NEGATIVE-INDEX METAMATERIALS

Sarah Harvey

Physics 486

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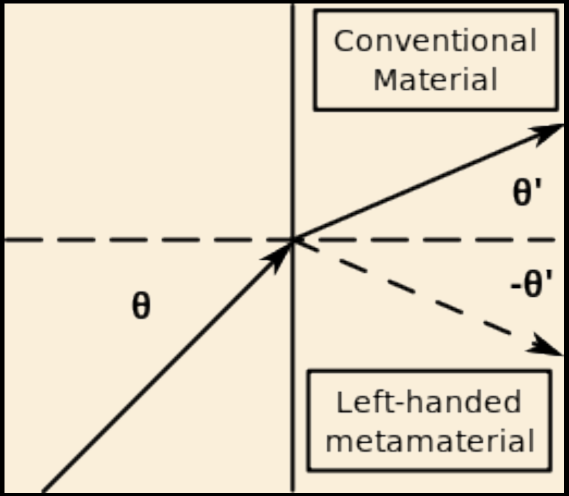
OUTLINE

- Introduction to metamaterials
- Review: index of refraction
- Theory of negative-index metamaterials
- Real negative-index materials
- Metamaterial cloaking
- Problems with negative-index metamaterials



WHAT ARE METAMATERIALS?

- Man-made materials, display properties that are not found in nature
- Can be designed with properties such that both the E and B components of EM radiation couple to the “meta-molecules,” enabling entirely new optical properties
  - Negative-index metamaterials (NIM)
- Accomplished by designing microstructures smaller than the wavelength of incident radiation such that the effective result is a material that displays bizarre electromagnetic properties
  - The homogenization hypothesis
  - Effective medium theory

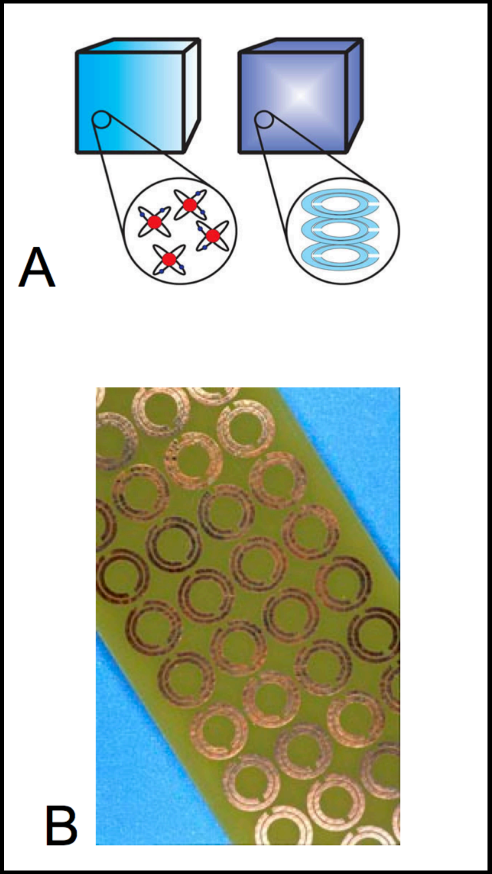


Figure A: In a metamaterial, subwavelength engineered units replace molecules as the dominant determinant of electromagnetic properties. Figure B: An early example of a metamaterial designed to give a magnetic response at around 10GHz. Image credit: Sir John Pendry.

REVIEW: INDEX OF REFRACTION

- Dimensionless number describing how electromagnetic radiation propagates through a medium

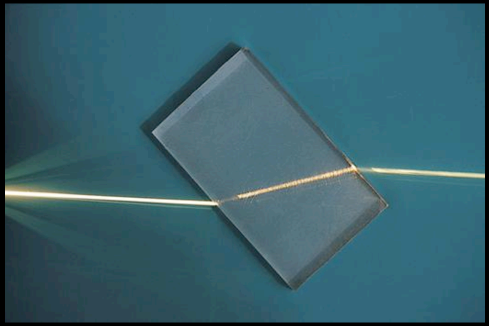
$$n = \frac{c}{v_{phase}}$$

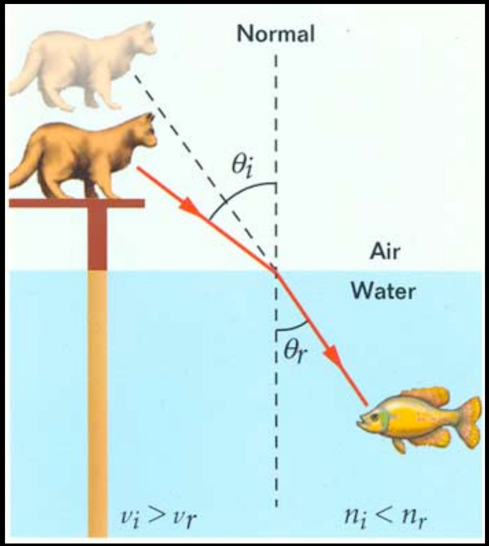
- Determines refraction of light in a material by **Snell's Law:**

$$n_1 \sin\theta_1 = \; n_2 \sin\theta_2$$

- Refractive index is related to the electric permittivity and magnetic permeability by the Maxwell relation:

$$n^2 = \mu_r \epsilon_r$$





Images: Wikimedia Commons

NEGATIVE INDEX OF REFRACTION

- Note: possibility of choosing the negative square root in the Maxwell relation $n^2 = \mu_r \epsilon_r$
- How can a dielectric material display a negative refractive index?
  - Lorentz dispersion law– can show that the permittivity (and similarly the permeability) are dependent on frequency of light in the medium

Know force on electrons due to electric and magnetic fields:

$$F = -e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

The electron in an atom or molecule can be assumed to be bound to the equilibrium position through an elastic restoring force

$$m\,\ddot{\boldsymbol{r}} + m\gamma\dot{\boldsymbol{r}} + m\omega_0^2\boldsymbol{r} = -e\boldsymbol{E_0}\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}$$

