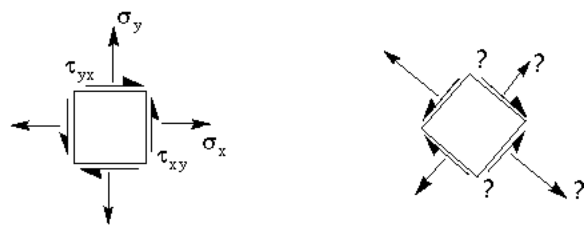
Stress Transformations



Stack Contents

- Problem Statement
- Formulation of the Problem
- Applying Equilibrium
- Transforming the Normal Stress
- Transforming the Shear Stress
- The Stress Transformation Equations

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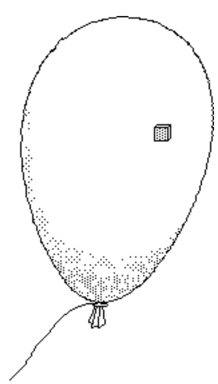
A Balloon



Imagine that we isolate a small element of an inflated balloon.

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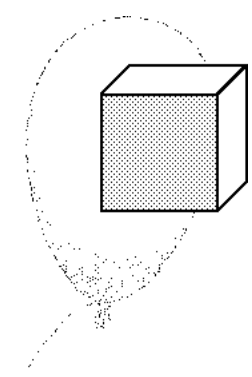
An Element from a Balloon



Most of the stresses found in a balloon run parallel to the surface of the balloon. Therefore, for this problem we will ignore any stress that occurs normal to the surface of the balloon.

Assume that we can calculate these stresses in the plane of the balloon.

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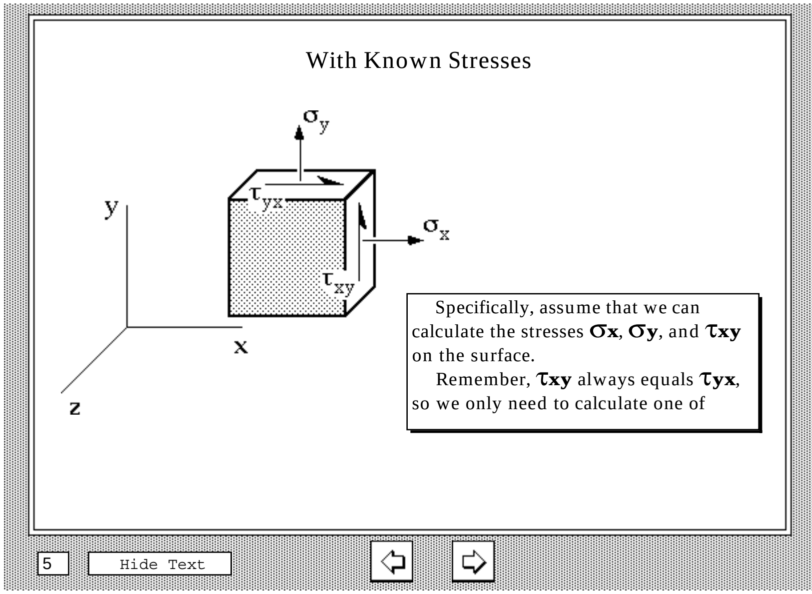
With Known Stresses
σ_y
τ_{yx}
σ_x
τ_{xy}
y
x
z
Specifically, assume that we can calculate the stresses σx, σy, and τxy on the surface.
Remember, τxy always equals τyx, so we only need to calculate one of
5
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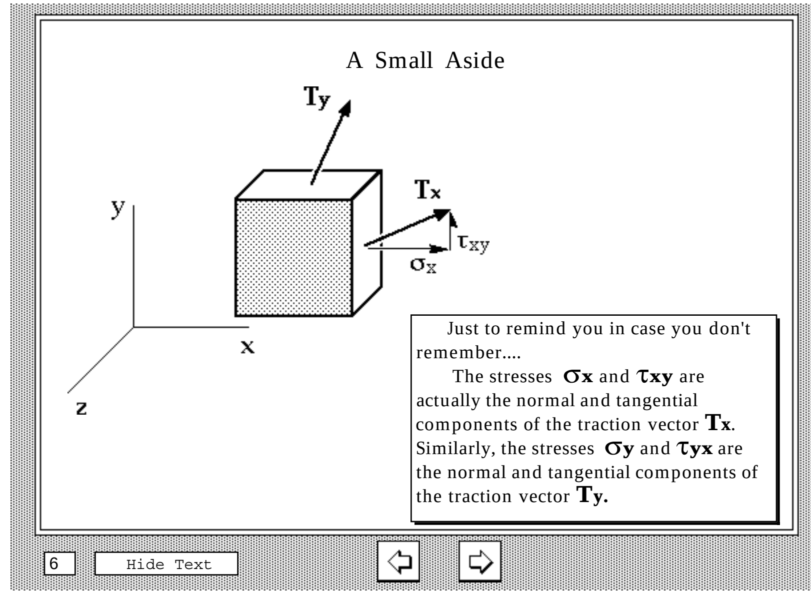
A Small Aside
Ty
Tx
τ_{xy}
σ_x
y
x
z
Just to remind you in case you don't remember....
The stresses σx and τxy are actually the normal and tangential components of the traction vector Tx. Similarly, the stresses σy and τyx are the normal and tangential components of the traction vector Ty.
6
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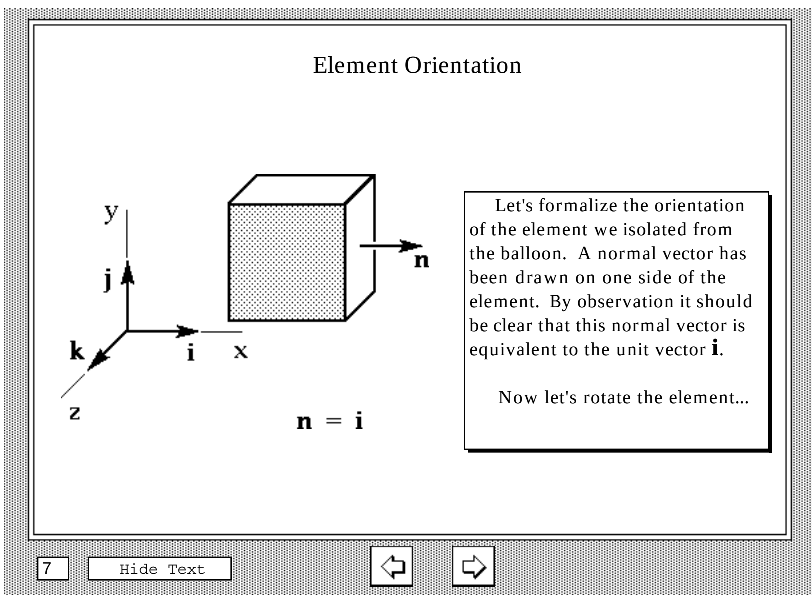
Element Orientation
y
j
k
i
x
z
n
n = i
Let's formalize the orientation of the element we isolated from the balloon. A normal vector has been drawn on one side of the element. By observation it should be clear that this normal vector is equivalent to the unit vector i.
Now let's rotate the element...
7
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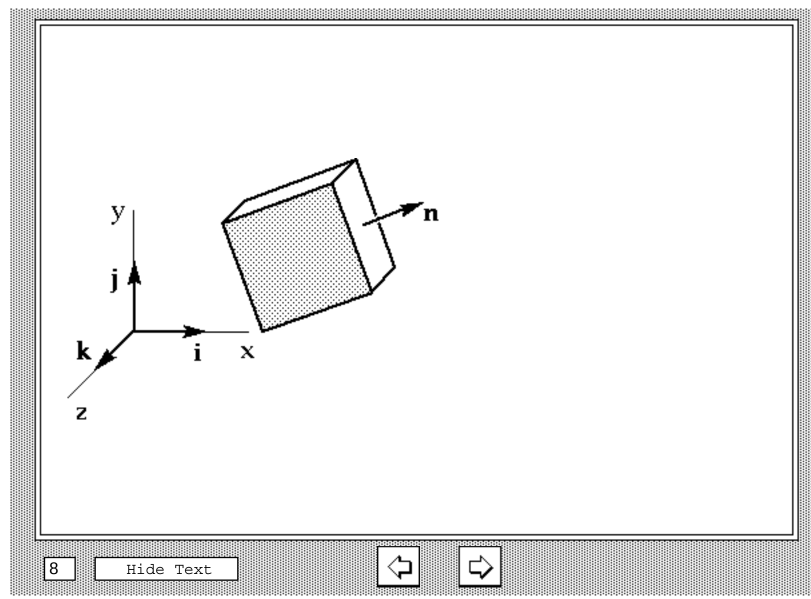
y
j
k
i
x
z
n
8
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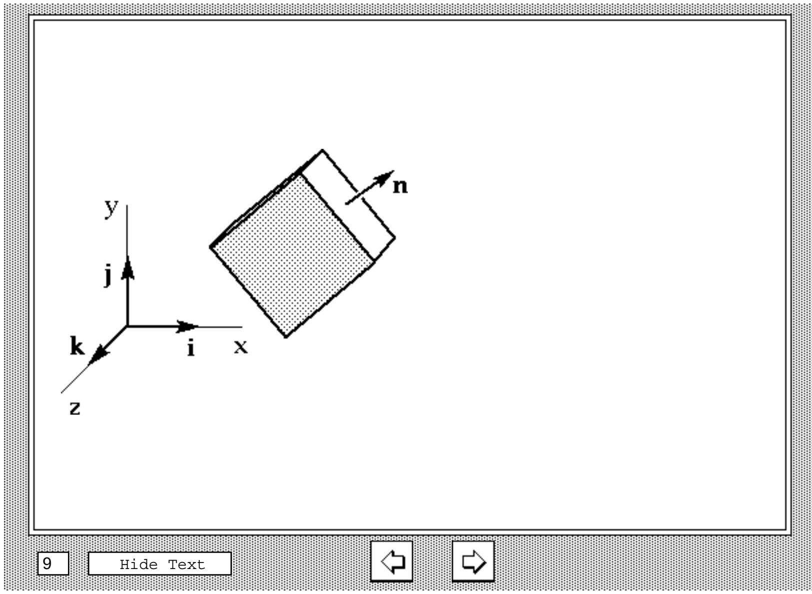

