An Introduction to the Concepts of Engineering Stress

Stack Contents

STRESS

Stress. Each of us has heard the word used in a number of contexts. Probably the most familiar use of the term "stress" is an emotional one. For example, you might hear someone say "I'm really stressed about this mechanics of materials final." Although this is not the interpretation of stress you will be using for this class, it is not as different as you may think.

Stress Analogies

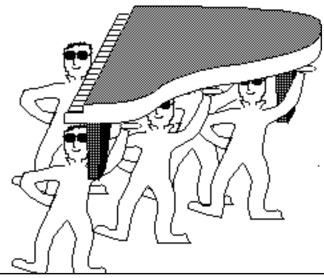
$$\frac{\text{Much Work}}{\text{Short Time}} = \text{High Stress} \qquad \frac{\text{Little Work}}{\text{Long Time}} = \text{Low Stress}$$

Let's consider emotional stress for a moment. What constitutes "high stress" and "low stress"? If we define emotional stress to be the ratio of work load to time allowed to complete the work, we could make comparisons of different stressful situations. If you had to complete three engineering assignments and a design project in two days, this would be highly stressful. If this stress became too great you might have a nervous breakdown or something -- failure. We wouldn't want that to happen, so what could we do to reduce your stress?

We could do one of two things. We could either reduce the number of assignments or give you more time to complete them. Either of these actions would reduce the "emotional stress ratio" and thereby prevent your demise.

Stress Analogies

$$\frac{\text{Heavy Object}}{\text{Many Hands}} = \text{Light Work (low stress)}$$



Let's consider another analogy to our stress concept -- "many hands make light work".
Imagine that your professor asked the students in your class to help her move a piano. Further, if you didn't help, you would not be invited to the year end Christmas Party featuring Barry Manilow at the piano.
You can imagine that everyone in class would jump to their feet and literally dance the piano to her home. With so much help each person would carry very little weight, and there would be little chance that an arm would fail.
In engineering terms we would say that each arm was experiencing "low stress".

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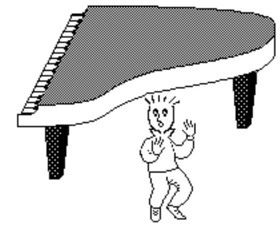
Stress Analogies

$$\frac{\text{Heavy Object}}{\text{Few Hands}} = \text{Heavy Work (high stress)}$$



If, however, the incentive of the sweet crooning Barry Manilow was not offered, you can imagine that perhaps the only person who would offer to help the professor would be her "significant other". In this case the load in each arm would be very high.
Unless the helper was a defensive lineman, you can imagine that the arms might fail.
In engineering terms we would say that each arm was experiencing "high stress".

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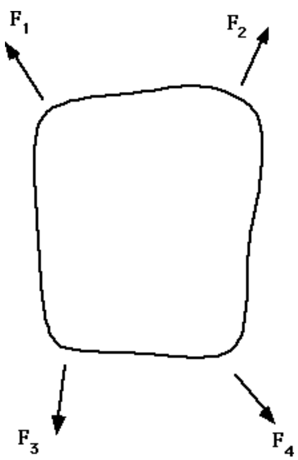
As demonstrated by the piano movers, stress may be thought of as a load divided over some number of things supporting that load. As engineers you will need a more rigorous definition of stress.

Consider the amorphous potato-like blob loaded by a set of external forces.



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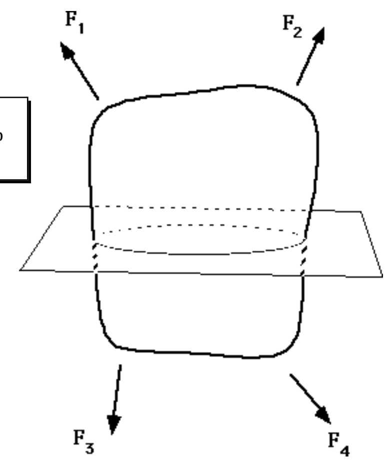
Imagine that we pass an imaginary plane through the blob as shown at the right.



