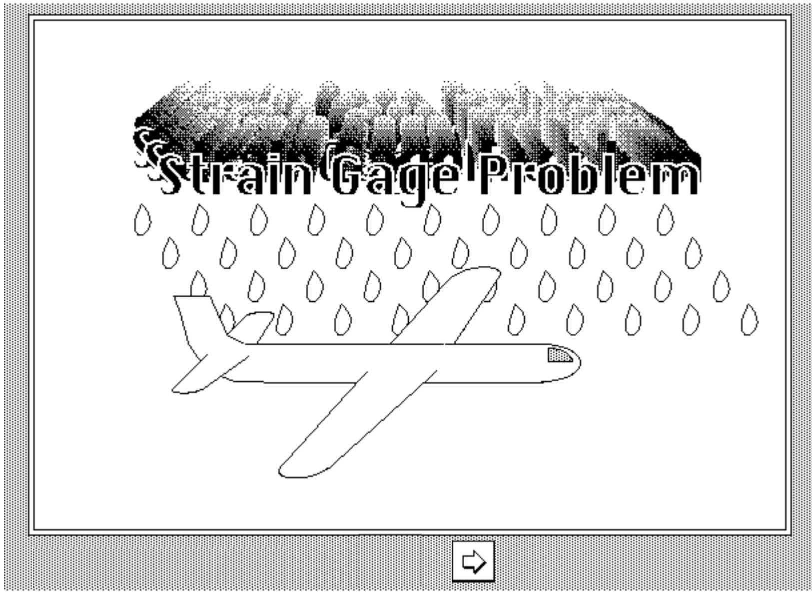
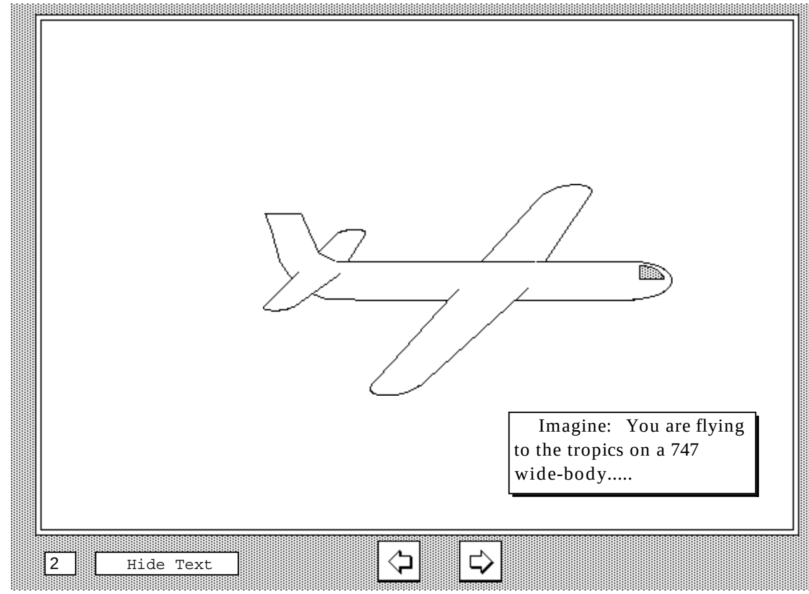
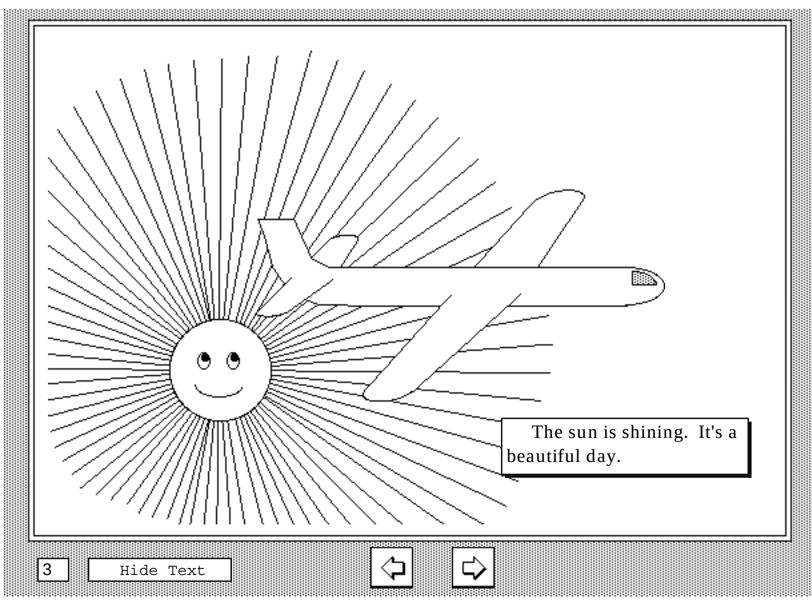
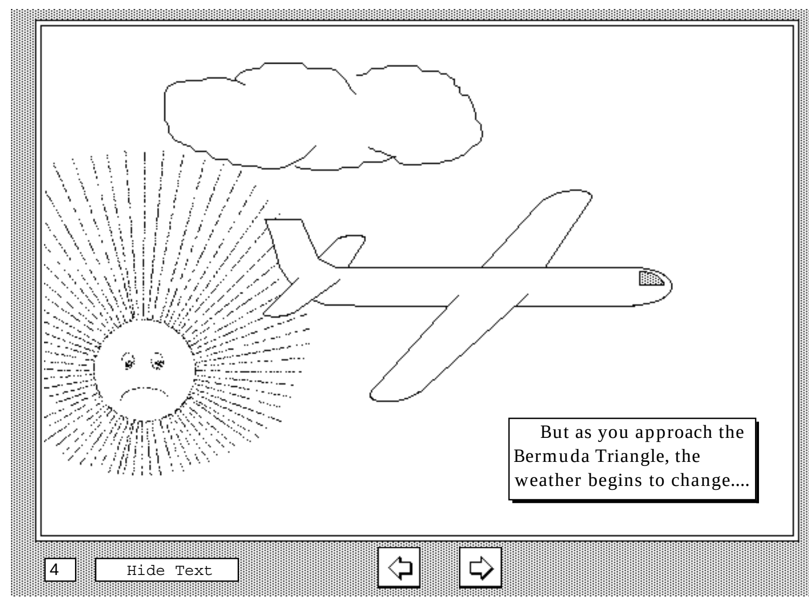
Strain Gage Problem: 1