y
j
k
i
x
z
n
9
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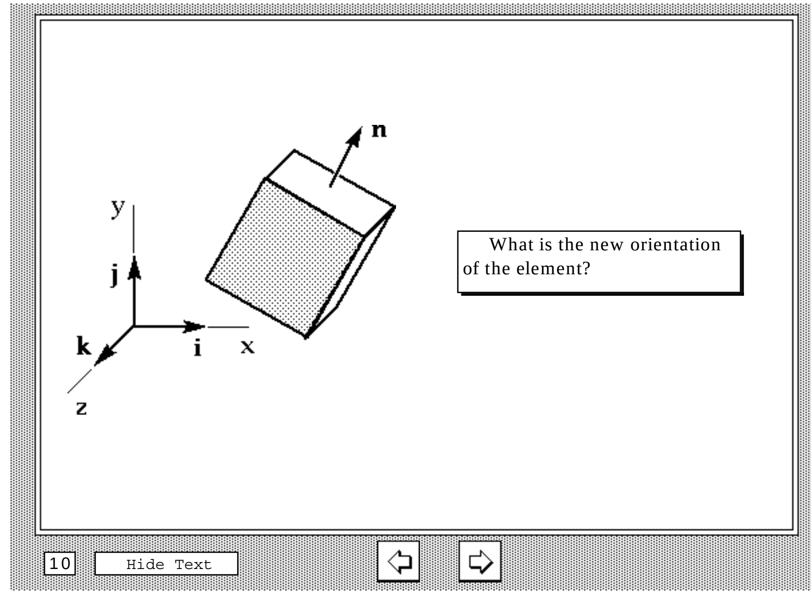

n
y
j
k
i
x
z
What is the new orientation of the element?
10
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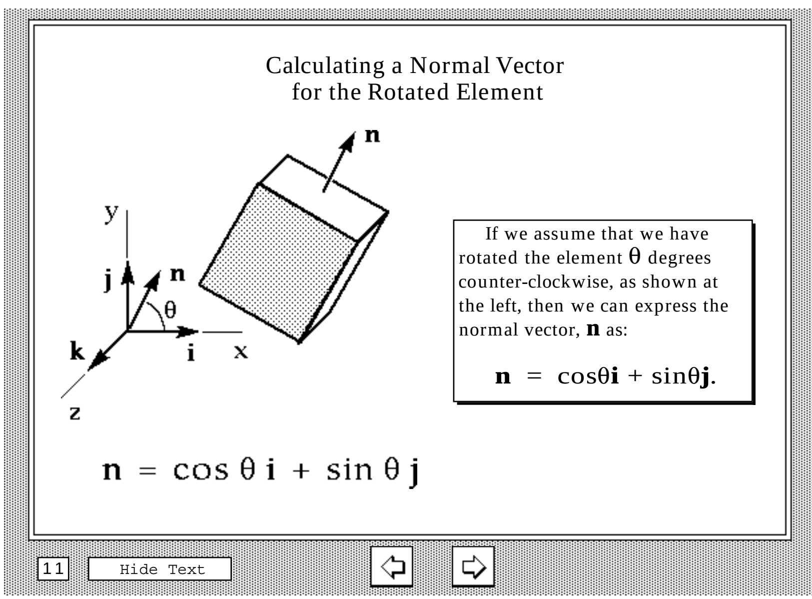

