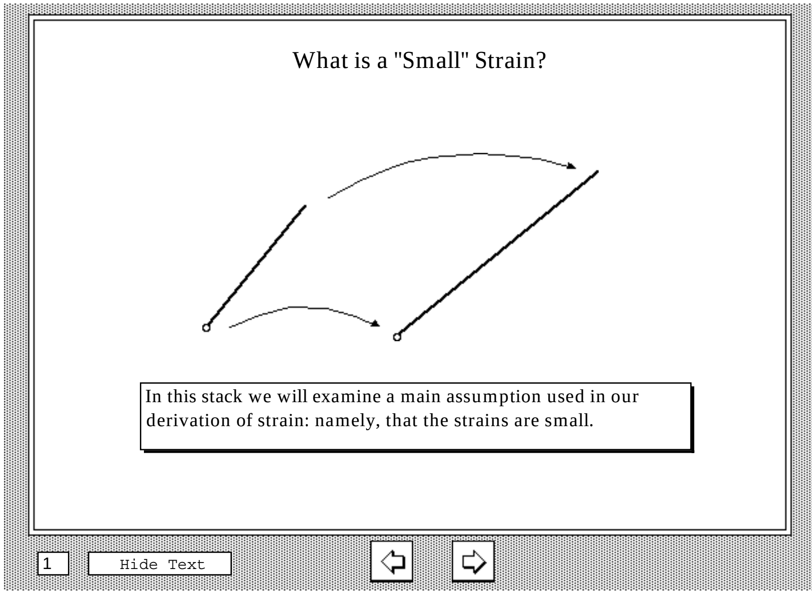
What is a "Small" Strain?
In this stack we will examine a main assumption used in our derivation of strain: namely, that the strains are small.
1
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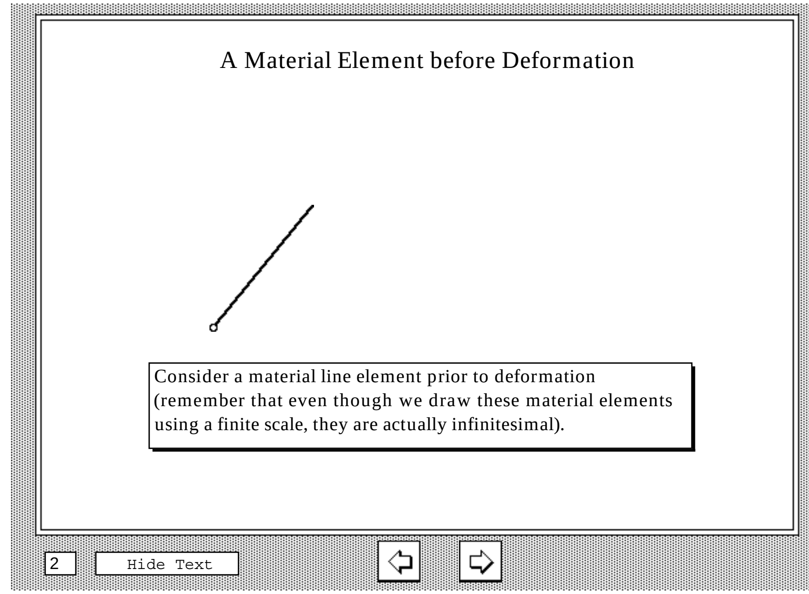
A Material Element before Deformation
Consider a material line element prior to deformation (remember that even though we draw these material elements using a finite scale, they are actually infinitesimal).
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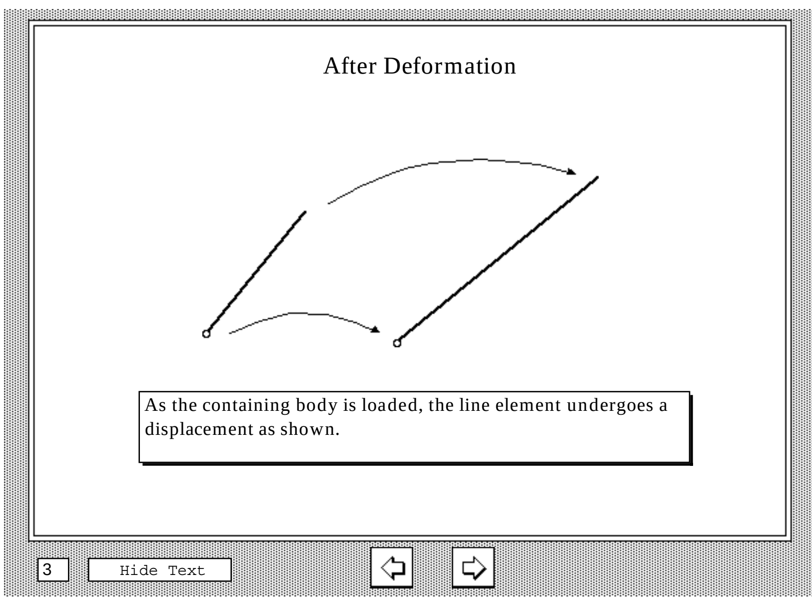
After Deformation
As the containing body is loaded, the line element undergoes a displacement as shown.
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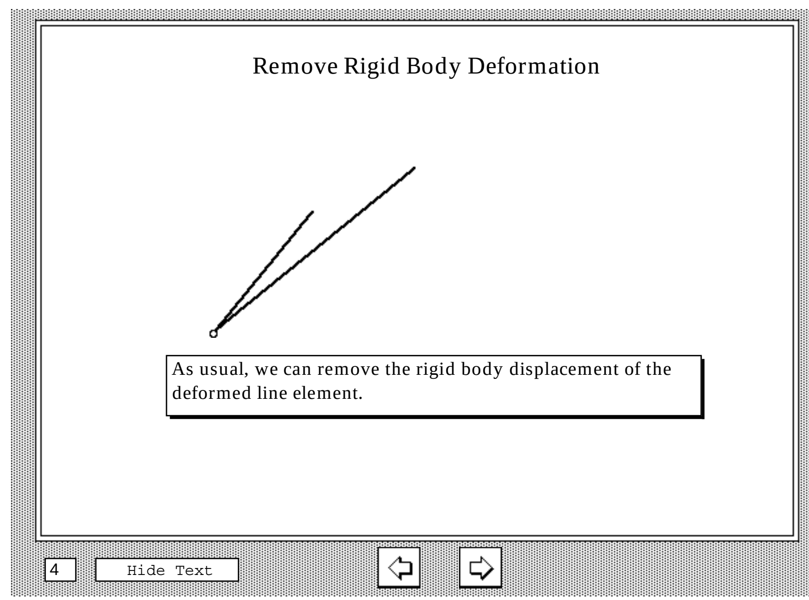
Remove Rigid Body Deformation
As usual, we can remove the rigid body displacement of the deformed line element.
4
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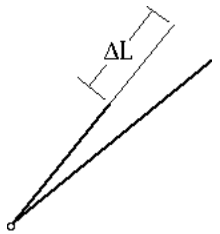
Decompose Deformation



