STRAIN & DEFORMATION

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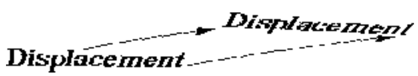


Whether we know it or not we are all intuitively familiar with the concept of *displacement*. Every time you move from one place to another your entire body is experiencing a displacement. Wiggle your finger; watch it displace.

When we use the word displacement in a technical context there is also an associated technical definition. This technical definition may be boiled down to:

A <u>DISPLACEMENT</u> is a vector quanitity used to measure movement of a point in an object from one location to the next.

This is gadget man. Drag him to a new location. Gadget man has a built in displacement meter on his utility belt, so he will tell you exactly how many pixels you have displaced him.


Gadget Man

<u>Rigid Body</u> & <u>Deformation</u>

A Moving Car — A Car Accident

For engineering purposes, we distinguish between two types of displacements. They are *rigid body displacements* and *deformations* .

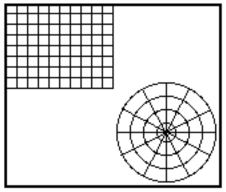
To demonstrate rigid body motion, imagine that you are speeding down the freeway in your AMC Pacer on your way to the Poconos. The car is experiencing displacement, but it is not changing shape. We say that the car is traveling as a "Rigid Body".

Now imagine that while traveling, the car plows into a charter bus. The remaining bits of twisted steel and broken glass are no longer in the same shape as the original car. We say that the car has been deformed.

In most cases, total displacement is composed of a rigid body translation / rotation and a deformation. For example, the distance from here to the Poconos would constitute the rigid body component of the radiator's displacement, and the distance from the front of the car to the right passenger's seat would constitute the deformation component of the radiator.

For this class we will be interested in <u>deformations only</u>. Rigid body motion is treated under a different subject known as dynamics or kinematics.

Rigid Body Translation



Rigid Body Motion | Deformation

As another example of rigid body motion versus deformation, consider the rubber sheet shown above. For reference we have drawn rectangular and polar grids on the sheet.

If you subject the sheet to a rigid body motion -- (go ahead and click the button) -- the grids remain undeformed.

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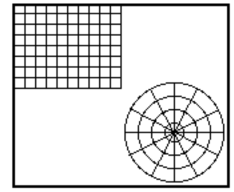
Deformation



Rigid Body Motion | Deformation

If, however, we fix the left edge of the sheet and then subject the sheet to the same loading, it will deform. This time the grids change

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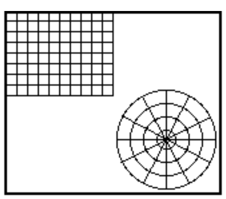
Rigid Body Rotation



Rigid Body Rotation | Deformation

It is important to note that rigid body motions include rotations as well as translations. Imagine we place the rubber sheet on a turntable and then apply the load...

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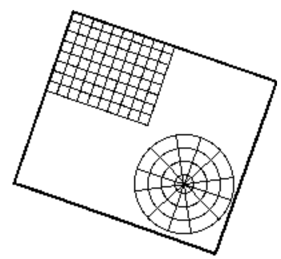
Rigid Body Rotation



As with the previous case of rigid body motion, the grids remain undeformed.

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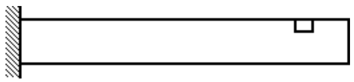
General Displacements



In general, displacement are a combination of rigid body motions and deformations. To see this, consider a block of material near the end of the cantilever beam above. Let's apply a load at the end of the beam...

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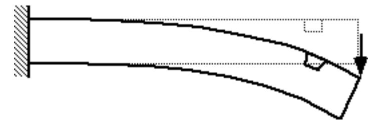
General Displacements



How can we describe the change in the block?

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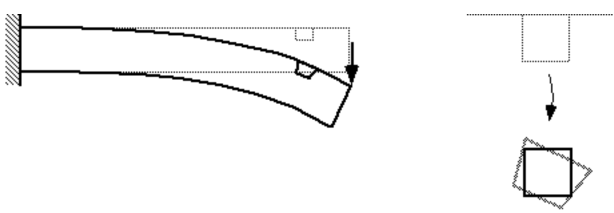
Rigid Body Translation



Translation

Part of the change was a rigid body translation of the block.

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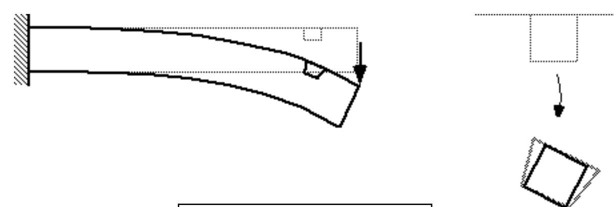
Rigid Body Rotation



Rotation

The block also experienced a rigid body

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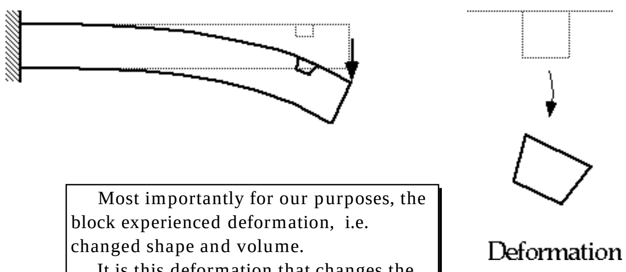
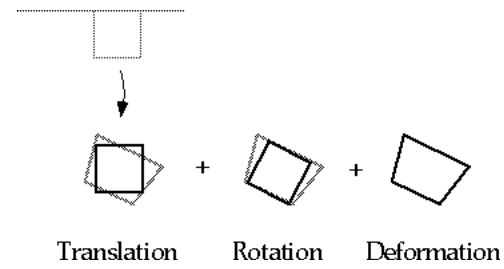
Deformation

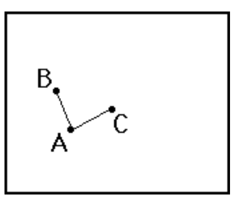


Most importantly for our purposes, the block experienced deformation, i.e. changed shape and volume.
It is this deformation that changes the material's internal state, causing stresses, cracks, failure, etc..

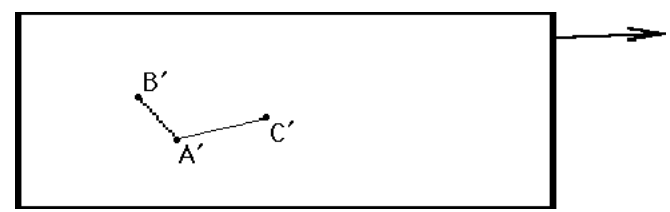
General Displacements



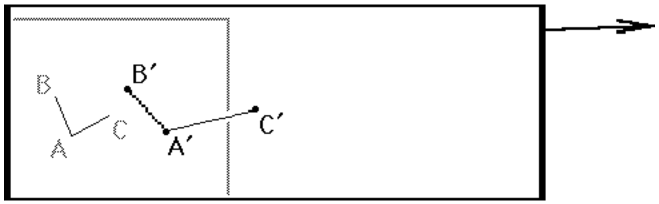
So we see that general displacements can be separated into a rigid body translation, a rigid body rotation, and a deformation.
Because we are studying the mechanics of <u>materials</u>, we will be interested in <u>deformations only</u> Rigid body motion is treated under a different subject known as dynamics or kinematics.



Since our focus is on the deformation experienced by materials, let's take a closer look at that case. Note that we have drawn two reference lines on our old friend, the rubber sheet. Now let's deform the sheet.



To see what has happened, let's superimpose the deformed sheet on the undeformed sheet...



....we can make two important observations. First, consider the lengths of the line segments.