Calculating a Normal Vector for the Rotated Element
n
y
j
n
θ
k
i
x
z
If we assume that we have rotated the element θ degrees counter-clockwise, as shown at the left, then we can express the normal vector, **n** as:
n = cosθ**i** + sinθ**j**.
n = cos θ i + sin θ j
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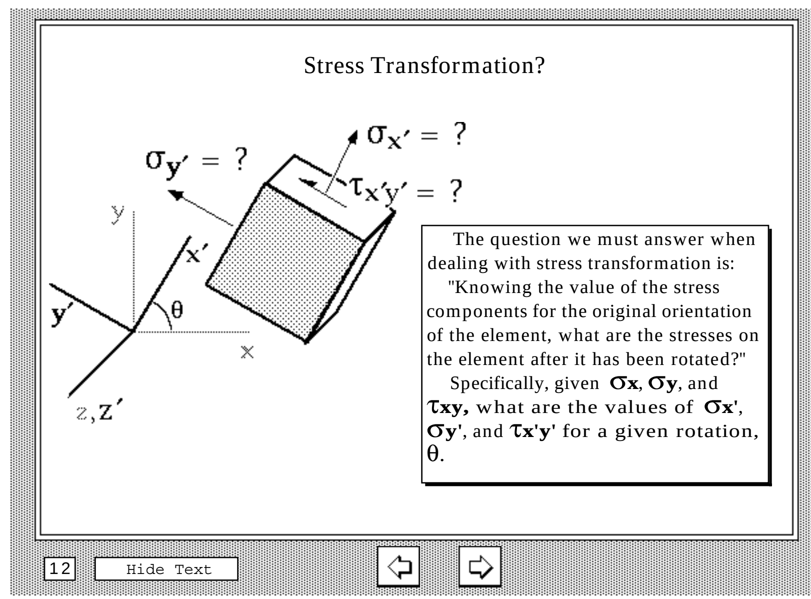

Stress Transformation?
σx′ = ?
σy′ = ?
τx′y′ = ?
y
x′
y′
θ
x
z,z′
The question we must answer when dealing with stress transformation is:
"Knowing the value of the stress components for the original orientation of the element, what are the stresses on the element after it has been rotated?"
Specifically, given σx, σy, and τxy, what are the values of σx', σy', and τx'y' for a given rotation, θ.
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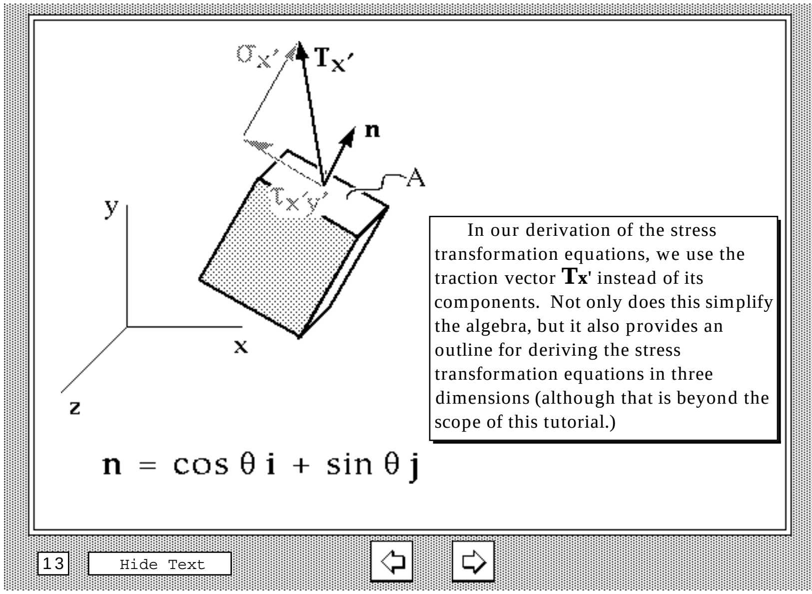
$\sigma_{x'}$
$\mathbf{T}_{x'}$
n
A
$\tau_{x'y'}$
y
x
z
In our derivation of the stress transformation equations, we use the traction vector $\mathbf{T}_{x'}$ instead of its components. Not only does this simplify the algebra, but it also provides an outline for deriving the stress transformation equations in three dimensions (although that is beyond the scope of this tutorial.)
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
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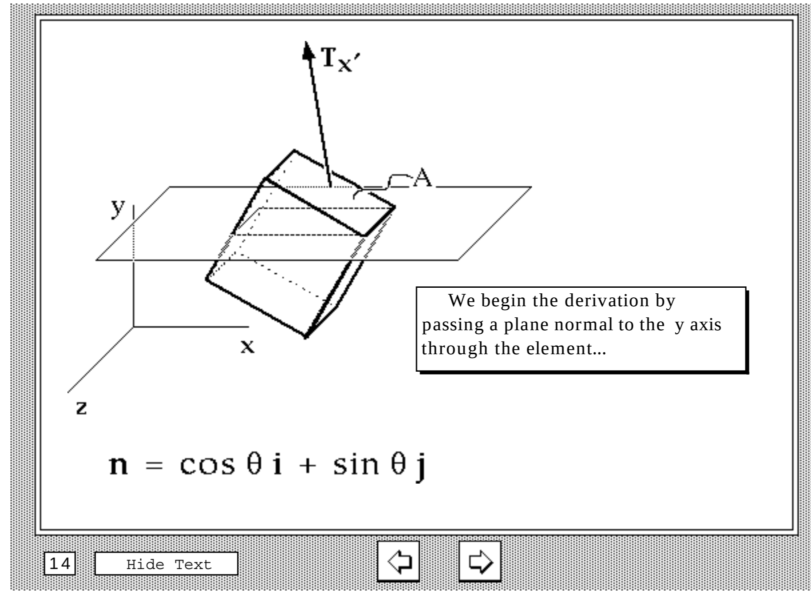
$\mathbf{T}_{x'}$
A
y
x
z
We begin the derivation by passing a plane normal to the y axis through the element...
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
14
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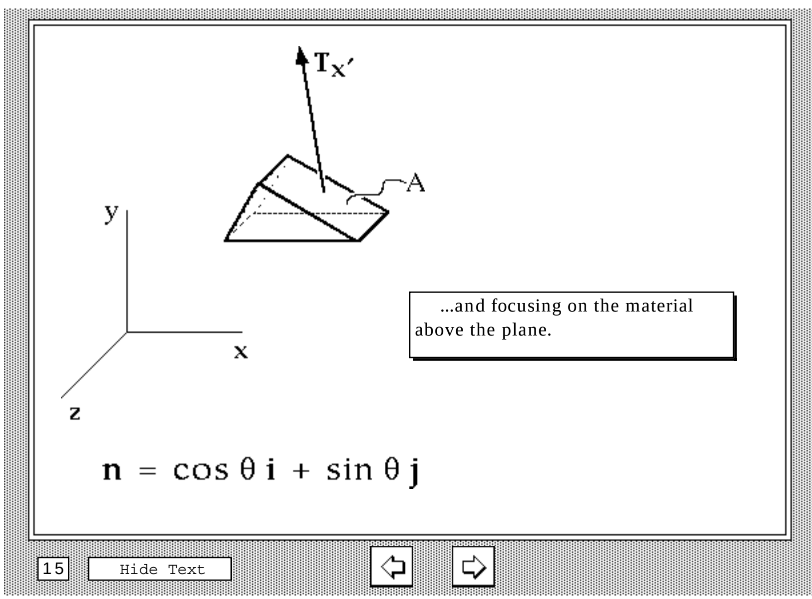
$\mathbf{T}_{x'}$
A
y
x
z
...and focusing on the material above the plane.
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
15
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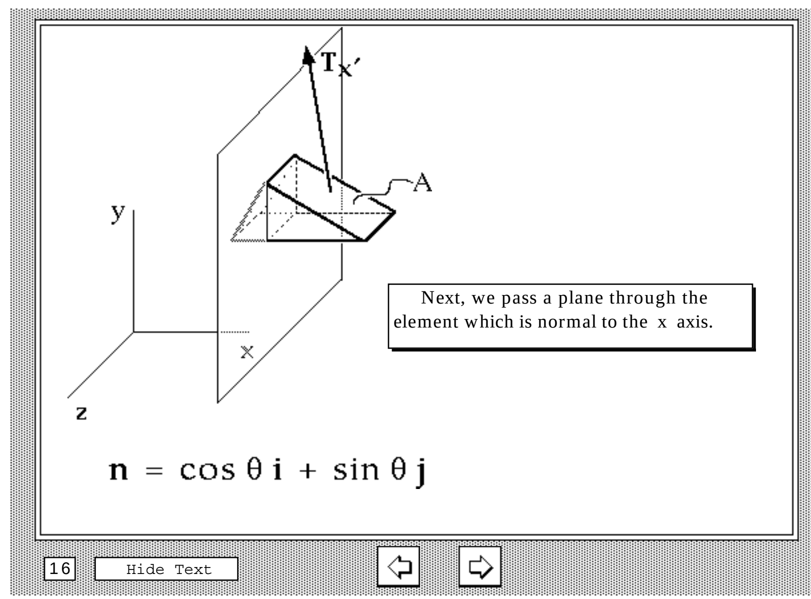
$\mathbf{T}_{x'}$
A
y
x
z
Next, we pass a plane through the element which is normal to the x axis.
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
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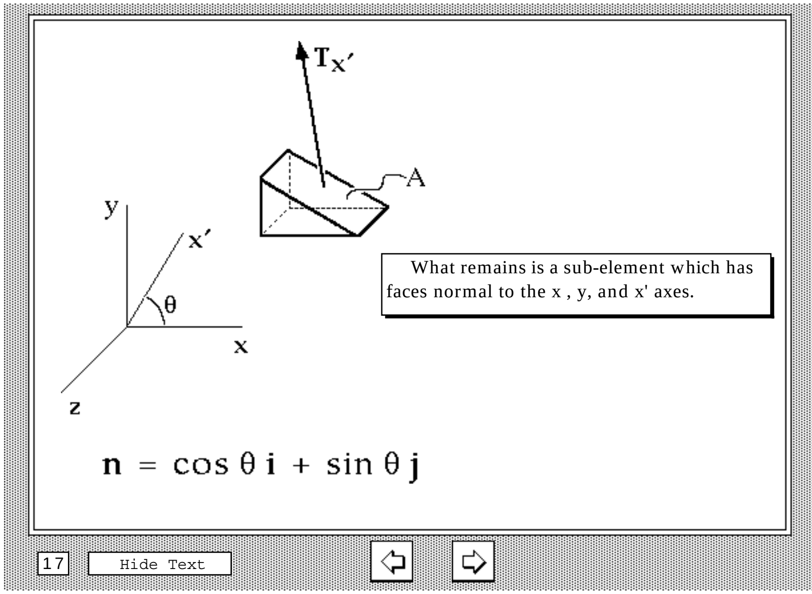