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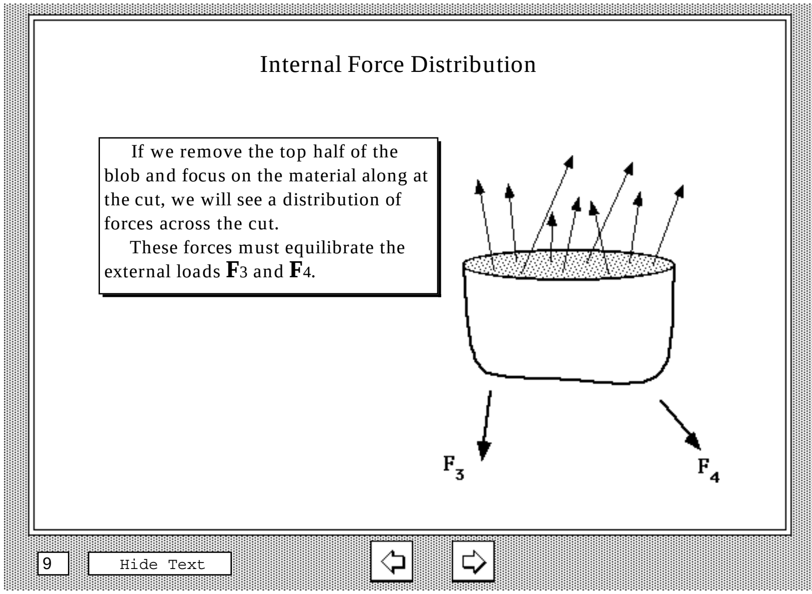
Internal Force Distribution
If we remove the top half of the blob and focus on the material along at the cut, we will see a distribution of forces across the cut.
These forces must equilibrate the external loads F3 and F4.
F3
F4
9
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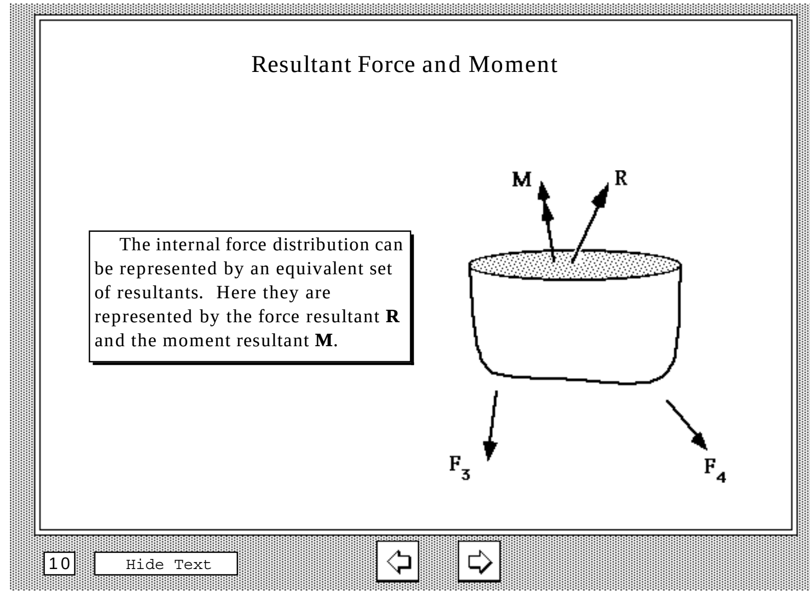
Resultant Force and Moment
The internal force distribution can be represented by an equivalent set of resultants. Here they are represented by the force resultant R and the moment resultant M.
M
R
F3
F4
10
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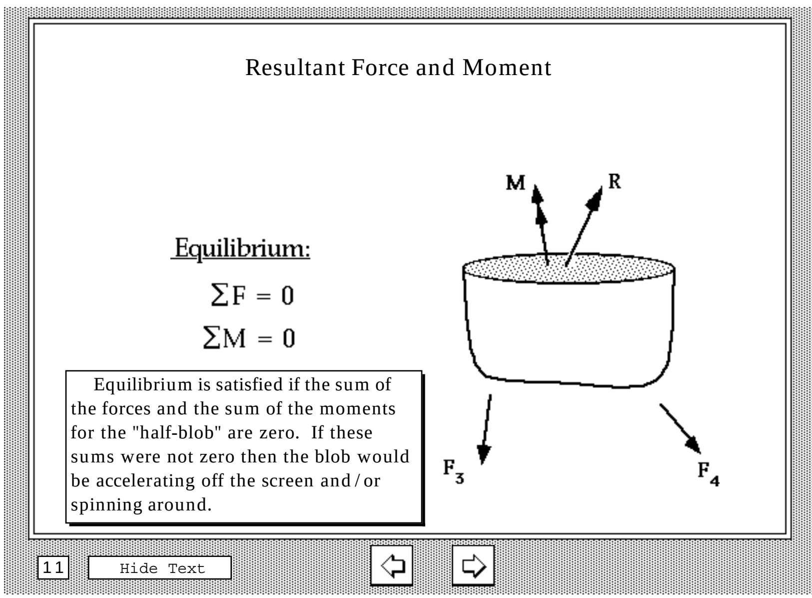
Resultant Force and Moment
Equilibrium:
ΣF = 0
ΣM = 0
Equilibrium is satisfied if the sum of the forces and the sum of the moments for the "half-blob" are zero. If these sums were not zero then the blob would be accelerating off the screen and/or spinning around.
