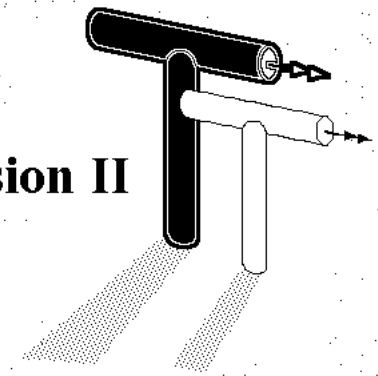

Torsion II

Displacements due to Torsion Loading

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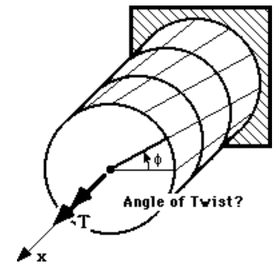
Angle of Twist



Often the design of a shaft will be based on limiting the amount of twist that occurs when the shaft is subjected to a torque. To address such problems we must derive a means to calculate the amount of twist in a shaft. Furthermore, being able to compute the angle of twist (kinematics) is important in analyzing statically indeterminate shafts.

In this section we will develop a formula for determing the *angle of twist*, ϕ, of one end of a shaft with respect to the other. As before, we assume a circular cross section and that the material is homogeneous and behaves in a linear-elastic manner.

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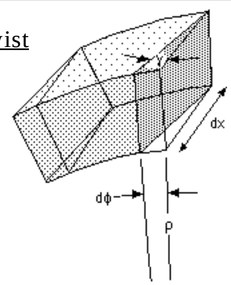
hear Strain vs. Angle of Twist

$$\gamma = \rho \frac{d\phi}{dx}$$



Recall, that by observing the deformed shape of an element we were able to derive the relationship between shear strain, γ, and angle of twist:

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Recall Shear Strain...

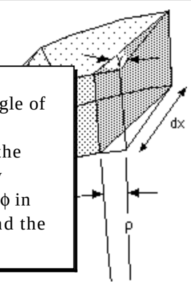
hear Strain vs. Angle of Twist

$$\gamma = \rho \frac{d\phi}{dx}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

Solving for $d\phi$ yields an expression for change of angle of twist.

Using Hooke's law and the torsion formula we can now develop an expression for $d\phi$ in terms of the applied load and the geometry of the section.



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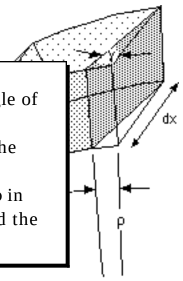
Recall Hooke's Law



Solving for $d\phi$, we get an expression for change of angle of twist.
Using **Hooke's law** and the torsion formula we can now develop an expression for $d\phi$ in terms of the applied load and the geometry of the section.

$$\gamma = \rho \frac{d\phi}{dx}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

Hooke's Law: $\gamma = \frac{\tau}{G}$

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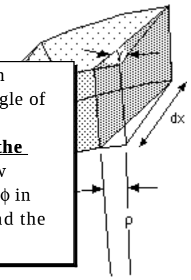
Recall Hooke's Law

Recall Torsion Formula



Solving for $d\phi$, we get an expression for change of angle of twist.
Using Hooke's law and **the torsion formula** we can now develop an expression for $d\phi$ in terms of the applied load and the geometry of the section.

$$\gamma = \rho \frac{d\phi}{dx}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

Hooke's Law: $\gamma = \frac{\tau}{G}$

Torsion Formula: $\tau = \frac{T(x)\,\rho}{J(x)}$

Substitute this expression for τ into Hooke's Law

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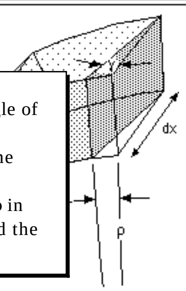
Recall Torsion Formula

More Substitution



Solving for $d\phi$, we get an expression for change of angle of twist.
Using Hooke's law and the torsion formula we can now develop an expression for $d\phi$ in terms of the applied load and the geometry of the section.

$$\gamma = \rho \frac{d\phi}{dx}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

Hooke's Law: $\gamma = \frac{\tau}{G} = \frac{T(x)\,\rho}{J(x)\,G}$

Use this value for γ in the equation at the top

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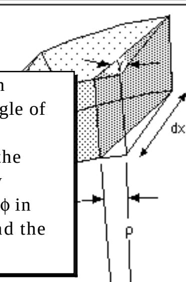
Cancel Like Terms...



Solving for $d\phi$, we get an expression for change of angle of twist.
Using Hooke's law and the torsion formula we can now develop an expression for $d\phi$ in terms of the applied load and the geometry of the section.

$$\gamma = \rho \frac{d\phi}{dx}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

$$d\phi = \frac{T(x)\,\rho}{J(x)\,G} \frac{dx}{\rho}$$

Cancel ρ from the expression →

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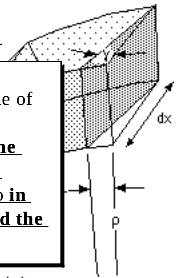
n Expression for Angle of Twist

$$\gamma = \rho \frac{d\phi}{dx}$$

$$d\phi = \gamma \frac{dx}{\rho}$$

Solving for dϕ, we get an expression for change of angle of twist.