We can view the deformation as being composed of two parts:

1. A stretch of ΔL as shown...

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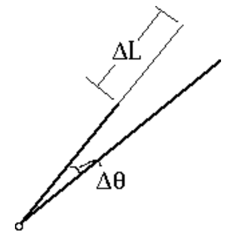
Decompose Deformation



We can view the deformation as being composed of two parts:

1. A stretch of ΔL as shown.
2. A rotation of $\Delta\theta$.

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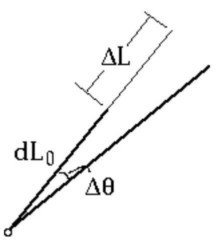
Small Strain Assumptions



We will denote the original length dL_0. An important ramification of our definition of strain is that we always refer to the undeformed configuration.

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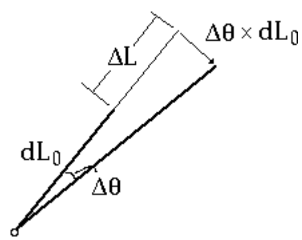
Small Strain Assumptions



One effect of this assumption is shown in the figure above. Rather than computing $\Delta\theta(dL_0 + \Delta L)$, we assume that $dL_0 + \Delta L \neq dL_0$ and use $\Delta\theta\ dL_0$ as shown. For small $\Delta\theta$ and ΔL this is a good approximation, but it is still an approximation. Using the original length, dL_0, rather than the final length $(dL_0 + \Delta L)$ is what we mean when we say "refer to the undeformed configuration".

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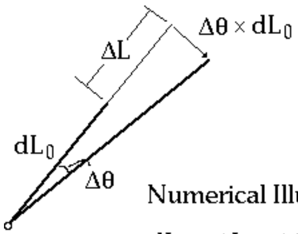
Small Strain Assumptions

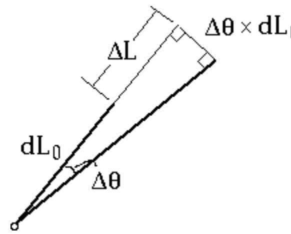


Numerical Illustration:

dL_0	ΔL	$\Delta\theta$	$\Delta\theta dL_0$	$\Delta\theta(dL_0 + \Delta L)$
1	0.5	0.05	0.05	0.075
1	0.1	0.05	0.05	0.055
1	0.001	0.05	0.05	0.05005

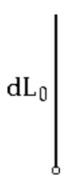
Here are some simple calculations showing how good this approximation is for several values of $\Delta L / dL_0$. For the majority of engineering applications, values of $\Delta L / dL_0$ tend to be below 0.001.

Small Strain Assumptions



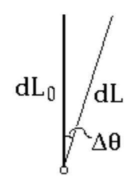
Another way of thinking about this approximation is to visualize it as requiring both the angles shown in the figure to be 90°. Obviously, this cannot be so, but for small strains the angles are close enough to 90° to ignore the difference.

Deformations can be Decoupled



We do not assume small strains simply because we like approximations. Rather, this assumption has a major impact on the complexity of our problem formulations. In particular, it keeps our problems *linear*, which means that we can use superposition. It also means that decouple shear and normal strain components.

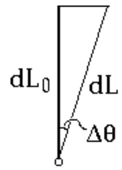
Deformations can be Decoupled



Shear without stretching : $dL \approx dL_0$

That is, we assume we can have a pure shear as shown, without stretching the line element. If we complete the triangle…

Deformations can be Decoupled



$$\frac{dL_0}{dL} = \cos \Delta\theta$$

Shear without stretching : $dL \approx dL_0$

…we can see that this is only possible in an approximate sense. The ratios of the lengths can be expressed in terms of the cosine as shown. We can expand the cosine for small , however, …

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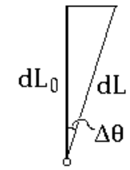
Deformations can be Decoupled



$$\frac{dL_0}{dL} = \cos \Delta\theta = 1 - \frac{\Delta\theta^2}{2!} + \ldots$$

Shear without stretching : $dL \approx dL_0$

…and we can see that the ratio will be essentially equal to 1 for sufficiently small $\Delta\theta$.

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Summary

- Small strains are strains for which we can refer to the undeformed configuration without introducing significant error. (For example, we can assume $L0/L \approx 1$)
- Small strains enable us to assume that normal and shear strains are independent, and to formulate our problems in a linear fashion.

The End

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