M
R
F3
F4
11
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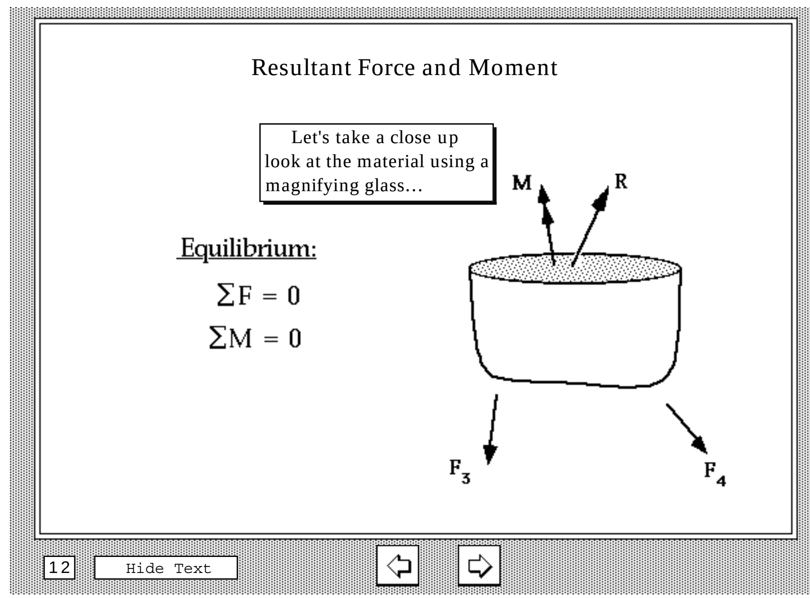
Resultant Force and Moment
Let's take a close up look at the material using a magnifying glass...
Equilibrium:
ΣF = 0
ΣM = 0
M
R
F3
F4
12
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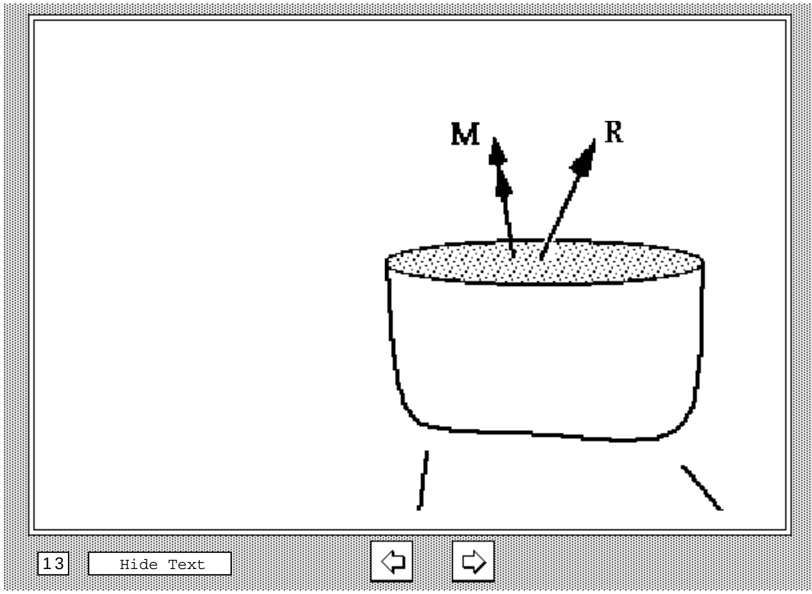
M
R
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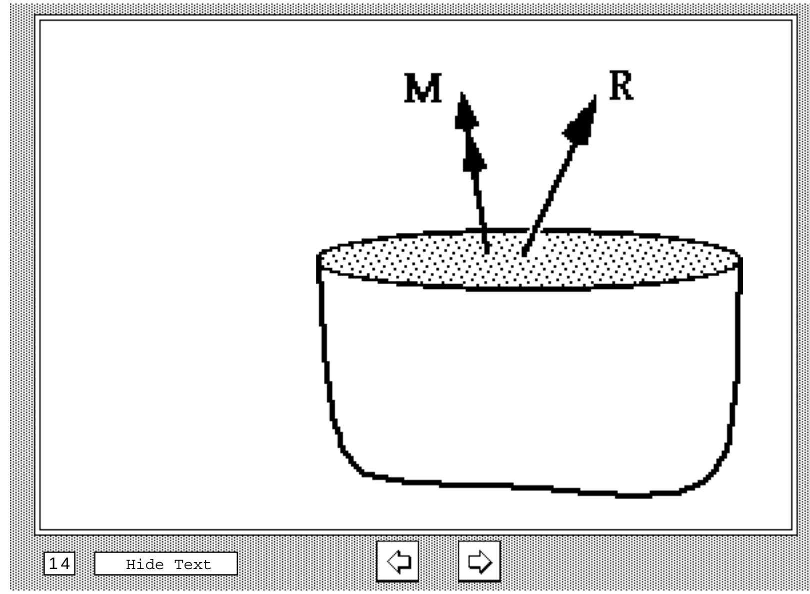
M
R
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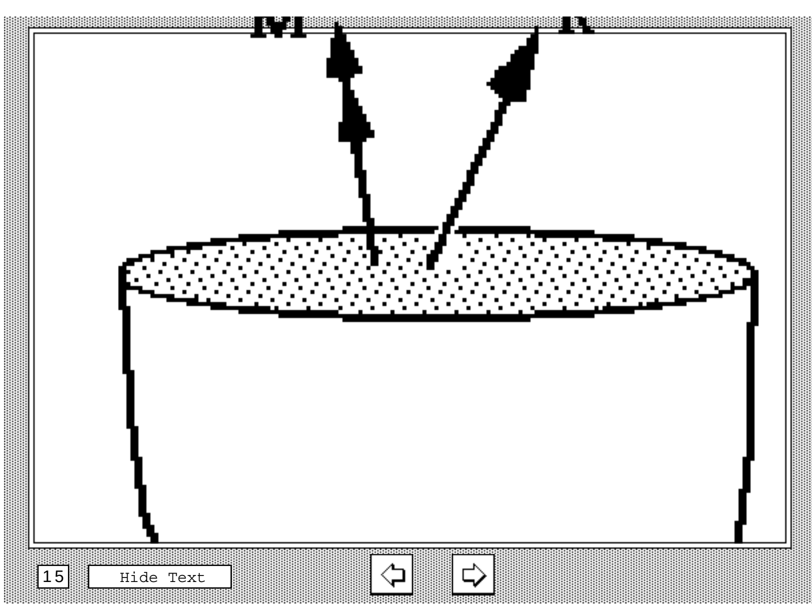
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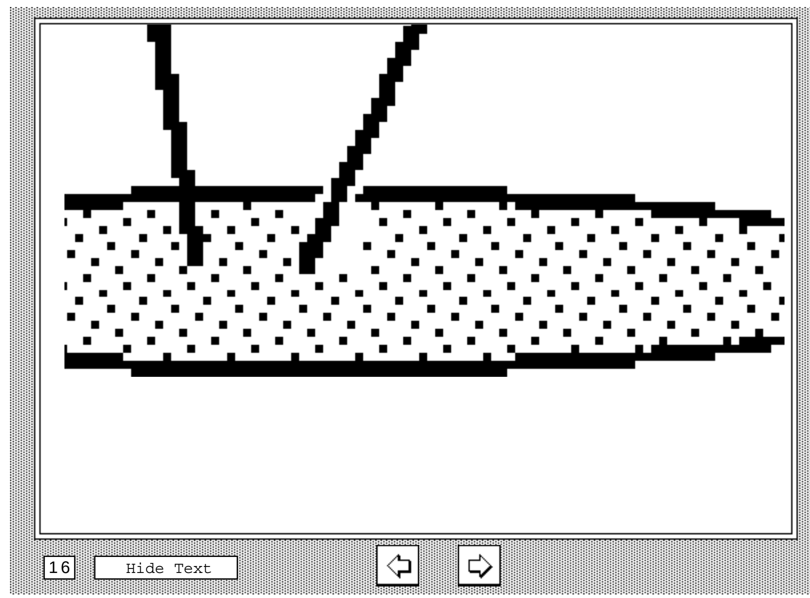
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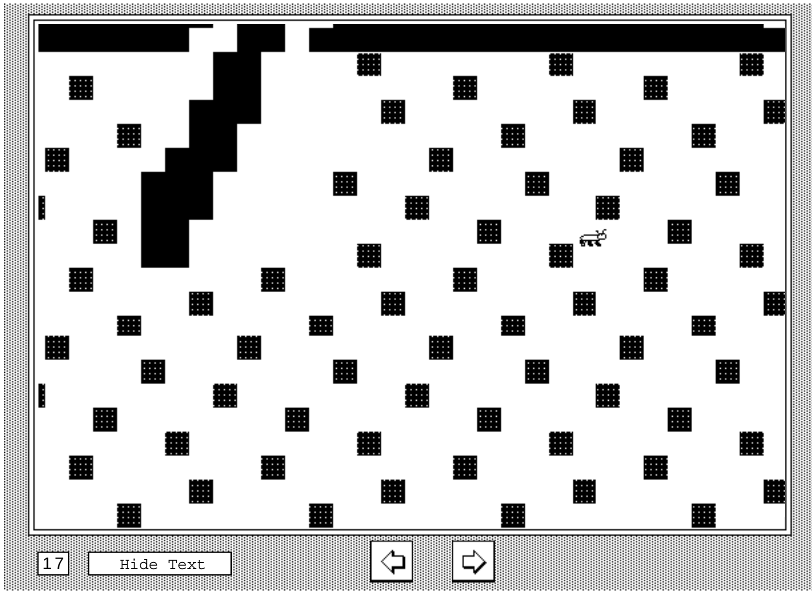
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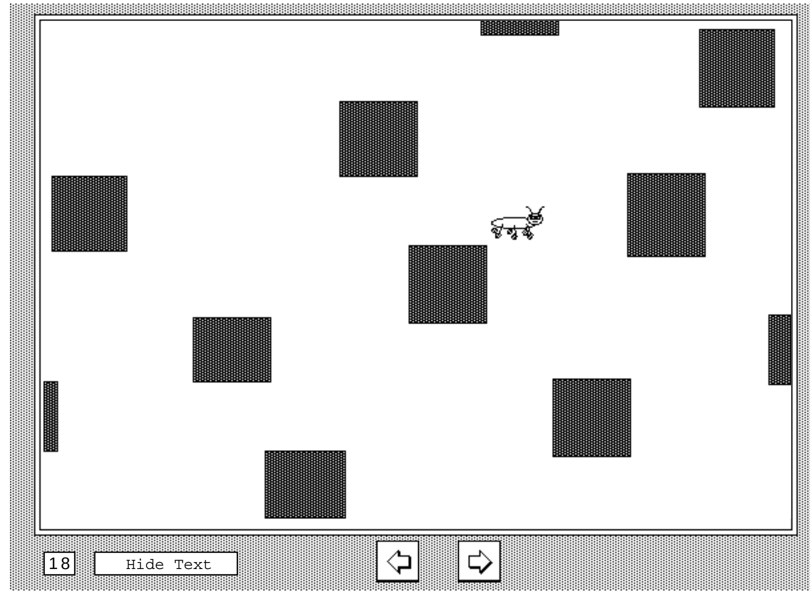
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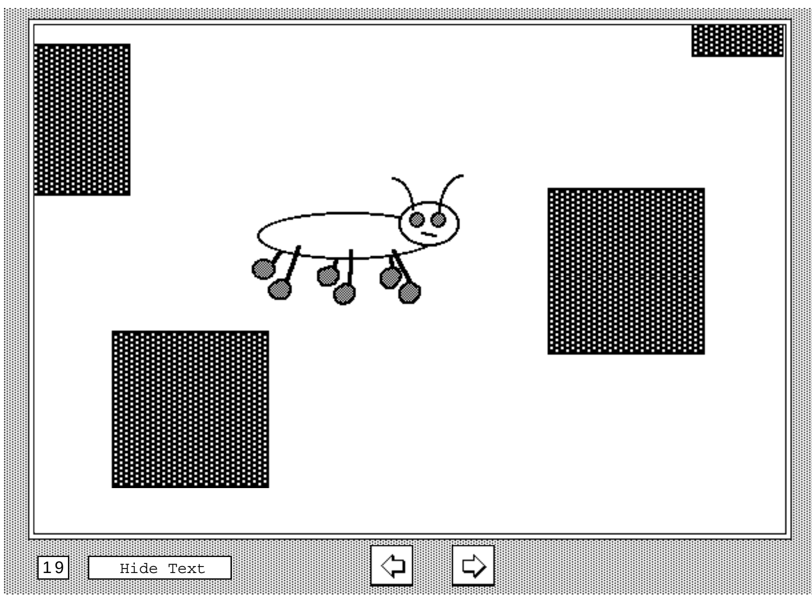