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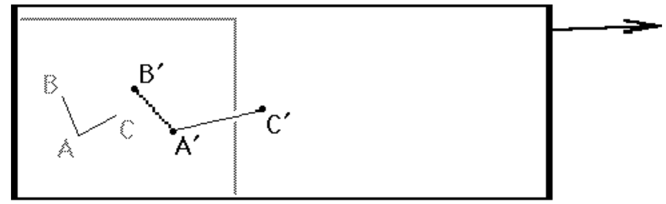

length A′C′ ≠ length AC

length A′B′ ≠ length AB

Note that the lengths have changed from the undeformed to the deformed sheet.

Second, consider the angle between the

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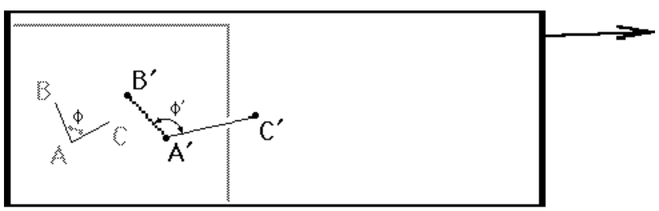

length A′C′ ≠ length AC

length A′B′ ≠ length AB

angle ϕ′ ≠ angle ϕ

The angle has also changed. Here is a timely question: If we want to fully describe the deformation of the rubber sheet, what factors should our description

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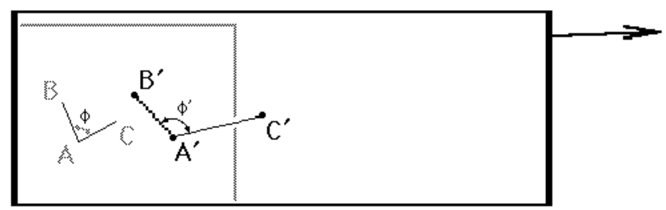

length A′C′ ≠ length AC

length A′B′ ≠ length AB

angle ϕ′ ≠ angle ϕ

This simple conclusion will be the basis for our detailed description of deformation.

A comprehensive measurement of deformation must account for both the <u>change in length of lines</u> drawn on the body, and the <u>change in angle</u> between them.

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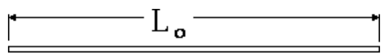
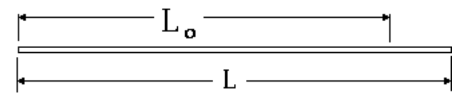
Change in Length -- Defining Average Normal Strain



Before we look at strain in depth, we will first consider some simple concepts of average strain.

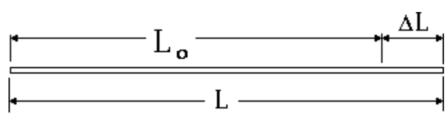
Consider first stretching or **normal** strain. The bar shown above is deformed by stretching...

Change in Length -- Defining Average Normal Strain



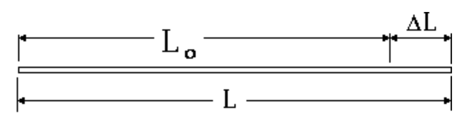
The final length is L. What we want to do is normalize this deformation, similar to our normalizing of forces in the case of stress.

Change in Length -- Defining Average Normal Strain



We accomplish this by quantifying the total change in length, ΔL, as shown, and then imagine this length change averaged over the entire length of the bar.

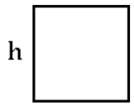
Change in Length -- Defining Average Normal Strain



The result is this simple definition for average normal strain. This definition is analogous to our earlier $\sigma = P / A$ result for normal stress. Note that the units of strain are dimensionless, i.e. Length / Length.

$$\varepsilon = \frac{\Delta L}{L}$$

Change in Angle -- Defining Average Shear Strain



Consider next the simple characterization of shape changing deformations: i.e. shear strain. In this case a block of material is sheared …

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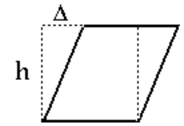
Change in Angle -- Defining Average Shear Strain



Note that the volume of the block does not change; only the shape has changed. To quantify this shape change, we identify the change in the 90° angles.

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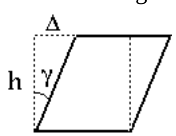
Change in Angle -- Defining Average Shear Strain



The angle change is the shear strain, γ, and can be computed easily in terms of Δ and h.

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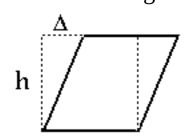
Change in Angle -- Defining Average Shear Strain



$$\gamma = \tan^{-1}\frac{\Delta}{h} \approx \frac{\Delta}{h}$$

As we will discuss in more detail later, we will always consider small strains, and so we can use the simplified expression without the inverse tangent.

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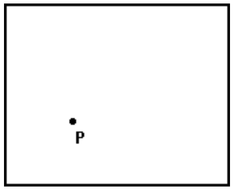
Calculating Shear Strain



Now that we have a basic idea about normal and shear strains, let's reconsider the rubber sheet. In particular, we will compute the shear strain at the point P. To do this we will draw two perpendicular reference lines...

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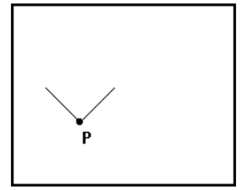
Calculating Shear Strain



Recall, that to compute the shear strain in the rubber sheet we simply calculate the angle change of the two reference lines after deformation.

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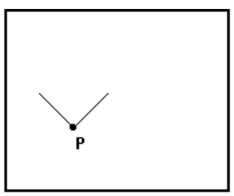
Calculating Shear Strain



Shear Strain = Original Angle − Deformed Angle

We begin our calculation of the shear strain by establishing the original angle between the reference lines..

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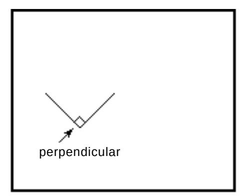
Calculating Shear Strain



Shear Strain = Original Angle − Deformed Angle

$\gamma = \quad \pi/2 \quad -$

In order to determine the deformed angle, we must first load the rubber sheet.

