EXPECTED VALUES OF SOME FUNCTIONS OF SLOPE AND DISTANCE ON A SETTING IN THE SHAPE OF A CIRCULAR SECTOR

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ABSTRACT

Previous studies have found yarding production to be a function of slope and distance parameters associated with a setting. Incorrect calculation of these parameters can lead to serious error in the production estimates. The correct procedure to be used is easily implemented once the problem is recognized. A test setting in the shape of a circular sector is used to demonstrate the potential magnitude of the error. The basis for the correct calculation of these slope and distance parameters is given.

INTRODUCTION

When estimates are to be made of cable system production on a setting in mountainous terrain, due consideration must be given to both the yarding distance and the slope of the unit. Production studies have consistently shown that these two factors are major determinants of yarding cycle time (Peters 1974). Failure to consider these two factors can lead to serious errors in production output estimation.

The purpose of this paper is to examine one possible source of estimation error, the incorrect quantification of slope and distance parameters for a setting. The magnitude of the error will be explored through the use of an analytical model. This model will be applied to a setting of simple geometric shape. In the final section of this paper these results are carried over to practical implementation.

TEST SETTING

The basis for the analytical model to be used here has been described elsewhere (Greulich 1980). In general the expected value of a function, denoted $E\{f(\cdot)\}$ is given as:

$$E\{f(\cdot)\} = \frac{1}{A}\int_A f(\cdot)\,dA \qquad (1)$$

For expository purposes the specific area over which integration is to be carried out is taken to be a circular sector of radius "R" with an interior angle "θ" ($\theta<\pi/2$). This sector is the horizontal plane projection of a setting located on a uniform side-hill slope. The landing is at the vertex of the sector. One edge of the setting is perpendicular to the contour and has a slope of "S" percent. Figure 1 gives a plan view of the setting to be analyzed.

The average yarding distance, AYD, is given by[1]:

$$AYD = E\{X\} = \frac{1}{A}\int_A x\,dA \qquad (2)$$

$$AYD = \left[\frac{2R}{3}\right]\left\{\left[\frac{.0075S^2+100}{(S^2+100^2)^{1/2}}\right] + \left[\frac{S^2\sin\theta\cos\theta}{400(S^2+100^2)^{1/2}\theta}\right]\right\} \qquad (3)$$

The average yarding slope, AYS, is given by:

$$AYS = E\{S\} = \frac{1}{A}\int_A s\,dA \qquad (4)$$

$$AYS = \frac{S\sin\theta}{\theta} \qquad (5)$$

The yarding cycle time function for a cable harvesting system may be written:

$$T = f(X, S, \ldots) \qquad (6)$$

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1. The italicized letters represent random variables; x and s respectively represent the straight line distance and the percent slope from the landing to a point in the setting.

The cycle time, T, is a function of the yarding distance, X, and slope, S. Other variables are important also and their inclusion has been noted by the series of commas. These other variables are extraneous to what follows and hereafter will be ignored. Observations are taken on T,X,S, and other important variables. The form of the function is specified, say,

$$T = B_0 + B_1 X + B_2 S + E \qquad (7)$$

where E is an error term with a mean of zero.

The parameters, B_i, are estimated[2]

$$T = b_0 + b_1 X + b_2 S \qquad (8)$$

Having estimated the function which relates yarding cycle time to distance and slope it is now applied to a setting for which a mean ("average") time must be estimated.

The expected cycle time is calculated as:

$$E\{T\} = \frac{1}{A}\int_A (b_0 + b_1 x + b_2 s)\,dA \qquad (9)$$

$$E\{T\} = b_0 + b_1 AYD + b_2 AYS \qquad (10)$$

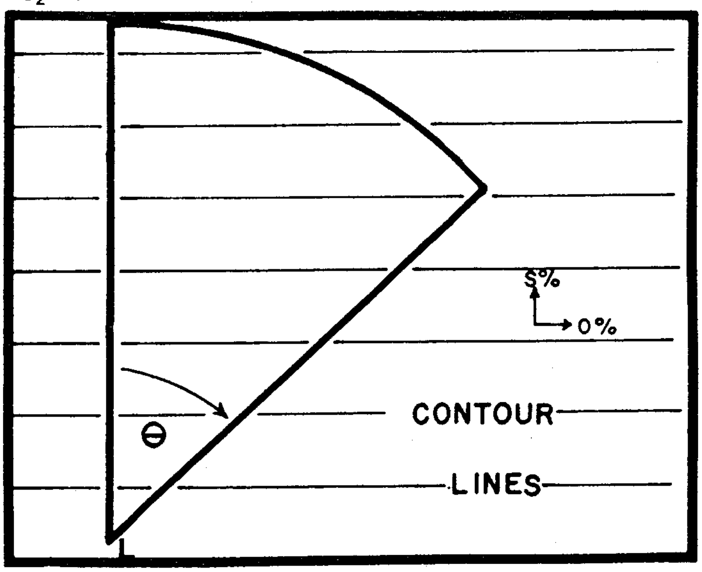

Figure 1. Diagram in plan view of the test setting.
One radius of the circular sector is fixed perpendicular to the contour of the side hill slope. The other radius is placed θ radians off the perpendicular with $\theta<\pi/2$. The central landing is located at "L". The maximum yarding slope is S% and the minimum is $S\cos\theta$%. The external yarding distance is $R/\cos[\tan^{-1}(S/100)]$. The horizontal distance from the landing to the back edge of the setting is a uniform distance R.