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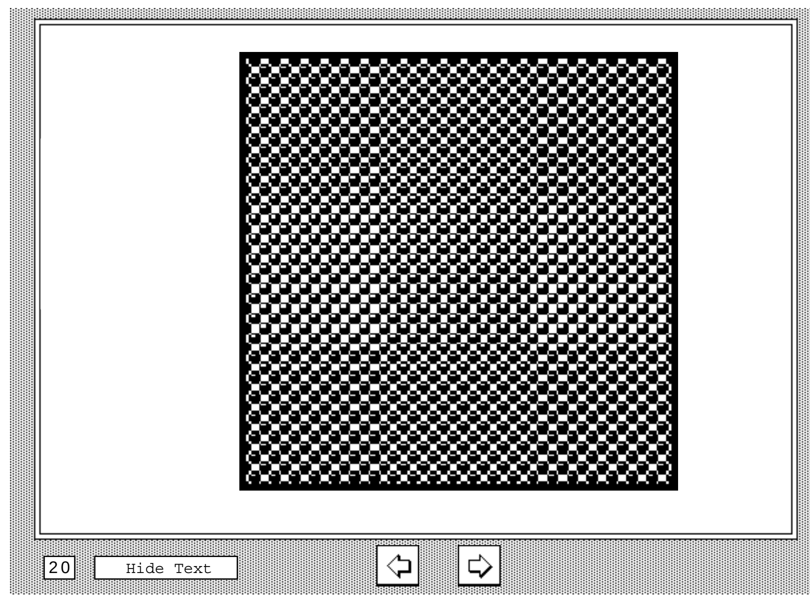
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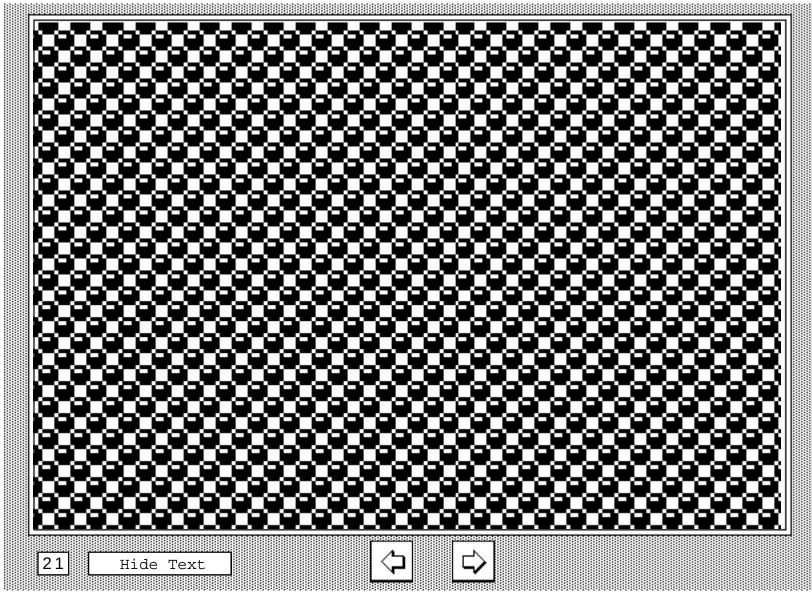
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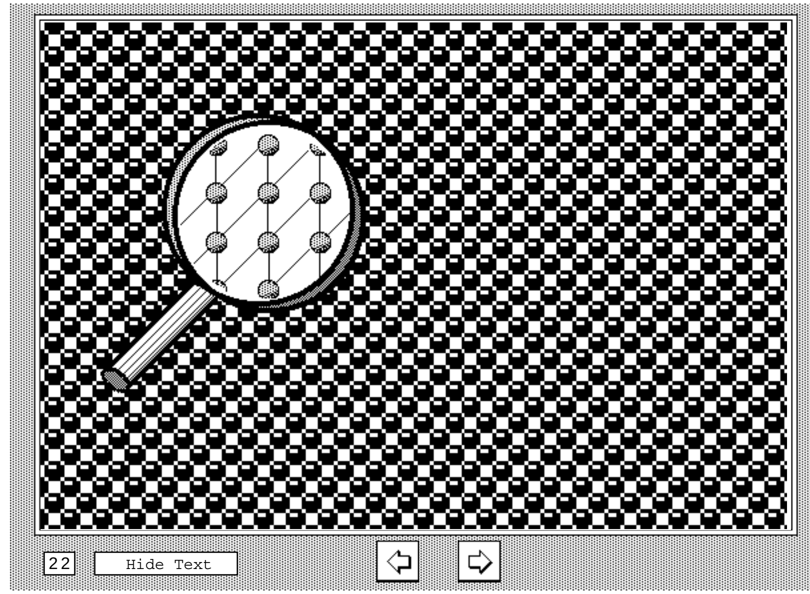
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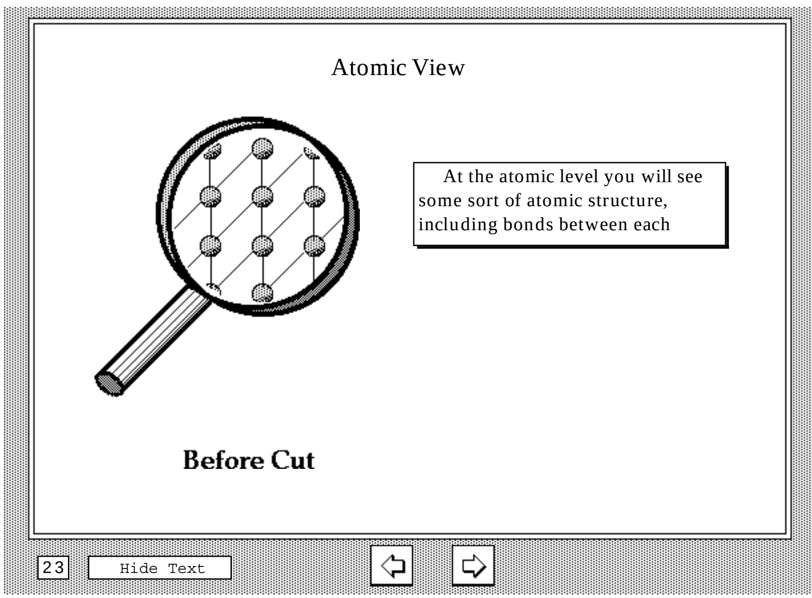
Atomic View
At the atomic level you will see some sort of atomic structure, including bonds between each
Before Cut
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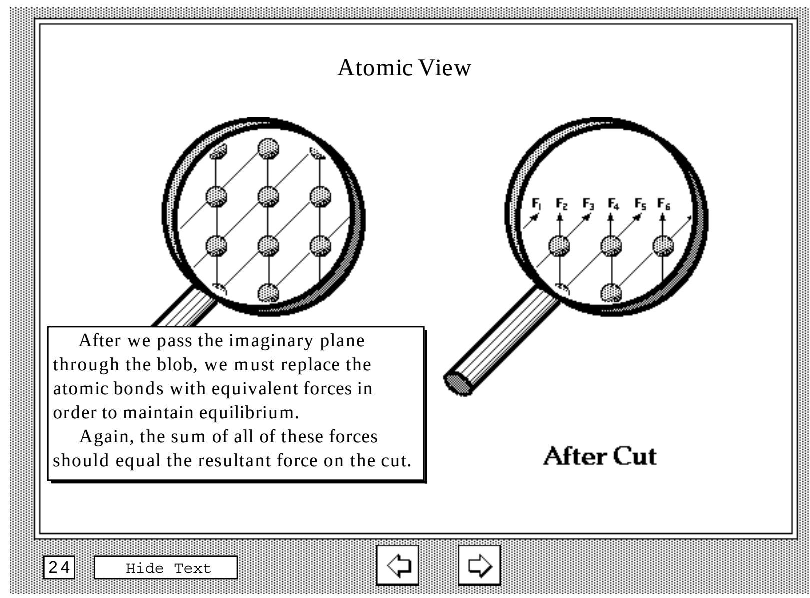
Atomic View
F_1 F_2 F_3 F_4 F_5 F_6
After we pass the imaginary plane through the blob, we must replace the atomic bonds with equivalent forces in order to maintain equilibrium.
Again, the sum of all of these forces should equal the resultant force on the cut.
After Cut
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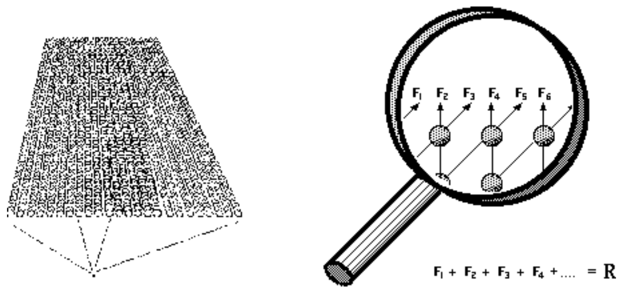
Trillions of Bonds per Unit Area