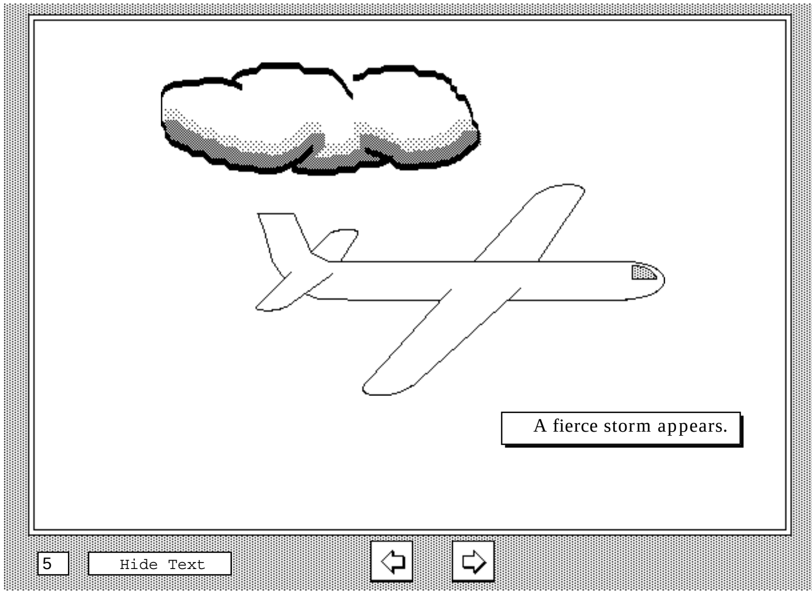


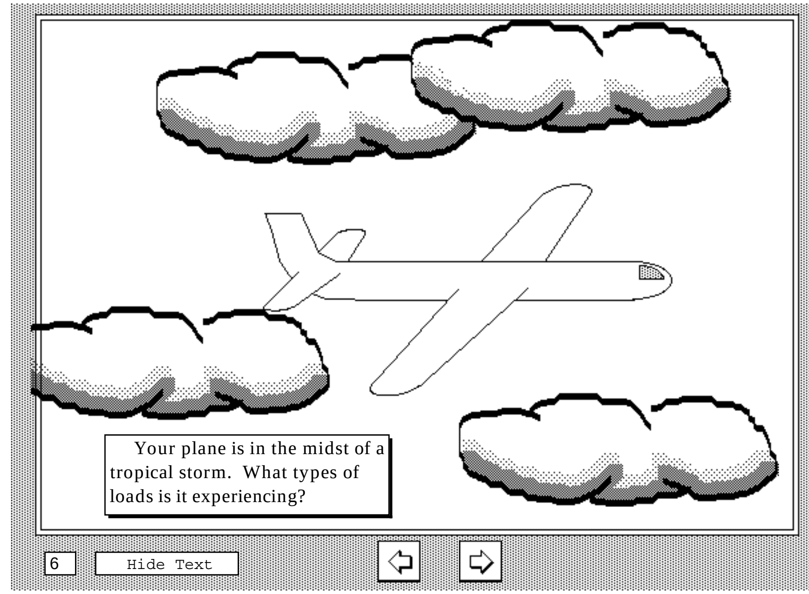


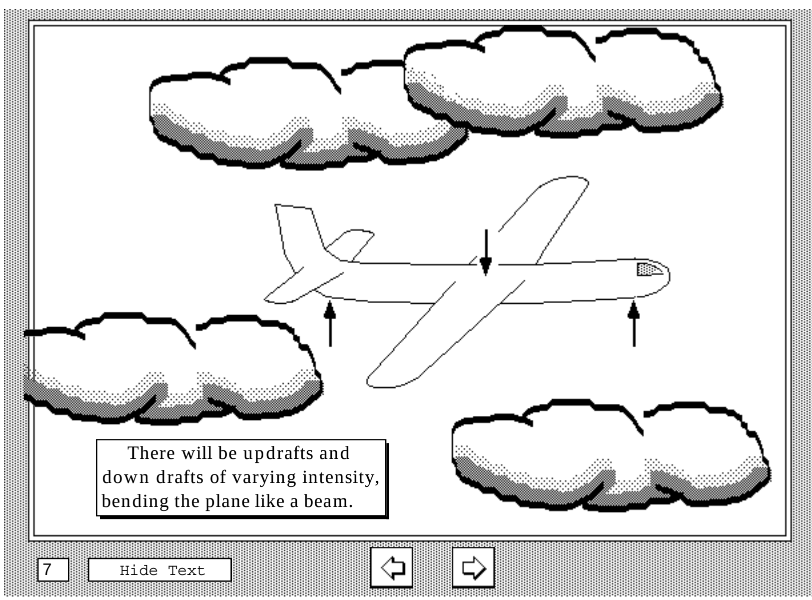


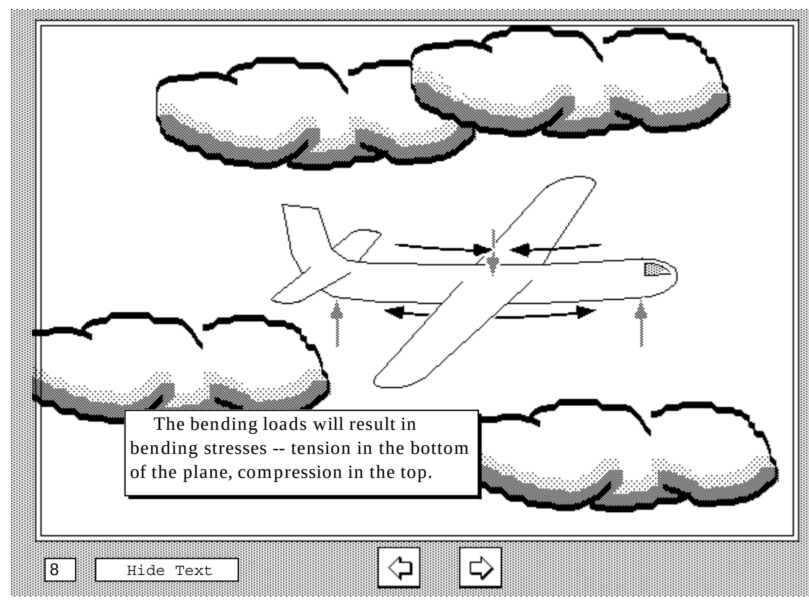


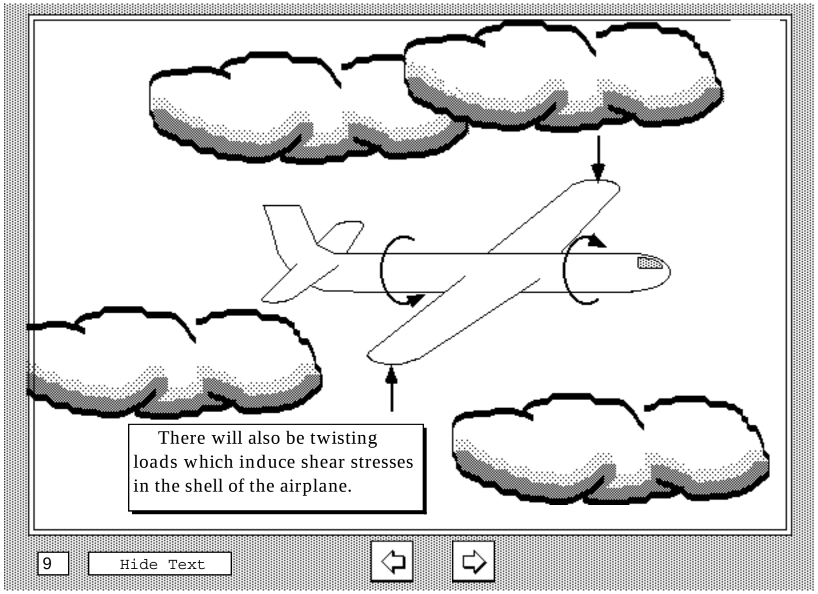
Strain Gage Problem: 2










There will also be twisting loads which induce shear stresses in the shell of the airplane.
9
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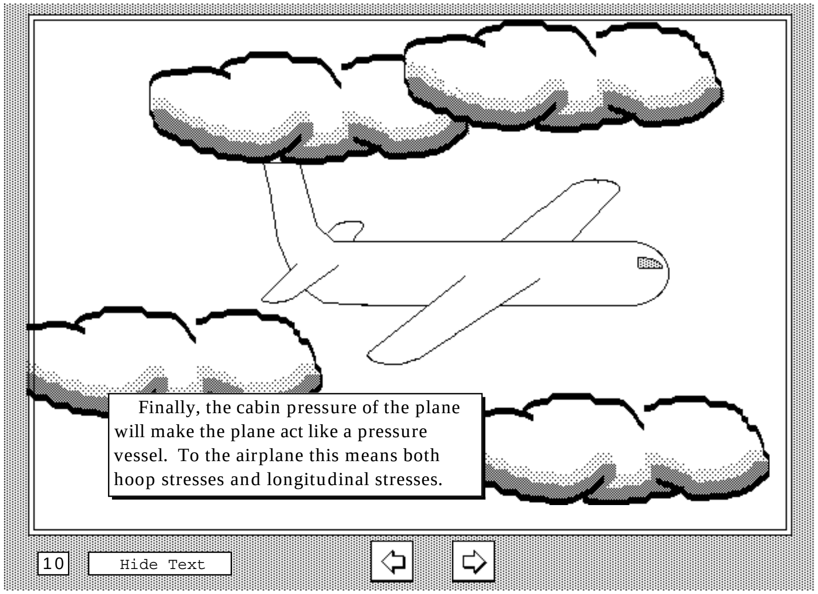
Finally, the cabin pressure of the plane will make the plane act like a pressure vessel. To the airplane this means both hoop stresses and longitudinal stresses.
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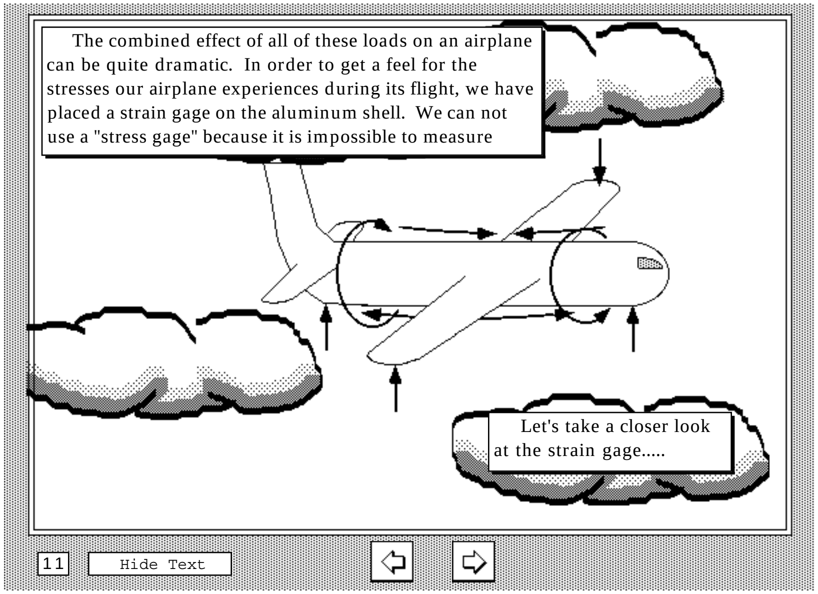
The combined effect of all of these loads on an airplane can be quite dramatic. In order to get a feel for the stresses our airplane experiences during its flight, we have placed a strain gage on the aluminum shell. We can not use a "stress gage" because it is impossible to measure
Let's take a closer look at the strain gage.....
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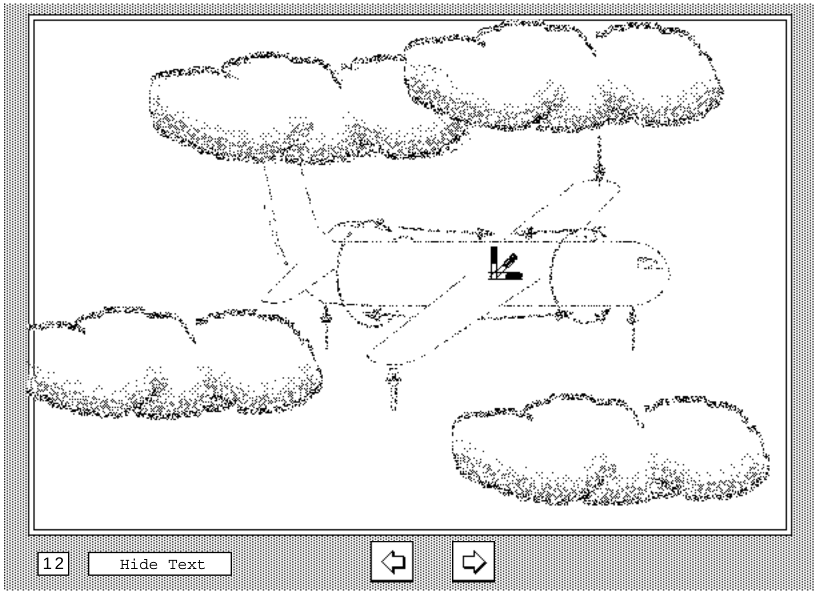
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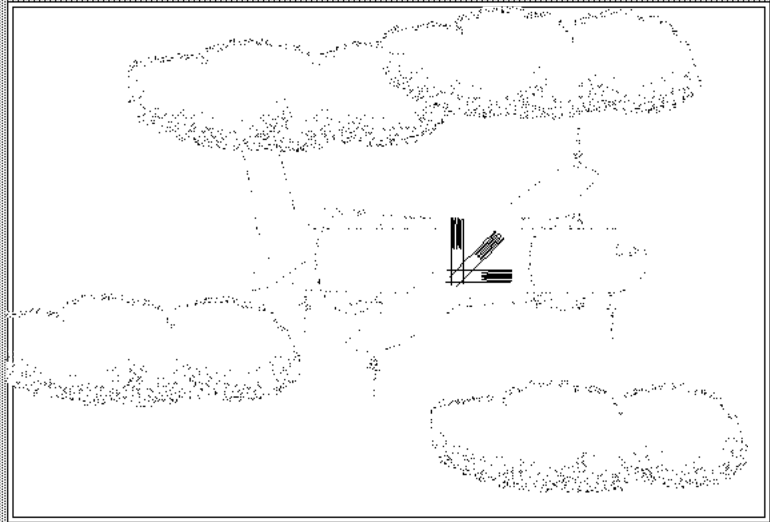

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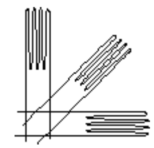
A Strain Gage Rosette

Our actual strain measuring device consists of three strain gages. Strain gages work on the principal that the resistance of a wire changes with the wire's length. If we stretch a wire, the resistance increases. We measure the change in resistance using a circuit known as a Wheatstone Bridge. Knowing the relationship between change in length and change in resistance for the strain gage material, we can calculate the stretch of our airplane, to which the gage is bonded.

Strain gages can only measure normal strain or "stretching" in the material. Can you guess why we have chosen this configuration of strain gages?



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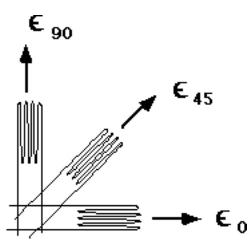
A 45° Strain Gage Rosette