Guess solution $\boldsymbol{r} = \boldsymbol{r_0}e^{-i\omega t}$

$$\boldsymbol{r_0} = \frac{-e\boldsymbol{E_0}/m}{\omega_0^2 - \omega(\omega + i\gamma)}$$

NEGATIVE INDEX OF REFRACTION

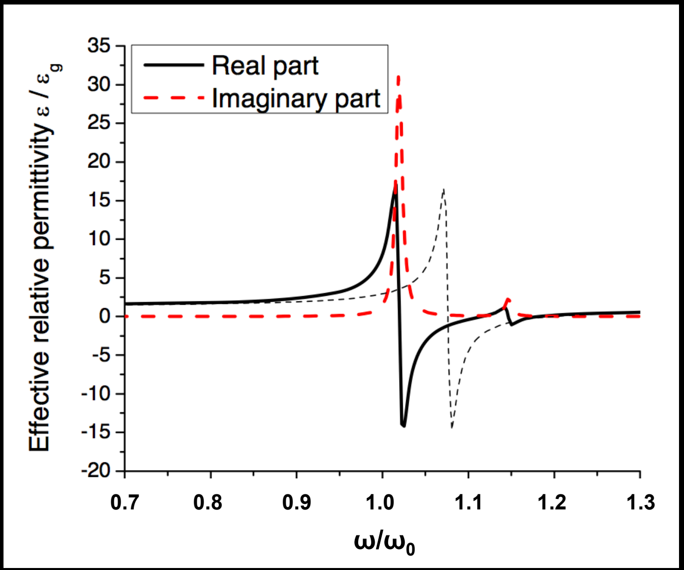
- The dipole moment due to each electron is $\boldsymbol{p} = -e\boldsymbol{r}$, and the total dipole moment per unit volume, **P**, is given by the vector sum of all the dipoles in a unit volume.
- If we assume one dipole per molecule and an average number density of N molecules per unit volume, we obtain

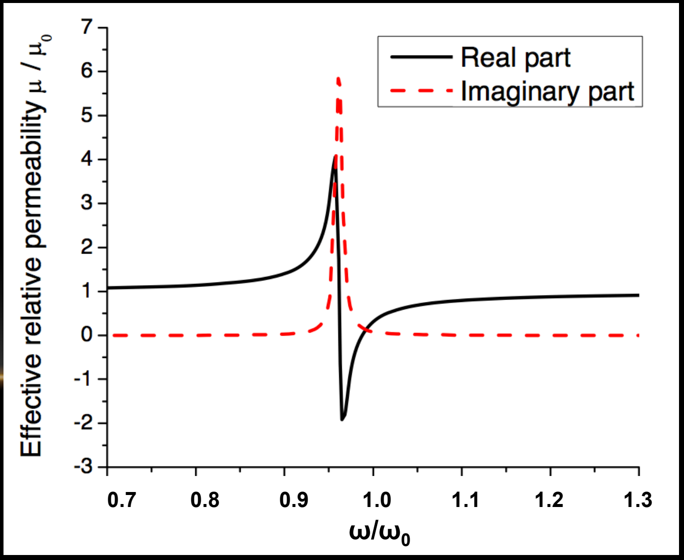
$$\boldsymbol{P} = N\boldsymbol{p} = \frac{Ne^2\boldsymbol{E}/m}{\omega_0^2 - \omega(\omega + i\gamma)} = \epsilon_0 \chi_e \boldsymbol{E}$$

- Rearranging:

$$\boxed{\epsilon(\omega) = 1 + \chi_e(\omega) = 1 + \frac{Ne^2/m\epsilon_0}{\omega_0^2 - \omega(\omega + i\gamma)}}$$

LORENTZ FORMULA FOR DISPERSION





NEGATIVE INDEX OF REFRACTION

- So if ε and μ < 0, we find an interesting result from Maxwell's equations in a time-harmonic plane wave in the medium:

$$\boldsymbol{k} \times \boldsymbol{E} = \omega\mu_0\mu\boldsymbol{H}$$
$$\boldsymbol{k} \times \boldsymbol{H} = -\omega\epsilon_0\epsilon\boldsymbol{E}$$

- **E**, **H**, and **k** form a left handed system! BUT Poynting vector is still

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$

- If we take the relationship between S and k in terms of index of refraction:

$$\boldsymbol{k} = \frac{\widehat{\boldsymbol{S}} n\omega}{c}$$

- We see that the index of refraction *n* must be negative for this relationship to hold!

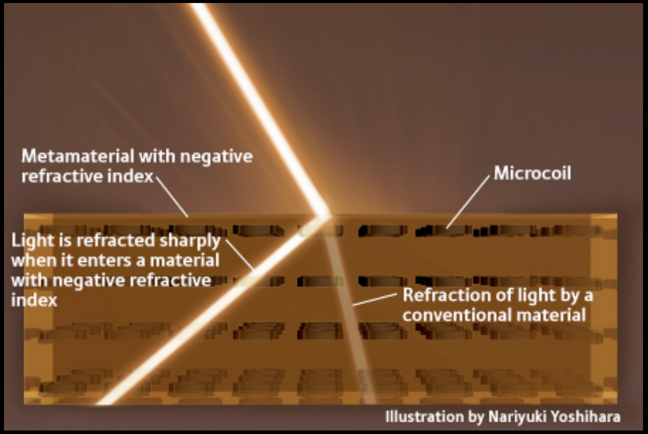
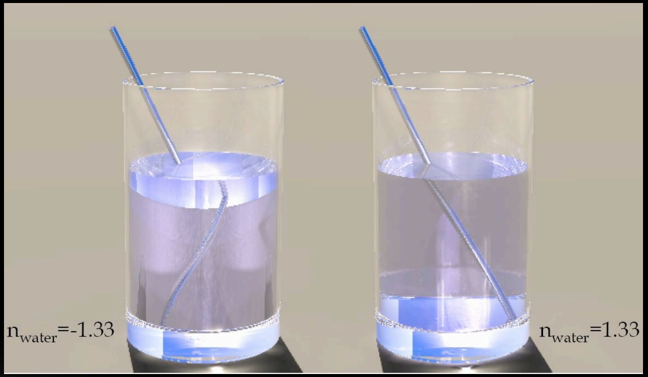