This atomic view of the material is impractical to use in modeling almost all typical engineered products. For example, a square inch of steel has approximately 1,200,000,000,000,000,000,000,000 atoms. Can you imagine keeping track of the bonds between 2.5 x 10 ^34 atoms, the number of atoms in a typical 5 story building?

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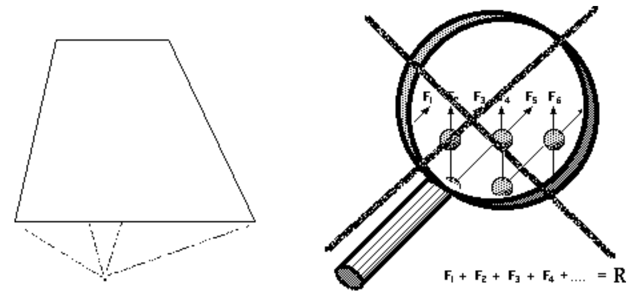
Adopt Continuum Model



Instead of analyzing the material at the atomic level, we will assume that materials can be modeled as continua. That is, no matter how close we look at the material, we never see the discrete structure of the atoms.

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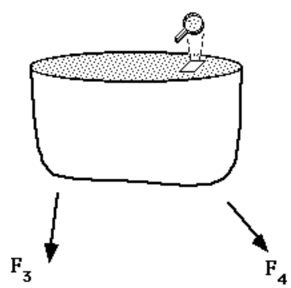
Reconsider the Magnified Patch...

Now let's magnify a small patch of the slice through the blob again, only this time we will use the assumption that the material is continuous.



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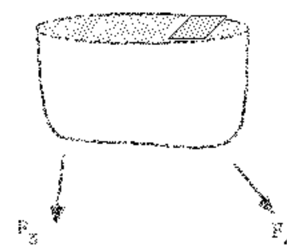

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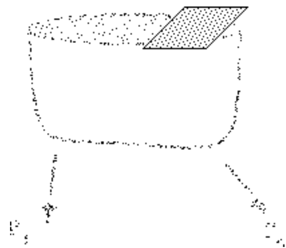

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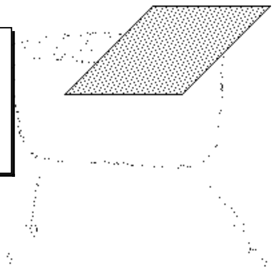
Magnified Patch

We have magnified the patch 20 times, but we see no change in the surface. Even if we magnified the path 100,000 times we would still see the same "stuff".



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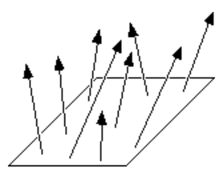
Force Distribution on Patch



This time, instead of seeing atomic bonds when we look closely at the surface we see a distribution of forces.

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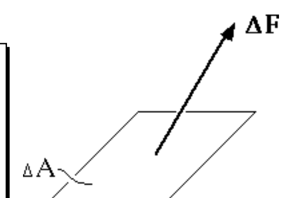
Resultant of Force Distribution on Patch with Area = ΔA

As before, it is possible to represent a system of forces by its resultant. Here, the resultant, Δ**F,** acts on a surface having area ΔA.

Recall that our goal at this point is a technical definition for the term "stress". The stress in our piano movers was a ratio of load (the piano) to the material that was supporting the load (the number of movers).

Similarly, a quantity that might fit this loose definition of stress is the ratio of the force acting on a small patch of area, Δ**F**, to the area of the small patch ΔA.



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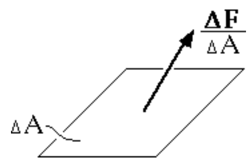
Force Resultant Divided by Area of Patch

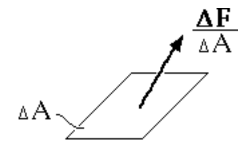
As you can see, dividing the resultant, $\Delta \mathbf{F}$, by the area over which it acts, ΔA, simply scales the vector.

Now imagine that we begin shrinking the area ΔA, and for each smaller area we calculate the resultant $\Delta \mathbf{F}$, and divide it by the new area.

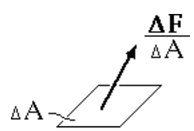
It turns out that instead of vanishing, the ratio $\Delta \mathbf{F} / \Delta A$ approaches a limit as the area approaches zero.



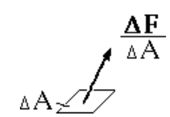
Force Resultant Divided by Area of Patch



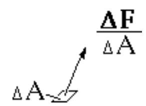
Force Resultant Divided by Area of Patch



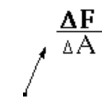
Force Resultant Divided by Area of Patch



Force Resultant Divided by Area of Patch

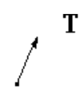


Force Resultant Divided by Area of Patch



This limit we call the "traction" vector, and it is denoted by the letter **T**.