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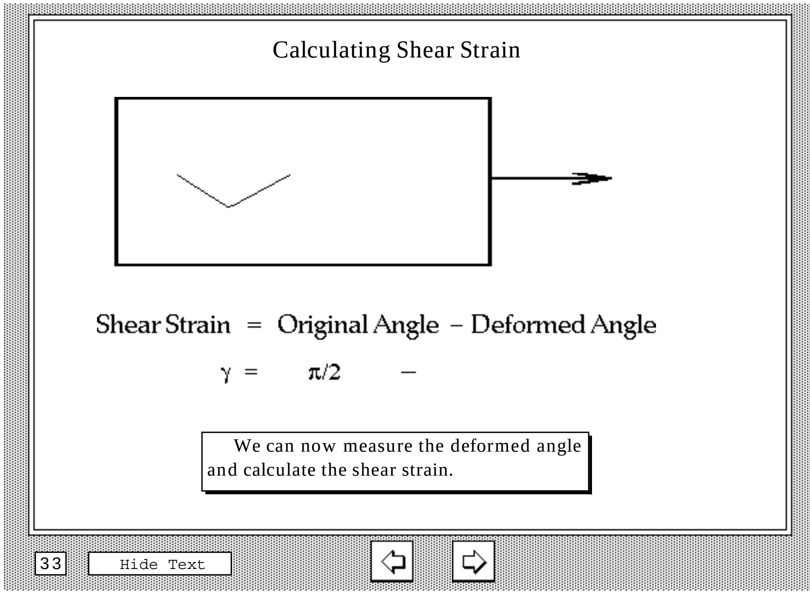
Calculating Shear Strain
Shear Strain = Original Angle − Deformed Angle
γ = π/2 −
We can now measure the deformed angle and calculate the shear strain.
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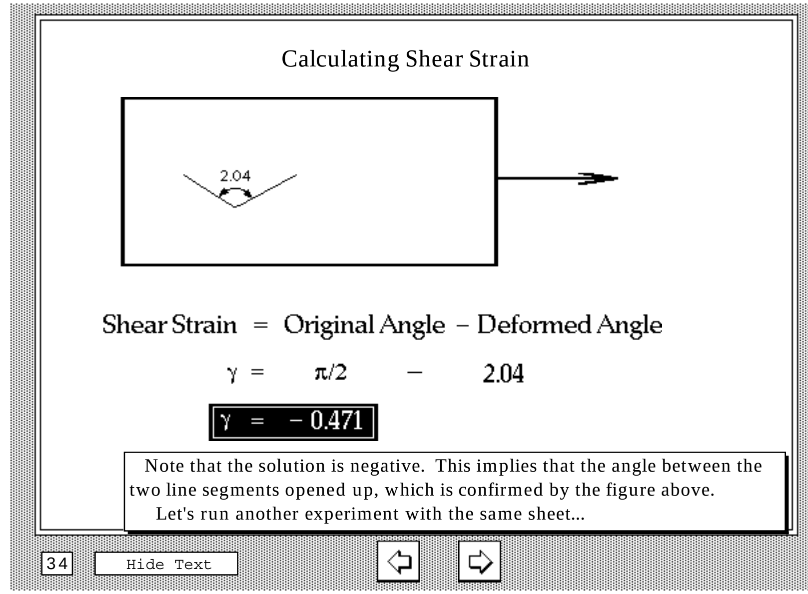
Calculating Shear Strain
2.04
Shear Strain = Original Angle − Deformed Angle
γ = π/2 − 2.04
γ = − 0.471
Note that the solution is negative. This implies that the angle between the two line segments opened up, which is confirmed by the figure above.
Let's run another experiment with the same sheet...
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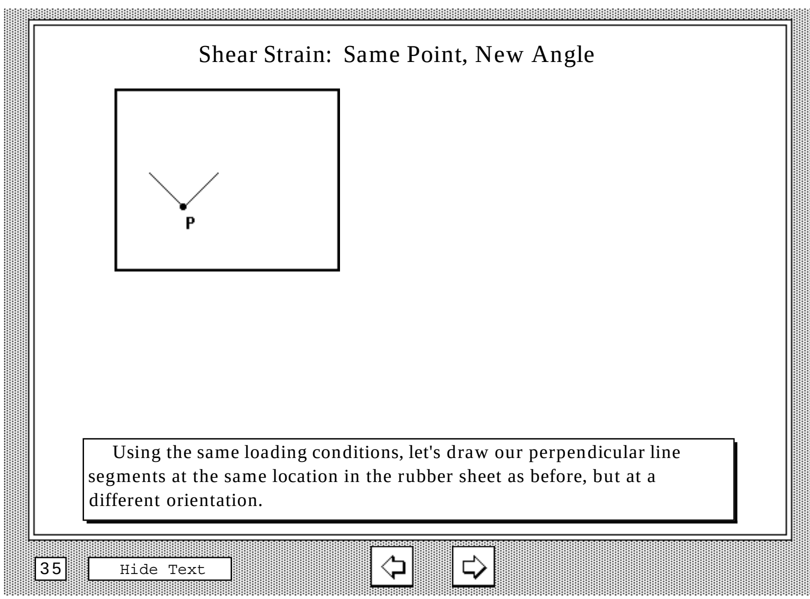
Shear Strain: Same Point, New Angle
P
Using the same loading conditions, let's draw our perpendicular line segments at the same location in the rubber sheet as before, but at a different orientation.
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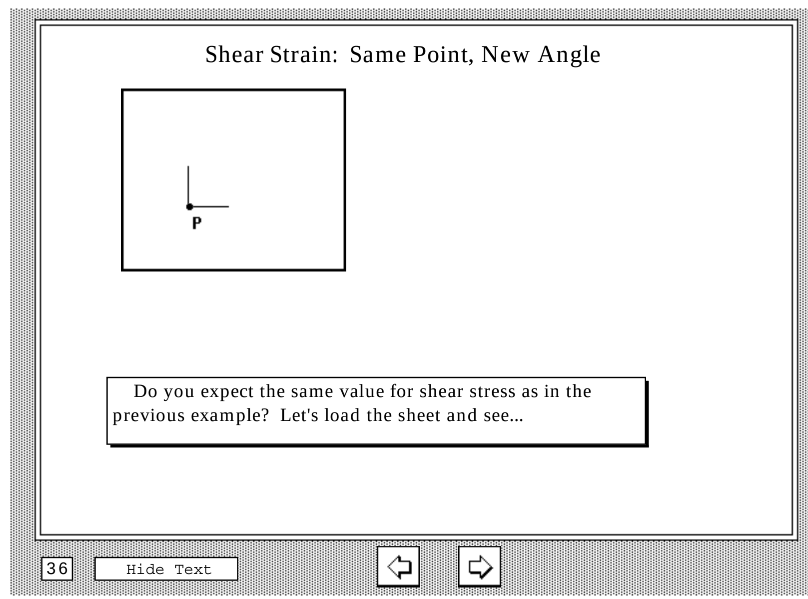
Shear Strain: Same Point, New Angle
P
Do you expect the same value for shear stress as in the previous example? Let's load the sheet and see...
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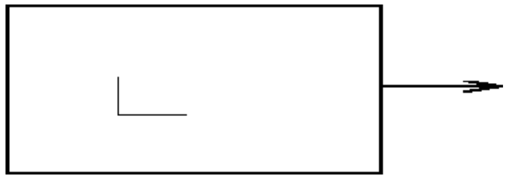
Shear Strain: Same Point, New Angle



How about that!? This time after deforming the sheet our reference lines are still perpendicular. Calculating the shear strain becomes trivial.

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Shear Strain: Same Point, New Angle

$$\text{Shear Strain} = \text{Original Angle} - \text{Deformed Angle}$$

Recall our expression for calculating shear strain. As before the original angle is $\pi/2$. However, in this case the deformed angle is also $\pi/2$ ...

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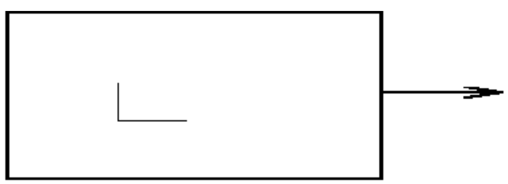
Shear Strain: Same Point, New Angle



$$\text{Shear Strain} = \text{Original Angle} - \text{Deformed Angle}$$

$$\gamma = \pi/2 - \pi/2$$

$$\gamma = 0$$

Yup, that's right, the shear strain now appears to be zero at the same point where it was previously non-zero..

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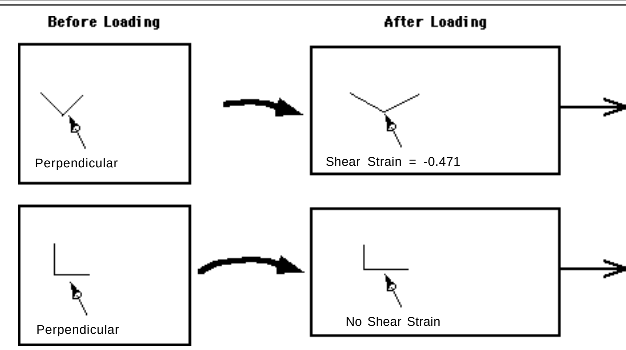

Let's review our recent discovery. When we drew the reference lines parallel to the edges of the rubber sheet we found that the shear strain was zero. Yet when the reference lines were oriented approximately 45° to the edges we calculated a shear strain of -0.471.

This means that the shear strain at a point varies with the orientation in which it is measured!! Let's investigate this further.