$T_{x'}$
A
y
x'
θ
x
z
What remains is a sub-element which has faces normal to the x , y, and x' axes.
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
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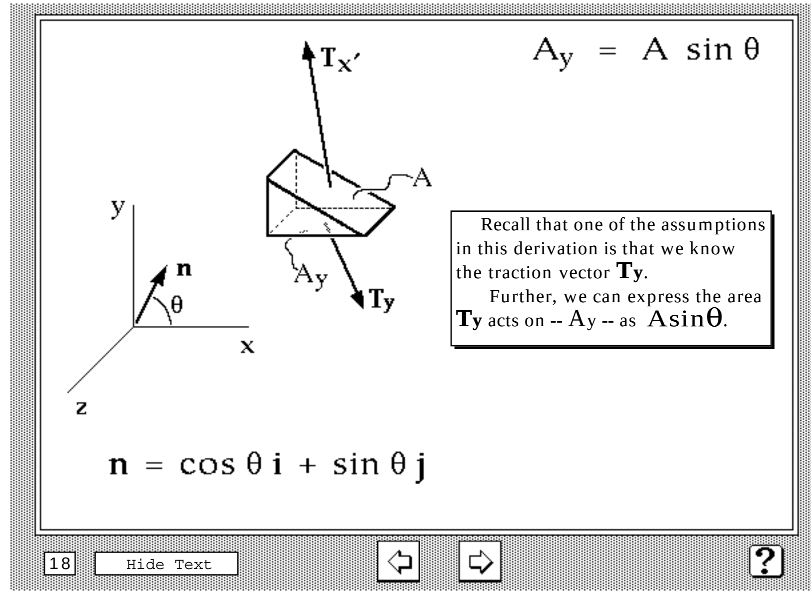

$A_y = A \sin\theta$
$T_{x'}$
A
y
n
θ
A_y
T_y
x
z
Recall that one of the assumptions in this derivation is that we know the traction vector $\mathbf{T_y}$.
Further, we can express the area $\mathbf{T_y}$ acts on -- A_y -- as $A\sin\theta$.
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
18
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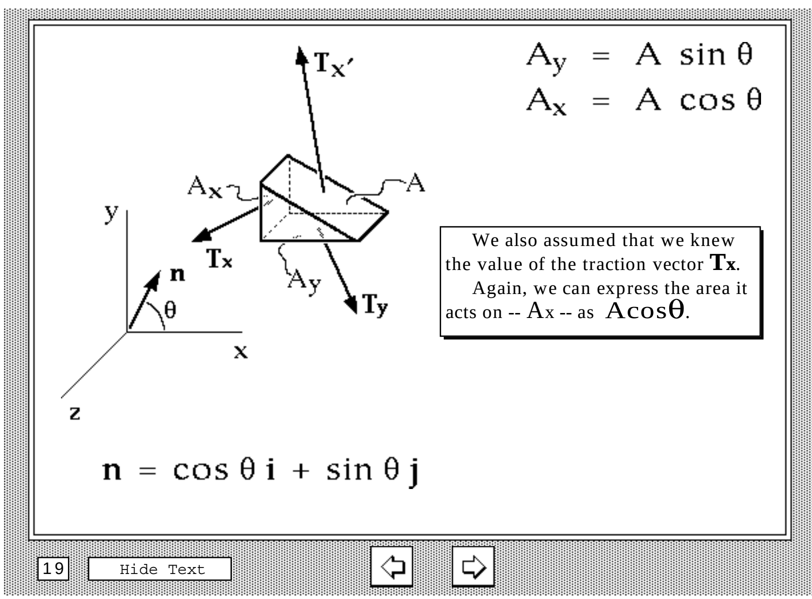
?