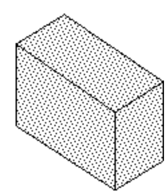
The Traction Vector



The traction vector, **T**, is defined exactly the way we calculated it. It is the limit of the ratio $\Delta\mathbf{F}/\Delta A$, as ΔA approaches zero, and it provides a measure of the direction and intensity of the loading exerted at a point on a

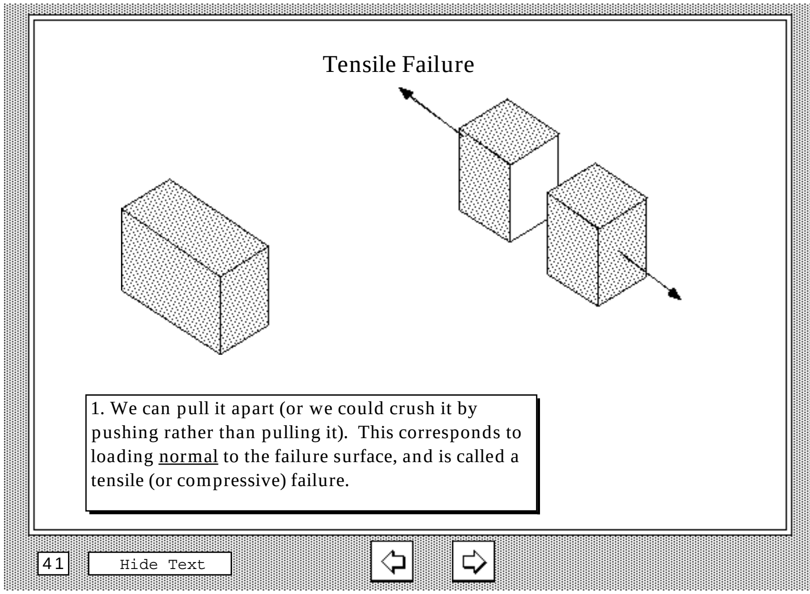
$$\mathbf{T} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A}$$

A Remark About Failure



We will return to the traction vector later, but first we need to think again about what we are trying to do.

One of the principal reasons for studying stress is to be able to predict when things will break or suffer damage due to loading. We therefore need to consider the two basic methods for breaking an object into two pieces ...


Tensile Failure
1. We can pull it apart (or we could crush it by pushing rather than pulling it). This corresponds to loading normal to the failure surface, and is called a tensile (or compressive) failure.
41
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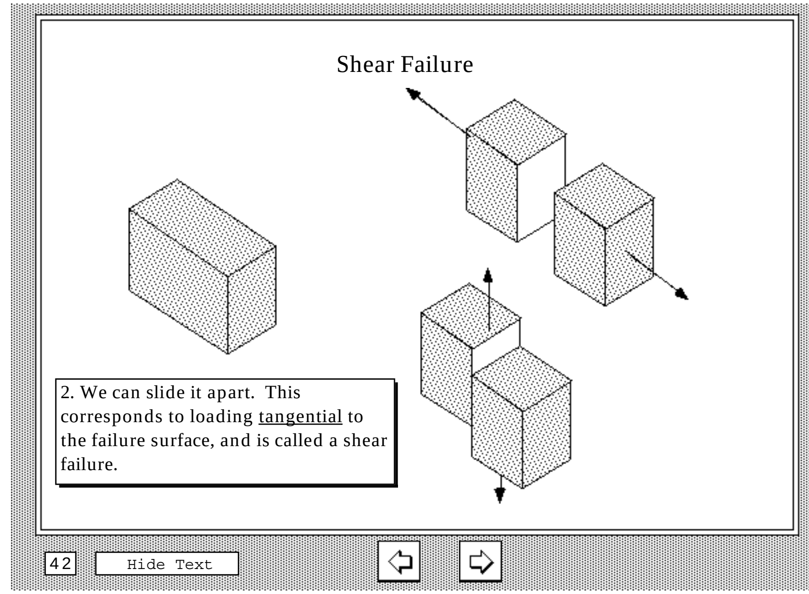
Shear Failure
2. We can slide it apart. This corresponds to loading tangential to the failure surface, and is called a shear failure.
42
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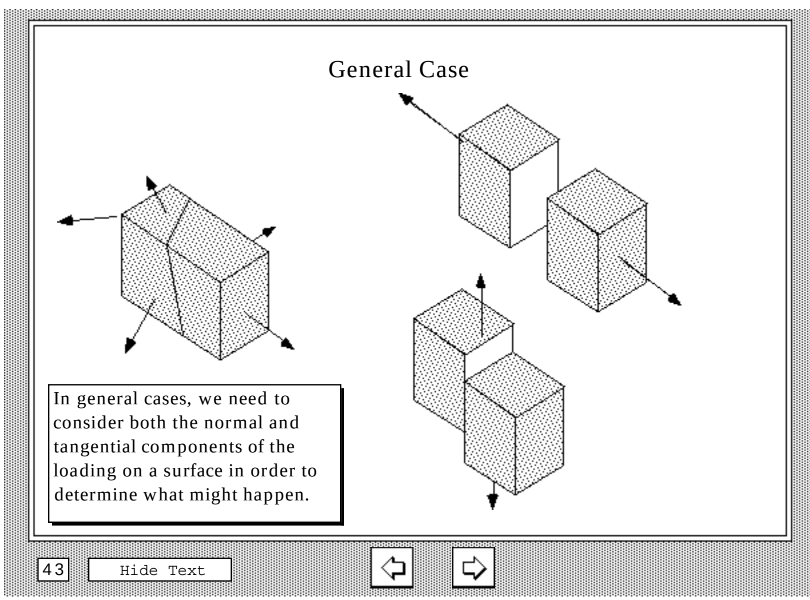
General Case
In general cases, we need to consider both the normal and tangential components of the loading on a surface in order to determine what might happen.
43
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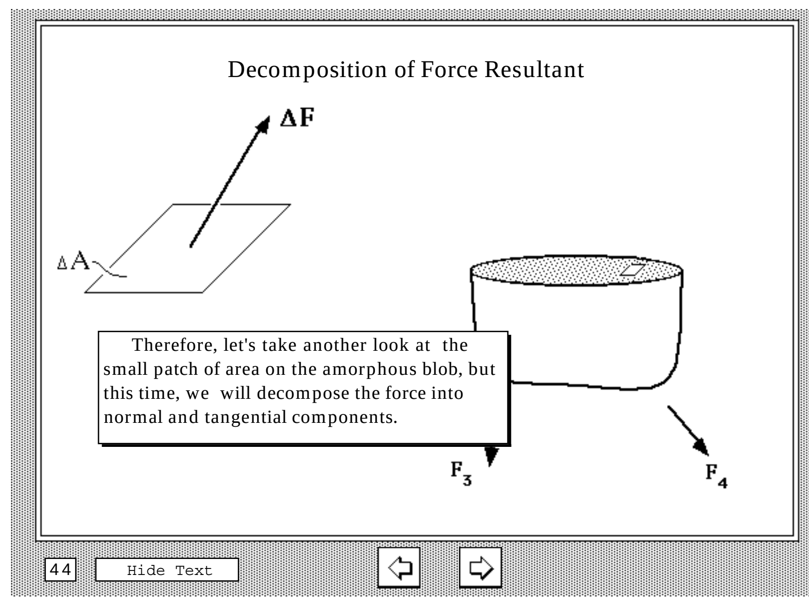
Decomposition of Force Resultant
ΔF
ΔA
Therefore, let's take another look at the small patch of area on the amorphous blob, but this time, we will decompose the force into normal and tangential components.
F3
F4
44
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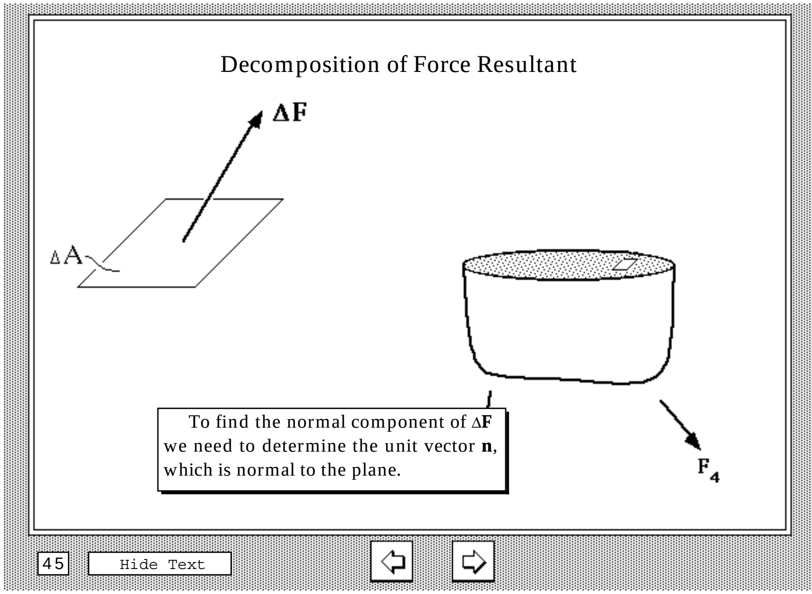
Decomposition of Force Resultant
ΔF
ΔA
To find the normal component of ΔF we need to determine the unit vector n, which is normal to the plane.
F4
45
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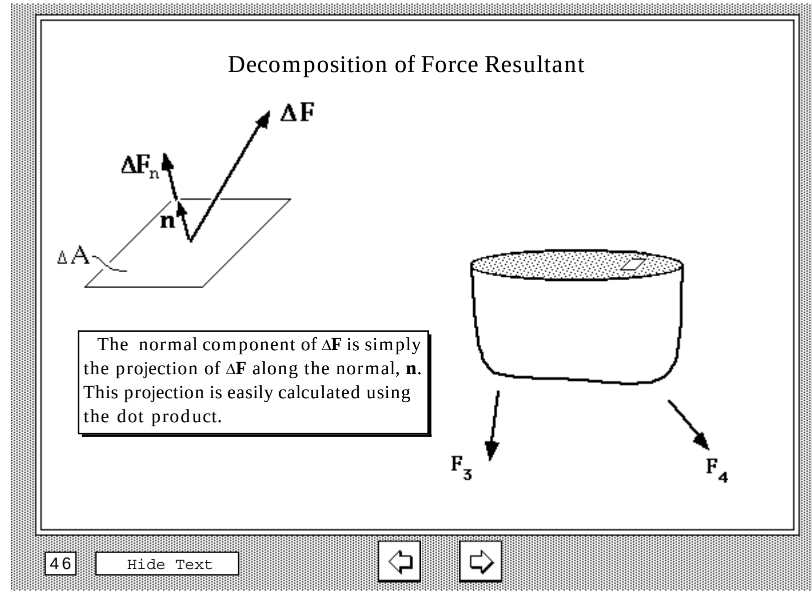
Decomposition of Force Resultant
ΔF
ΔFn
n
ΔA
The normal component of ΔF is simply the projection of ΔF along the normal, n. This projection is easily calculated using the dot product.
F3
F4
46
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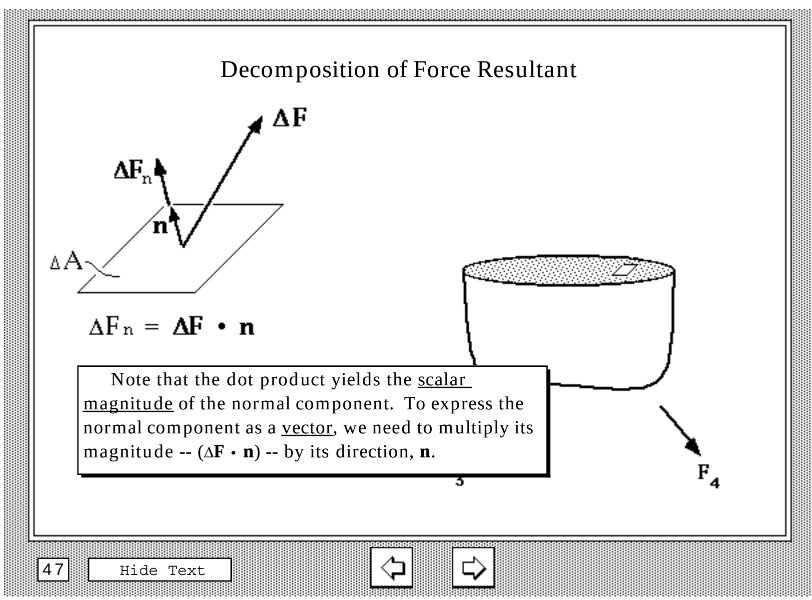
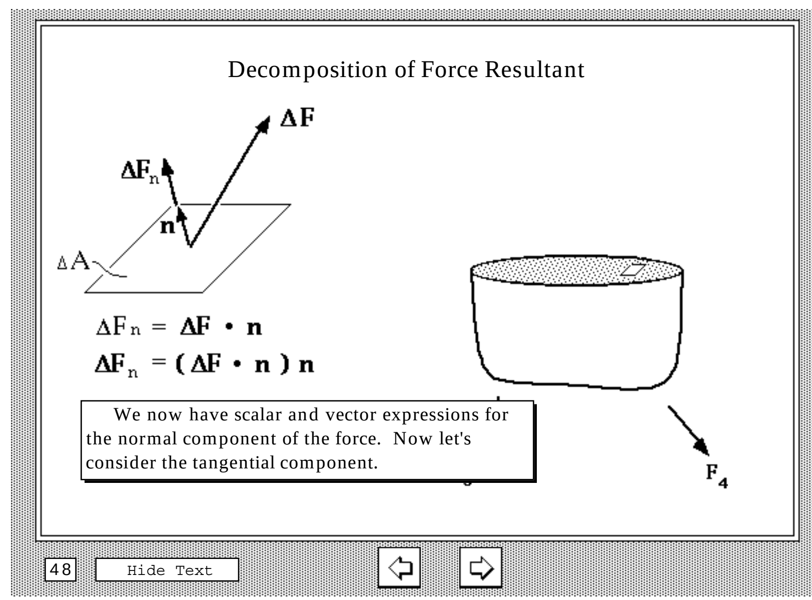

