. [7 pts] Does the net flux **through the left face** of the Gaussian surface *increase, decrease,* or *remain the same* when $-q_o$ is added? Explain your reasoning.

*The net flux through the left face* increases *when $-q_o$ is added. Superposition tells us that to determine the change, we only need to consider the contribution from the new charge. The area vector on the left face must point left everywhere, outward from the enclosed region, since the Gaussian surface is a closed surface. The new field points directly toward –qo everywhere, and this field will have a component in the same direction as the area vector everywhere on the left face, so the new contribution to the flux is positive. This will* increase *the net flux through the surface.*