We have oriented the strain gages at 0°, 45° and 90°. These three strain gages allow us to measure normal strain the corresponding directions. From this information it is possible to calculate all three components for two-dimensional strain: ε_x, ε_y, and ε_{xy}.

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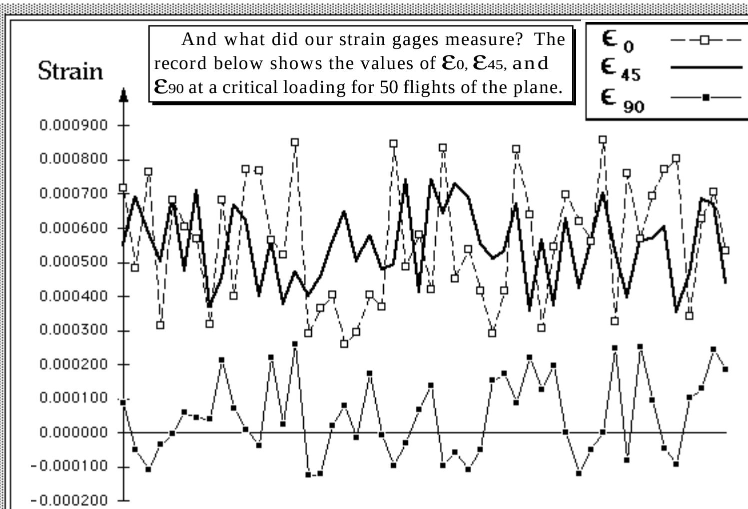
And what did our strain gages measure? The record below shows the values of ε_0, ε_{45}, and ε_{90} at a critical loading for 50 flights of the plane.



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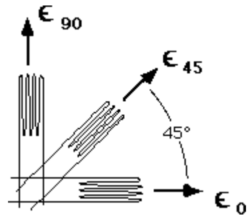

We just stated that one can to calculate ε_x, ε_y, and ε_{xy} from the measured normal strains, ε_0, ε_{45}, and ε_{90}. How is this done?

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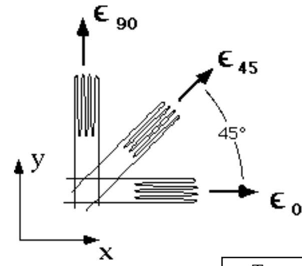

$\epsilon_x = \epsilon_0$

ϵ_{xy} ? ϵ_{45}

$\epsilon_y = \epsilon_{90}$

To make it easy on ourselves, we will assume that the x-y axes line up with the 0° and 90° strain gages. We can now calculate ε_x and ε_y directly. How do we calculate the shear strain ε_{xy}? Do not make the mistake of trying to relate it directly to the normal strain ε_{45}.

This seems like a good time to recall our strain transformation equations.

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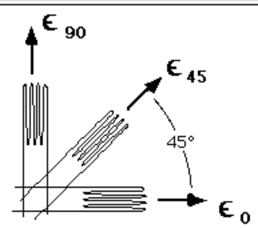

$\epsilon_x = \epsilon_0$

ϵ_{xy} ? ϵ_{45}

$\epsilon_y = \epsilon_{90}$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \epsilon_{xy}\sin 2\theta$$

For our purposes we will need only one of the transformation equations. Look closely at the equation. We already know the values of ε_x and ε_y. If we choose θ to be 45° then $\varepsilon_{x'}$ is equal to ε_{45}. The only unknown in the equation is ε_{xy}. One equation, one unknown. Isn't that nice.

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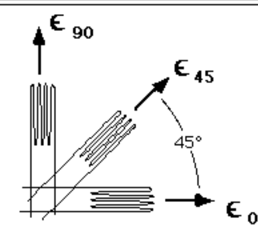

$\epsilon_x = \epsilon_0$

ϵ_{xy} ? ϵ_{45}

$\epsilon_y = \epsilon_{90}$

$\theta = 45°$

$$\epsilon_{45} = \frac{\epsilon_0 + \epsilon_{90}}{2} + \frac{\epsilon_0 - \epsilon_{90}}{2}\underbrace{\cos 2\theta}_{0} + \epsilon_{xy}\underbrace{\sin 2\theta}_{1}$$

We can simplify our equation by noting that when θ is 45°, $\sin 2\theta = 1$ and $\cos 2\theta = 0$.