$A_y = A \sin\theta$
$A_x = A \cos\theta$
$T_{x'}$
A_x
A
y
T_x
n
θ
A_y
T_y
x
z
We also assumed that we knew the value of the traction vector $\mathbf{T_x}$.
Again, we can express the area it acts on -- A_x -- as $A\cos\theta$.
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
19
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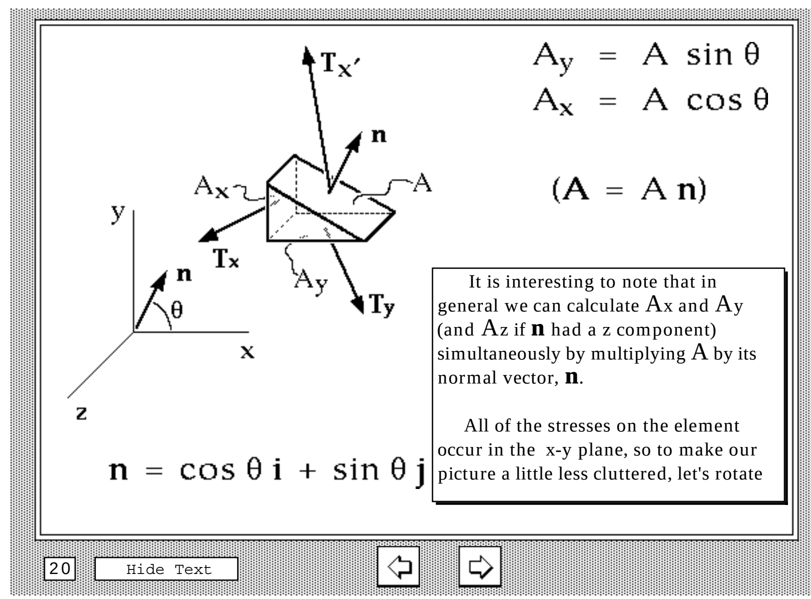

$A_y = A \sin\theta$
$A_x = A \cos\theta$
$(\mathbf{A} = A\,\mathbf{n})$
$T_{x'}$
n
A_x
A
y
T_x
n
θ
A_y
T_y
x
z
It is interesting to note that in general we can calculate A_x and A_y (and A_z if $\mathbf{n}$ had a z component) simultaneously by multiplying A by its normal vector, $\mathbf{n}$.
All of the stresses on the element occur in the x-y plane, so to make our picture a little less cluttered, let's rotate
$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$
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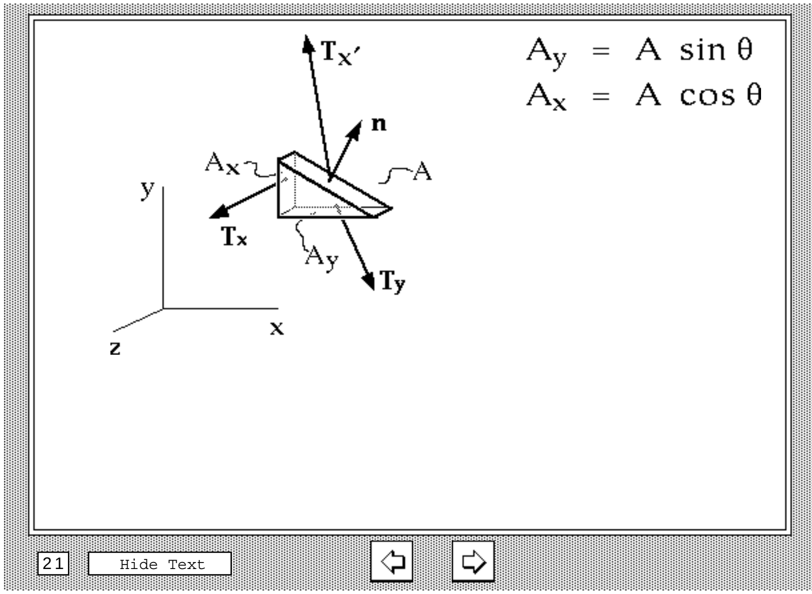
$A_y = A \sin\theta$
$A_x = A \cos\theta$
Tx'
n
Ax
A
Tx
Ay
Ty
y
x
z
21
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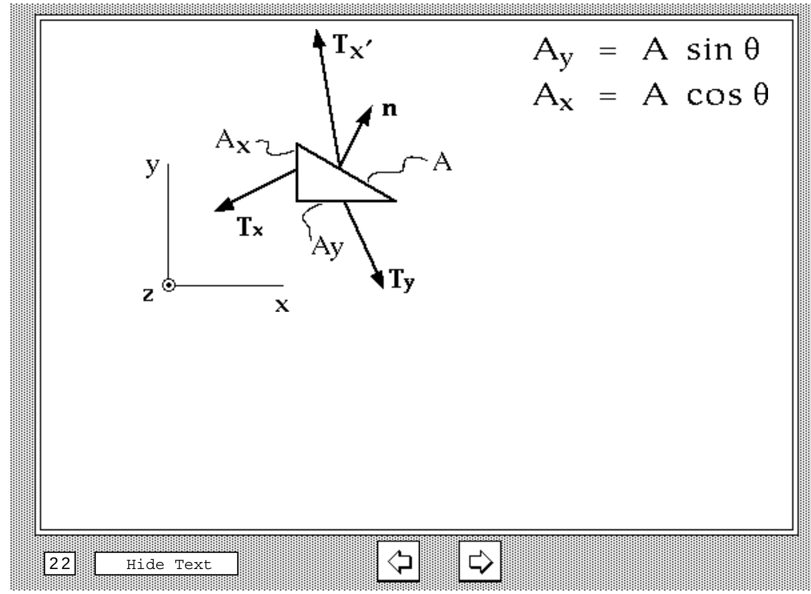
$A_y = A \sin\theta$
$A_x = A \cos\theta$
Tx'
n
Ax
A
Tx
Ay
Ty
y
x
z
22
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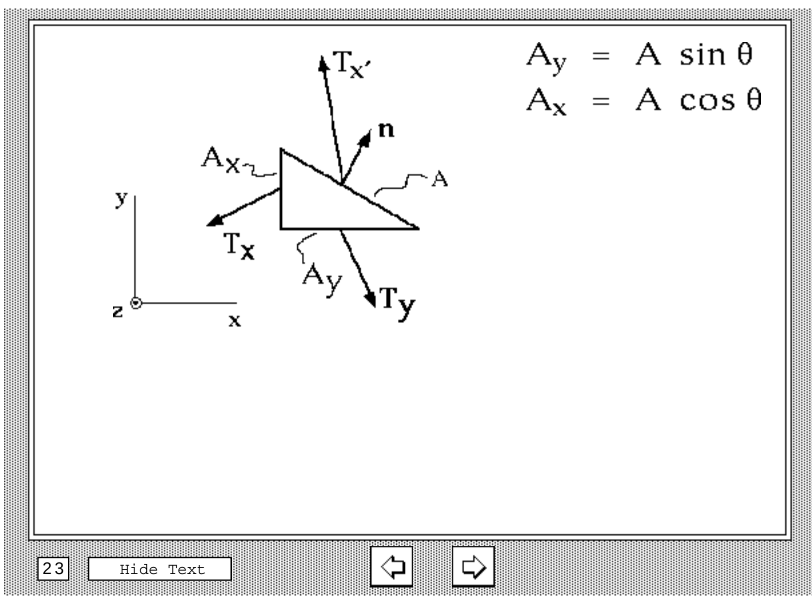
$A_y = A \sin\theta$
$A_x = A \cos\theta$
Tx'
n
Ax
A
Tx
Ay
Ty
y
x
z
23
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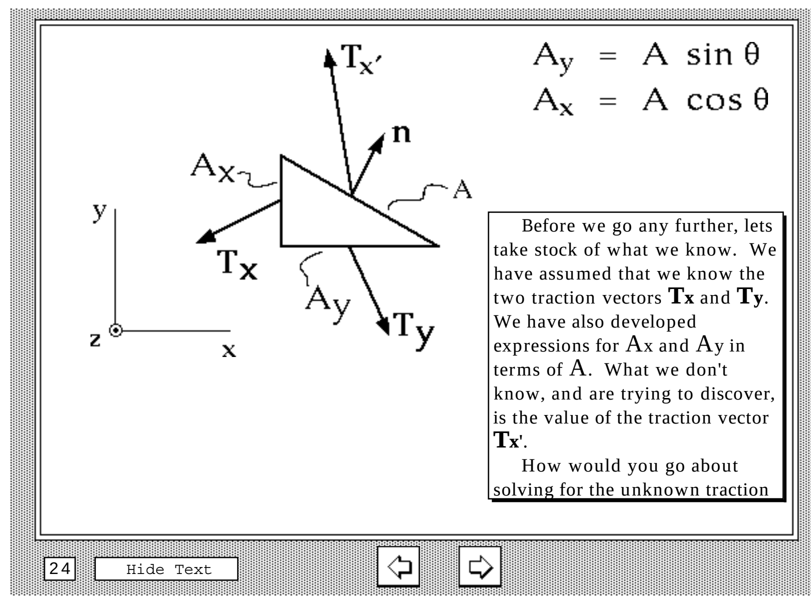
$A_y = A \sin\theta$
$A_x = A \cos\theta$
Tx'
n
Ax
A
Tx
Ay
Ty
y
x
z
Before we go any further, lets take stock of what we know. We have assumed that we know the two traction vectors **Tx** and **Ty**. We have also developed expressions for Ax and Ay in terms of A. What we don't know, and are trying to discover, is the value of the traction vector **Tx'**.
How would you go about solving for the unknown traction
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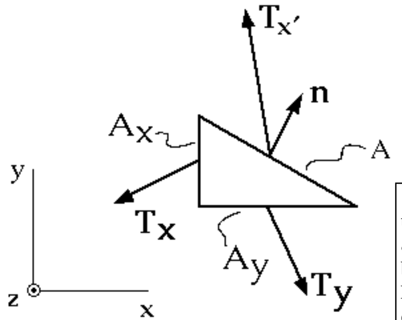
$$A_y = A \sin\theta$$
$$A_x = A \cos\theta$$