Using Hooke's law and the torsion formula we can now develop an expression for dϕ in terms of the applied load and the geometry of the section.



$$d\phi = \frac{T(x)}{J(x)\,G}\,dx$$

And here it is, just as advertised! Let's save this result

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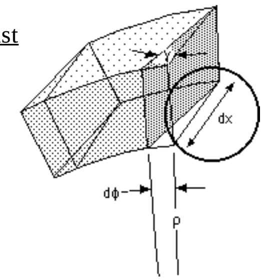
Differential Twist

$$d\phi = \frac{T(x)}{J(x)\,G}\,dx$$



In the this equation, dϕ represents the angle of twist for a differential element of length dx. In order to obtain the angle of twist for the entire shaft we must integrate dϕ over the entire length, L, of the shaft.

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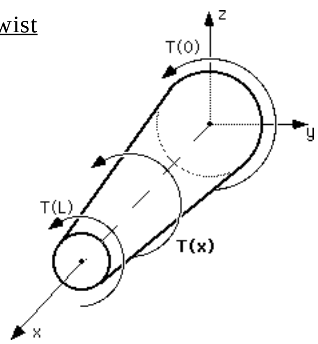
Total Angle of Twist

$$d\phi = \frac{T(x)}{J(x)\,G}\,dx$$

$$\phi = \int_0^L \frac{T(x)}{J(x)\,G}\,dx$$



We have arrived!! An expression for angle of twist in terms of loading, section geometry, and the shear modulus of the material.

Two things to remember: (1) If the internal torque or the polar moment of inertia vary down the length of the shaft, they must be expressed as functions of x and included in the integral, and (2) the resulting angle of twist will be in radians.

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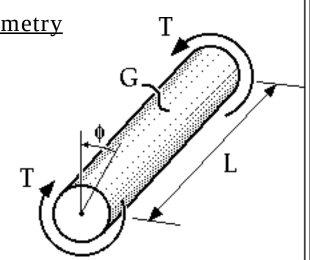
Constant Torque / Geometry

$$\phi = \int_0^L \frac{T(x)}{J(x)\,G}\,dx$$

$$\boxed{\phi = \frac{T\,L}{J\,G}}$$



The most commonly encountered shafts in engineering practice are those in which the applied torque and the cross-sectional area are constant along the length of the shaft.

If this is the case then then internal torque T(x) = T, and the polar moment of inertia J(x) = J, and the integral expression for angle of twist can be integrated to yield ϕ =TL/JG. This is form in which you are most likely to encounter angle of twist in the future.

Remember, this formula only applies to those shafts which have constant cross section and constant internal torque!

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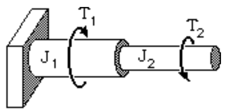
Discrete Changes in Internal Torque and / or Cross-Section



$$\phi_{total} = \sum \frac{TL}{JG}$$

If the value of internal torque and / or the cross sectional area changes abruptly from one region of the shaft to the next, the equation shown above can be applied to each segment of the shaft where all quantities in the equation are constant. The angle of twist for the entire shaft is then found by performing the vector addition of the angle of twist of each segment.

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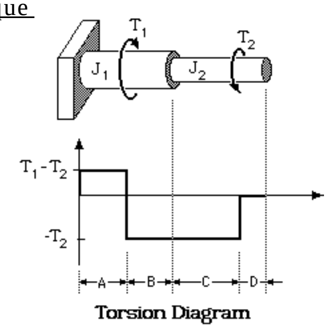
Discrete Changes in Internal Torque and / or Cross-Section

$$\phi_{total} = \sum \frac{TL}{JG}$$



In the example shown here, the shaft has one change in geometry and two changes in loading. Using this information we can divide the shaft into four sections with constant torque and cross-section.

$$\phi_{TOT} = \phi_A + \phi_B + \phi_C + \phi_D$$

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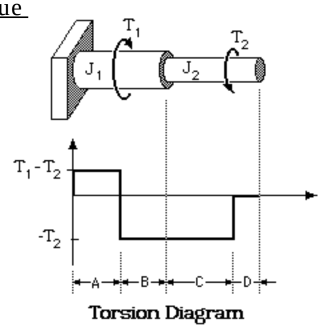
Discrete Changes in Internal Torque and / or Cross-Section



$$\phi_{total} = \sum \frac{TL}{JG}$$

The angle of twist for each section is calculated as $\phi = TL / JG$.

The total angle of twist from one end of the shaft to the other is found by summing up the twists for the individual sections.

$$\phi_{TOT} = \phi_A + \phi_B + \phi_C + \phi_D$$

$$\phi_{TOT} = \frac{(T_1 - T_2)L_A}{J_1 G} - \frac{T_2 L_B}{J_1 G} - \frac{T_2 L_C}{J_2 G} + \frac{(0)L_D}{J_2 G}$$

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The End

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