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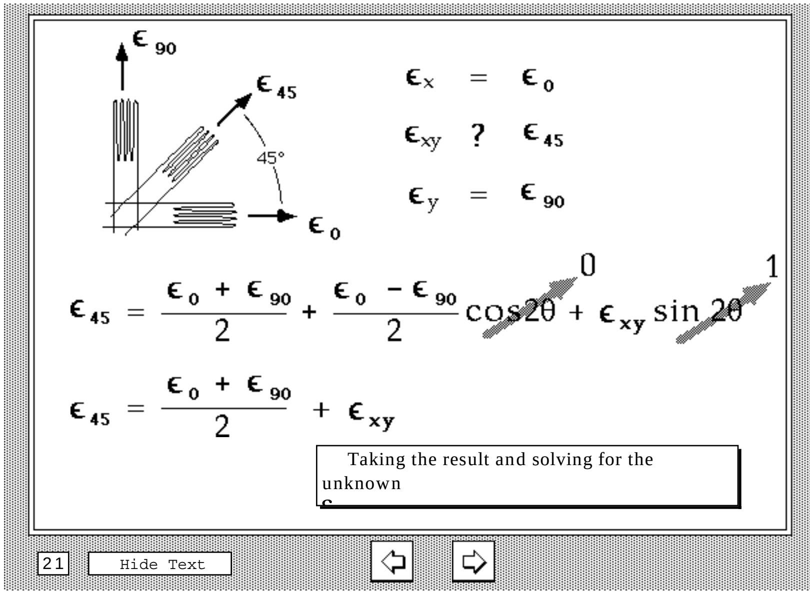
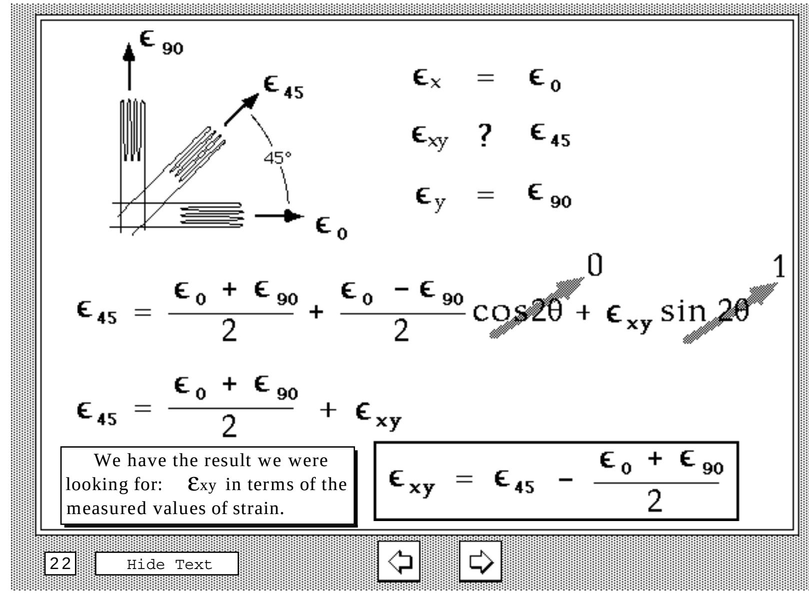
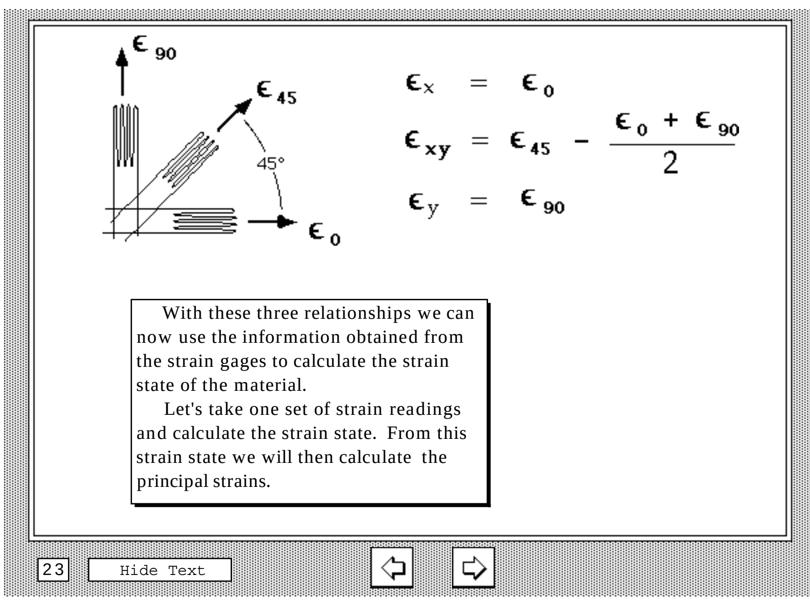
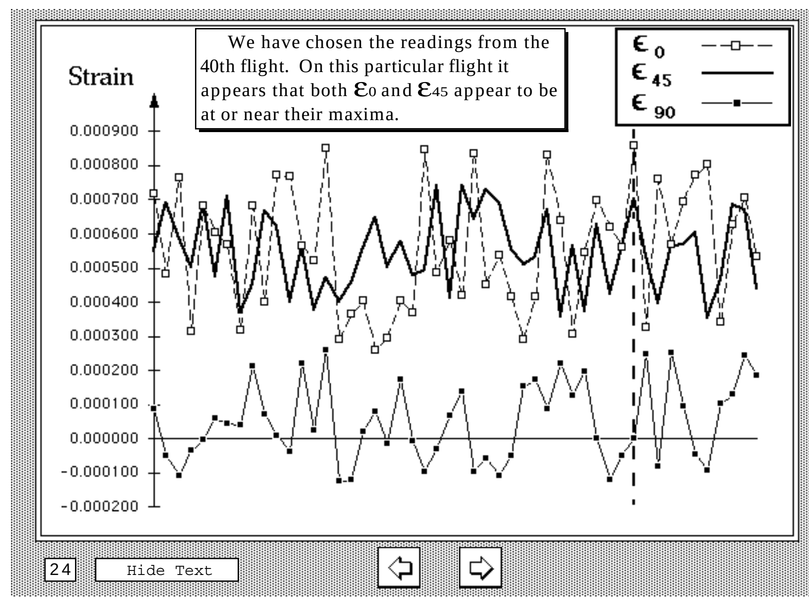
Strain Gage Problem: 6

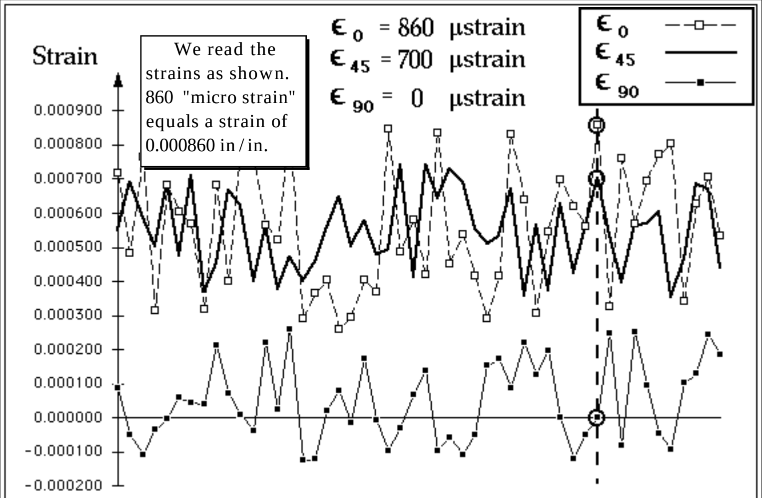








ϵ_0 = 860 μstrain
ϵ_{45} = 700 μstrain
ϵ_{90} = 0 μstrain



We read the strains as shown. 860 "micro strain" equals a strain of 0.000860 in / in.

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ϵ_0 = 860 μstrain
ϵ_{45} = 700 μstrain
ϵ_{90} = 0 μstrain

$$\epsilon_x = \epsilon_0$$
$$\epsilon_{xy} = \epsilon_{45} - \frac{\epsilon_0 + \epsilon_{90}}{2}$$
$$\epsilon_y = \epsilon_{90}$$

Using our latest conquest -- the strain conversion equations for a 45° strain rosette --we can calculate the three strains ε_x, ε_y, and ε_{xy}.

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ϵ_0 = 860 μstrain
ϵ_{45} = 700 μstrain
ϵ_{90} = 0 μstrain

$$\epsilon_x = \epsilon_0 = 860\,\mu\text{strain}$$
$$\epsilon_{xy} = \epsilon_{45} - \frac{\epsilon_0 + \epsilon_{90}}{2} = 270\,\mu\text{strain}$$
$$\epsilon_y = \epsilon_{90} = 0\ \mu\text{strain}$$

That was easy enough. Now how about calculating the principal strains?