As with most problems of this nature, we begin by applying equilibrium to the free body diagram. To keep our little element from accelerating off the page, all of the forces acting on it must sum to zero.

$$\Sigma F = 0$$

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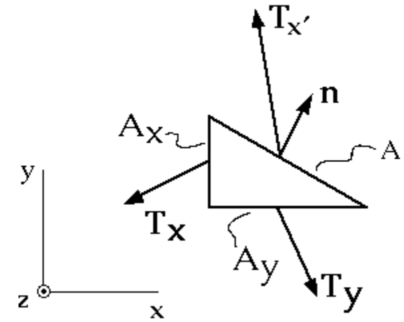

Very Important! Remember that stress is force divided by area. Therefore if you are going to use stress in a force equilibrium equation you must first multiply the stress by the area on which it acts.

The negative signs in front of the second two terms comes from the fact that the traction vectors **Tx** and **Ty** act on faces whose normals are in the negative x and y directions.

Let's make a little room on the screen before performing some algebra tricks....

$$\Sigma F = 0$$

$$T_{x'}A - T_x A_x - T_y A_y = 0$$

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$$A_y = A \sin\theta$$
$$A_x = A \cos\theta$$

$$T_{x'}A - T_x A_x - T_y A_y = 0$$

We begin by substituting our values for A_x and A_y into the equilibrium equation.

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$$A_y = A \sin\theta$$
$$A_x = A \cos\theta$$

$$T_{x'}A - T_x A_x - T_y A_y = 0$$

$$T_{x'}A - T_x A \cos\theta - T_y A \sin\theta = 0$$

Next, we divide through by A....

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$$A_y = A \sin\theta$$
$$A_x = A \cos\theta$$

$$T_{x'}A - T_x A_x - T_y A_y = 0$$
$$T_{x'}A - T_x A \cos\theta - T_y A \sin\theta = 0$$
$$T_{x'} - T_x \cos\theta - T_y \sin\theta = 0$$

Finally, we solve for the unknown traction vector **Tx'**.

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$$A_y = A \sin\theta$$
$$A_x = A \cos\theta$$

$$T_{x'}A - T_x A_x - T_y A_y = 0$$
$$T_{x'}A - T_x A \cos\theta - T_y A \sin\theta = 0$$
$$T_{x'} - T_x \cos\theta - T_y \sin\theta = 0$$
$$\boxed{T_{x'} = T_x \cos\theta + T_y \sin\theta}$$

Ta Da!

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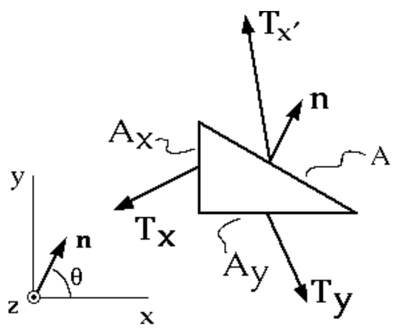

$$\boxed{T_{x'} = T_x \cos\theta + T_y \sin\theta}$$

We now have an expression for our unknown traction vector, **Tx'**, in terms of the two known traction vectors.

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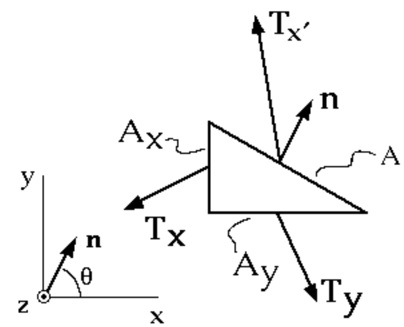

$$T_x = \sigma_x \mathbf{i} + \tau_{xy} \mathbf{j}$$
$$T_y = \tau_{yx} \mathbf{i} + \sigma_y \mathbf{j}$$

What do we do now? You should remember that by definition the traction vectors **Tx** and **Ty** are related to their component stresses as shown above.

Before we make the obvious substitution, lets clean up the screen again...

$$T_{x'} = T_x \cos\theta + T_y \sin\theta$$

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$$T_{x'} = T_x \cos\theta + T_y \sin\theta \qquad \begin{aligned} T_x &= \sigma_x \mathbf{i} + \tau_{xy} \mathbf{j} \\ T_y &= \tau_{yx} \mathbf{i} + \sigma_y \mathbf{j} \end{aligned}$$

We now proceed by substituting the expressions for $\mathbf{T_x}$ and $\mathbf{T_y}$ into the equilibrium equation.

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$$T_{x'} = T_x \cos\theta + T_y \sin\theta \qquad \begin{aligned} T_x &= \sigma_x \mathbf{i} + \tau_{xy} \mathbf{j} \\ T_y &= \tau_{yx} \mathbf{i} + \sigma_y \mathbf{j} \end{aligned}$$

$$T_{x'} = (\sigma_x \mathbf{i} + \tau_{xy} \mathbf{j}) \cos\theta + (\tau_{yx} \mathbf{i} + \sigma_y \mathbf{j}) \sin\theta$$

Grouping the $\mathbf{i}$ and $\mathbf{j}$ terms...