Consider now a specification which includes terms which are nonlinear in the variables, for example:

$$T = b_0 + b_1 X + b_2 S + b_3 X^2 + b_4 XS + b_5 S^{-1} \qquad (11)$$

Clearly the first three terms are evaluated as previously done, however the last three terms require some additional considerations. An immediate temptation is the use of AYD and AYS wherever X and S respectively appear. Some reflection, however, reveals that such an approach will in general give erroneous results since:

$$E\{X^2\} = E\{X\}^2 + \mathrm{Var}\{X\} \qquad (12)$$

$$E\{XS\} = E\{X\}E\{S\} + \mathrm{Cov}\{X,S\} \qquad (13)$$

$$E\{S^{-1}\} = E\{S\}^{-1} - E\{S\}^{-1}\mathrm{Cov}\{S^{-1},S\} \qquad (14)$$

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2. Least squares regression is typically used at this point.

In the first instance the use of AYD^2 gives an underestimation. In the second case the use of AYD·AYS gives either an over- or an underestimation depending on the sign of the covariance term. If slope and distance are uncorrelated then there is no error in this case. In the third situation the use of AYS^{-1} gives an underestimation.

In order to gain some appreciation for the magnitude of the errors involved with the incorrect use of AYD and AYS in non-linear functions the previously described circular sector is evaluated. For this setting the following exact results are obtained:

$$E\{X^2\} = [\frac{R^2}{2}] + [\frac{R^2S^2}{200^2}] + [\frac{R^2S^2\sin\theta\cos\theta}{200^2\theta}] \quad (16)$$

$$E\{XS\} = [\frac{(100^2+S^2)^{1/2}}{100}] [\frac{2RS}{3\theta}] [\frac{\sin^2\theta}{4} - \frac{S^2\sin^4\theta}{4(100^2+S^2)}]^{1/2} + [\frac{(100^2+S^2)^{1/2}}{2S}] [\sin^{-1} \frac{S\sin\theta}{(100^2+S^2)^{1/2}}] \quad (17)$$

$$E\{\frac{1}{S}\} = [\frac{1}{2\theta S}] [\log_e \frac{1+\sin\theta}{1-\sin\theta}] \quad (18)$$

The relative errors for different values of θ and S are given in tables 1, 2, and 3. An examination of these tables reveals that the potential for very serious error exists when AYD and AYS are used incorrectly. Since it is difficult to predict when such an error will occur, the indiscrimate use of AYD and AYS should be avoided. If these parameters cannot be safely used in the evaluation of non-linear terms, then an alternative must be found. Fortunately, there is an alternative procedure which is statistically correct and in most cases easily applied. This alternative will be discussed and then applied to the circular sector test problem.

ALTERNATIVE PROCEDURE

The following procedure is recommended as being a statistically correct and easily implemented alternative to the use of AYD and AYS in non-linear terms. In the same program which is used to numerically estimate AYD and AYS.provisions can be easily made to concurrently estimate non-linear functions of yarding distance and yarding slope. As in the case of AYD and AYS estimation, a discrete approximation is made to the definition of the expected value, formula (1). For example, if the setting of area "A" is divided into "n" sub-areas, a_i, which are mutually exclusive and,

$$\sum_{i=1}^{n} a_i = A \quad (19)$$

then an estimate of the AYD is,

$$\frac{1}{A} \sum_{i=1}^{n} x_i a_i \quad (20)$$

An estimate of $E\{XS\}$ is

$$\frac{1}{A} \sum_{i=1}^{n} x_i s_i a_i \quad (21)$$

The exact answer is the limit as $n \to \infty$ and $a_i \to 0$.[3]

This numerical procedure is the basis for almost all existing programs which evaluate AYD. The inclusion of two or three additional program steps in these existing programs will easily and quickly yield statistically correct estimates of non-linear terms.

VERIFICATION

A program to verify these predicted results has been written for the Hewlett-Packard 9831A desktop computer used in conjunction with the 9874A digitizer and the 9871A printer. In this program a somewhat different procedure from that outlined in formulae 19-21 has been used.