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ϵ_x = 860μstrain
ϵ_{xy} = 270μstrain
ϵ_y = 0 μstrain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x + \epsilon_y}{2}\right)^2 - \left(\epsilon_x \epsilon_y - \epsilon_{xy}^2\right)}$$

Recall our equation for calculating principal stresses and principal strains in two-dimensions. Finding the principal strains is as easy as plugging in the numbers and chugging through the calculations.

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$\epsilon_x = 860\,\mu\text{strain}$

$\epsilon_{xy} = 270\,\mu\text{strain}$

$\epsilon_y = 0\ \mu\text{strain}$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x + \epsilon_y}{2}\right)^2 - \left(\epsilon_x\,\epsilon_y - \epsilon_{xy}^2\right)}$$

$$\epsilon_{1,2} = \frac{860 + 0}{2} \pm \sqrt{\left(\frac{860 + 0}{2}\right)^2 - \left(860 \cdot 0 - 270^2\right)}$$

The appropriate substitution lead to

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$\epsilon_x = 860\,\mu\text{strain}$

$\epsilon_{xy} = 270\,\mu\text{strain}$

$\epsilon_y = 0\ \mu\text{strain}$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x + \epsilon_y}{2}\right)^2 - \left(\epsilon_x\,\epsilon_y - \epsilon_{xy}^2\right)}$$

$$\epsilon_{1,2} = \frac{860 + 0}{2} \pm \sqrt{\left(\frac{860 + 0}{2}\right)^2 - \left(860 \cdot 0 - 270^2\right)}$$

$\epsilon_{1,2} = -78\,\mu\text{strain}\ ,\ 938\,\mu\text{strain}$

The Principal Strains!

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$\epsilon_x = 860\,\mu\text{strain}$

$\epsilon_{xy} = 270\,\mu\text{strain}$

$\epsilon_y = 0\ \mu\text{strain}$

$\epsilon_{1,2} = -78\,\mu\text{strain}\ ,\ 938\,\mu\text{strain}$

It will require very little additional work at this point to calculate the maximum in-plane shear strain.

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$\epsilon_x = 860\,\mu\text{strain}$

$\epsilon_{xy} = 270\,\mu\text{strain}$

$\epsilon_y = 0\ \mu\text{strain}$

$\epsilon_{1,2} = -78\,\mu\text{strain}\ ,\ 938\,\mu\text{strain}$

$$\max \epsilon_{x'y'} = \sqrt{\left(\frac{\epsilon_x + \epsilon_y}{2}\right)^2 - \left(\epsilon_x\,\epsilon_y - \epsilon_{xy}^2\right)}$$

Recall the equation to calculate maximum in-plane shear strain...

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$\epsilon_x = 860\mu\text{strain}$

$\epsilon_{xy} = 270\mu\text{strain}$

$\epsilon_y = 0\ \mu\text{strain}$

$\epsilon_{1,2} = -78\ \mu\text{strain}\ ,\ 938\ \mu\text{strain}$

$$\max \epsilon_{x'y'} = \sqrt{\left(\frac{\epsilon_x + \epsilon_y}{2}\right)^2 - \left(\epsilon_x\,\epsilon_y - \epsilon_{xy}^2\right)}$$

$$\max \epsilon_{x'y'} = 508\ \mu\text{strain}$$

We plug in the numbers, and out pops the solution. Ta Da!

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Principal Strains

$\epsilon_{1,2} = -78\ \mu\text{strain}\ ,\ 938\ \mu\text{strain}$

Maximum In-Plane Shear Strain

$\max \epsilon_{x'y'} = 508\ \mu\text{strain}$

In summary, we were able to calculate principal and maximum shear strains from our strain gage readings by first establishing a relationship between the strain gage readings and the in-plane strains. We then used two formulae to calculate the principal and maximum shear strains.

We have certainly been successful, however, we were interested in the *stresses* experienced by the airplane, not the strains. So let's forge ahead....

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$\epsilon_0 = 860\ \mu\text{strain}$

$\epsilon_{45} = 700\ \mu\text{strain}$

$\epsilon_{90} = 0\ \mu\text{strain}$

$$\epsilon_x = \epsilon_0 = 860\mu\text{strain}$$

$$\epsilon_{xy} = \epsilon_{45} - \frac{\epsilon_0 + \epsilon_{90}}{2} = 270\mu\text{strain}$$

$$\epsilon_y = \epsilon_{90} = 0\ \mu\text{strain}$$

Recall the point at which we calculated the in-plane strain components from the readings on the strain gages. Previously we proceeded from this point by calculating the principal and maximum shear strains. Now we are interested in stresses. To calculate stresses from strains we will work with Hooke's Law.

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Recall Hooke's Law

$\epsilon_x = 860\mu\text{strain}$

$\epsilon_y = 0\ \mu\text{strain}$

$\epsilon_{xy} = 270\mu\text{strain}$

Hooke's Law

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\gamma_{xy} = 2\,\varepsilon_{xy} = \frac{\tau_{xy}}{G}$$

We previously formulated Hooke's Law to relate stress to strain using the material properties E, G, and ν.

Notice that in their present form, our equations include stresses in three dimensions. For this problem, we have been treating the aircraft skin as a shell. Therefore, we will require Hooke's Law for two dimensions only.

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Hooke's Law for Two Dimensions

$\epsilon_x = 860 \mu strain$

$\epsilon_y = 0 \quad \mu strain$

$\epsilon_{xy} = 270 \mu strain$

To formulate Hooke's Law for two dimensions we simply dropped the stress component in the z direction.

There is a second problem we must address. As we have written them, the equations express strain in terms of stress. In this problem we want to calculate stress in terms of strain.

For the shear equation the inversion is simple...

Hooke's Law (2D)

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu \ \sigma_y]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu \ \sigma_x]$$

$$\gamma_{xy} = 2\,\epsilon_{xy} = \frac{\tau_{xy}}{G}$$

Inverting the Shear Relationship

$\epsilon_x = 860 \mu strain$

$\epsilon_y = 0 \quad \mu strain$

$\epsilon_{xy} = 270 \mu strain$

To invert the equations relating the normal stresses to the normal strains, we must essentially invert a two by two matrix. The procedure is not difficult, and the results are shown on the next card.

Hooke's Law (2D)

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu \ \sigma_y]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu \ \sigma_x]$$

$$\tau_{xy} = G\,\gamma_{xy} = 2G\epsilon_{xy}$$

Inverting the Normal Relationships

$\epsilon_x = 860 \mu strain$

$\epsilon_y = 0 \quad \mu strain$

$\epsilon_{xy} = 270 \mu strain$

Let's take stock of where we are. We wish to calculate the stresses σ_x, σ_y, and τ_{xy}. We already know the strains ϵ_x, ϵ_y, and ϵ_{xy}. We do not yet know the material properties E, G, and ν.

At this point we can proceed no further until we choose a material!

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}[\epsilon_x + \nu\,\epsilon_y]$$

$$\sigma_y = \frac{E}{(1-\nu^2)}[\epsilon_y + \nu\,\epsilon_x]$$

$$\tau_{xy} = G\,\gamma_{xy} = 2G\epsilon_{xy}$$

Material Properties of Aluminum

$\epsilon_x = 860 \mu strain$

$\epsilon_y = 0 \quad \mu strain$

$\epsilon_{xy} = 270 \mu strain$

$E = 10{,}000$ ksi

$\nu = 0.35$

The outer shell of the aircraft we have been testing is made of aluminum. The fabricators of our airplane have told us that the aluminum has a Young's modulus of 10,000 ksi and a Poisson's ratio of 0.35.

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}[\epsilon_x + \nu\,\epsilon_y]$$

$$\sigma_y = \frac{E}{(1-\nu^2)}[\epsilon_y + \nu\,\epsilon_x]$$

$$\tau_{xy} = G\,\gamma_{xy} = 2G\epsilon_{xy}$$

Material Properties of Aluminum

$\epsilon_x = 860 \mu strain$
$\epsilon_y = 0 \quad \mu strain$
$\epsilon_{xy} = 270 \mu strain$

$$\frac{E}{(1-\nu^2)} = \frac{10{,}000\ ksi}{(1-0.35^2)}$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}[\epsilon_x + \nu\,\epsilon_y]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}[\epsilon_y + \nu\,\epsilon_x]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

Using this information we can calculate the coefficient on the two normal stress equations.

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Material Properties of Aluminum

$\epsilon_x = 860 \mu strain$
$\epsilon_y = 0 \quad \mu strain$
$\epsilon_{xy} = 270 \mu strain$

$$\frac{E}{(1-\nu^2)} = 11{,}400\ ksi$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}[\epsilon_x + \nu\,\epsilon_y]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}[\epsilon_y + \nu\,\epsilon_x]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

We now have all the information we need to solve the first two equations. We must still calculate the shear modulus, G, to solve for the

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Material Properties of Aluminum

$\epsilon_x = 860 \mu strain$
$\epsilon_y = 0 \quad \mu strain$
$\epsilon_{xy} = 270 \mu strain$

$$\frac{E}{(1-\nu^2)} = 11{,}400\ ksi$$
$$G = \frac{E}{2(1+\nu)}$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}[\epsilon_x + \nu\,\epsilon_y]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}[\epsilon_y + \nu\,\epsilon_x]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

Recall the relationship between the shear modulus, and E and ν.