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$$T_{x'} = T_x \cos\theta + T_y \sin\theta \qquad \begin{aligned} T_x &= \sigma_x \mathbf{i} + \tau_{xy} \mathbf{j} \\ T_y &= \tau_{yx} \mathbf{i} + \sigma_y \mathbf{j} \end{aligned}$$

$$T_{x'} = (\sigma_x \mathbf{i} + \tau_{xy} \mathbf{j}) \cos\theta + (\tau_{yx} \mathbf{i} + \sigma_y \mathbf{j}) \sin\theta$$

$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\mathbf{j}$$

....we arrive at an expression for the unknown traction $\mathbf{T_{x'}}$ in terms of the known stress components $\boldsymbol{\sigma_x}$, $\boldsymbol{\sigma_y}$, and $\boldsymbol{\tau_{xy}}$.

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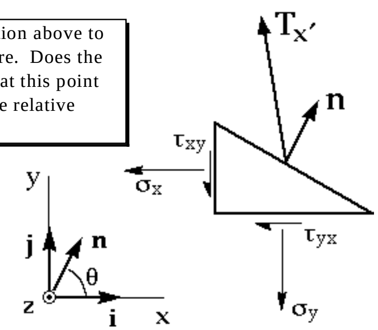
$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\mathbf{j}$$

Compare the terms in the equation above to the quantities depicted on the figure. Does the equation make sense? Remember, at this point the sines and cosines come from the relative areas on which the stresses act.



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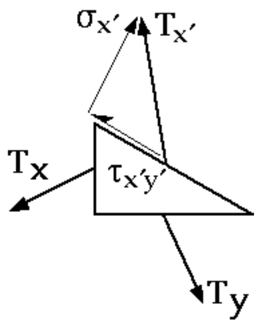
$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\,\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\,\mathbf{j}$$

$$\sigma = \mathbf{T} \cdot \mathbf{n}$$

The final step in deriving our stress transformation equations is to calculate the two stress components of the traction vector **Tx'**. They are **σx'** and **τx'y'**.

Recall the general relationship between normal stress and the traction vector:

$$\sigma = \mathbf{T} \bullet \mathbf{n}$$



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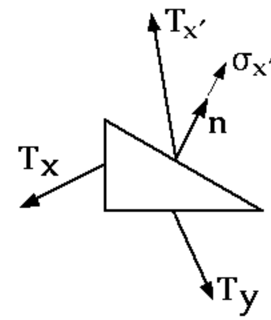
$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\,\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\,\mathbf{j}$$

$$\sigma = \mathbf{T} \cdot \mathbf{n}$$

$$\sigma_{x'} = \mathbf{T}_{x'} \cdot \mathbf{n}$$

We can write a similar expression for the normal stress **σx'**.

Recalling our expression for the normal vector **n**, we can substitute it into the equation above....



$$\mathbf{n} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$$

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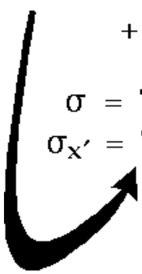
$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\,\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\,\mathbf{j}$$

$$\sigma = \mathbf{T} \cdot \mathbf{n}$$

$$\sigma_{x'} = \mathbf{T}_{x'} \cdot (\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$



....and then substitute in the expression for **Tx'** at the top of the page. At this point we will skip a step, but if you wish you can perform the dot product yourself and verify the result.

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Finally! We have an expression for the normal stress on the rotated element in terms of the stresses on the un-rotated element. This is an important result! It would not be half bad if you could produce this result yourself.

$$\sigma = \mathbf{T} \cdot \mathbf{n}$$

$$\sigma_{x'} = \mathbf{T}_{x'} \cdot (\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

$$\boxed{\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta}$$

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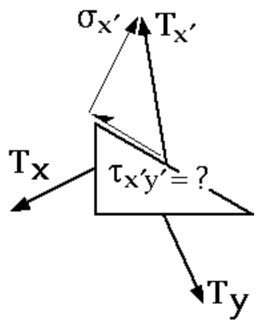
$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\,\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\,\mathbf{j}$$

We now have an expression for **σx'**, but what about the shear component **τx'y'** ?



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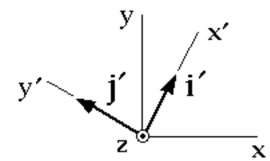
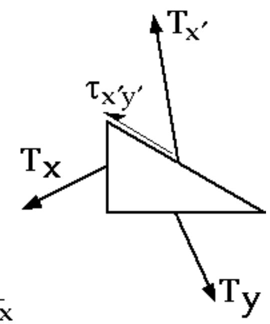
$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\,\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\,\mathbf{j}$$

$$\tau_{x'y'} = T_{x'} \cdot \mathbf{j}'$$

Since we are working in only two dimensions, **τx'y'** must act in the **y'** direction. To solve for **τx'y'** we need only perform the simple dot product shown above.





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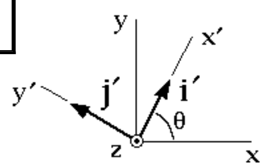
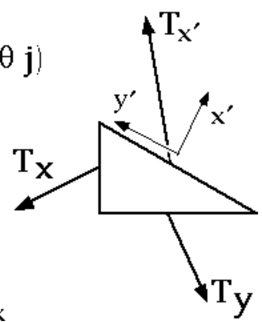
$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\,\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\,\mathbf{j}$$

$$\tau_{x'y'} = T_{x'} \cdot \mathbf{j}'$$

$$\tau_{x'y'} = T_{x'} \cdot (-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$$

Confirm for yourself that the **j'** is expanded as -sinθ**i** + cosθ**j**.





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$$T_{x'} = (\sigma_x \cos\theta + \tau_{yx} \sin\theta)\,\mathbf{i} + (\tau_{xy} \cos\theta + \sigma_y \sin\theta)\,\mathbf{j}$$

$$\tau_{x'y'} = T_{x'} \cdot \mathbf{j}'$$

$$\tau_{x'y'} = T_{x'} \cdot (-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$$

$$\boxed{\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)}$$

Lastly, if we substitute in the expression for the traction **Tx'** at the top of the page and perform the dot product we will arrive at the boxed result.

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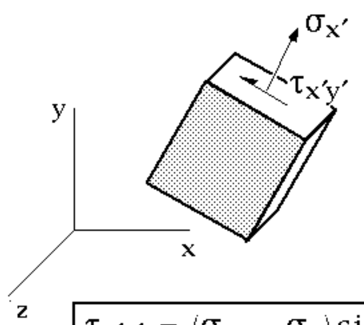
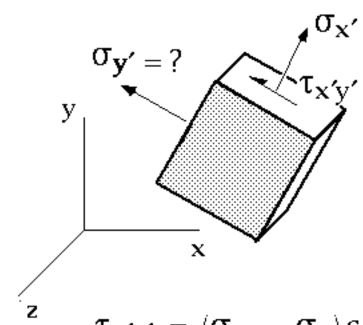


These two equations are known as the <u>two-dimensional stress transformation equations</u>. Given the stresses at one orientation of the element you can use these equations to calculate the stresses for any other orientation.

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\,\tau_{xy}\sin\theta\cos\theta$$

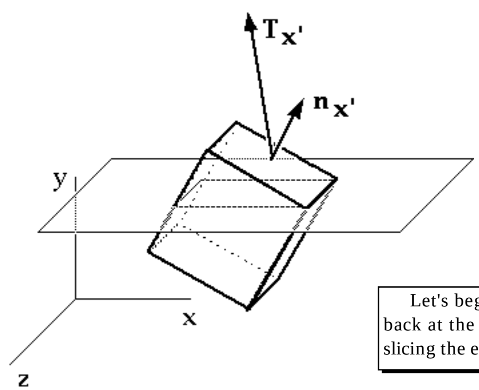


We could end our derivation here, but for completeness we will go on and solve for the unknown normal stress in the y' direction.