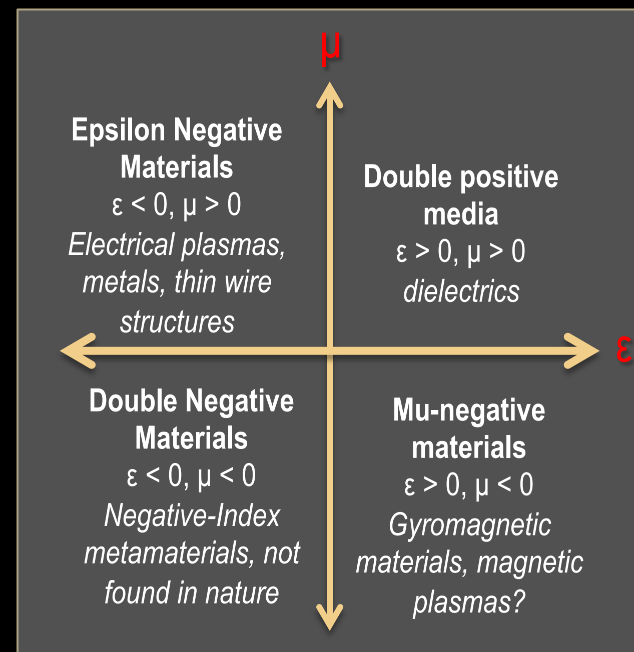
Image credit: Riken Research



Simulated image from Dolling et al., Optics Express 2006.

NEGATIVE-INDEX METAMATERIALS

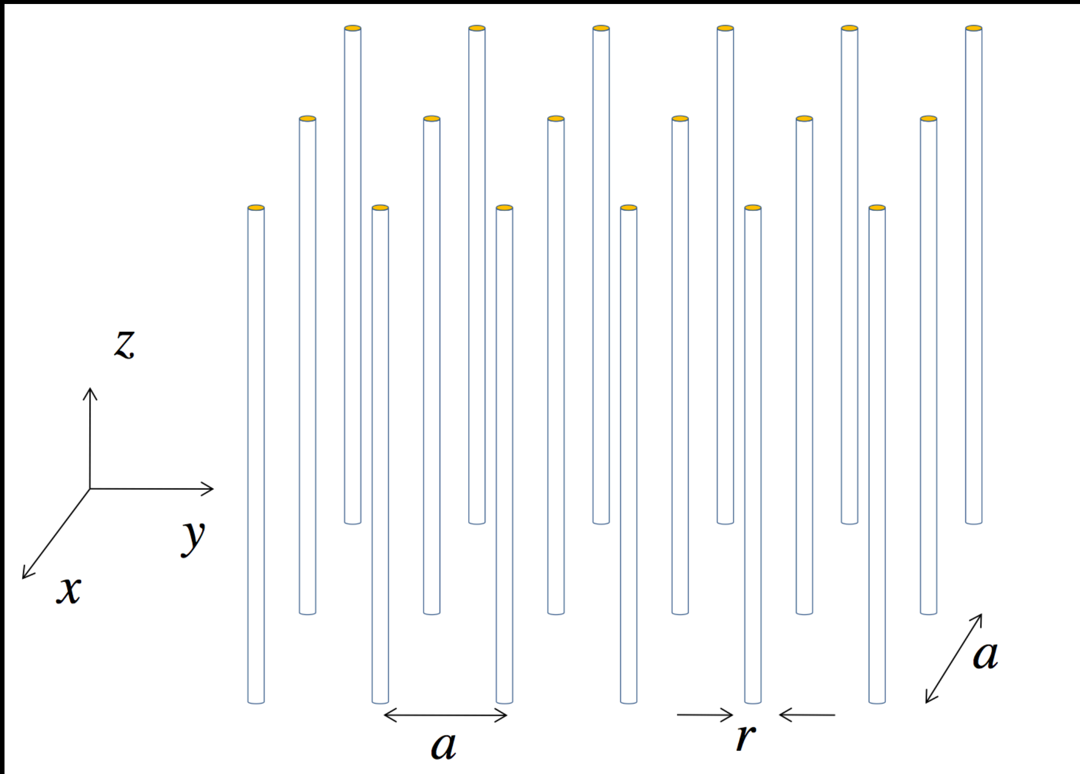
- Optical properties of materials are governed by the electric permittivity ε and the magnetic permeability μ
- Any collection of objects with size and spacing << λ can be described by an ε and μ by the homogenization hypothesis
- Resonances in metamaterials can induce large amounts of dispersion in the effective medium parameters at frequencies close to resonance
  - By properly driving these resonances, one can cause the parameters ε and μ to be simultaneously negative



Classification of materials based on dielectric and magnetic properties

REAL NEGATIVE-INDEX METAMATERIALS

- No naturally occurring NIM has ever been discovered!
- Can combine two regions to store electrical and magnetic energy into a single cell of metamaterial
    - Inductor-capacitor resonance circuits
- A theoretical curiosity until Smith et al experimentally demonstrated a negative refraction in the microwave using an array of copper strips and split-ring resonators in 2000

