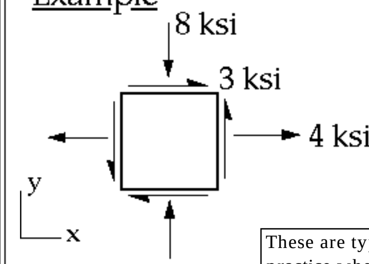
Stress Transformation Examples

☐ Example I

☐ Example II

☐ Special Cases of Mohr's Circ

Example



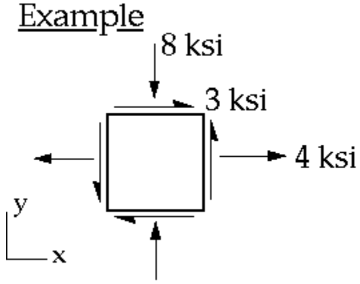
(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

These are typical questions that need to be answered in practice when working with stress. (The questions in part (b) are generally the most common and most important, since they deal with maximums and minimums.) Following the general advice we received earlier, we will solve part (a) using the stress transformation equations and part (b) using Mohr's circle.

Consider first part (a) ...

Example



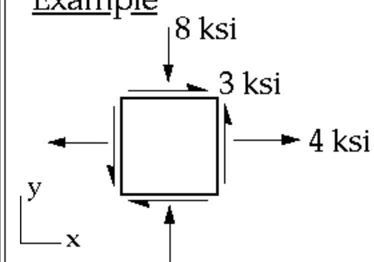
(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

To use the stress transformation equations, we need to identify the proper angle to use. The only trick is to get the sign right, which is not difficult since you can rely on the basic right-hand rule. In this case we have 25° clockwise, so the angle is negative and the corresponding sine and cosine are calculated as shown.

Example



(a) Determine the stress components for an element rotated 25° clockwise.

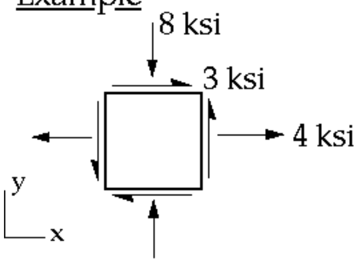
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

To compute $\sigma_{x'}$, we use the equation we derived earlier. To evaluate this equation we need to identify σ_x, σ_y and τ_{xy} from the figure. Again, the only hard part is getting the signs right.

Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

$\sigma_x =$

$\sigma_y =$

$\tau_{xy} =$

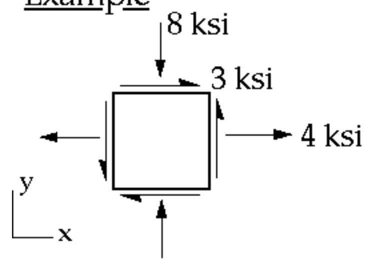
(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

Before you go on, see if you can correctly identify the components on the figure. Remember that tension is positive, and compression is negative for normal components, while positive shears consist of positive directions on positive faces.

5 | Hide Text

Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

$\sigma_x = 4$ ksi

$\sigma_y = -8$ ksi

$\tau_{xy} = 3$ ksi

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

Once we have the original stress components and the angle, we simply plug the relevant values into the equation.

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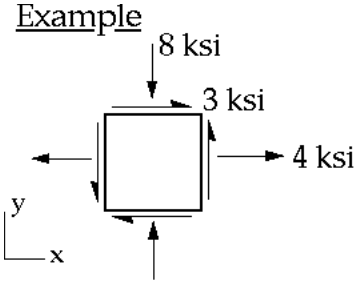
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

$\sigma_x = 4$ ksi

$\sigma_y = -8$ ksi

$\tau_{xy} = 3$ ksi

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$$\sigma_{x'} = -0.441$$

This is the result for $\sigma_{x'}$.

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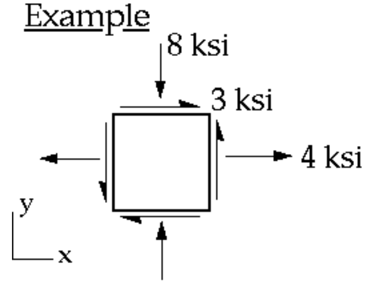
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

$\sigma_x = 4$ ksi

$\sigma_y = -8$ ksi

$\tau_{xy} = 3$ ksi

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$$\sigma_{x'} = -0.441$$

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

This is the transformed shear equation. Plugging in values as before ...

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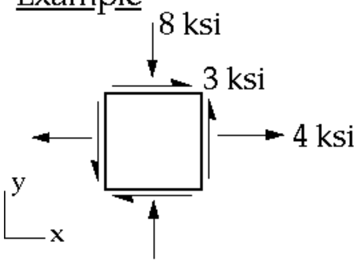
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

$\sigma_x = 4$ ksi
$\sigma_y = -8$ ksi
$\tau_{xy} = 3$ ksi

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$\sigma_{x'} = -0.441$
$\tau_{x'y'} = 6.52$

...gives this result. We now repeat the process for $\sigma_{y'}$.

