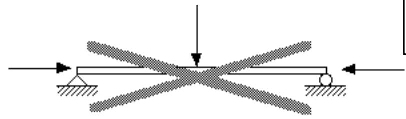
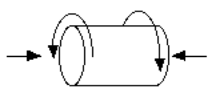
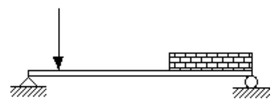
Superposition

Superposition is a powerful tool for solving complex problems, using solutions to simpler problems. This stack provides an overview of superposition and its limitations.







Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

In a mathematical sense, superposition follows from one of the basic definitions of linearity: the sum of a linear function of multiple values is equal to the linear function of the sum of the values.

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Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x$$

Consider an example.

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Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x$$

$$f(1) = 3$$
$$f(2) = 6$$

These are the functions of the values.

4 Hide Text

Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x$$

$$f(1) = 3$$

$$f(2) = 6$$

$$f(1+2) = f(3) = 9$$

This is the function of the sum of the values. Since 9 = 6 + 3, this function works.

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Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x + 4$$

Let's try another.

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Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x + 4$$

$$f(1) = 3(1) + 4 = 7$$

$$f(2) = 3(2) + 4 = 10$$

The functions of the values.

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Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x + 4$$

$$f(1) = 3(1) + 4 = 7$$

$$f(2) = 3(2) + 4 = 10$$

$$f(1+2) = f(3) = 3(3) + 4 = 13$$

The function of the sum of the values. Note that since 13 ≠ + 7 + 10, this function does not work! Even though it has a linear term, it also has a constant term and the function is not purely linear.

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Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x^2$$

One more example.

Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x^2$$

$$f(1) = 3(1^2) = 3$$

$$f(2) = 3(2^2) = 12$$

As before.

Linearity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(x) = 3x^2$$

$$f(1) = 3(1^2) = 3$$

$$f(2) = 3(2^2) = 12$$

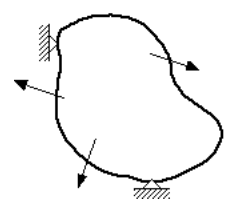
$$f(1+2) = f(3) = 3(3^2) = 27$$

Not surprisingly, this function does not work either. The moral of the story is that linearity of the kind necessary for superposition is more an exception than a rule. Always make sure your problem is appropriately linear before using superposition.

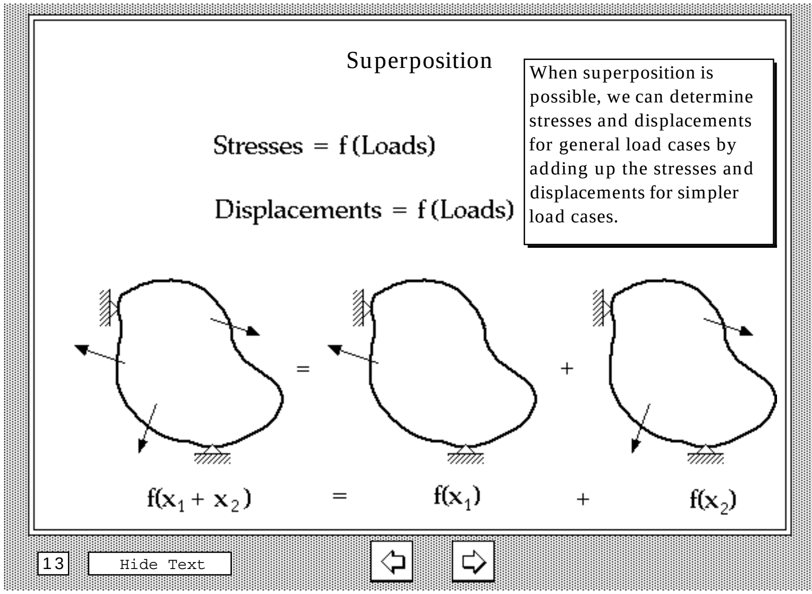
Application

Stresses = f (Loads)

Displacements = f (Loads)



In the context of mechanics of solids, we are typically interested in two functional relations. Given loads, we try to determine stresses and / or displacements.