Decomposition of Force Resultant
ΔF
ΔFn
n
ΔA
ΔFn = ΔF • n
Note that the dot product yields the scalar magnitude of the normal component. To express the normal component as a vector, we need to multiply its magnitude -- (ΔF • n) -- by its direction, n.
F4
47
Hide Text


Decomposition of Force Resultant
ΔF
ΔFn
n
ΔA
ΔFn = ΔF • n
ΔFn = (ΔF • n) n
We now have scalar and vector expressions for the normal component of the force. Now let's consider the tangential component.
F4
48
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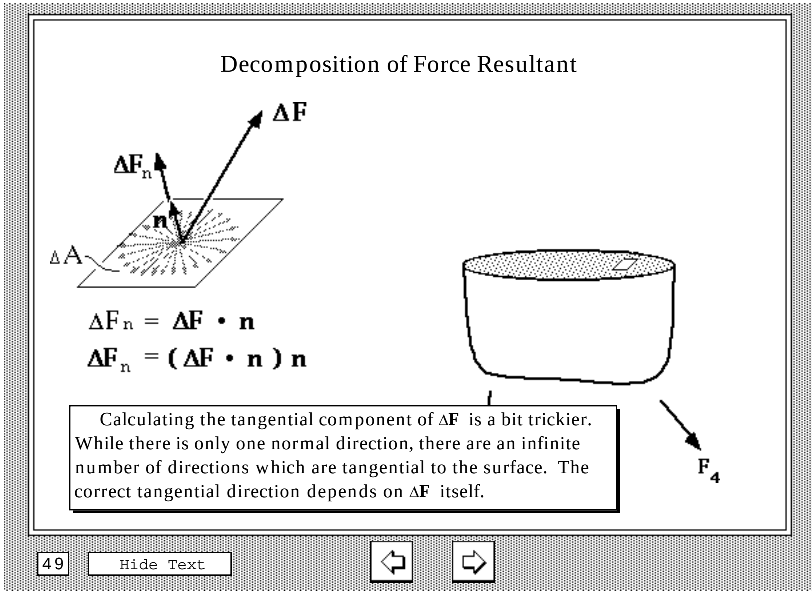
Decomposition of Force Resultant
ΔF
ΔFn
n
ΔA
ΔFn = ΔF • n
ΔFn = (ΔF • n) n
Calculating the tangential component of ΔF is a bit trickier. While there is only one normal direction, there are an infinite number of directions which are tangential to the surface. The correct tangential direction depends on ΔF itself.
F4
49
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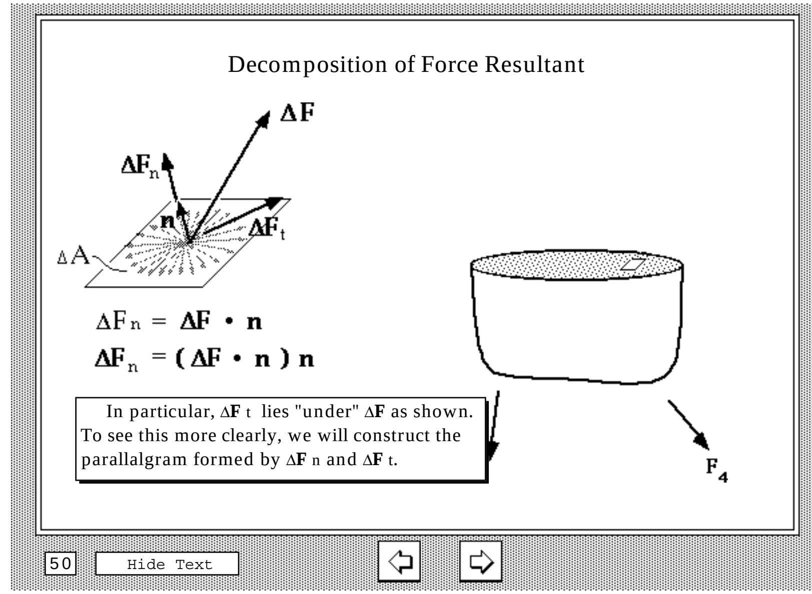
Decomposition of Force Resultant
ΔF
ΔFn
n
ΔFt
ΔA
ΔFn = ΔF • n
ΔFn = (ΔF • n) n
In particular, ΔF t lies "under" ΔF as shown. To see this more clearly, we will construct the parallalgram formed by ΔF n and ΔF t.
F4
50
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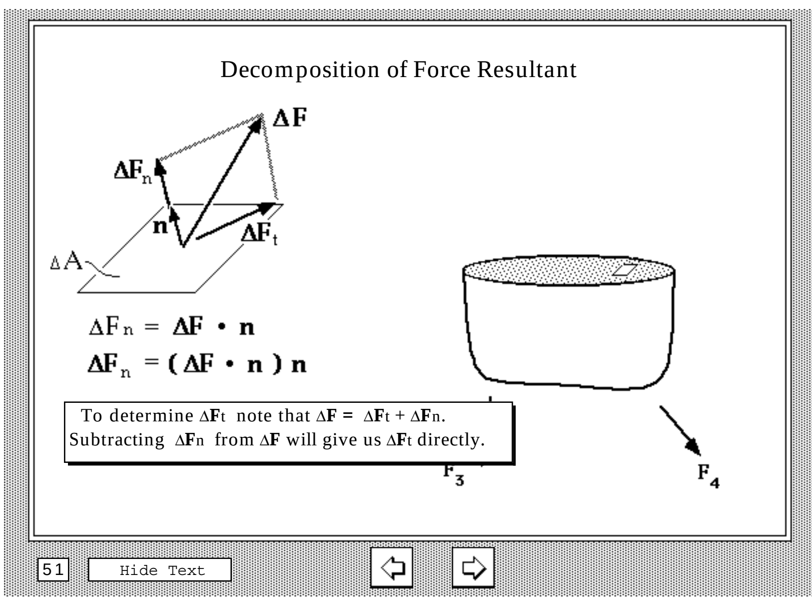
Decomposition of Force Resultant
ΔF
ΔFn
n
ΔFt
ΔA
ΔFn = ΔF • n
ΔFn = (ΔF • n) n
To determine ΔFt note that ΔF = ΔFt + ΔFn.
Subtracting ΔFn from ΔF will give us ΔFt directly.
F3
F4
51
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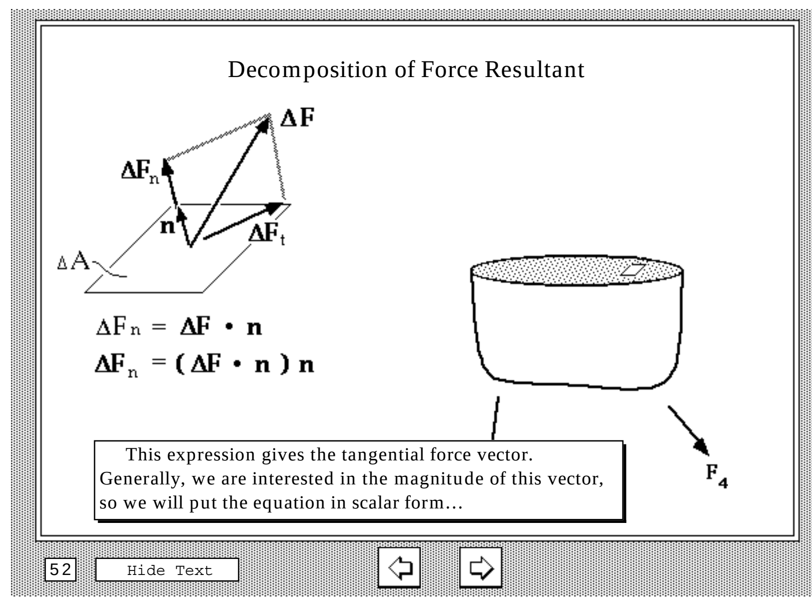
Decomposition of Force Resultant
ΔF
ΔFn
n
ΔFt
ΔA
ΔFn = ΔF • n
ΔFn = (ΔF • n) n
This expression gives the tangential force vector. Generally, we are interested in the magnitude of this vector, so we will put the equation in scalar form…
F4
52
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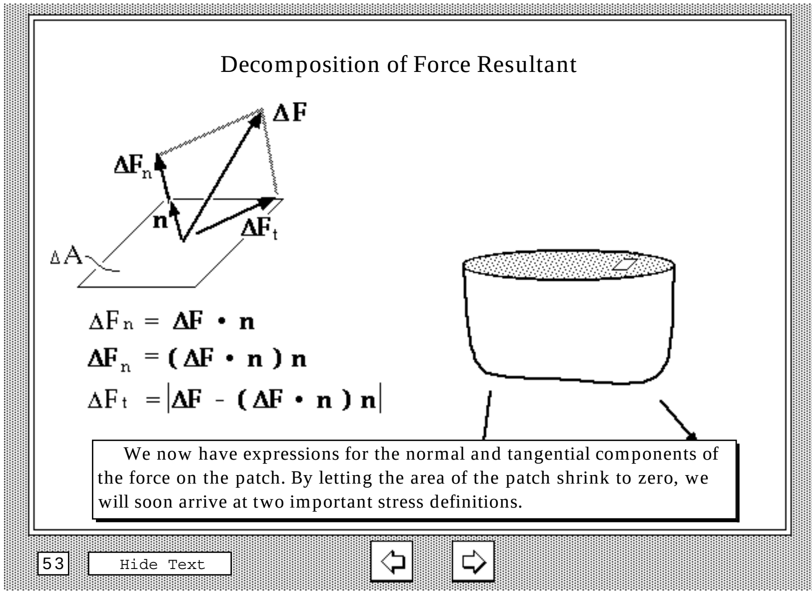
Decomposition of Force Resultant
$\Delta \mathbf{F}$
$\Delta \mathbf{F}_n$
$\mathbf{n}$
$\Delta \mathbf{F}_t$
ΔA
$\Delta F_n = \Delta \mathbf{F} \bullet \mathbf{n}$
$\Delta \mathbf{F}_n = (\Delta \mathbf{F} \bullet \mathbf{n})\,\mathbf{n}$
$\Delta F_t = |\Delta \mathbf{F} - (\Delta \mathbf{F} \bullet \mathbf{n})\,\mathbf{n}|$
We now have expressions for the normal and tangential components of the force on the patch. By letting the area of the patch shrink to zero, we will soon arrive at two important stress definitions.
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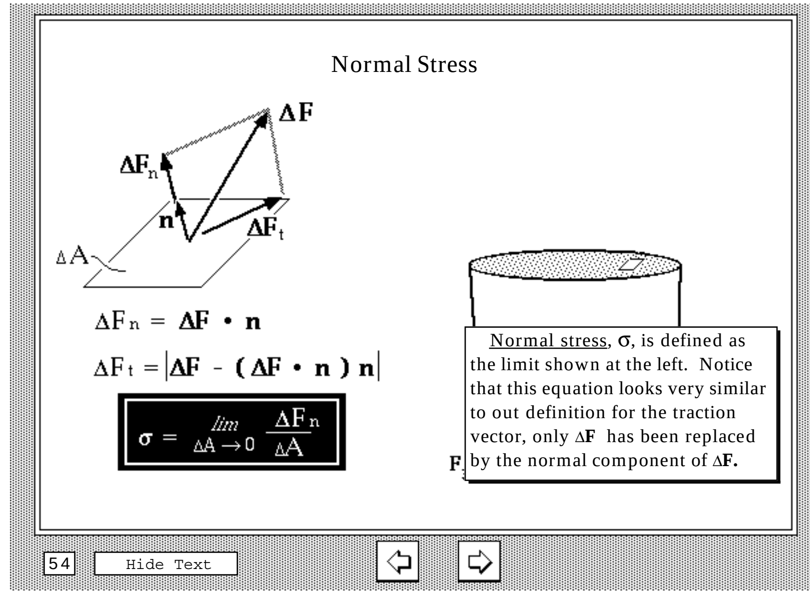
Normal Stress
$\Delta \mathbf{F}$
$\Delta \mathbf{F}_n$
$\mathbf{n}$
$\Delta \mathbf{F}_t$
ΔA
$\Delta F_n = \Delta \mathbf{F} \bullet \mathbf{n}$
$\Delta F_t = |\Delta \mathbf{F} - (\Delta \mathbf{F} \bullet \mathbf{n})\,\mathbf{n}|$
$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$
Normal stress, σ, is defined as the limit shown at the left. Notice that this equation looks very similar to out definition for the traction vector, only ΔF has been replaced by the normal component of ΔF.
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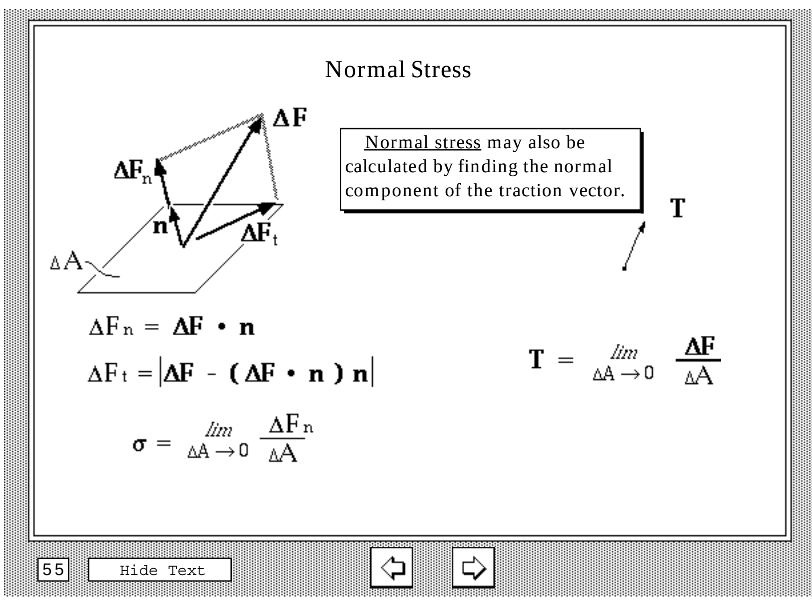
Normal Stress
$\Delta \mathbf{F}$
$\Delta \mathbf{F}_n$
$\mathbf{n}$
$\Delta \mathbf{F}_t$
ΔA
Normal stress may also be calculated by finding the normal component of the traction vector.
T
$\Delta F_n = \Delta \mathbf{F} \bullet \mathbf{n}$
$\Delta F_t = |\Delta \mathbf{F} - (\Delta \mathbf{F} \bullet \mathbf{n})\,\mathbf{n}|$
$\mathbf{T} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A}$
$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$
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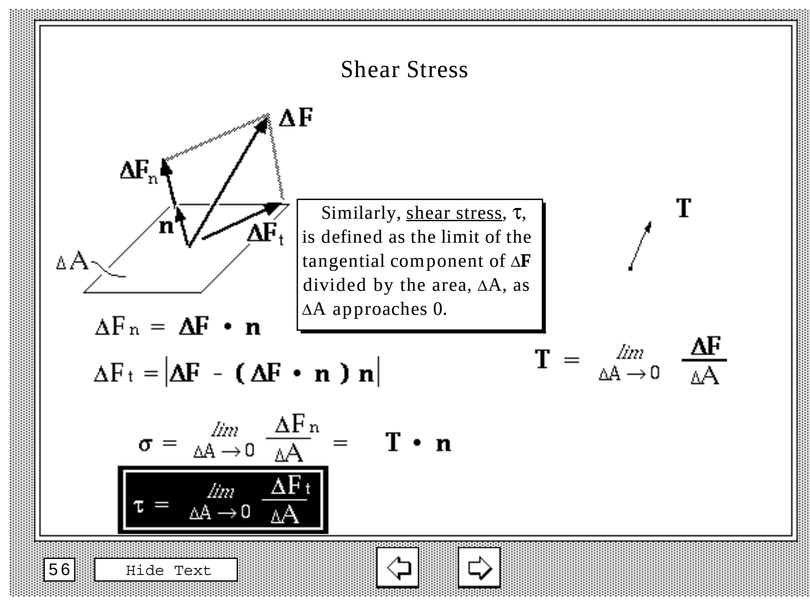
Shear Stress
$\Delta \mathbf{F}$
$\Delta \mathbf{F}_n$
$\mathbf{n}$
$\Delta \mathbf{F}_t$
ΔA
Similarly, shear stress, τ, is defined as the limit of the tangential component of ΔF divided by the area, ΔA, as ΔA approaches 0.
T
$\Delta F_n = \Delta \mathbf{F} \bullet \mathbf{n}$
$\Delta F_t = |\Delta \mathbf{F} - (\Delta \mathbf{F} \bullet \mathbf{n})\,\mathbf{n}|$
$\mathbf{T} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A}$
$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} = \mathbf{T} \bullet \mathbf{n}$
$\tau = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A}$
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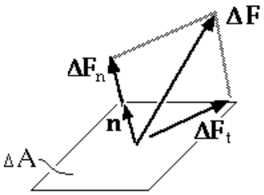
Shear Stress