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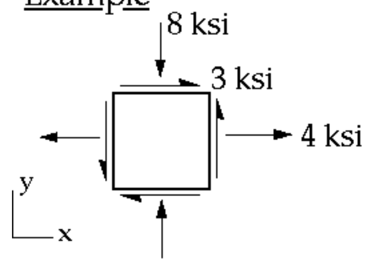
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

$\sigma_x = 4$ ksi
$\sigma_y = -8$ ksi
$\tau_{xy} = 3$ ksi

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$\sigma_{x'} = -0.441$

Here's the equation:

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\,\tau_{xy} \sin\theta\cos\theta$$

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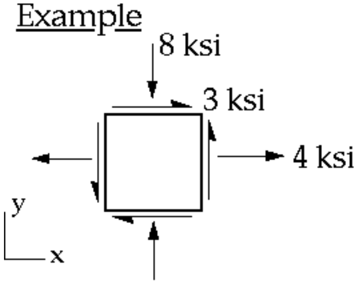
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.

$\sigma_x = 4$ ksi
$\sigma_y = -8$ ksi
$\tau_{xy} = 3$ ksi

(a) $\theta = -25°$ $\begin{cases} \sin\theta = -0.4226 \\ \cos\theta = 0.9063 \end{cases}$

$\sigma_{x'} = -0.441$
$\tau_{x'y'} = 6.52$
$\sigma_{y'} = -3.56$

And here's the result.

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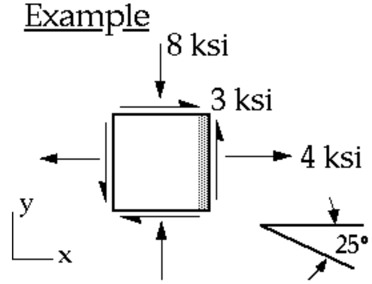
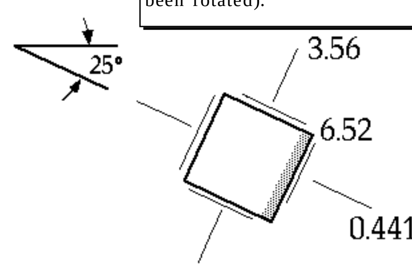
Example



It's not a bad idea to show the transformed state on a sketch. You should be able to see where the arrowheads go on this figure so it is consistent with the calculated results. (Note that the x-face has been shaded so you don't lose track of how the block has been rotated).