Superposition
When superposition is possible, we can determine stresses and displacements for general load cases by adding up the stresses and displacements for simpler load cases.
Stresses = f (Loads)
Displacements = f (Loads)
=
+
$f(x_1 + x_2)$ = $f(x_1)$ + $f(x_2)$
13
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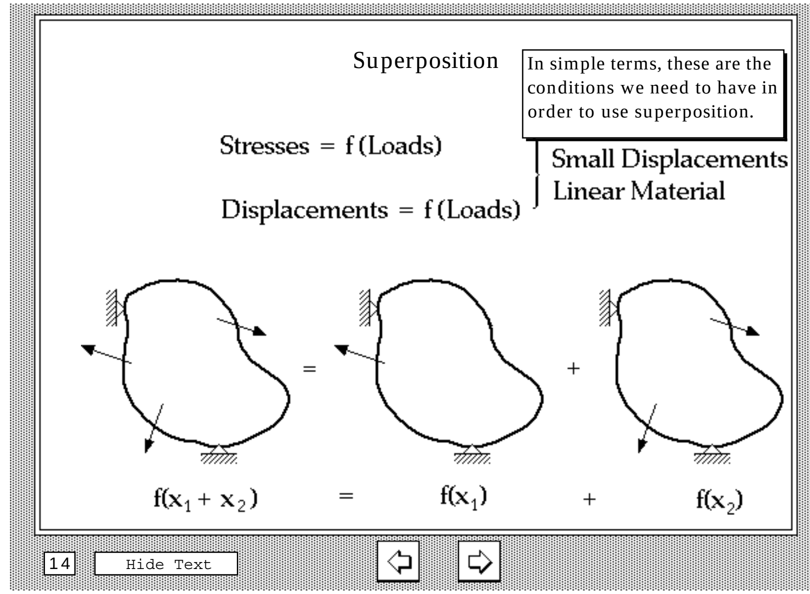
Superposition
In simple terms, these are the conditions we need to have in order to use superposition.
Stresses = f (Loads)
Displacements = f (Loads)
Small Displacements
Linear Material
=
+
$f(x_1 + x_2)$ = $f(x_1)$ + $f(x_2)$
14
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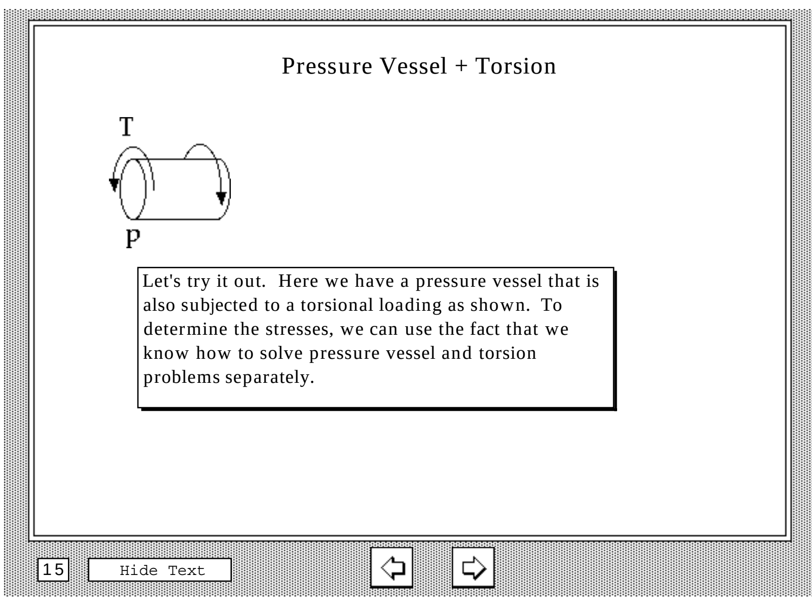
Pressure Vessel + Torsion