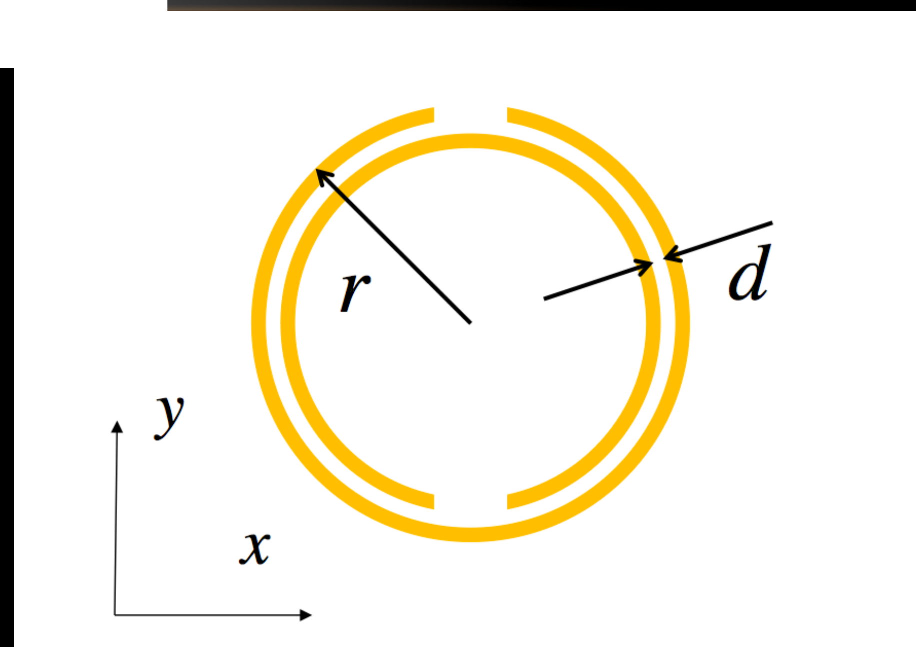


Split-ring resonators are often used in metamaterials to induce strong magnetic coupling to the B field and provide negative μ by acting as a resonant LC circuit

$$\mu(\omega) = 1 - \frac{\frac{\pi r^2}{a^2}\omega^2}{\omega_0^2 - \omega^2}$$

Wire mesh mimics a diluted metal and can provide negative ε with proper geometrical choices

$$\epsilon(\omega) = 1 - \frac{2\pi c^2}{\omega^2 a^2 \ln\left(\frac{\mathrm{a}}{\mathrm{r}}\right)}$$