(a) $\theta = -25°$

$\sigma_{x'} = -0.441$
$\tau_{x'y'} = 6.52$
$\sigma_{y'} = -3.56$

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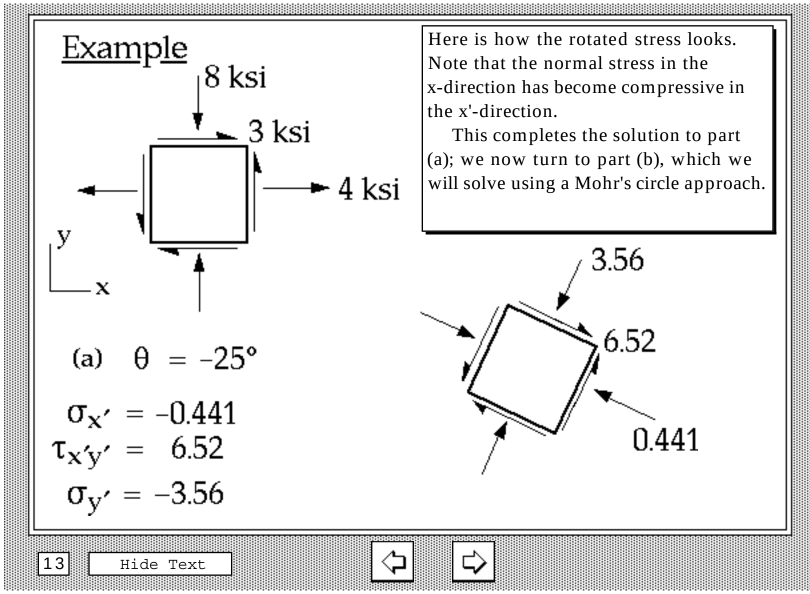
Example
8 ksi
3 ksi
4 ksi
y
x
(a) θ = –25°
σx′ = –0.441
τx′y′ = 6.52
σy′ = –3.56
Here is how the rotated stress looks. Note that the normal stress in the x-direction has become compressive in the x'-direction.
This completes the solution to part (a); we now turn to part (b), which we will solve using a Mohr's circle approach.
3.56
6.52
0.441
13
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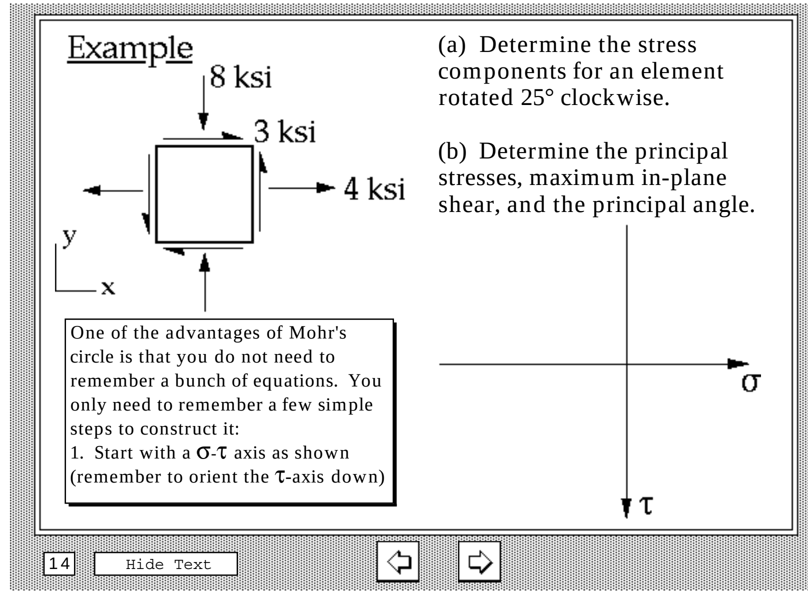
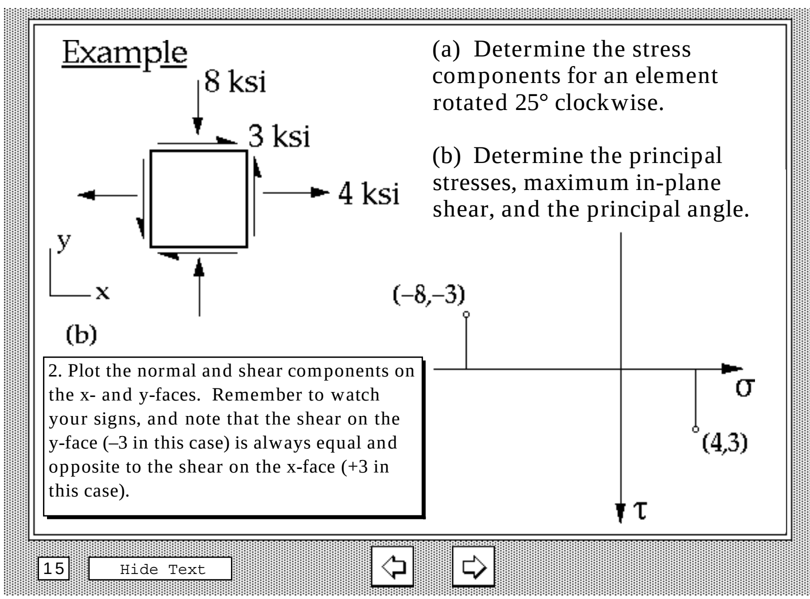
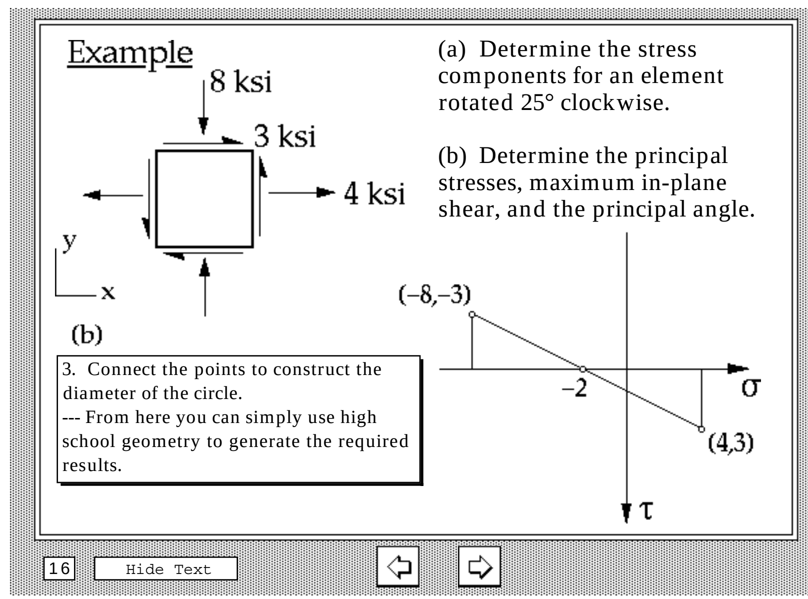

Example
8 ksi
3 ksi
4 ksi
y
x
(a) Determine the stress components for an element rotated 25° clockwise.
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.
One of the advantages of Mohr's circle is that you do not need to remember a bunch of equations. You only need to remember a few simple steps to construct it:
1. Start with a σ-τ axis as shown (remember to orient the τ-axis down)
σ
τ
14
Hide Text


Example
8 ksi
3 ksi
4 ksi
y
x
(a) Determine the stress components for an element rotated 25° clockwise.
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.
(b)
2. Plot the normal and shear components on the x- and y-faces. Remember to watch your signs, and note that the shear on the y-face (–3 in this case) is always equal and opposite to the shear on the x-face (+3 in this case).
(–8,–3)
(4,3)
σ
τ
15
Hide Text