T
P
Let's try it out. Here we have a pressure vessel that is also subjected to a torsional loading as shown. To determine the stresses, we can use the fact that we know how to solve pressure vessel and torsion problems separately.
15
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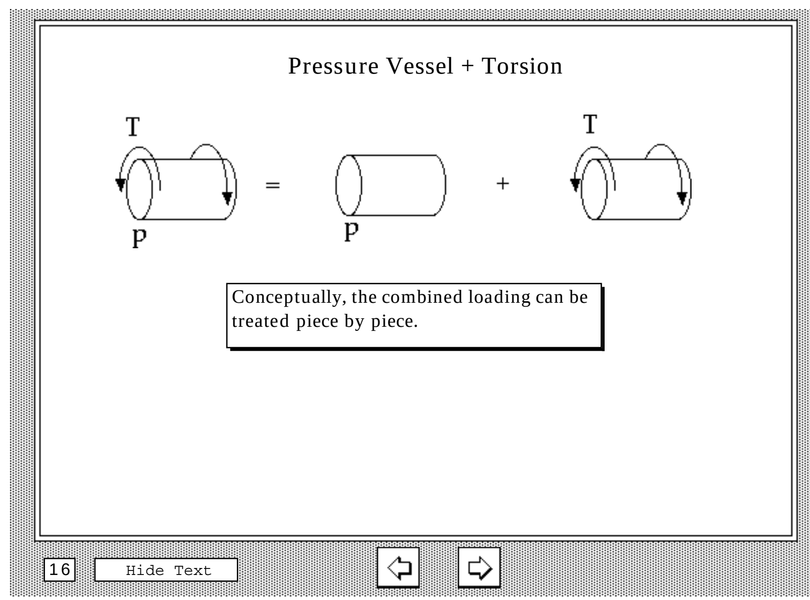
Pressure Vessel + Torsion
T
P
=
P
+
T
Conceptually, the combined loading can be treated piece by piece.
16
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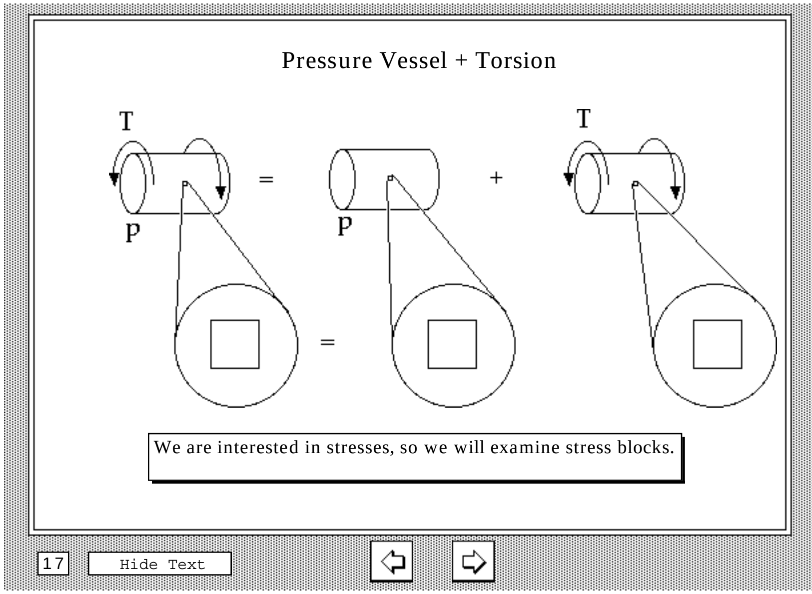
Pressure Vessel + Torsion
T
P
=
P
+
T
=
We are interested in stresses, so we will examine stress blocks.
17
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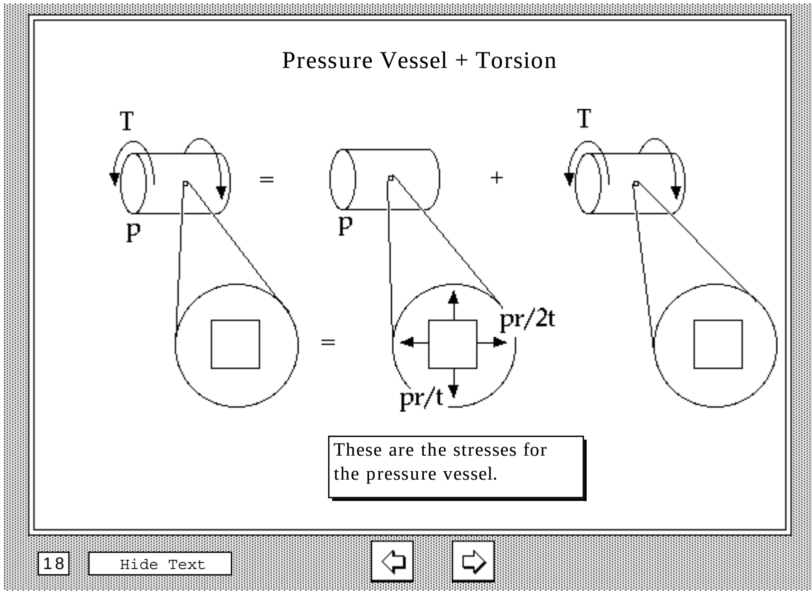