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Material Properties of Aluminum

$\epsilon_x = 860 \mu strain$
$\epsilon_y = 0 \quad \mu strain$
$\epsilon_{xy} = 270 \mu strain$

$$\frac{E}{(1-\nu^2)} = 11{,}400\ ksi$$
$$G = 3{,}700\ ksi$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}[\epsilon_x + \nu\,\epsilon_y]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}[\epsilon_y + \nu\,\epsilon_x]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

Using 10,000 ksi for E and 3.5 for ν we calculate the value of G = 3,700 ksi.

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Calculating the Stresses

$\epsilon_x = 860 \mu\text{strain}$
$\epsilon_y = 0 \quad \mu\text{strain}$
$\epsilon_{xy} = 270 \mu\text{strain}$

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$
$$G = 3{,}700 \text{ ksi}$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\epsilon_x + \nu\, \epsilon_y\right]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}\left[\epsilon_y + \nu\, \epsilon_x\right]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

$\sigma_x =$
$\sigma_y =$
$\tau_{xy} =$

We now have all the information we need to calculate the three unknown stresses.

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Calculating σ_x

$\epsilon_x = 860 \mu\text{strain}$
$\epsilon_y = 0 \quad \mu\text{strain}$
$\epsilon_{xy} = 270 \mu\text{strain}$

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$
$$G = 3{,}700 \text{ ksi}$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\epsilon_x + \nu\, \epsilon_y\right]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}\left[\epsilon_y + \nu\, \epsilon_x\right]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

$\sigma_x = 11{,}400 \text{ ksi}\left[0.000860 + 0.35\,(0)\right]$
$\sigma_y =$
$\tau_{xy} =$

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Calculating σ_x

$\epsilon_x = 860 \mu\text{strain}$
$\epsilon_y = 0 \quad \mu\text{strain}$
$\epsilon_{xy} = 270 \mu\text{strain}$

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$
$$G = 3{,}700 \text{ ksi}$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\epsilon_x + \nu\, \epsilon_y\right]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}\left[\epsilon_y + \nu\, \epsilon_x\right]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

$\sigma_x = 11{,}400 \text{ ksi}\left[0.000860 + 0.35\,(0)\right] = 9.8 \text{ ksi}$
$\sigma_y =$
$\tau_{xy} =$

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Calculating σ_y

$\epsilon_x = 860 \mu\text{strain}$
$\epsilon_y = 0 \quad \mu\text{strain}$
$\epsilon_{xy} = 270 \mu\text{strain}$

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$
$$G = 3{,}700 \text{ ksi}$$

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\epsilon_x + \nu\, \epsilon_y\right]$$
$$\sigma_y = \frac{E}{(1-\nu^2)}\left[\epsilon_y + \nu\, \epsilon_x\right]$$
$$\tau_{xy} = G\,\gamma_{xy} = 2G\varepsilon_{xy}$$

$\sigma_x = 11{,}400 \text{ ksi}\left[0.000860 + 0.35\,(0)\right] = 9.8 \text{ ksi}$
$\sigma_y = 11{,}400 \text{ ksi}\left[0 + 0.35\,(0.000860)\right]$
$\tau_{xy} =$

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Calculating σ_y

ϵ_x = 860 μstrain
ϵ_y = 0 μstrain
ϵ_{xy} = 270 μstrain

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$

G = 3,700 ksi

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\epsilon_x + \nu\,\epsilon_y\right]$$

$$\sigma_y = \frac{E}{(1-\nu^2)}\left[\epsilon_y + \nu\,\epsilon_x\right]$$

$$\tau_{xy} = G\,\gamma_{xy} = 2G\,\varepsilon_{xy}$$

$$\sigma_x = 11{,}400 \text{ ksi}\left[0.000860 + 0.35\,(0)\right] = 9.8 \text{ ksi}$$

$$\sigma_y = 11{,}400 \text{ ksi}\left[0 + 0.35\,(0.000860)\right] = 3.4 \text{ ksi}$$

τ_{xy} =

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Calculating τ_{xy}

ϵ_x = 860 μstrain
ϵ_y = 0 μstrain
ϵ_{xy} = 270 μstrain

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$

G = 3,700 ksi

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\epsilon_x + \nu\,\epsilon_y\right]$$

$$\sigma_y = \frac{E}{(1-\nu^2)}\left[\epsilon_y + \nu\,\epsilon_x\right]$$

$$\tau_{xy} = G\,\gamma_{xy} = 2G\,\varepsilon_{xy}$$

$$\sigma_x = 11{,}400 \text{ ksi}\left[0.000860 + 0.35\,(0)\right] = 9.8 \text{ ksi}$$

$$\sigma_y = 11{,}400 \text{ ksi}\left[0 + 0.35\,(0.000860)\right] = 3.4 \text{ ksi}$$

$$\tau_{xy} = 3{,}700 \text{ ksi}\,(2)(0.000270)$$

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Calculating τ_{xy}

ϵ_x = 860 μstrain
ϵ_y = 0 μstrain
ϵ_{xy} = 270 μstrain

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$

G = 3,700 ksi

Hooke's Law (2D)

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\epsilon_x + \nu\,\epsilon_y\right]$$

$$\sigma_y = \frac{E}{(1-\nu^2)}\left[\epsilon_y + \nu\,\epsilon_x\right]$$

$$\tau_{xy} = G\,\gamma_{xy} = 2G\,\varepsilon_{xy}$$

$$\sigma_x = 11{,}400 \text{ ksi}\left[0.000860 + 0.35\,(0)\right] = 9.8 \text{ ksi}$$

$$\sigma_y = 11{,}400 \text{ ksi}\left[0 + 0.35\,(0.000860)\right] = 3.4 \text{ ksi}$$

$$\tau_{xy} = 3{,}700 \text{ ksi}\,(2)(0.000270) = 2.0 \text{ ksi}$$

We have now calculated the stresses on the airplane in the direction of our strain rosette. Are these the highest stresses the plane will experience?

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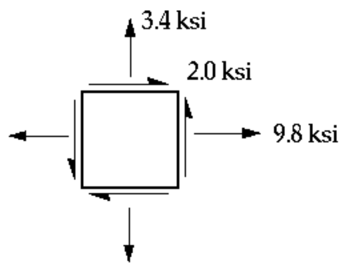

If we could isolate a small element of material from the shell of the airplane we would see the stress state shown at the left.

From our knowledge of stress we can conclude that since these are not the principal stresses (the shear stress is not zero) they do not reflect the maximum normal stress experienced by the stress block.

Also, since the normal stresses are not equal, this particular orientation does not reflect the maximum shear stress in the material (recall Mohr's circle).

$$\sigma_x = 11{,}400 \text{ ksi}\left[0.000860 + 0.35\,(0)\right] = 9.8 \text{ ksi}$$

$$\sigma_y = 11{,}400 \text{ ksi}\left[0 + 0.35\,(0.000860)\right] = 3.4 \text{ ksi}$$