Split-ring resonators

Wires

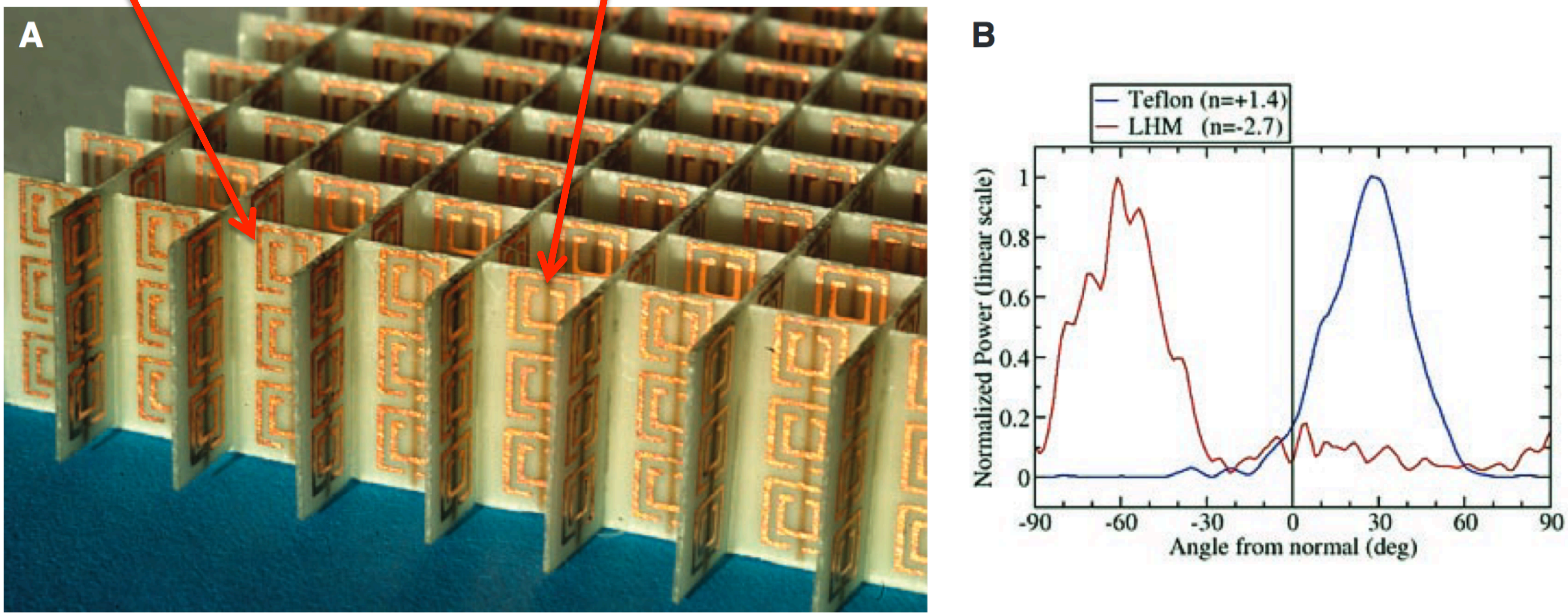
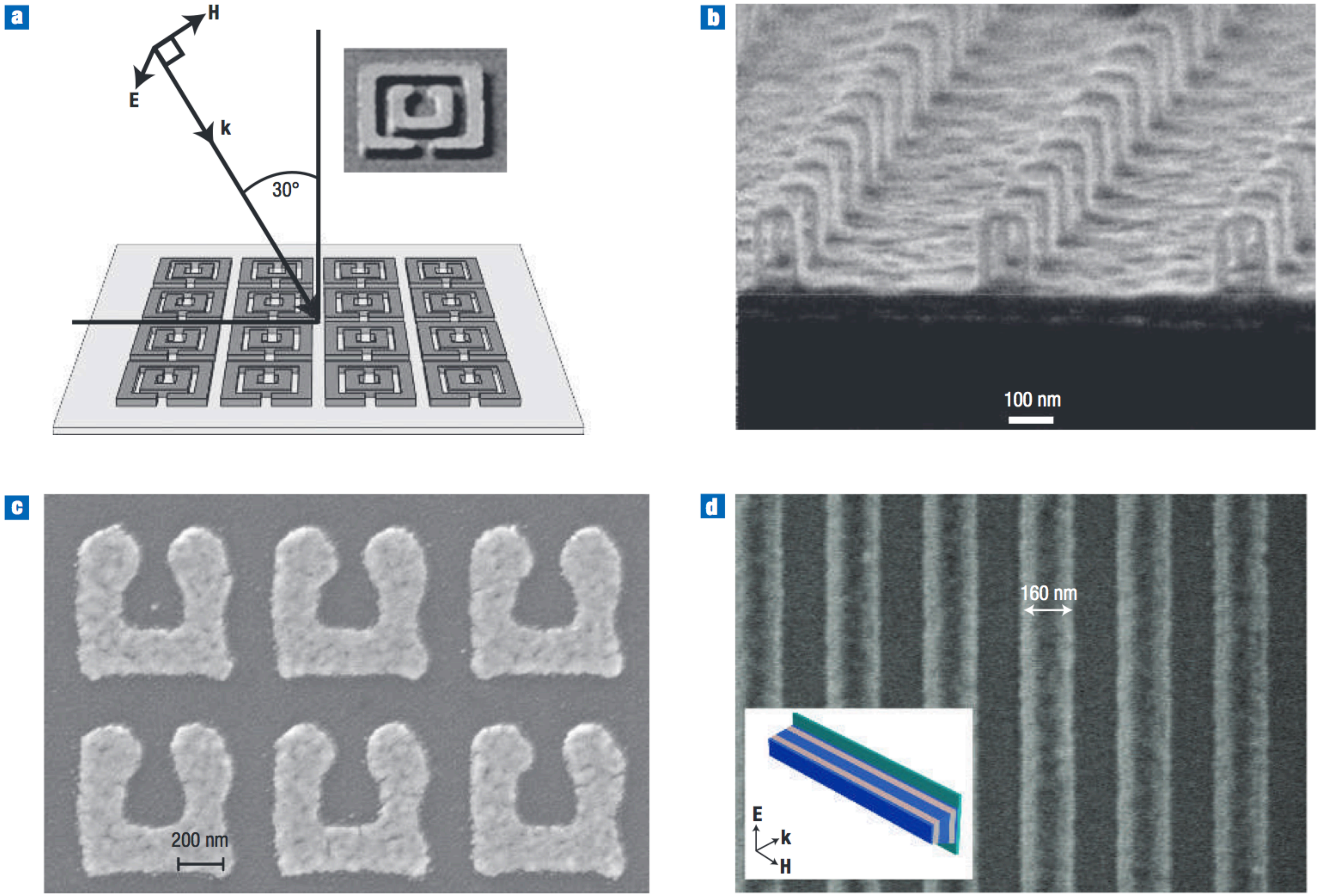


Fig. 3. **(A)** A negative index metamaterial formed by SRRs and wires deposited on opposite sides lithographically on standard circuit board. The height of the structure is 1 cm. **(B)** The power detected as a function of angle in a Snell's law experiment performed on a Teflon sample (blue curve) and a negative index sample (red curve).

Microwave frequency NIM

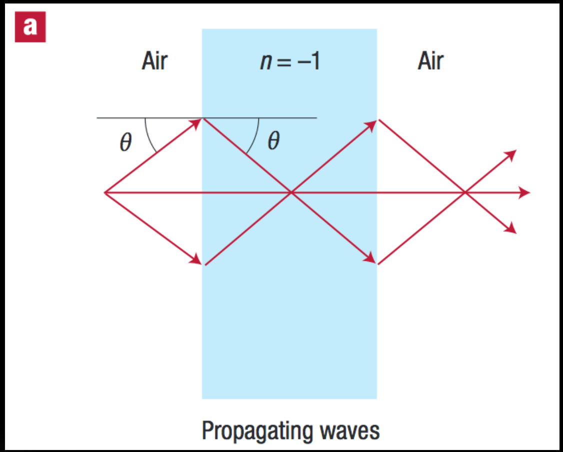
Smith, D., Pendry, J., & Wiltshire, M. (2004). Metamaterials and Negative Refractive Index. Science, 305, 788-792.



Terahertz and optical magnetic metamaterials. a) double SSR with THz response. b) staple metamaterial with mid-IR magnetic resonance. c) single SRRs with negative permeability in the IR. d) Paired silver strips with negative permeability at 725 nm. Shalaev, V. (2007). Optical negative-index metamaterials. Nature Photonics, 1, 41-48.