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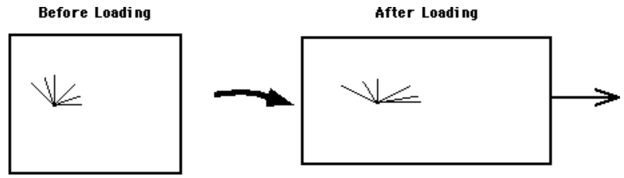

If the strains vary with orientation, then somehow we must check every possible orientation of our reference axes to determine maxima and minima. This is analogous to our experience with stress, and the motivation is similar: the behavior of many materials depends on how they are strained, and if we expect to predict how such materials will behave in applications, then we must be able to calculate the extremes of what the material experiences.

We will do a more detailed experiment with our rubber sheet to investigate the case of general orientations.

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An Experiment in Strain

In this experiment, we will draw several lines on the rubber sheet. Each line will start at the point where we wish to calculate the strain, and will be drawn $\pi/12$ radians counter-clockwise from the previous line.→ →

After loading, your job is to calculate the shear and normal strain for each line segment drawn on the rubber sheet.→

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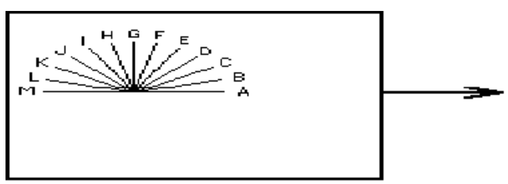
Calculating the Normal Strain



We begin our analysis of the deformed rubber sheet by calculating the normal strain at the point in the B direction. To acquire the data necessary to calculate this strain, click on the letter labeling line segment B. →

Recall the equation for normal strain.→ From the information provided by Gizmo Woman it is now possible to calculate the normal strain in direction B. → →

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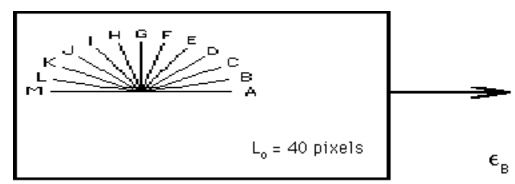
Calculating the Shear Strain



$$\epsilon_B = \frac{\Delta L}{L_o} = \frac{65-40}{40}$$

$$\epsilon_B = 0.625$$

To calculate the shear strain we must determine the new angle between two lines which were originally perpendicular. Recall our equation for calculating shear strain. → The line originally perpendicular to B is H. Therefore, the "New Angle" is the angle between lines B and H.→ The value of this new angle, α, can be calculated from the orientations of lines B and H. → → → Substituting the value of α back into the equation, we arrive at the value for shear strain in the B and H directions.→ →

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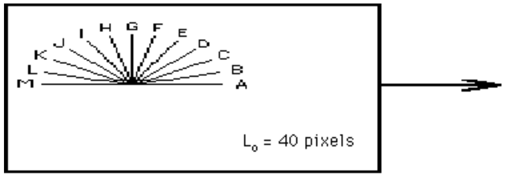
Calculating the Strains in the Remaining Directions



$$\epsilon = \frac{\Delta L}{L_o}$$

$$\gamma = \frac{\pi}{2} - \alpha$$

Your job is to calculate the normal and shear strains for the directions indicated by line segments A to G and enter them in the appropriate box at the right. Remember, the original length of each line segment was 40 pixels.

If you get stuck at any point, review the previous pages to see how the normal and shear strains were calculated for line segment B.

	ϵ	γ
A		
B	0.625	-0.3871
C		
D		
E		
F		
G		

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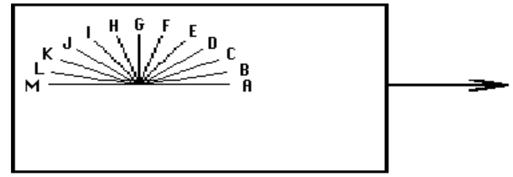
Analyzing the Strain Data



Let's continue analyzing the data you have just recorded.

To begin, we divide the shear strain by

	ϵ	γ
A	0.675	0.0
B	0.625	-0.387
C	0.4825	-0.671
D	0.2875	-0.775
E	0.095	-0.671
F	-0.047	-0.387
G	-0.10	0.0

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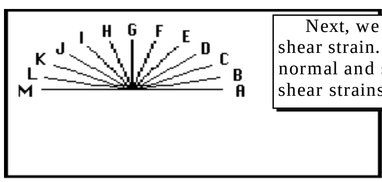
Plotting Normal Strain vs Shear Strain



Next, we plot the normal strain vs. the shear strain. We begin by plotting the normal and shear axes.→ Note that positive shear strains are plotted downward.

	ϵ	$\gamma/2$
A	0.675	0.0
B	0.625	-0.1936
C	0.4825	-0.3356
D	0.2875	-0.3875
E	0.095	-0.3356
F	-0.047	-0.1936
G	-0.10	0.0

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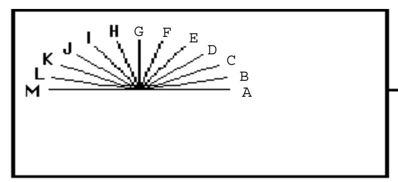
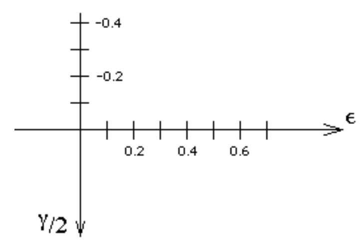
Plotting Normal Strain vs Shear Strain

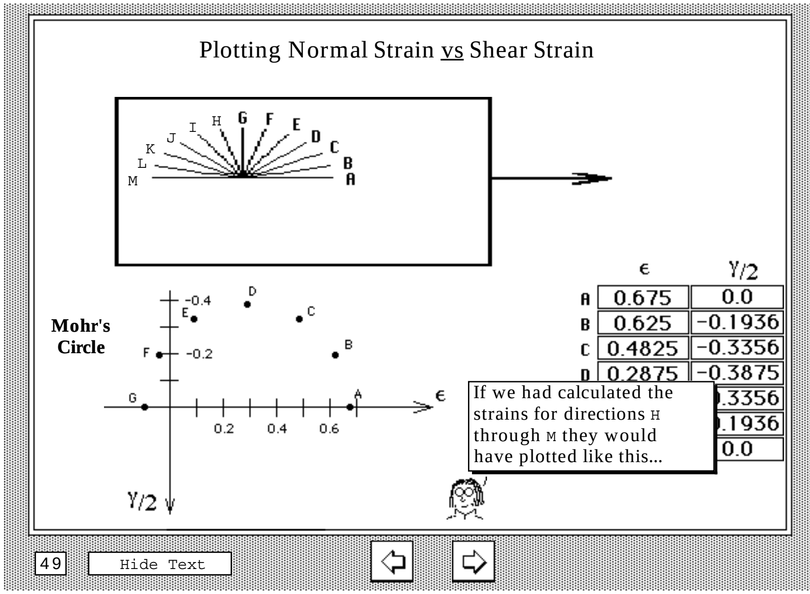


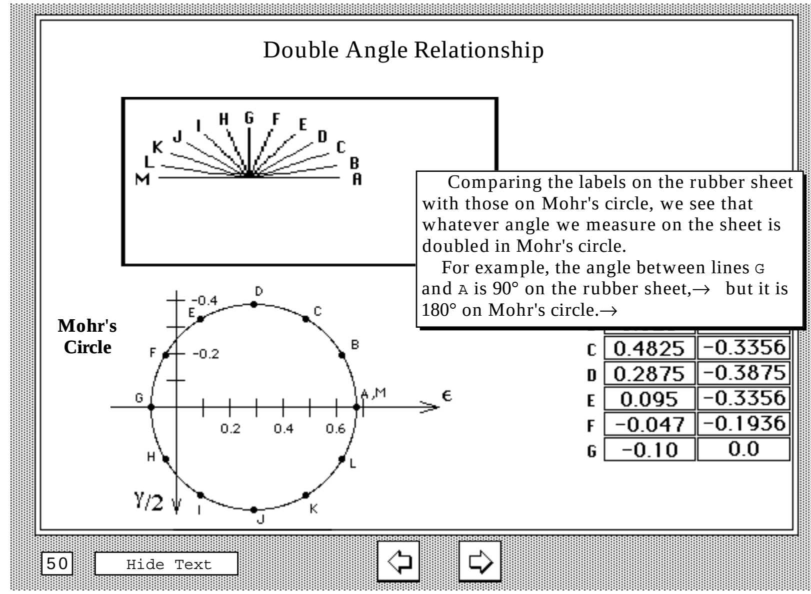
Let's now plot points A through G on the graph we have just created.