Pressure Vessel + Torsion
T
P
=
P
+
T
=
pr/2t
pr/t
These are the stresses for the pressure vessel.
18
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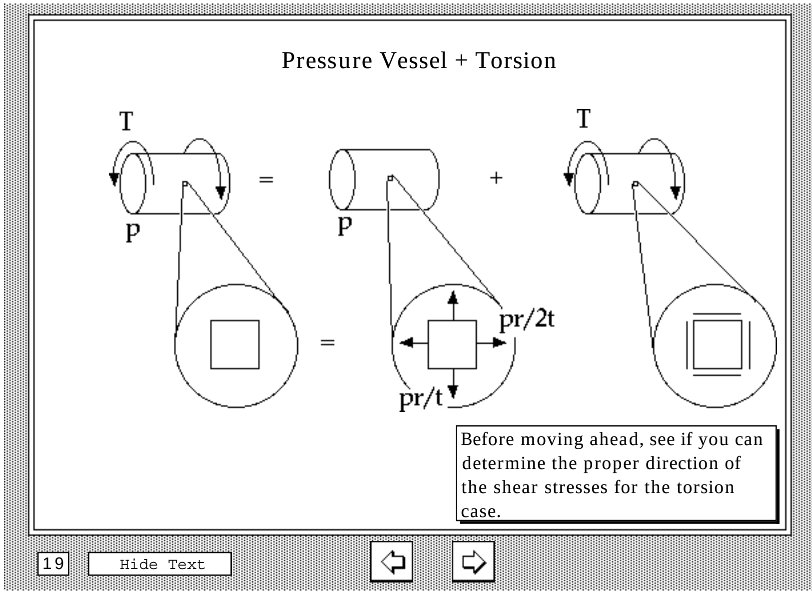
Pressure Vessel + Torsion
T
P
=
P
+
T
=
pr/2t
pr/t
Before moving ahead, see if you can determine the proper direction of the shear stresses for the torsion case.
19
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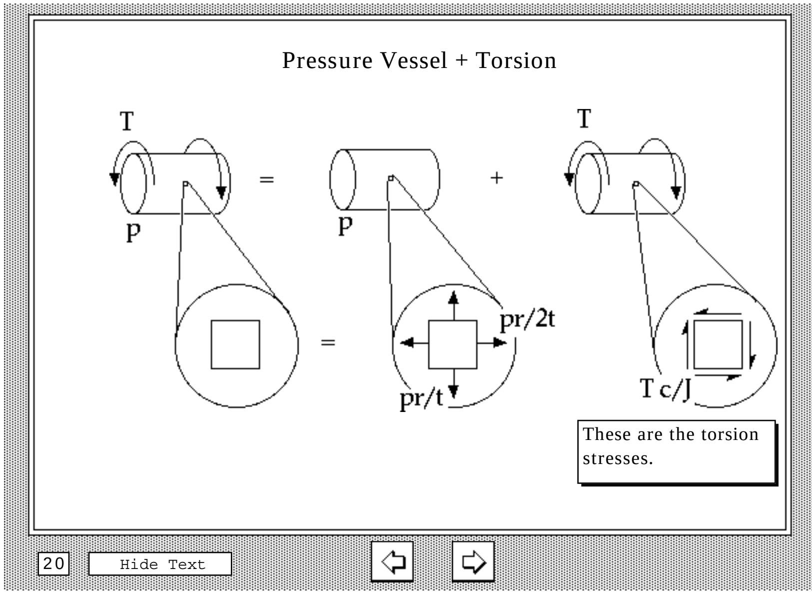
Pressure Vessel + Torsion
T
P
=
P
+
T
=
pr/2t
pr/t
T c/J
These are the torsion stresses.
20
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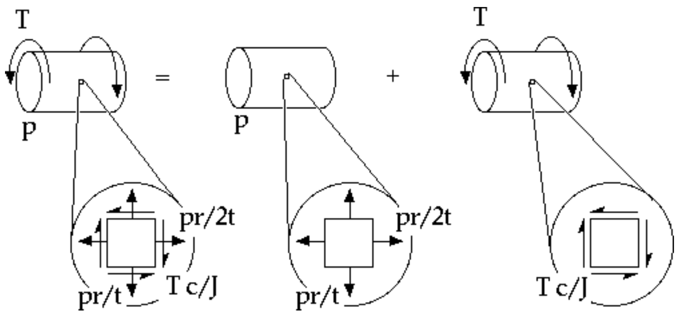
Pressure Vessel + Torsion