We can also calculate τ from the traction vector, **T**, by subtracting out the normal component of **T**.

$$\mathbf{T} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A}$$

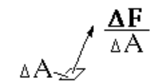
$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} = \mathbf{T} \cdot \mathbf{n}$$

$$\tau = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A} = \left| \mathbf{T} - \sigma \mathbf{n} \right|$$

Conclusion

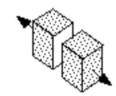
Traction Vector

$$\mathbf{T} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A}$$



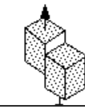
Normal Stress

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} = \mathbf{T} \cdot \mathbf{n}$$



Shear Stress

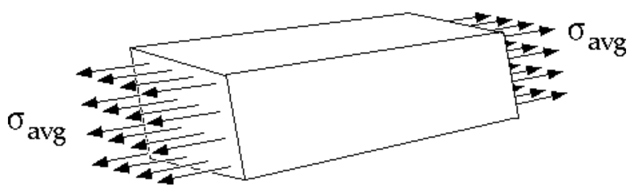
$$\tau = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A} = \left| \mathbf{T} - \sigma \mathbf{n} \right|$$



We have seen that stress may be thought of as a ratio of load to area. A more rigorous derivation of stress led us to an expression for the traction vector. Finally, we found that we could decompose the traction vector into two components; one which was **normal** to the surface of the material which we termed σ, and one which was **tangential** to the surface, which we termed τ. We call σ the "normal stress" and τ the

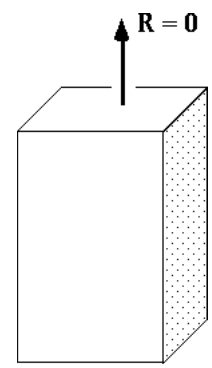
A Small Addendum:

The Problem with Using "Average Stress"



Zero Force Resultant

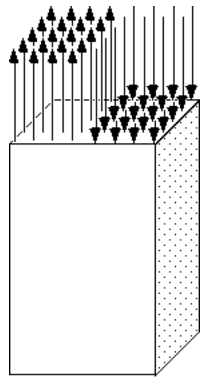
At the right is a free body diagram drawn for a segment of a square rod. It is unloaded and therefore the average stress across the section is zero. Does this mean that the actual stress in the material is zero?



Non-Zero Stress Distribution

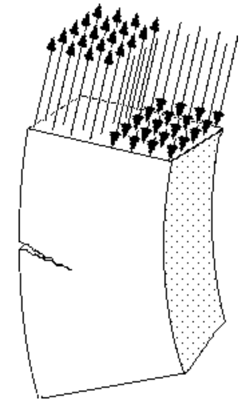
No! The actual stress distribution across the section might be like the one shown here.

Certainly, if we summed up all of the stresses in the vertical direction we would find that the resultant is zero. Therefore, the average stress on the cut is also zero. Yet the stresses that are actually present will cause the rod to bend, and may be large enough to fail the rod.



Failure

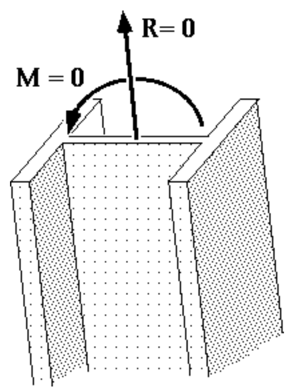
If you were the engineer who designed this rod using average stresses, you would now have a serious law suit on your hands.



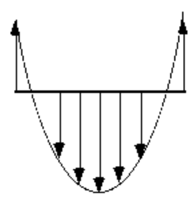
Zero Force and Moment Resultants

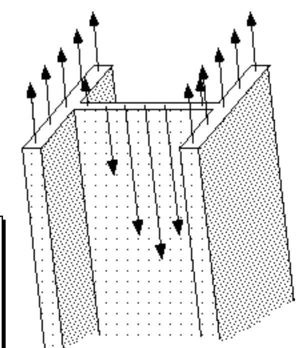
In the previous example, the net moment on the section was not zero. If the rod were made of a ductile material like steel, you would have observed large deformations before the rod failed.

In the example shown on this page, both the resultant force and the resultant moment are zero on the cut section of the I-beam. In this case you would not observe a bending or stretching of the beam. Are the internal stresses equal to zero in this case?



Stress is NOT Zero





No, they don't have to be. The parabolic stress distribution shown here will result in no net force and no net moment on the section. Yet the tensile stresses on the outside of the section may be high enough to yield the material. Stress distributions such as this are common in hot rolled steel sections.

What Did I Just Learn?

The lesson you should have just learned is that you can not use average stresses with reckless abandon.

Always be aware of the loading conditions which create the stresses. As you become more familiar with different loading conditions you will gain knowledge of when it is valid to use average stresses in your calculations.

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The End