Example
8 ksi
3 ksi
4 ksi
y
x
(a) Determine the stress components for an element rotated 25° clockwise.
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.
(b)
3. Connect the points to construct the diameter of the circle.
--- From here you can simply use high school geometry to generate the required results.
(–8,–3)
–2
(4,3)
σ
τ
16
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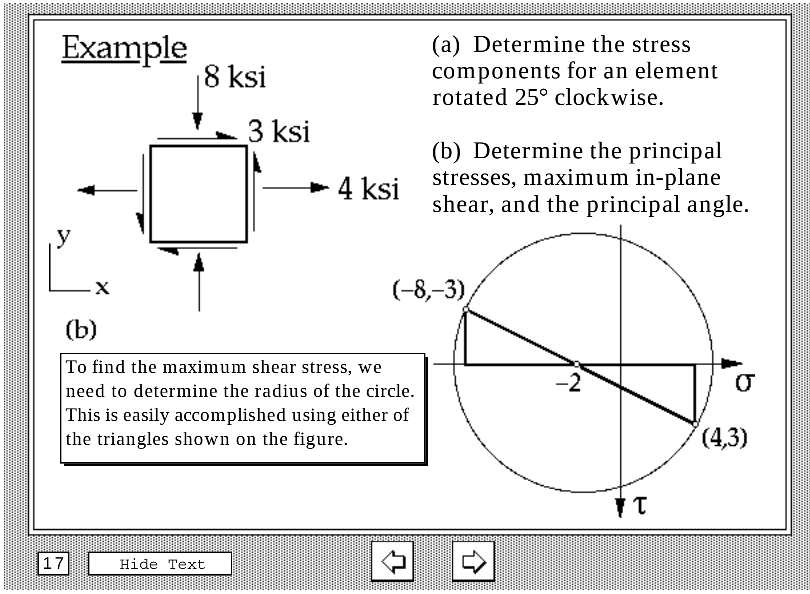
Example
8 ksi
3 ksi
4 ksi
y
x
(a) Determine the stress components for an element rotated 25° clockwise.
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.
(–8,–3)
–2
σ
(4,3)
τ
(b)
To find the maximum shear stress, we need to determine the radius of the circle. This is easily accomplished using either of the triangles shown on the figure.
17
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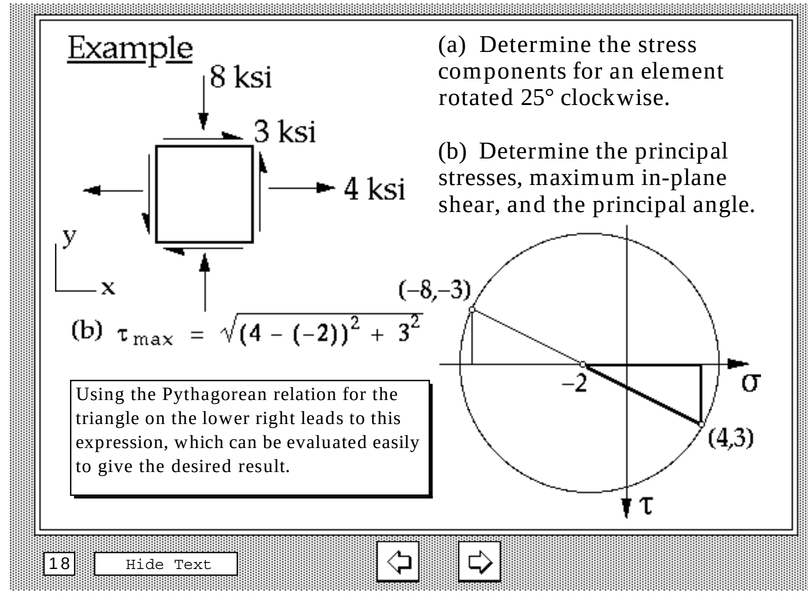
Example
8 ksi
3 ksi
4 ksi
y
x
(a) Determine the stress components for an element rotated 25° clockwise.
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.
(–8,–3)
–2
σ
(4,3)
τ
(b) $\tau_{max} = \sqrt{(4 - (-2))^2 + 3^2}$
Using the Pythagorean relation for the triangle on the lower right leads to this expression, which can be evaluated easily to give the desired result.
18
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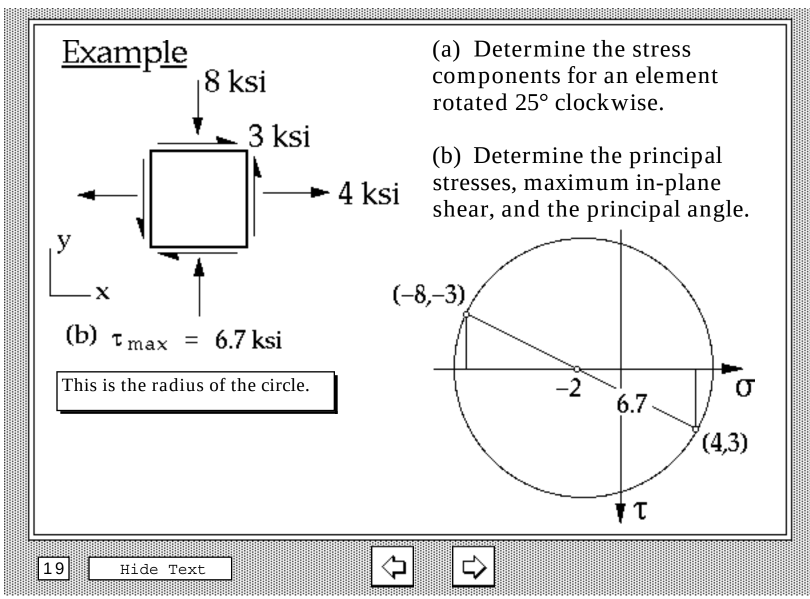
Example
8 ksi
3 ksi
4 ksi
y
x
(a) Determine the stress components for an element rotated 25° clockwise.
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.
(–8,–3)
–2
6.7
σ
(4,3)
τ
(b) τ_{max} = 6.7 ksi
This is the radius of the circle.
19
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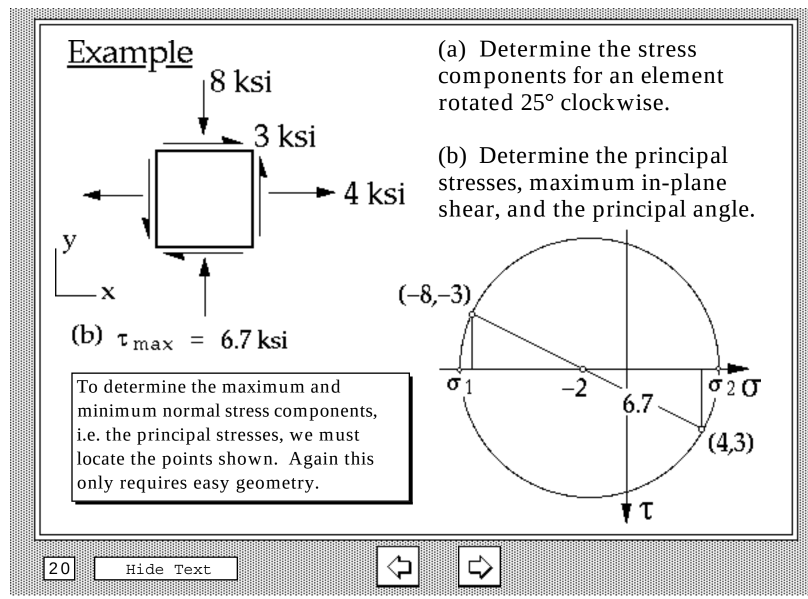
Example
8 ksi
3 ksi
4 ksi
y
x
(a) Determine the stress components for an element rotated 25° clockwise.
(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.
(–8,–3)
σ_1
–2
6.7
σ_2
σ
(4,3)
τ
(b) τ_{max} = 6.7 ksi
To determine the maximum and minimum normal stress components, i.e. the principal stresses, we must locate the points shown. Again this only requires easy geometry.
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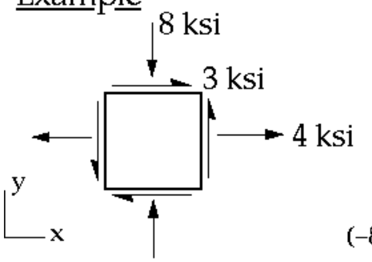
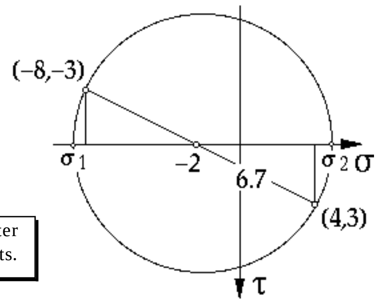
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.