By superposition, the net stress is simply the sum of the torsion and pressure vessel contributions.

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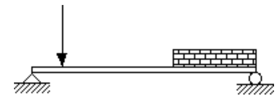
Transversely-Loaded Beams



It is common to use superposition in the analysis of beams.

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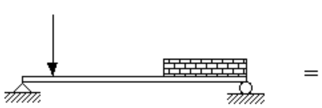
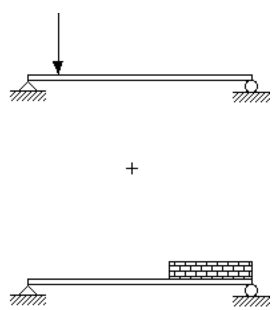
Transversely-Loaded Beams





Loads can be considered separately and their effects added. This holds for shears, moments, stresses and deflections.

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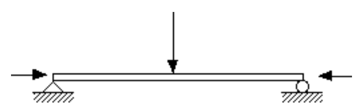
Transversely and Axially-Loaded Beams



Let's look at a case where superposition does not work, lest we think it always does.

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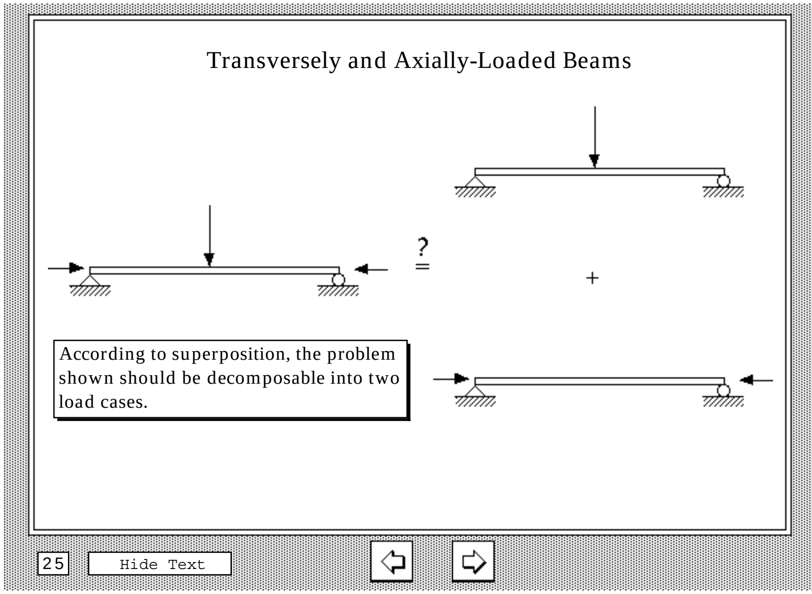
Transversely and Axially-Loaded Beams
?