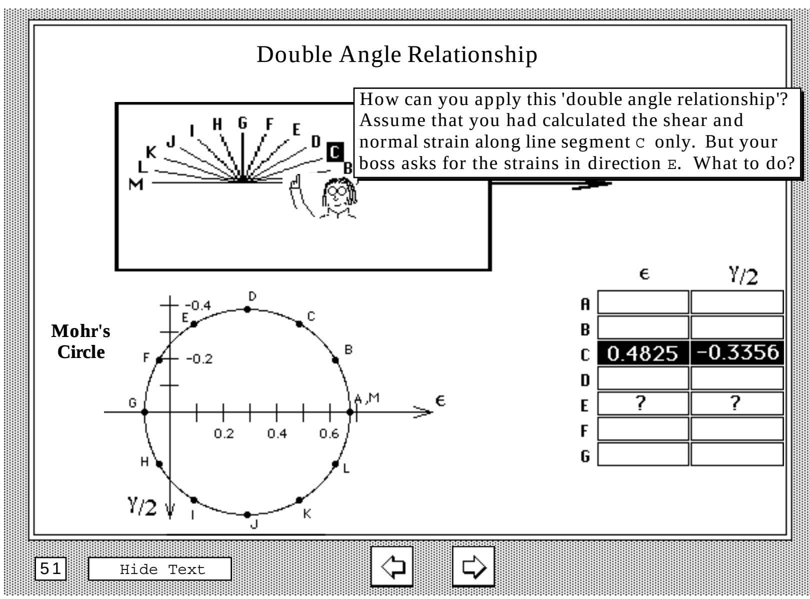


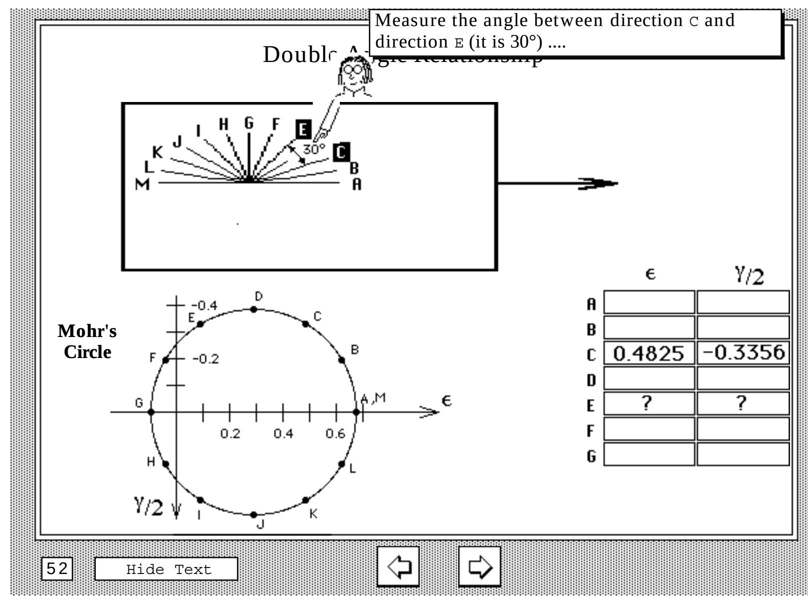
	ϵ	$\gamma/2$
A	0.675	0.0
B	0.625	-0.1936
C	0.4825	-0.3356
D	0.2875	-0.3875
E	0.095	-0.3356
F	-0.047	-0.1936
G	-0.10	0.0

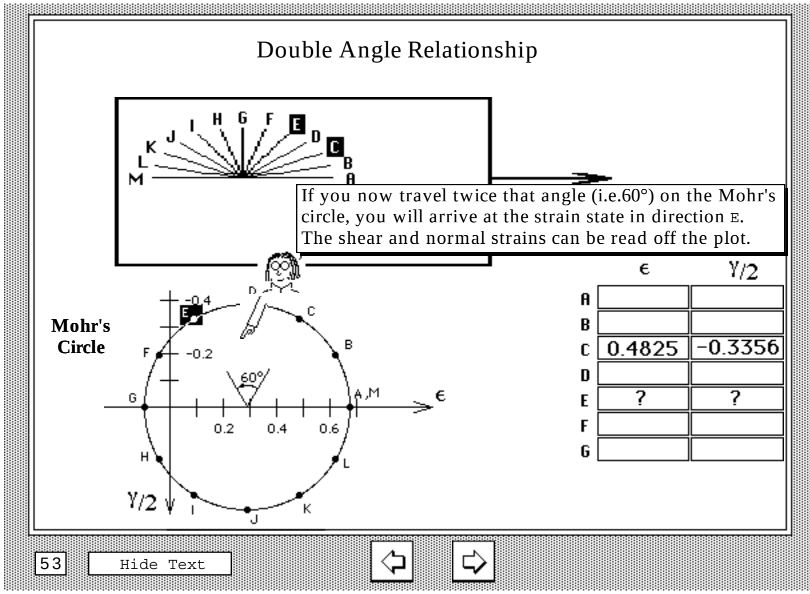
48 Hide Text

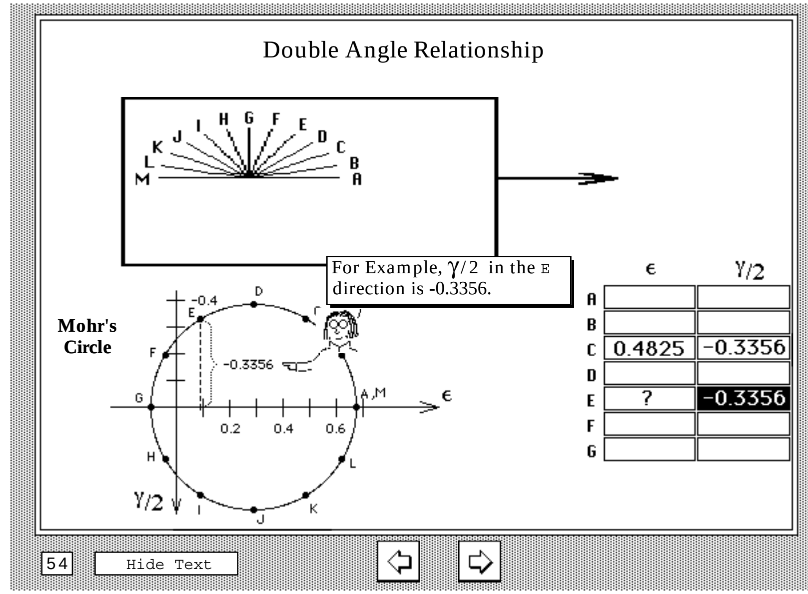

Plotting Normal Strain vs Shear Strain
Mohr's Circle
-0.4
-0.2
0.2
0.4
0.6
ϵ
γ/2
A 0.675 0.0
B 0.625 -0.1936
C 0.4825 -0.3356
D 0.2875 -0.3875
If we had calculated the strains for directions H through M they would have plotted like this...
49
Hide Text