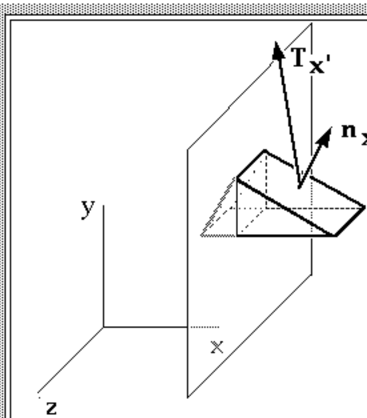
This is not completely necessary, however, for we could also solve for the stress in the y' direction simply by rotating the element 90° ccw and solving for **σx'**.

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\,\tau_{xy}\sin\theta\cos\theta$$



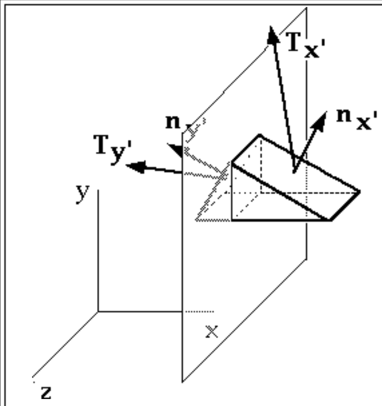
Let's begin our quest for **σy'** back at the point when we were slicing the element.



$$\mathbf{n}_{x'} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$$

Recall that we developed the expression for the normal vector shown above.

This time instead of ignoring the material to the left of the plane, let's look at the stresses that occur on this portion of the element.



$$n_{x'} = \cos\theta\, \mathbf{i} + \sin\theta\, \mathbf{j}$$
$$n_{y'} = -\sin\theta\, \mathbf{i} + \cos\theta\, \mathbf{j}$$

The left surface is normal to the y' axis so the traction vector that acts on it is **Ty'**.

Compare the normal to this surface, **ny'**, to the normal **nx'**. Notice that since these two vectors are oriented at 90° to each other we can get from one to the other by making the following replacements:

cosθ is replaced by -sinθ, and
sinθ is replaced by cosθ.

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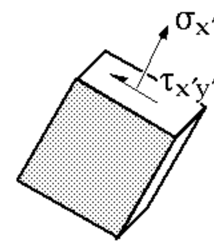

$$n_{x'} = \cos\theta\, \mathbf{i} + \sin\theta\, \mathbf{j}$$
$$n_{y'} = -\sin\theta\, \mathbf{i} + \cos\theta\, \mathbf{j}$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

If we recall the stress transformation equation for **σx'**.....

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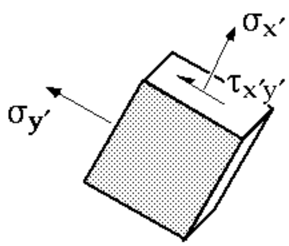

$$n_{x'} = \cos\theta\, \mathbf{i} + \sin\theta\, \mathbf{j}$$
$$n_{y'} = -\sin\theta\, \mathbf{i} + \cos\theta\, \mathbf{j}$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\,\tau_{xy} \sin\theta \cos\theta$$

....we can extrapolate to the stress transformation equation for **σy'** simply by replacing cosθ with -sinθ, and sinθ with cosθ.

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Stress Transformation Equations

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$
$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\,\tau_{xy} \sin\theta \cos\theta$$
$$\tau_{x'y'} = (\sigma_y - \sigma_x) \sin\theta \cos\theta + \tau_{xy} (\cos^2\theta - \sin^2\theta)$$

These are the complete results of the preceding derivation. Here we call them the stress transformation equations, but they are also the transformation equations for any second-order tensor in two dimensions.

As a final addendum, if we use the following identities:

sin2θ = 2sinθ cosθ
cos2θ = sinθ^2 - cosθ^2

we can write the transformation equations in terms of double angles as follows.

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Stress Transformation Equations

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\,\tau_{xy} \sin\theta \cos\theta$$

$$\tau_{x'y'} = (\sigma_y - \sigma_x) \sin\theta \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

OR

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy} \cos 2\theta$$

Plotting the Equations

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\,\tau_{xy} \sin\theta \cos\theta$$

$$\tau_{x'y'} = (\sigma_y - \sigma_x) \sin\theta \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

Now that we have these equations, let's plot them to see if anything interesting happens. Rather than plotting the shear and normal components with respect to θ, let's plot shear versus normal stress with θ as a parameter.

Plotting the Equations

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cos\theta$$

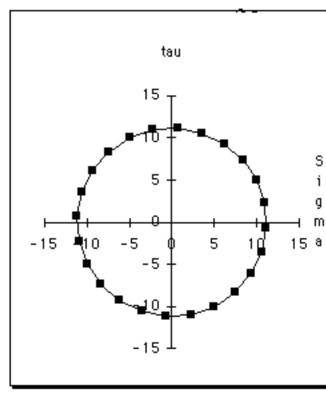
$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\,\tau_{xy} \sin\theta \cos\theta$$

$$\tau_{x'y'} = (\sigma_y - \sigma_x) \sin\theta \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

Here is a part of a spreadsheet set up to do this plotting (you should do this yourself if you can). Columns D and E contain the x' and x'y' stress components calculated for $0 < \theta < 180°$. Guess what happens when we plot τ vs. σ.

	A	B	C	D	E
1	sx =	10			
2	txy =	5			
3	sy =	-10		$\sigma_{x'}$	$\tau_{x'y'}$
4					
5	theta	sin theta	cos theta	sigma	tau
6	0	0	1	10	5
7	7.5	0.1305	0.99144	10.9533535	2.24143868
29	172.5	0.1305	-0.9914	8.36516304	7.41781958
30	180	0	-1	10	5

Plotting the Equations



We get a circle! If you make your own spreadsheet, you will be able to see that this is always true (just be sure to get equal scales on your axes). Note also that θ went from 0 to 180°, but the plot is a full 360°. These are important observations, so let's state them clearly...

A	B	C	D	E
sx =	10			
txy =	5			
sy =	-10		$\sigma_{x'}$	$\tau_{x'y'}$
theta	sin theta	cos theta	sigma	tau
0	0	1	10	5
7.5	0.1305	0.99144	10.9533535	2.24143868
172.5	0.1305	-0.9914	8.36516304	7.41781958
180	0	-1	10	5

Important Facts

1. The normal and shear components of stress acting on planes with aribitrary orientations form a circle when plotted with respect to one another.

2. A 180° range of plane orientations corresponds to a 360° circle.

It is possible to show these results algebraically by manipulating the transformation equations, but we will not take the time to do that here.

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Concluding Remarks

As engineers, we need to be able to determine the stress components in a material on any plane so that we can find critical cases (or else the material will find it for us with potentially dire consequences). The tranformation equations provide the needed tool.

We also learned that a stress tensor has associated invariant properties analogous to the magnitude of a vector. This fact is useful in more advanced contexts, so you may encounter it again.

Finally, we saw that the stress transformation equations plot as a circle. This amazing but true result will allow us to construct another useful tool for analyzing stress. Stay tuned.