=
+
According to superposition, the problem shown should be decomposable into two load cases.
25
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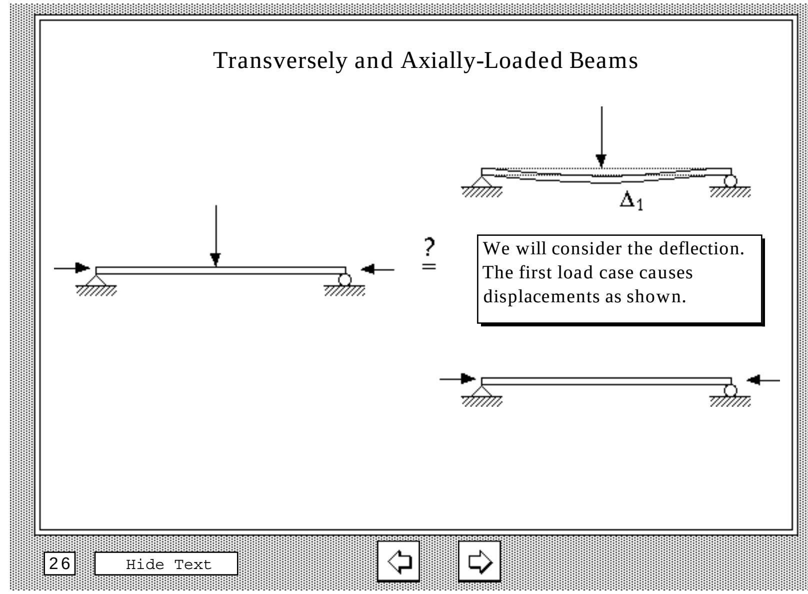
Transversely and Axially-Loaded Beams
Δ1
?
=
We will consider the deflection. The first load case causes displacements as shown.
26
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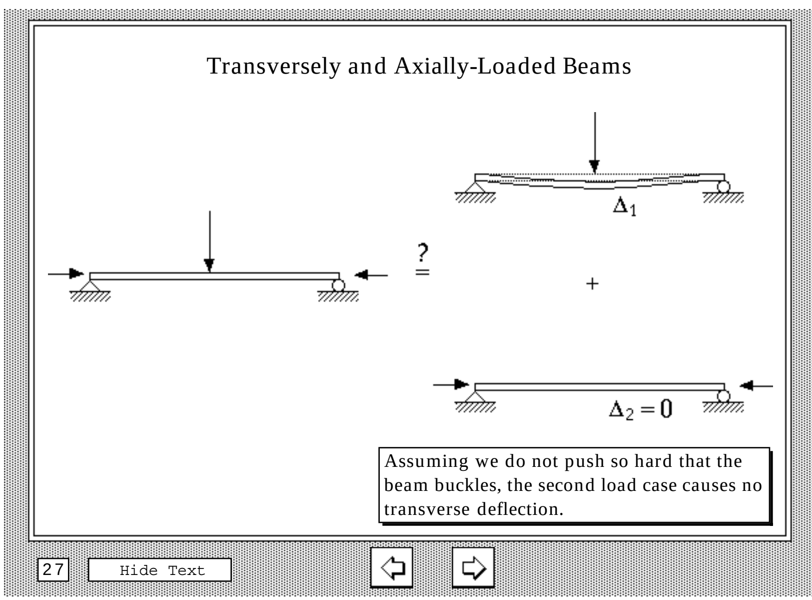
Transversely and Axially-Loaded Beams
Δ1
?
=
+
Δ2 = 0
Assuming we do not push so hard that the beam buckles, the second load case causes no transverse deflection.
27
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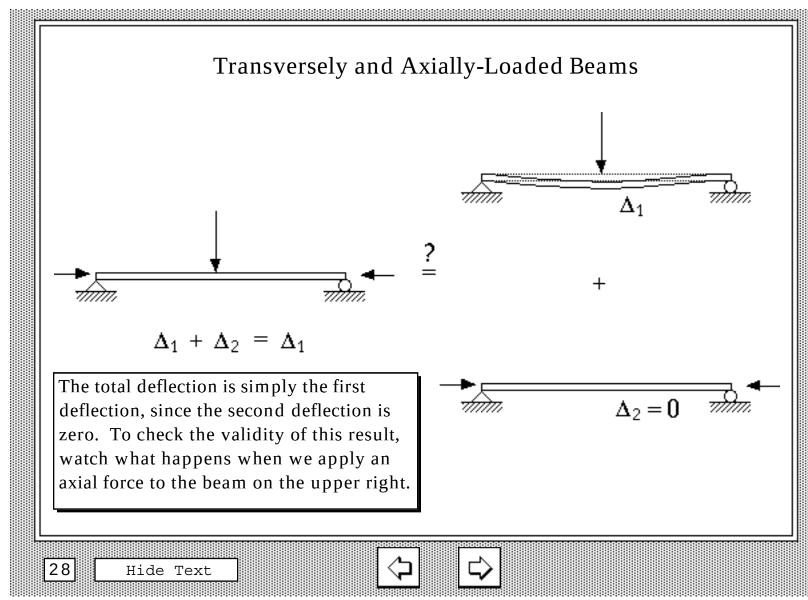
Transversely and Axially-Loaded Beams
Δ1
?
=
+
Δ1 + Δ2 = Δ1
Δ2 = 0
The total deflection is simply the first deflection, since the second deflection is zero. To check the validity of this result, watch what happens when we apply an axial force to the beam on the upper right.