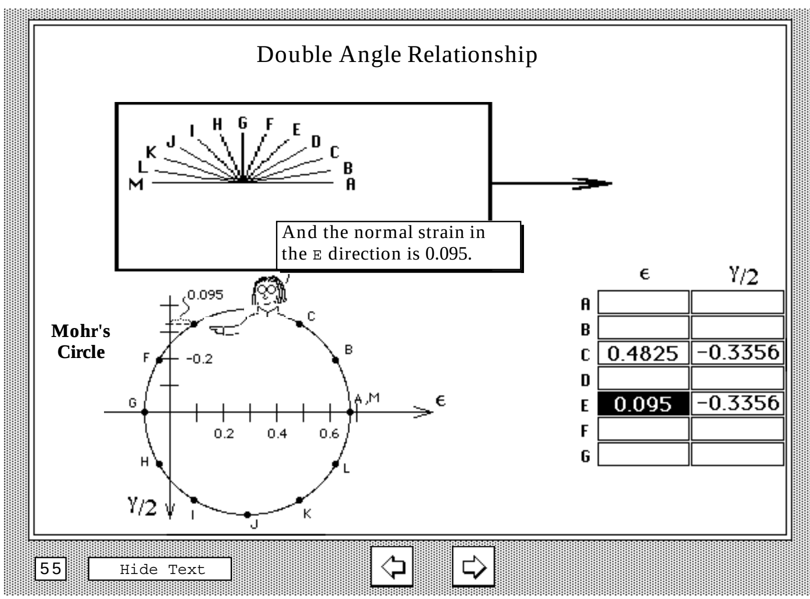

Double Angle Relationship
Comparing the labels on the rubber sheet with those on Mohr's circle, we see that whatever angle we measure on the sheet is doubled in Mohr's circle.
For example, the angle between lines G and A is 90° on the rubber sheet,→ but it is 180° on Mohr's circle.→
Mohr's Circle
C 0.4825 -0.3356
D 0.2875 -0.3875
E 0.095 -0.3356
F -0.047 -0.1936
G -0.10 0.0
50
Hide Text

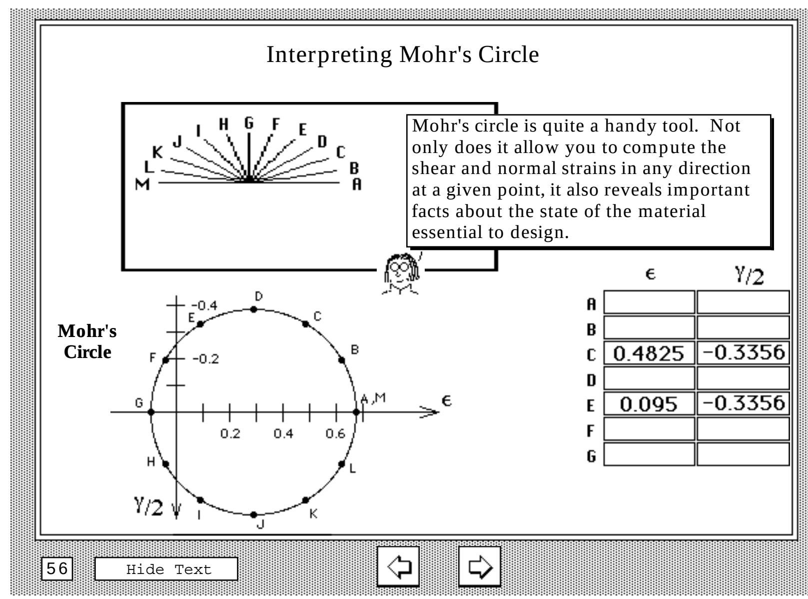

Double Angle Relationship
How can you apply this 'double angle relationship'? Assume that you had calculated the shear and normal strain along line segment C only. But your boss asks for the strains in direction E. What to do?
Mohr's Circle
ϵ
γ/2
C 0.4825 -0.3356
E ? ?
51
Hide Text

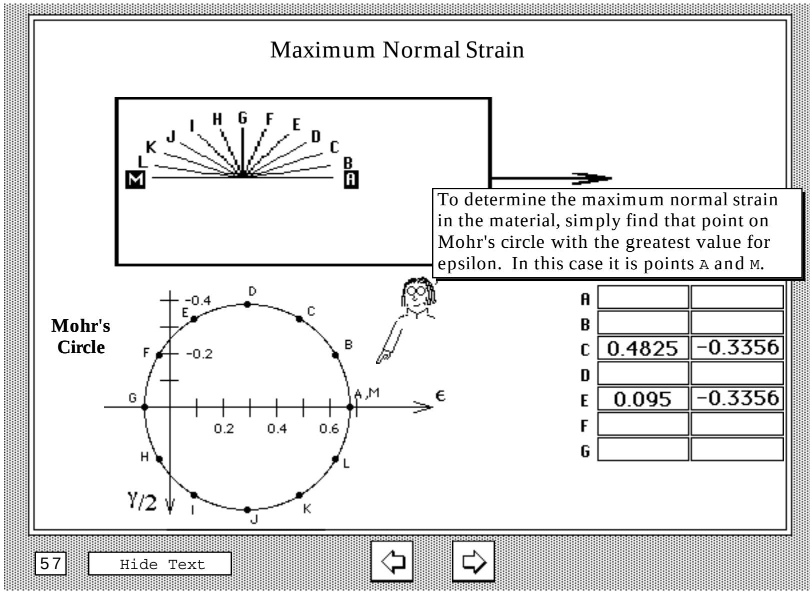

Double Angle Relationship
Measure the angle between direction C and direction E (it is 30°) ....
30°
Mohr's Circle
ϵ
γ/2
C 0.4825 -0.3356
E ? ?
52
Hide Text

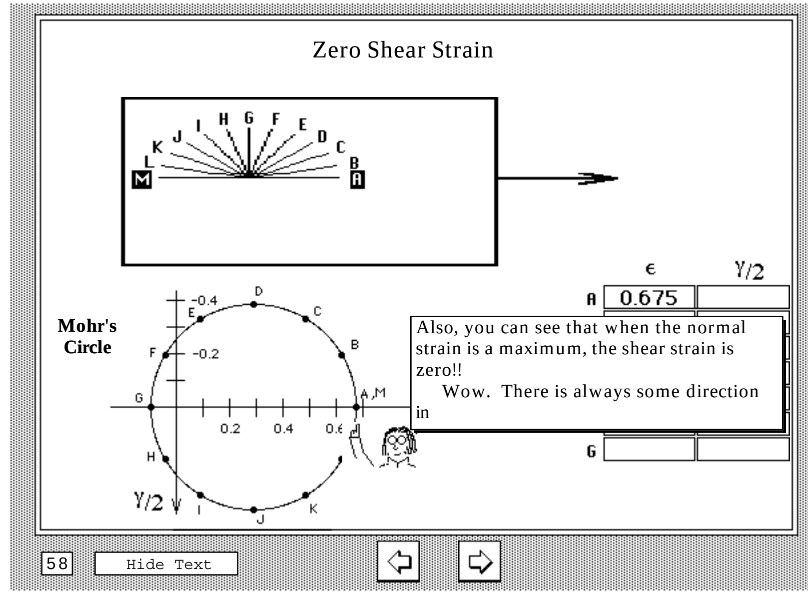

Double Angle Relationship
If you now travel twice that angle (i.e.60°) on the Mohr's circle, you will arrive at the strain state in direction E. The shear and normal strains can be read off the plot.
Mohr's Circle
60°
ϵ
γ/2
C 0.4825 -0.3356
E ? ?
53
Hide Text

