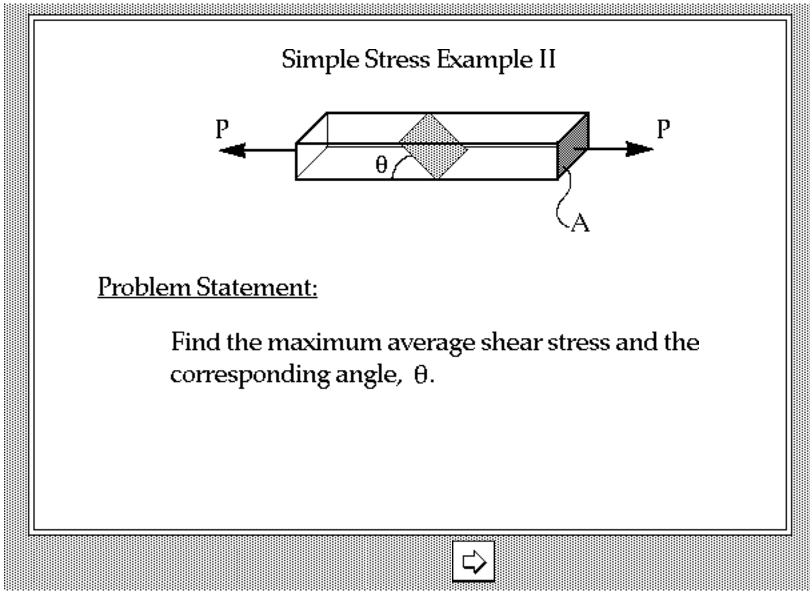
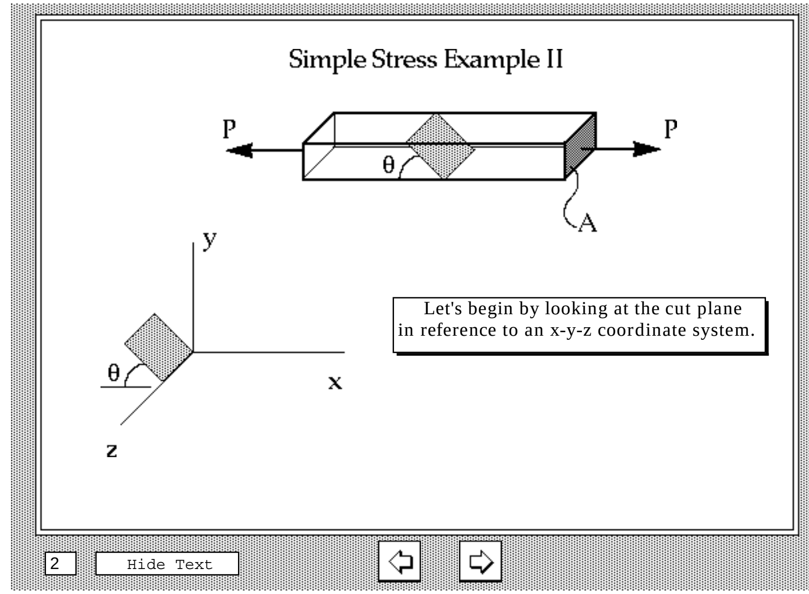

Simple Stress Example II
P
P
θ
A
Problem Statement:
Find the maximum average shear stress and the corresponding angle, θ.


Simple Stress Example II
P
P
θ
A
y
θ
x
z
Let's begin by looking at the cut plane in reference to an x-y-z coordinate system.
2
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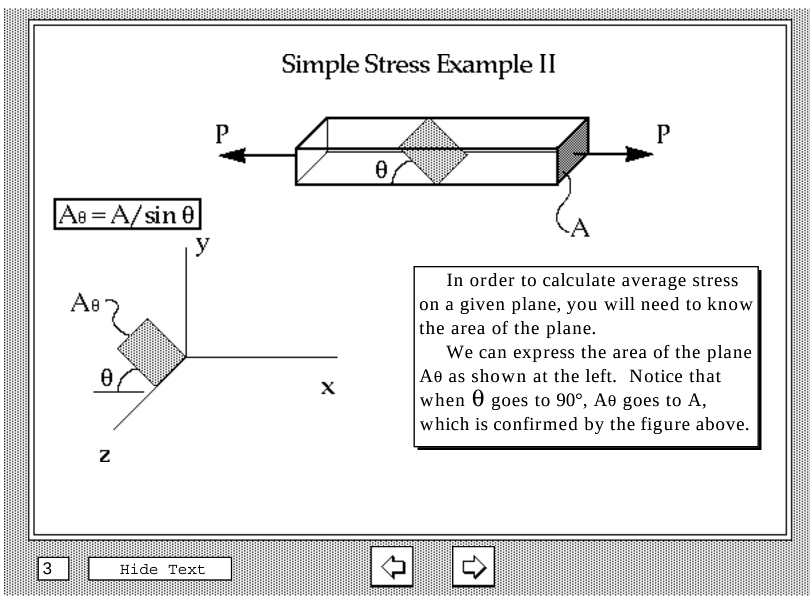
Simple Stress Example II
P
P
θ
A
Aθ = A/ sin θ
y
Aθ
θ
x
z
In order to calculate average stress on a given plane, you will need to know the area of the plane.