$$\tau_{xy} = 3{,}700 \text{ ksi}\,(2)(0.000270) = 2.0 \text{ ksi}$$

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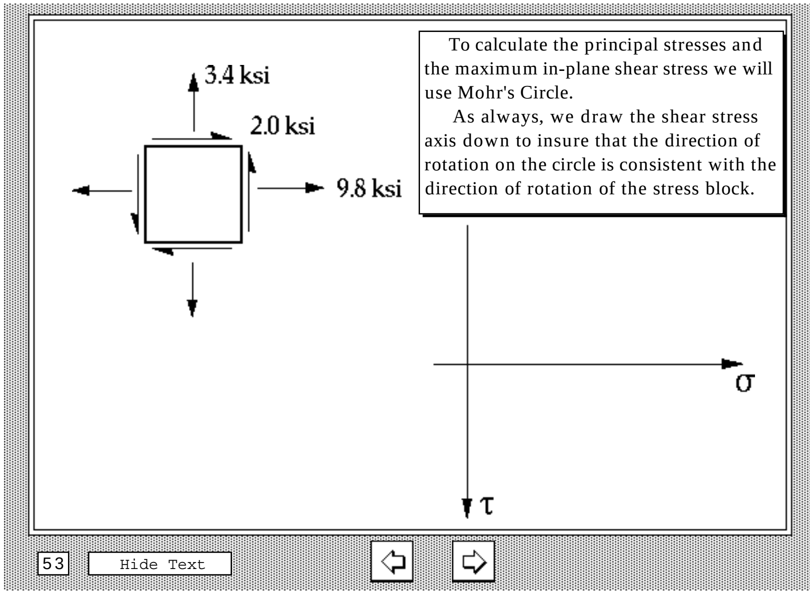
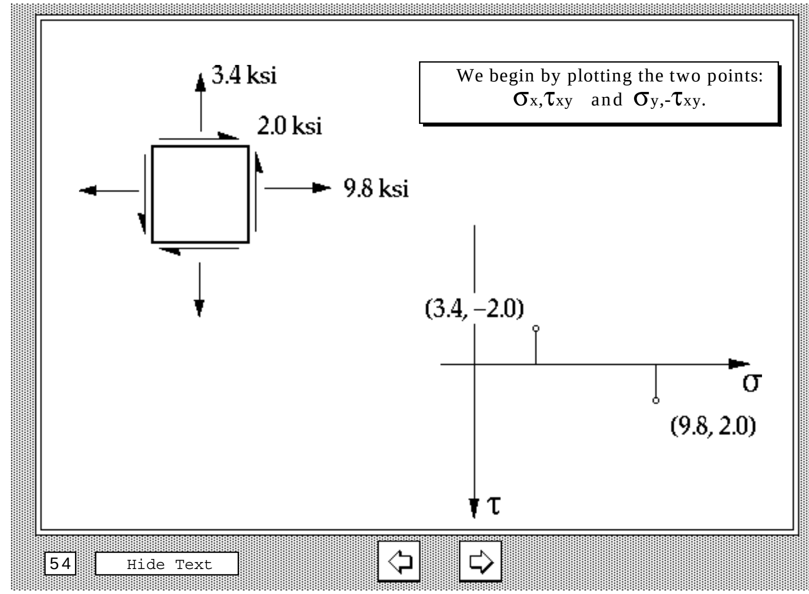
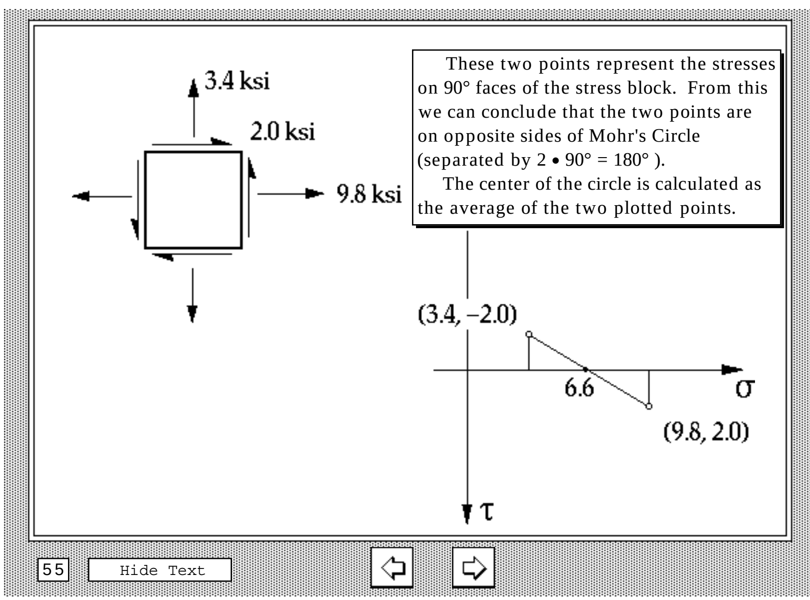
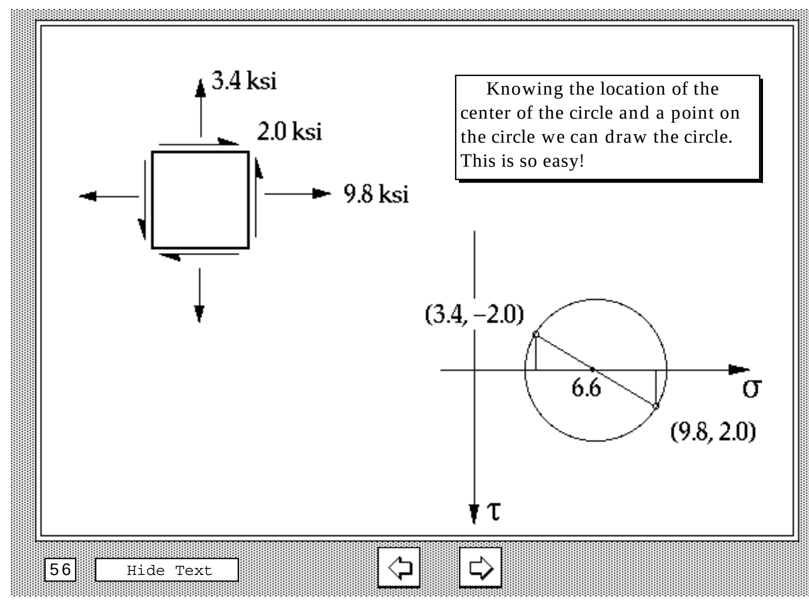
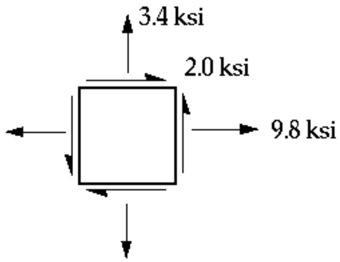
Strain Gage Problem: 14



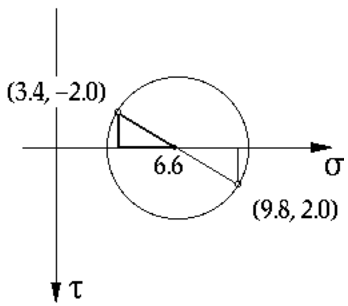








The maximum shear stress is calculated as the radius of the circle using the Pythagorean Theorem.



$$\tau_{max} = \sqrt{(6.6-3.4)^2 + (-2.0)^2}$$

$$\boxed{\tau_{max} = 3.8 \text{ ksi}}$$

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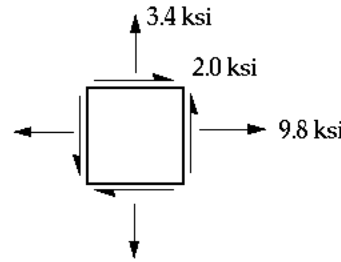
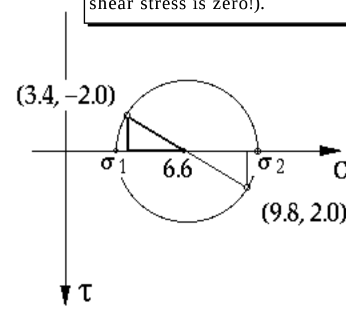


The principal stress values σ_1 and σ_2 occur where the circle crosses the σ axis (where the shear stress is zero!).



$$\tau_{max} = \sqrt{(6.6-3.4)^2 + (-2.0)^2}$$

$$\tau_{max} = 3.8 \text{ ksi}$$

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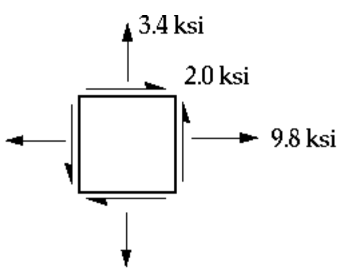
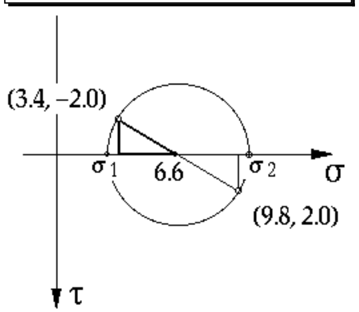


The location of the left intercept of the circle is the center minus the radius; the right intercept is the center plus the radius.

And there you have it. The maximum shear and normal stresses present in the airplane under the strain



$$\tau_{max} = \sqrt{(6.6-3.4)^2 + (-2.0)^2}$$

$$\tau_{max} = 3.8 \text{ ksi}$$

$$\sigma_1 = 6.6 - 3.8 = 2.8 \text{ ksi}$$

$$\sigma_2 = 6.6 + 3.8 = 10.4 \text{ ksi}$$

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Another way to reach this same solution is to calculate the principal stresses in the aircraft shell using Hooke's Law and the principal strains we calculated earlier.

<u>Hooke's Law</u> (2D)

$$\sigma_x = \frac{E}{(1 - \nu^2)} [\epsilon_x + \nu \epsilon_y]$$

$$\sigma_y = \frac{E}{(1 - \nu^2)} [\epsilon_y + \nu \epsilon_x]$$

$$\tau_{xy} = G \gamma_{xy} = 2G \varepsilon_{xy}$$

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We substitute in the indices 1 and 2 when referring to principal strains and principal stresses.