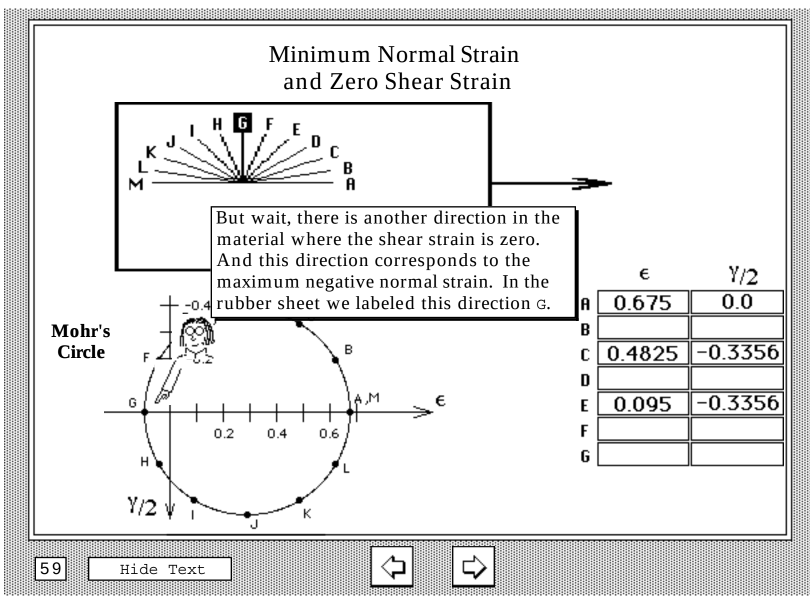

Double Angle Relationship
For Example, γ/2 in the E direction is -0.3356.
Mohr's Circle
-0.3356
ϵ
γ/2
C 0.4825 -0.3356
E ? -0.3356
54
Hide Text

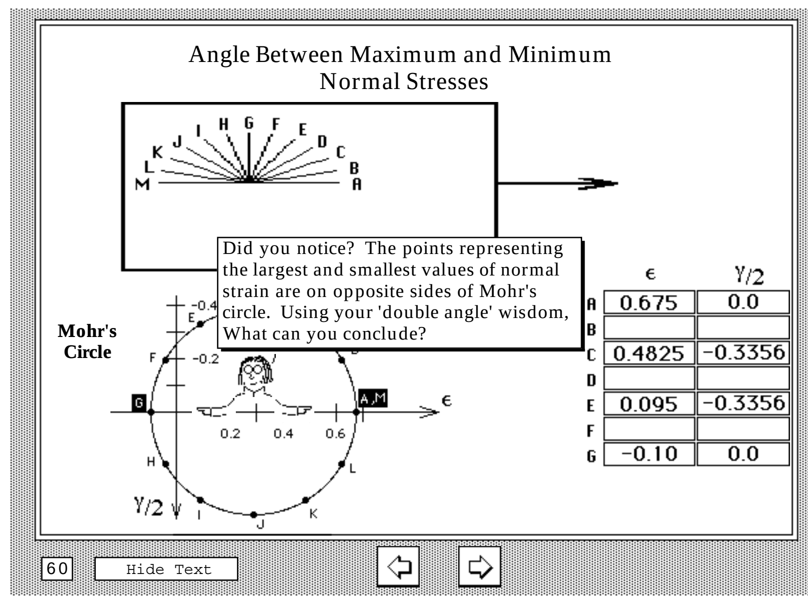

Double Angle Relationship
And the normal strain in the E direction is 0.095.
Mohr's Circle
0.095
ϵ
γ/2
C 0.4825 -0.3356
E 0.095 -0.3356
55
Hide Text

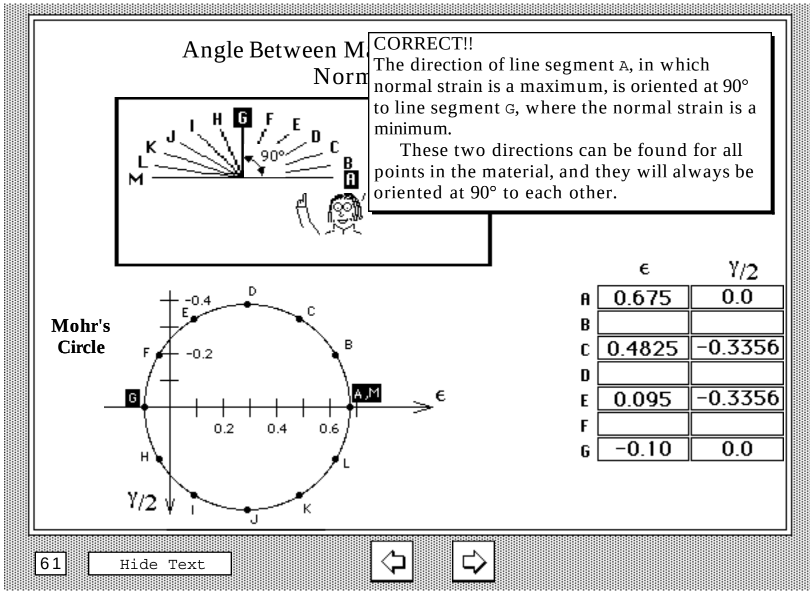

Interpreting Mohr's Circle
Mohr's circle is quite a handy tool. Not only does it allow you to compute the shear and normal strains in any direction at a given point, it also reveals important facts about the state of the material essential to design.
Mohr's Circle
ϵ
γ/2
C 0.4825 -0.3356
E 0.095 -0.3356
56
Hide Text

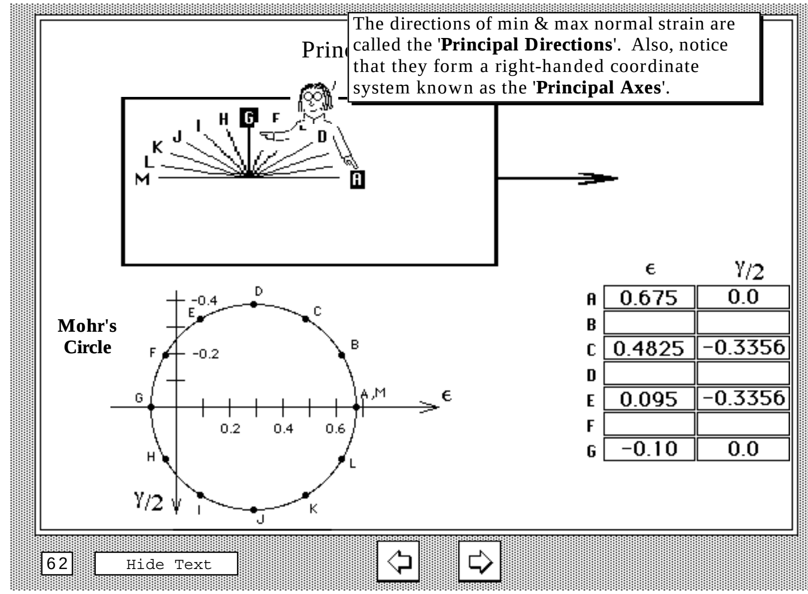

Maximum Normal Strain
To determine the maximum normal strain in the material, simply find that point on Mohr's circle with the greatest value for epsilon. In this case it is points A and M.
Mohr's Circle
C 0.4825 -0.3356
E 0.095 -0.3356
57
Hide Text