(b) $\tau_{max} = 6.7$ ksi

$\sigma_1 = -2 - 6.7 = -8.7$ ksi

$\sigma_2 = -2 + 6.7 = 4.7$ ksi

The center plus the radius and the center minus the radius give the needed results.

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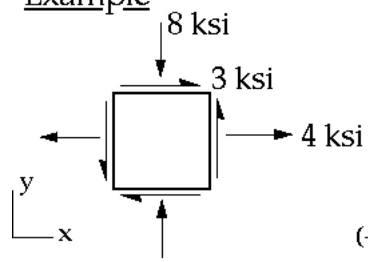
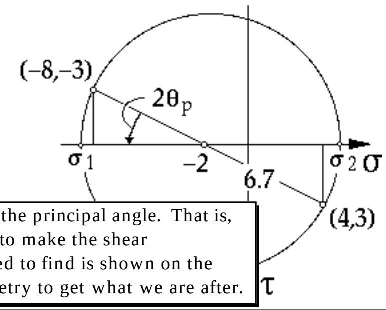
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.



(b) $\tau_{max} = 6.7$ ksi

$\sigma_1 = -2 - 6.7 = -8.7$ ksi

$\sigma_2 = -2 + 6.7 = 4.7$ ksi

The final part of the question concerns the principal angle. That is, how much should we rotate the block to make the shear components vanish? The angle we need to find is shown on the figure, and we can use basic trigonometry to get what we are after.

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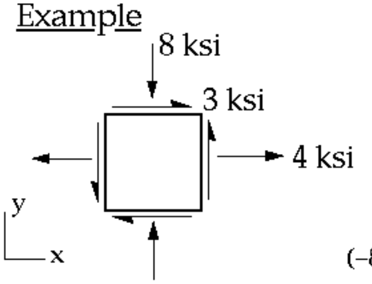
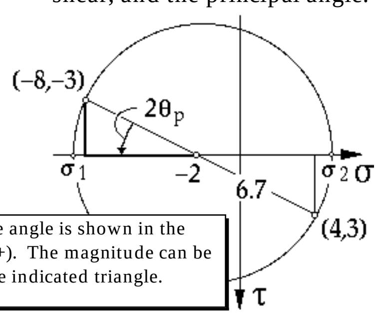
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.