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu \epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu \epsilon_1]$$

$$\tau_{12} = G \gamma_{12} = 2G\varepsilon_{12}$$

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Recall that in the principal direction, the shear strain is zero. From this we can conclude the shear stress is also zero!

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu \epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu \epsilon_1]$$

$$\tau_{12} = G \gamma_{12} = 2G\varepsilon_{12} = 0$$

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Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu \epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu \epsilon_1]$$

$$\tau_{max} = G \gamma_{max}$$

Actually, we are interested in the maximum shear stress, which occurs at a 45° orientation from the principal directions.

The maximum shear stress may be calculated directly from the maximum shear strain as shown above.

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$$\epsilon_1 = -78 \ \mu\text{strain}$$

$$\epsilon_2 = 938 \ \mu\text{strain}$$

$$\gamma_{max} = 1016 \ \mu\text{strain}$$

At this point we need to recall the principal strains we calculated earlier in this example.....

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu \epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu \epsilon_1]$$

$$\tau_{max} = G \gamma_{max}$$

64 Hide Text Recall Principal Strai

$\epsilon_1 = -78$ µstrain

$\epsilon_2 = 938$ µstrain

$\gamma_{max} = 1016$ µstrain

$$\frac{E}{(1 - \nu^2)} = 11{,}400 \text{ ksi}$$

$G = 3{,}700$ ksi

...and the material constants for the aluminum used in the aircraft.

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu\,\epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu\,\epsilon_1]$$

$$\tau_{max} = G\,\gamma_{max}$$

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$\epsilon_1 = -78$ µstrain

$\epsilon_2 = 938$ µstrain

$\gamma_{max} = 1016$ µstrain

$$\frac{E}{(1 - \nu^2)} = 11{,}400 \text{ ksi}$$

$G = 3{,}700$ ksi

$\sigma_1 =$

$\sigma_2 =$

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu\,\epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu\,\epsilon_1]$$

$$\tau_{max} = G\,\gamma_{max}$$

Substituting the values for the principal strains and the material constants into the equations above, we can now solve for the principal stresses.

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$\epsilon_1 = -78$ µstrain

$\epsilon_2 = 938$ µstrain

$\gamma_{max} = 1016$ µstrain

$$\frac{E}{(1 - \nu^2)} = 11{,}400 \text{ ksi}$$

$G = 3{,}700$ ksi

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu\,\epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu\,\epsilon_1]$$

$$\tau_{max} = G\,\gamma_{max}$$

$\sigma_1 = 11{,}400 \text{ ksi} [-0.000078 + (0.35)(0.000938)]$

$\sigma_2 =$

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$\epsilon_1 = -78$ µstrain

$\epsilon_2 = 938$ µstrain

$\gamma_{max} = 1016$ µstrain

$$\frac{E}{(1 - \nu^2)} = 11{,}400 \text{ ksi}$$

$G = 3{,}700$ ksi

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1 - \nu^2)} [\epsilon_1 + \nu\,\epsilon_2]$$

$$\sigma_2 = \frac{E}{(1 - \nu^2)} [\epsilon_2 + \nu\,\epsilon_1]$$

$$\tau_{max} = G\,\gamma_{max}$$

$\sigma_1 = 11{,}400 \text{ ksi} [-0.000078 + (0.35)(0.000938)] = 2.8 \text{ ksi}$

$\sigma_2 =$

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$\epsilon_1 = -78\ \mu\text{strain}$

$\epsilon_2 = 938\ \mu\text{strain}$

$\gamma_{max} = 1016\ \mu\text{strain}$

$\frac{E}{(1-\nu^2)} = 11{,}400\text{ ksi}$

$G = 3{,}700\text{ ksi}$

<u>Hooke's Law</u> (2D)

$\sigma_1 = \frac{E}{(1-\nu^2)}[\epsilon_1 + \nu\,\epsilon_2]$

$\sigma_2 = \frac{E}{(1-\nu^2)}[\epsilon_2 + \nu\,\epsilon_1]$

$\tau_{max} = G\,\gamma_{max}$

$\sigma_1 = 11{,}400\text{ ksi}\,[-0.000078 + (0.35)(0.000938)] = 2.8\text{ ksi}$

$\sigma_2 = 11{,}400\text{ ksi}\,[0.000938 + (0.35)(-0.000078)]$

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$\epsilon_1 = -78\ \mu\text{strain}$

$\epsilon_2 = 938\ \mu\text{strain}$

$\gamma_{max} = 1016\ \mu\text{strain}$

$\frac{E}{(1-\nu^2)} = 11{,}400\text{ ksi}$

$G = 3{,}700\text{ ksi}$

<u>Hooke's Law</u> (2D)

$\sigma_1 = \frac{E}{(1-\nu^2)}[\epsilon_1 + \nu\,\epsilon_2]$

$\sigma_2 = \frac{E}{(1-\nu^2)}[\epsilon_2 + \nu\,\epsilon_1]$

$\tau_{max} = G\,\gamma_{max}$

$\sigma_1 = 11{,}400\text{ ksi}\,[-0.000078 + (0.35)(0.000938)] = 2.8\text{ ksi}$

$\sigma_2 = 11{,}400\text{ ksi}\,[0.000938 + (0.35)(-0.000078)] = 10.4\text{ ksi}$

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$\epsilon_1 = -78\ \mu\text{strain}$

$\epsilon_2 = 938\ \mu\text{strain}$

$\gamma_{max} = 1016\ \mu\text{strain}$

$\frac{E}{(1-\nu^2)} = 11{,}400\text{ ksi}$

$G = 3{,}700\text{ ksi}$

<u>Hooke's Law</u> (2D)

$\sigma_1 = \frac{E}{(1-\nu^2)}[\epsilon_1 + \nu\,\epsilon_2]$

$\sigma_2 = \frac{E}{(1-\nu^2)}[\epsilon_2 + \nu\,\epsilon_1]$

$\tau_{max} = G\,\gamma_{max}$

$\sigma_1 = 2.8\text{ ksi}$

$\sigma_2 = 10.4\text{ ksi}$

Finally, we calculate the maximum in-plane shear stress.

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$\epsilon_1 = -78\ \mu\text{strain}$

$\epsilon_2 = 938\ \mu\text{strain}$

$\gamma_{max} = 1016\ \mu\text{strain}$

$\frac{E}{(1-\nu^2)} = 11{,}400\text{ ksi}$

$G = 3{,}700\text{ ksi}$

<u>Hooke's Law</u> (2D)

$\sigma_1 = \frac{E}{(1-\nu^2)}[\epsilon_1 + \nu\,\epsilon_2]$

$\sigma_2 = \frac{E}{(1-\nu^2)}[\epsilon_2 + \nu\,\epsilon_1]$

$\tau_{max} = G\,\gamma_{max}$

$= (3{,}700\text{ ksi})\,(.001016)$

$\sigma_1 = 2.8\text{ ksi}$

$\sigma_2 = 10.4\text{ ksi}$

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$\epsilon_1 = -78\ \mu$strain

$\epsilon_2 = 938\ \mu$strain

$\gamma_{max} = 102\ \mu$strain

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$

$G = 3{,}700$ ksi

$\sigma_1 = 2.8$ ksi

$\sigma_2 = 10.4$ ksi

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1-\nu^2)}[\epsilon_1 + \nu\,\epsilon_2]$$

$$\sigma_2 = \frac{E}{(1-\nu^2)}[\epsilon_2 + \nu\,\epsilon_1]$$

$$\tau_{max} = G\,\gamma_{max} = (3{,}700 \text{ ksi})(.001016) = 3.8 \text{ ksi}$$

$\epsilon_1 = -78\ \mu$strain

$\epsilon_2 = 938\ \mu$strain

$\gamma_{max} = 102\ \mu$strain

$$\frac{E}{(1-\nu^2)} = 11{,}400 \text{ ksi}$$

$G = 3{,}700$ ksi

$\sigma_1 = 2.8$ ksi

$\sigma_2 = 10.4$ ksi

Hooke's Law (2D)

$$\sigma_1 = \frac{E}{(1-\nu^2)}[\epsilon_1 + \nu\,\epsilon_2]$$

$$\sigma_2 = \frac{E}{(1-\nu^2)}[\epsilon_2 + \nu\,\epsilon_1]$$

$\tau_{max} = 3.8$ ksi

If you compare these results with the results we calculated using Mohr's Circle you will see that they are identical.

Conclusion

This stack demonstrated how to use Hooke's Law to calculate stresses from strains. We found that to calculate the principal stresses we could follow one of two proceedures:

(A) - calculate the stresses from the measured strains
- calculate the principal stresses

(B) - calculate the principal strains
- calculate the principal stresses from the principal strains.

We also saw that we could use either the transformation equations or Mohr's Cirlce to calculate the principal values.

The End