3. There are additional restrictions on how $a_i \to 0$.

Table 1. Relative errors for different values of θ and S, example a.

$$\frac{E\{X\}\,E\{S\} - E\{XS\}}{E\{XS\}}$$

		θ (5π/180)	θ (45π/180)	θ (85π/180)
S	40%	(neg)	(neg)	(1%L)
	60%	(neg)	(neg)	(2%L)
	80%	(neg)	(neg)	(3%L)

neg = negligible
L = low
H = high

Table 2. Relative errors for different values of θ and S, example b.

$$\frac{E\{X\}^2 - E\{X^2\}}{E\{X^2\}}$$

		θ (5π/180)	θ (45π/180)	θ (85π/180)
S	40%	(11%L)	(11%L)	(11%L)
	60%	(11%L)	(11%L)	(11%L)
	80%	(11%L)	(11%L)	(10%L)

Table 3. Relative errors for different values of θ and S, example c.

$$\frac{\frac{1}{E\{S\}} - E\{\frac{1}{S}\}}{E\{\frac{1}{S}\}}$$

		θ (5π/180)	θ (45π/180)	θ (85π180)
S	40%	(neg)	(1%L)	(29%L)
	60%	(neg)	(1%L)	(29%L)
	80%	(neg)	(1%L)	(29%L)

The procedure employed in this program is based on a systematic random pattern of turns being placed across the setting (figure 2). Since the turns are assumed to have a uniform distribution over the area of the setting, every point within the area is given the same probability of being selected as the location of a turn.[4] In order to obtain confidence intervals for the estimates three systematic pattern overlays with random starting points were taken. The program output is given in Table 4. Table 5 gives the exact results for this setting (calculated using formulae 3, 5, 16, 17, 18), the program estimates, and estimates based on the incorrect use of AYD and AYS.

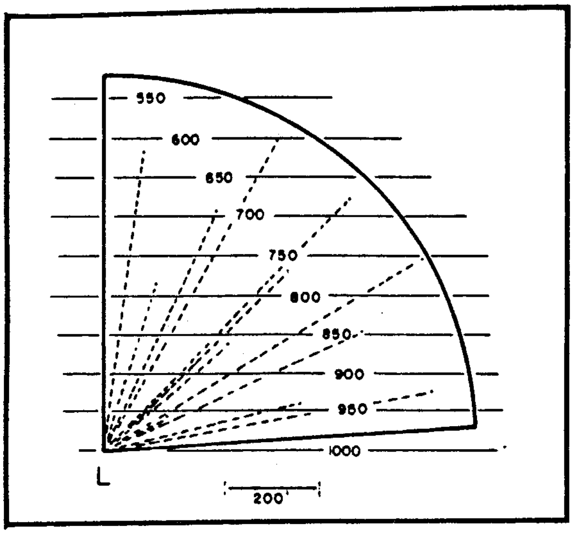

Figure 2. Plan view of the test example setting.
A systematic pattern of points has been randomly placed across the setting. These points, representing turns, are used to obtain estimates of slope and distance parameters. Dashed lines indicate the paths followed by each sample turn from its location in the setting to the landing. In this specific example an evaluation of the twelve points shown on the diagram gives AYD and AYS estimates of 591 feet and 39 percent respectively.

Table 4. Computer program estimates

LANDING NUMBER 999
ESTIMATED AREA OF UNIT 444061
NUMBER OF OVERLAYS 3
NUMBER OF POINTS 10 11 12 0 0
0 0 0 0 0

SQUARE OVERLAY GRID

PARAMETER	ESTIMATE	SE
AYD	583	22.2
AYS	39.01	0.6
AY(D*S)	23868.76	
AY(D/S)	20.00	
AY(S/D)	0.07515	
AY(D^2)	380463.55	
AY(S^2)	1828.07	
AY(1/S)	0.03811	
AY(1/D)	0.00214	
MIN(|S|)	6.98	
MAX(D)	910.48	
MIN(D)	110.42	
MAX(S)	59.60	
MIN(S)	6.98	

Three randomly located overlays of systematically spaced points have been used to estimate several setting parameters. Standard errors of the estimate for AYD and AYS are also calculated.

4. This procedure can be easily extended through stratification to include situations where turns, though not uniformly distributed over the entire setting, are uniformly distributed within mutually exclusive and exhaustive sub-divisions of the setting area.

Table 5. Circular sector test program results

R = 800 feet, θ = 85π/180 radians, S = 60 percent.

	True Values	Program Estimate	Estimates Based on AYD & AYS
$E\{X\}$	583	583 (neg)	---
$E\{S\}$	40.3	39.0 (3%L)	---
$E\{\frac{1}{S}\}$	.03518	.03811 (8%H)	.02563 (27%L)
$E\{X^2\}$	380971	380464 (neg)	339889 (11%L)
$E\{XS\}$	23914	23869 (neg)	22743 (5%L)

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