(b) $\tau_{max} = 6.7$ ksi

$\sigma_1 = -2 - 6.7 = -8.7$ ksi

$\sigma_2 = -2 + 6.7 = 4.7$ ksi

$$2\left|\theta_p\right| = \tan^{-1}\frac{3}{6}$$

The sign of the angle is shown in the figure (ccw = +). The magnitude can be found from the indicated triangle.

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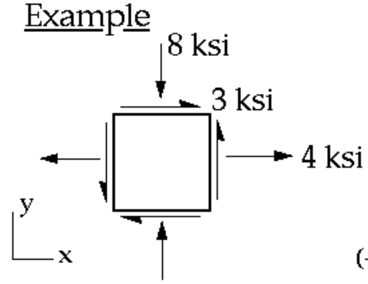
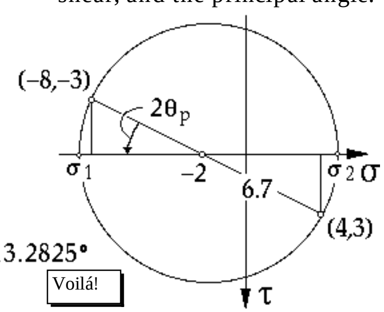
Example



(a) Determine the stress components for an element rotated 25° clockwise.

(b) Determine the principal stresses, maximum in-plane shear, and the principal angle.



(b) $\tau_{max} = 6.7$ ksi

$\sigma_1 = -2 - 6.7 = -8.7$ ksi

$\sigma_2 = -2 + 6.7 = 4.7$ ksi

$$2\left|\theta_p\right| = \tan^{-1}\left(\frac{3}{6}\right) \longrightarrow \left|\theta_p\right| = 13.2825°$$

Voilá!

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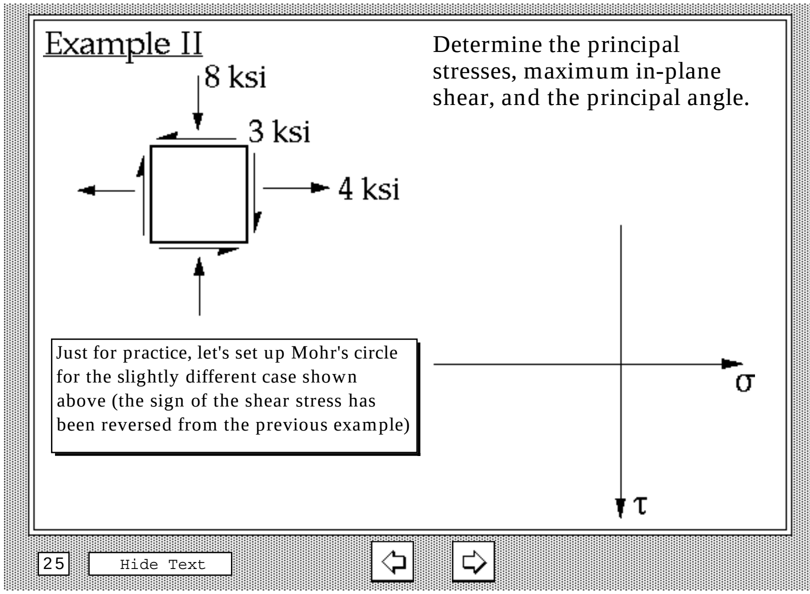
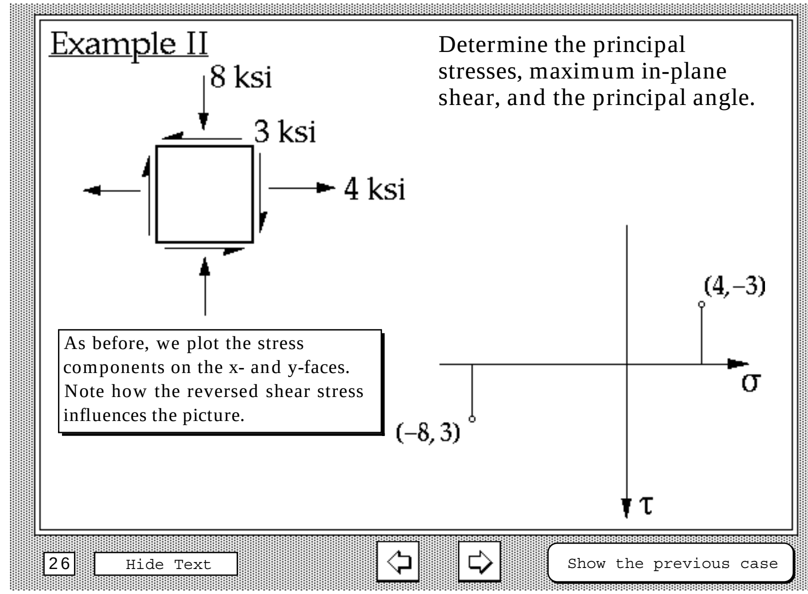
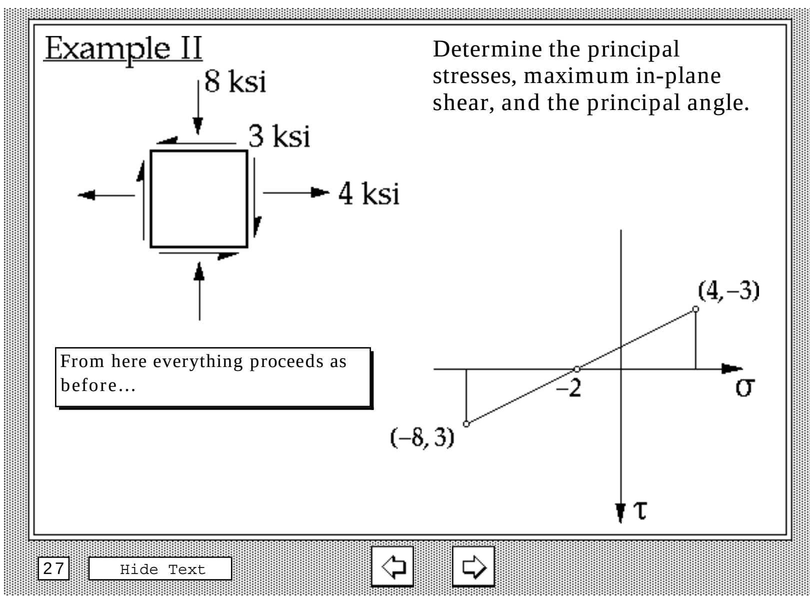
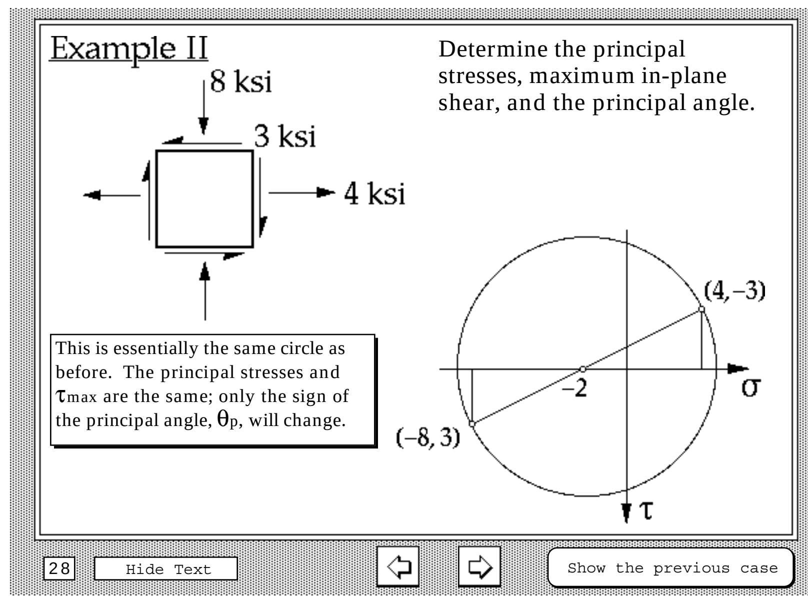
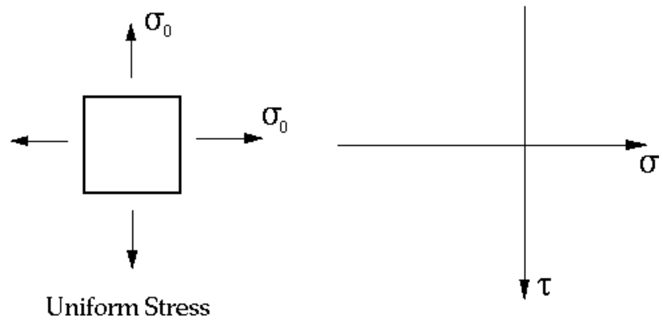