APPLICATIONS OF NIM

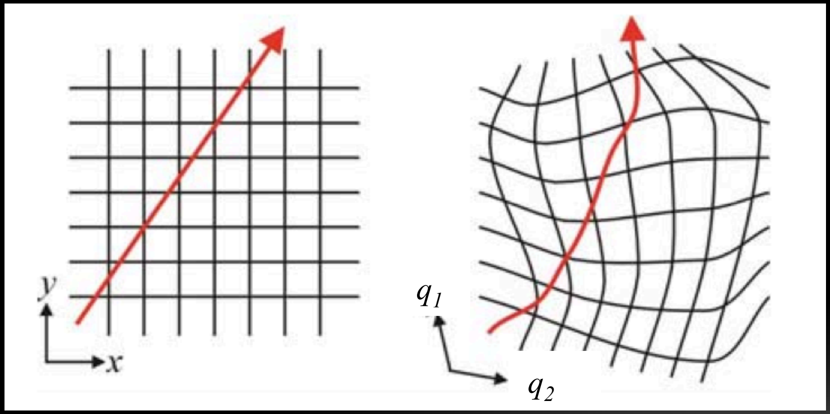
- Provides opportunities to realize physical phenomena that were previously only theoretical exercises
- Improvements in magnetic resonance imaging (MRI)
- Super-resolution microscopy (metamaterial lenses and superlenses)
- **Cloaking and transformation optics**



NIM flat lens (Image: Zhang et al.)

TRANSFORMATION OPTICS AND METAMATERIAL CLOAKING

- John Pendry proposed in 2006 a way of controlling electromagnetic radiation using metamaterials that could obscure objects– transformation optics was born
- The mathematical technique of transformation optics shows media with gradients in optical properties are equivalent to curved geometries of spacetime for light propagation
  - Uses coordinate transformations to predict optical properties required for particular ray manipulations



$$q_1(x,y,z), q_2(x,y,z), q_3(x,y,z)$$

$$\epsilon^{i'i'} = \epsilon^{ii}\frac{Q_1Q_2Q_3}{Q_i^2}$$

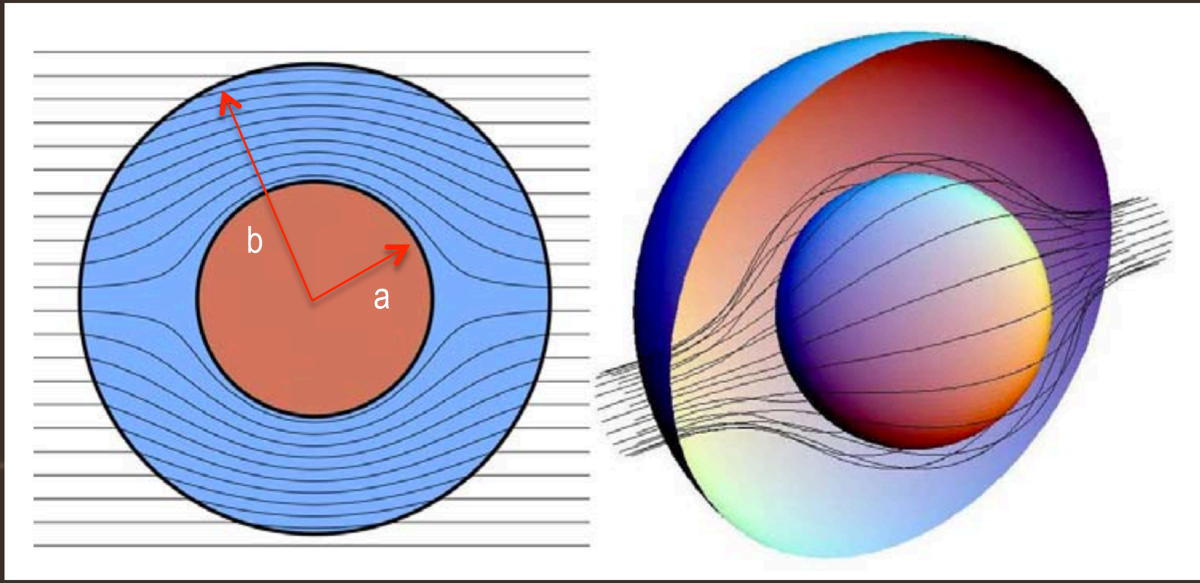
$$\mu^{i'i'} = \mu^{ii}\frac{Q_1Q_2Q_3}{Q_i^2}$$

Where i = 1, 2, 3 and

$$Q_i^2 = \left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2$$

TRANSFORMATION OPTICS AND METAMATERIAL CLOAKING

- Goal: use transformation optics and metamaterials to direct EM radiation smoothly around an object to render it invisible
  - Fine print: object would be plunged into darkness and would not be able to see out…
- Pendry et al. showed this theoretically and experimentally in 2010 but there are still many issues before perfect cloaking can be realized
- Potential in digital metamaterials? (Giovampaola 2014)

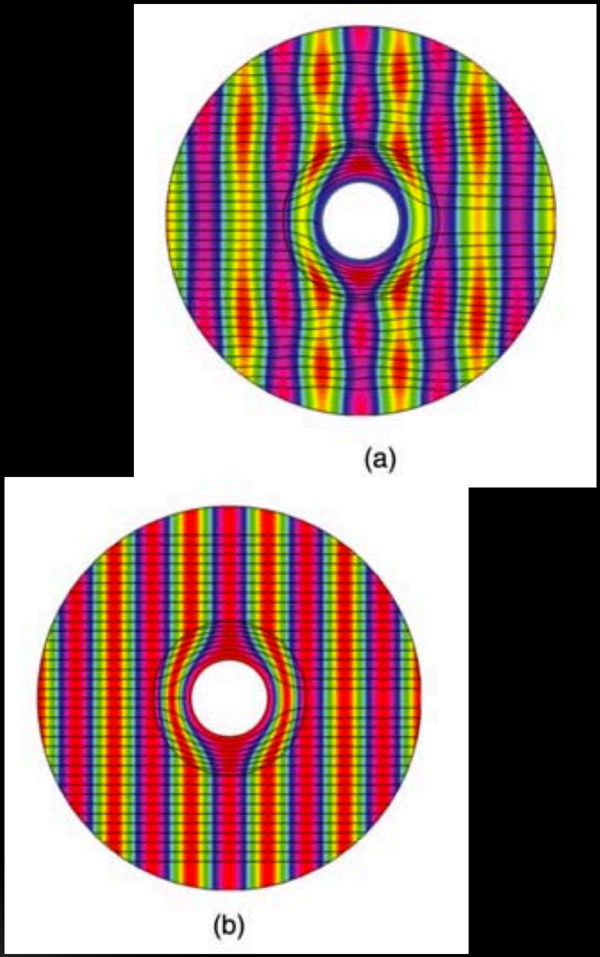


Pendry, J., Schurig, D., & Smith, D. (2006). Controlling Electromagnetic Fields. Science, 1780-1782.