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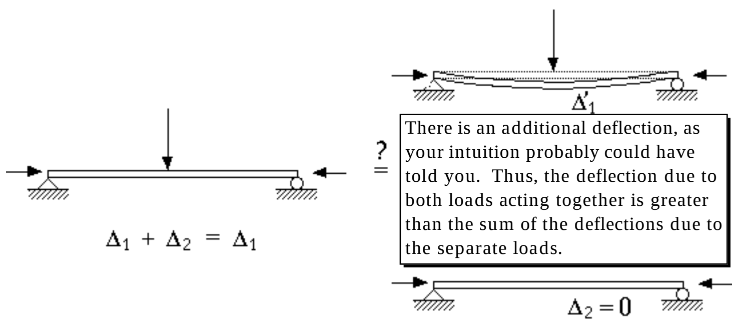
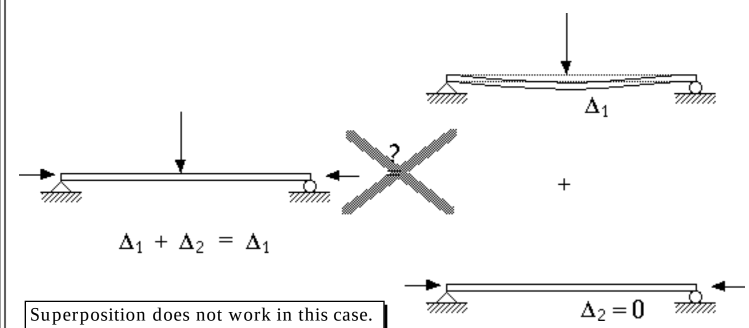
Transversely and Axially-Loaded Beams



There is an additional deflection, as your intuition probably could have told you. Thus, the deflection due to both loads acting together is greater than the sum of the deflections due to the separate loads.

Transversely and Axially-Loaded Beams



Superposition does not work in this case.

Summary

The ability to construct solutions to complicated problems by simply adding together basic solutions is extremely useful in almost all fields of engineering. The main thing to remember is that it only works when it works: i.e. the system must be linear. Fortunately, a large number of engineering systems fall into this category, and so superposition is a powerful concept well worth learning thoroughly.

That's All for Now