Example II
Determine the principal stresses, maximum in-plane shear, and the principal angle.
8 ksi
3 ksi
4 ksi
Just for practice, let's set up Mohr's circle for the slightly different case shown above (the sign of the shear stress has been reversed from the previous example)
σ
τ
25
Hide Text


Example II
Determine the principal stresses, maximum in-plane shear, and the principal angle.
8 ksi
3 ksi
4 ksi
As before, we plot the stress components on the x- and y-faces. Note how the reversed shear stress influences the picture.
(4, –3)
(–8, 3)
σ
τ
26
Hide Text
Show the previous case


Example II
Determine the principal stresses, maximum in-plane shear, and the principal angle.
8 ksi
3 ksi
4 ksi
From here everything proceeds as before…
(4, –3)
–2
(–8, 3)
σ
τ
27
Hide Text


Example II
Determine the principal stresses, maximum in-plane shear, and the principal angle.
8 ksi
3 ksi
4 ksi
This is essentially the same circle as before. The principal stresses and τmax are the same; only the sign of the principal angle, θp, will change.
(4, –3)
–2
(–8, 3)
σ
τ
28
Hide Text
Show the previous case

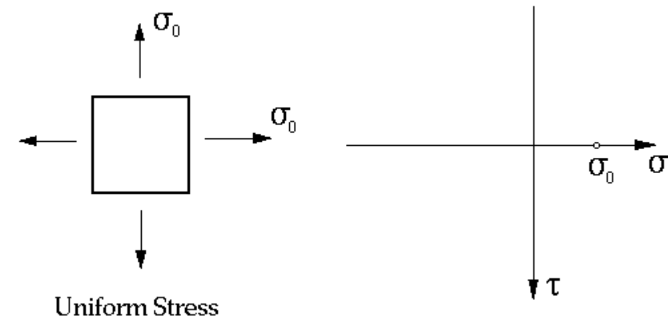
Mohr's Circle for some special cases:



In these final few cards we will consider Mohr's circle for some important special cases. You should be able to construct these simple cases yourself.