CHALLENGES WITH METAMATERIALS

- Metamaterials remain impractical for most applications
- Broadband optical metamaterials require resonant structures ~ 1 nm!
  - Best results attained in the microwave range
- Ohmic losses to the metal resonators make metamaterials notoriously "lossy"
  - Problem for any device that requires high efficiency (microscopy, solar cells, cloaking)

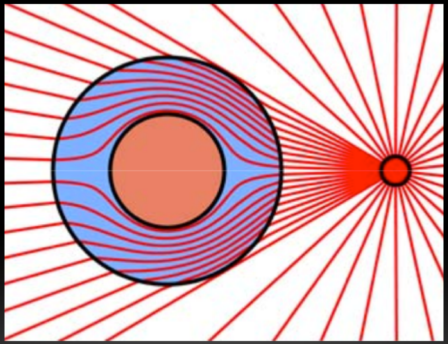
But there might still be hope…



Two-dimensional cloaks. Images: WH Wee and JB Pendry

SUMMARY AND CONCLUSION

- There is no theoretical restriction on the sign of refractive index– negative indices are not observed in nature but can be fabricated using metamolecules and the homogenization hypothesis
- Metamaterials research offers both opportunities to study optical systems that were once purely theoretical
- Incredible applications for technology if they could be made feasible
- Many hurdles before metamaterials could be seen in widespread application



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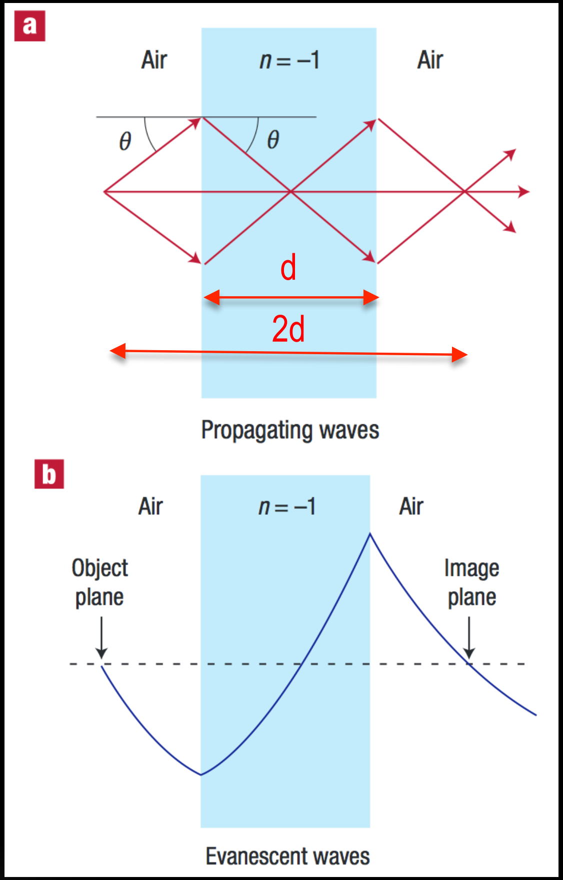
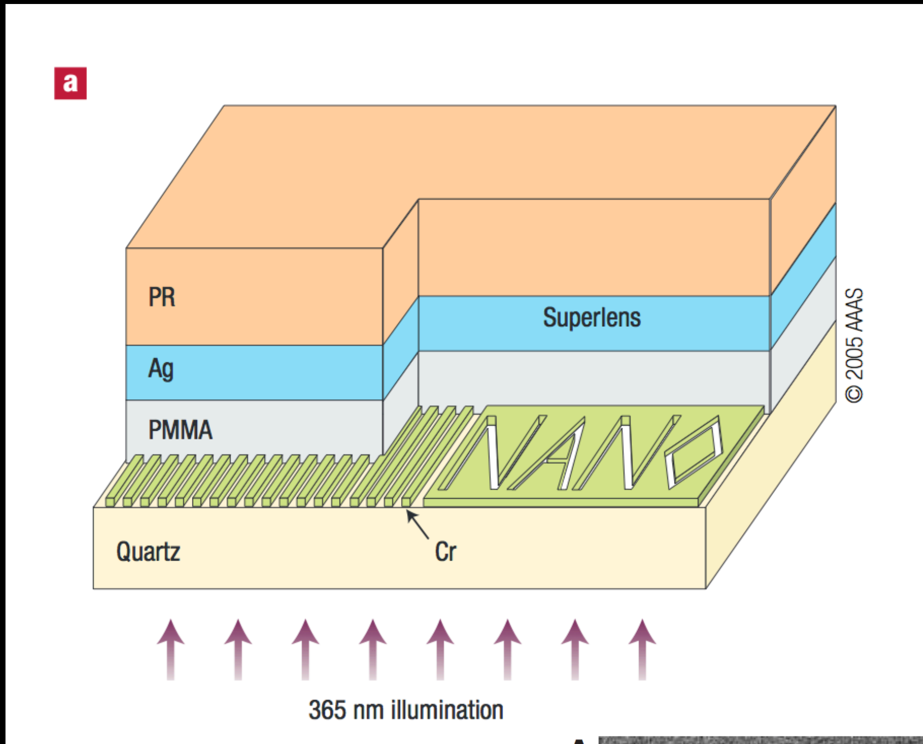
SUPER-RESOLUTION MICROSCOPY

- “Superlens” first proposed by Sir John Pendry in 2000
  - Flat lens with focus 2d from the object
- Effectively creates a region of “negative space”
- Conventional diffraction limit caused by exponential decay of evanescent waves in normal materials
- Can show that evanescent waves exponentially enhanced through the slab, can theoretically recover all information about object at focus
- Drawback: image is same size as object



NIM flat lens (Image: Zhang et al.)