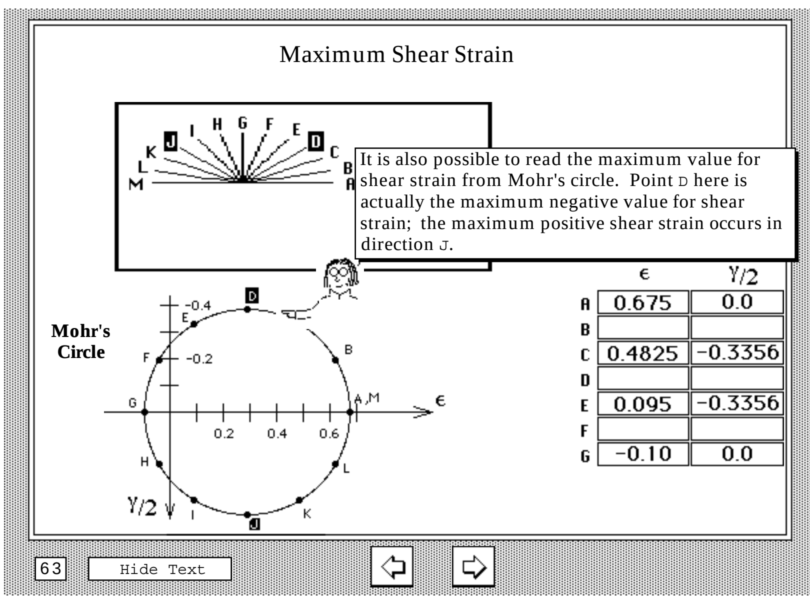

Zero Shear Strain
Also, you can see that when the normal strain is a maximum, the shear strain is zero!!
Wow. There is always some direction in
Mohr's Circle
ϵ γ/2
A 0.675
58
Hide Text

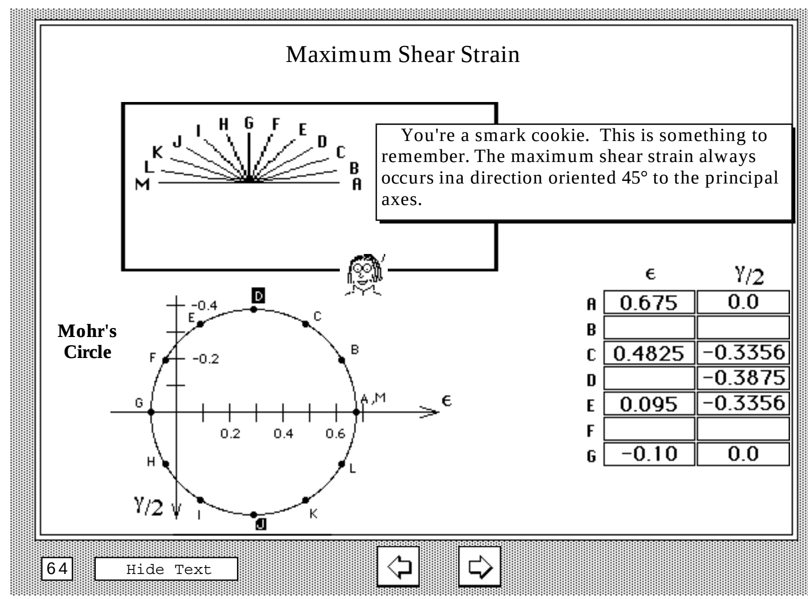

Minimum Normal Strain and Zero Shear Strain
But wait, there is another direction in the material where the shear strain is zero. And this direction corresponds to the maximum negative normal strain. In the rubber sheet we labeled this direction G.
Mohr's Circle
ϵ γ/2
A 0.675 0.0
C 0.4825 -0.3356
E 0.095 -0.3356
59
Hide Text


Angle Between Maximum and Minimum Normal Stresses
Did you notice? The points representing the largest and smallest values of normal strain are on opposite sides of Mohr's circle. Using your 'double angle' wisdom, What can you conclude?
Mohr's Circle
ϵ γ/2
A 0.675 0.0
C 0.4825 -0.3356
E 0.095 -0.3356
G -0.10 0.0
60
Hide Text

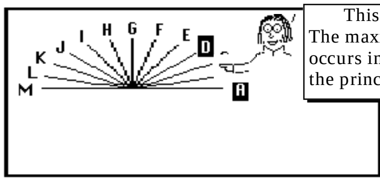

Angle Between M
Norm
CORRECT!!
The direction of line segment A, in which normal strain is a maximum, is oriented at 90° to line segment G, where the normal strain is a minimum.
These two directions can be found for all points in the material, and they will always be oriented at 90° to each other.
90°
Mohr's Circle
ϵ γ/2
A 0.675 0.0
B
C 0.4825 -0.3356
D
E 0.095 -0.3356
F
G -0.10 0.0
61
Hide Text


Prin
The directions of min & max normal strain are called the 'Principal Directions'. Also, notice that they form a right-handed coordinate system known as the 'Principal Axes'.
Mohr's Circle
ϵ γ/2
A 0.675 0.0
B
C 0.4825 -0.3356
D
E 0.095 -0.3356
F
G -0.10 0.0
62
Hide Text

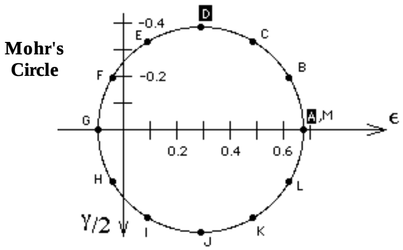

Maximum Shear Strain
It is also possible to read the maximum value for shear strain from Mohr's circle. Point D here is actually the maximum negative value for shear strain; the maximum positive shear strain occurs in direction J.
Mohr's Circle
ϵ γ/2
A 0.675 0.0
B
C 0.4825 -0.3356
D
E 0.095 -0.3356
F
G -0.10 0.0
63
Hide Text


Maximum Shear Strain
You're a smark cookie. This is something to remember. The maximum shear strain always occurs ina direction oriented 45° to the principal axes.
Mohr's Circle
ϵ γ/2
A 0.675 0.0
B
C 0.4825 -0.3356
D -0.3875
E 0.095 -0.3356
F
G -0.10 0.0
64
Hide Text

Maximum Shear Strain vs Principal Directions



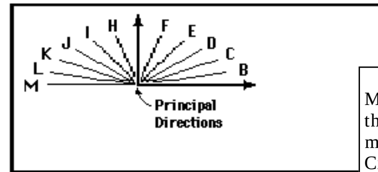
This is something to remember. The maximum shear strain always occurs in a direction oriented 45° to the principal axes.

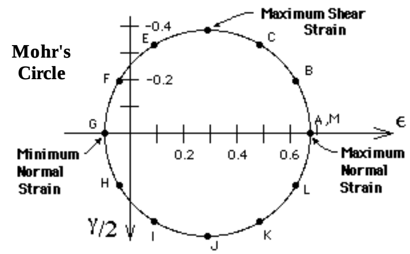


	ϵ	γ/2
A	0.675	0.0
B		
C	0.4825	-0.3356
D		-0.3875
E	0.095	-0.3356
F		
G	-0.10	0.0

65 Hide Text Undeformed Configuration

A Brief Review



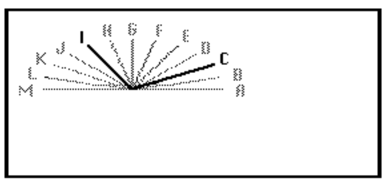


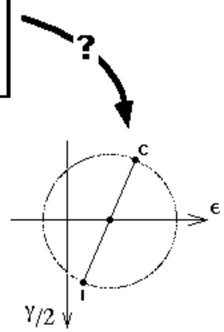
What have we learned about Mohr's Circle so far? We can read the values for maximum and minimum normal strain from Mohr's Circle. The directions of maximum and minimum normal strain form the principal directions in the material, and in these directions the shear strains are zero.

Further, we can also read the values for the maximum shear stress from the circle. The maximum shear strains (positive and negative) are oriented at 45° from the principal directions in the material.

66 Hide Text

Constructing Mohr's Circle





As in the case of stress, to construct Mohr's Circle we only need to know the normal and shear strains associated with any two orthogonal directions.

If you need a refresher on the process involved in constructing the circle, click on the button below.

67 Hide Text Constructing Mohr's Circl

Summary

In this stack we have presented the basic definitions relevant to describing internal deformations in solids. Normal strains measure stretching, while shear strains measure angle changes.

We have also seen how similar strain is to stress from the perspective of transformations. We will explore this further in Strain II.

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