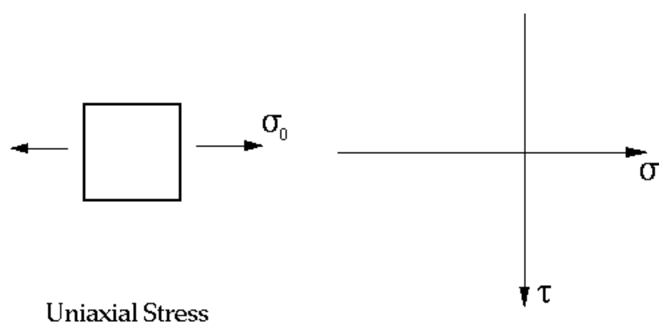
Consider first uniform stress. What do you think Mohr's circle will look like?

Mohr's Circle for some special cases:



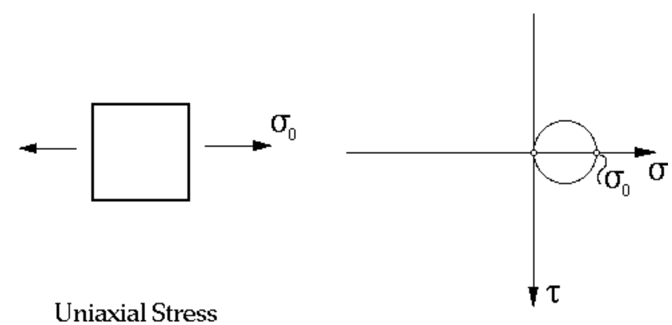
If we plot the stress components on the x- and y-faces, they fall on the same point as shown. The diameter of the circle is zero in this case, and so the "circle" is just a single point. This means that no matter how the block is rotated, the stress components are always the same.

Mohr's Circle for some special cases:



When there is only a single normal component of stress as shown, it is called uniaxial stress. To construct Mohr's circle for this case we plot points at $(\sigma_0, 0)$ and at (0, 0).

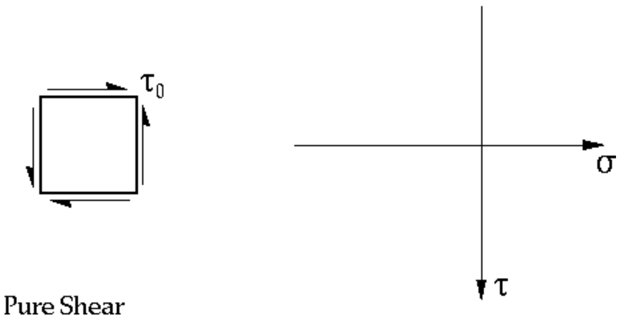
Mohr's Circle for some special cases:



The corresponding circle is as shown. Note that τ_{max} (i.e. the radius of the circle) is 1/2 the given normal stress, σ_0, and τ_{max} occurs 90° (= 45° in the material) from the orientation of the uniaxial stress. This is important for many metals, because as we will see later, they tend to fail in shear.

Compare this result with that of the earlier Simple Stress Example II.

Mohr's Circle for some special cases:



Pure Shear

Our final special case is pure shear. We will see that this type of stress state arises when we apply torsion to a shaft. Can you guess how the circle will look in this

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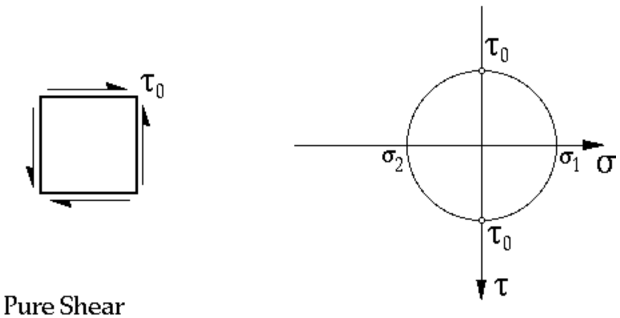
Mohr's Circle for some special cases:



Pure Shear

The plotted points are as shown. Note that the principal normal stresses are equal in magnitude to the given shear stress, and that these normal stresses occur on planes oriented 45° from the original x-y axes.

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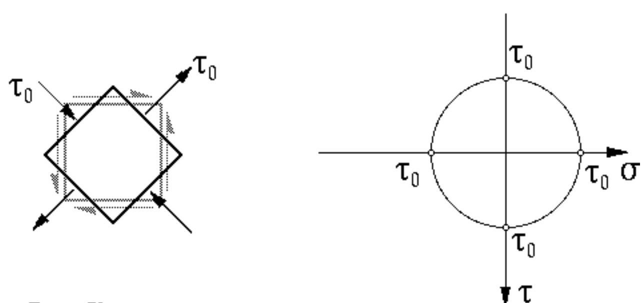
Mohr's Circle for some special cases:



Pure Shear

This shows that even when we think we are only applying shear, the material can actually experience both tension and compression with equal magnitude. To see the effect this can have, take a piece of chalk and twist it until it breaks. Note how the failure occurs at 45° to the axis of the chalk. This is because the chalk is weakest in tension, and the tension is maximum at 45°.

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That's all folks