See Griffiths Electrodynamics Problem 9.39



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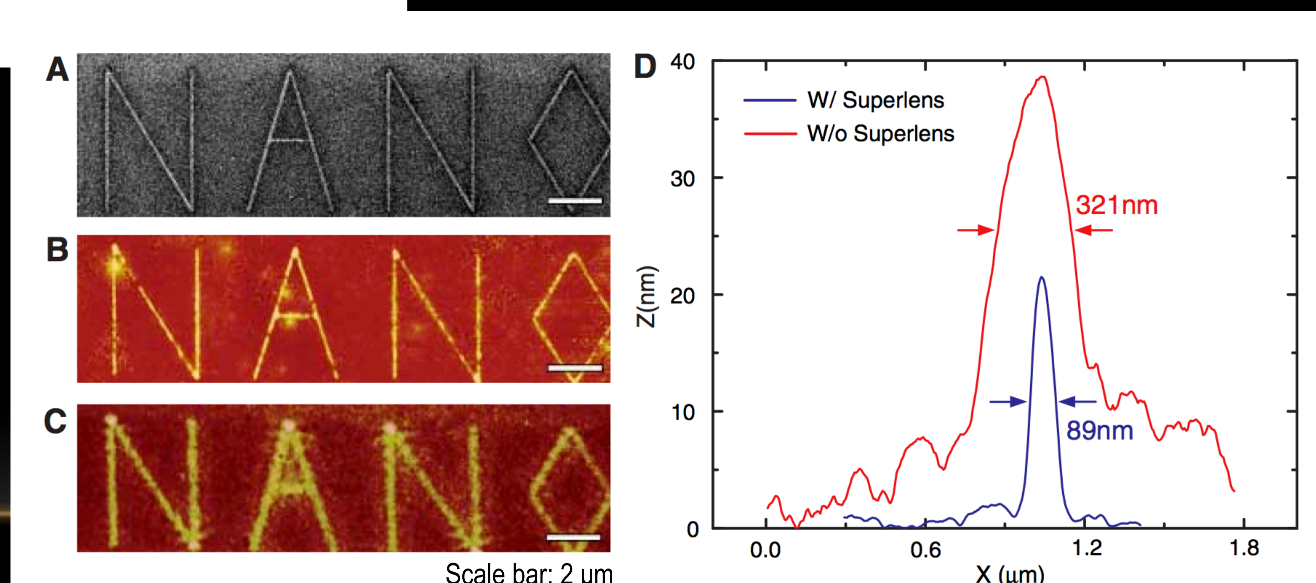

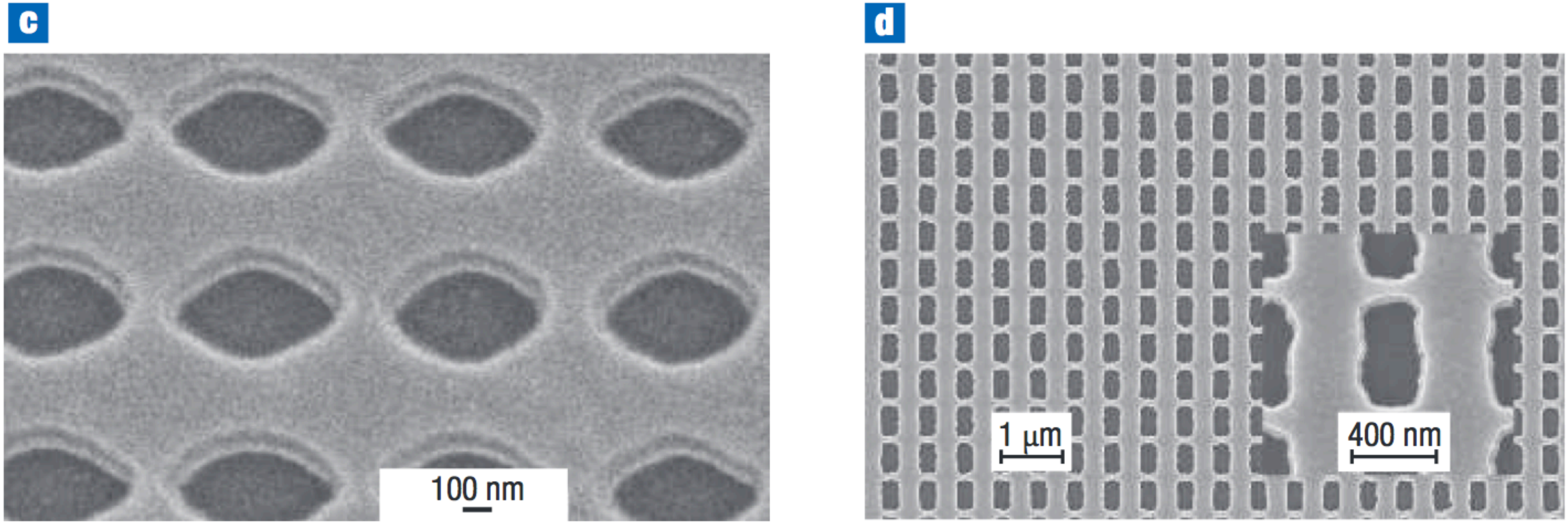

Negative index metamaterials c) at λ = 2 microns and d) at λ = 1.4 microns.

Shalaev, V. (2007). Optical negative-index metamaterials. Nature Photonics, 1, 41-48.