We can express the area of the plane Aθ as shown at the left. Notice that when θ goes to 90°, Aθ goes to A, which is confirmed by the figure above.
3
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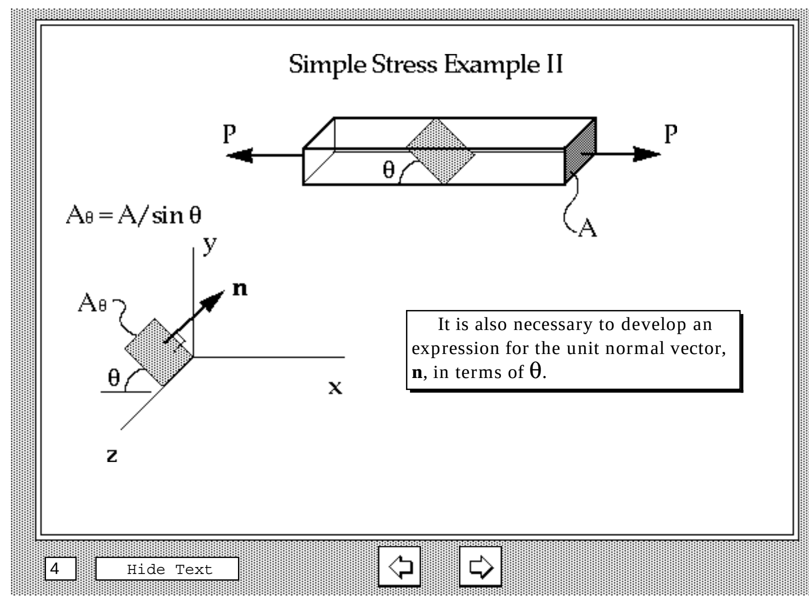
Simple Stress Example II
P
P
θ
A
Aθ = A/ sin θ
y
n
Aθ
θ
x
z
It is also necessary to develop an expression for the unit normal vector, n, in terms of θ.
4
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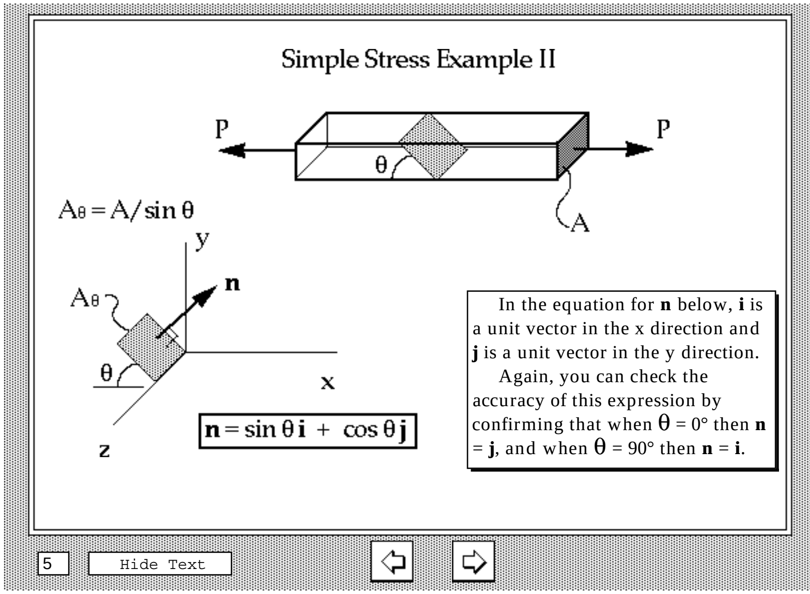
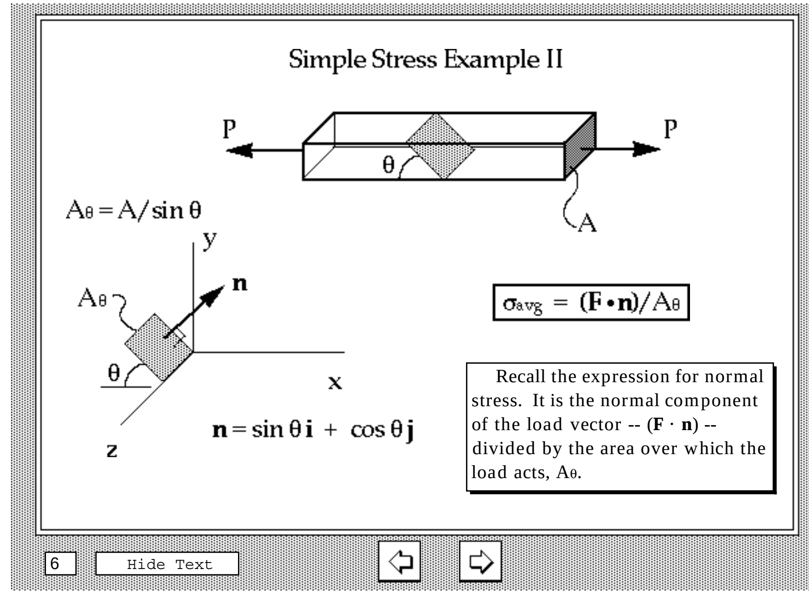
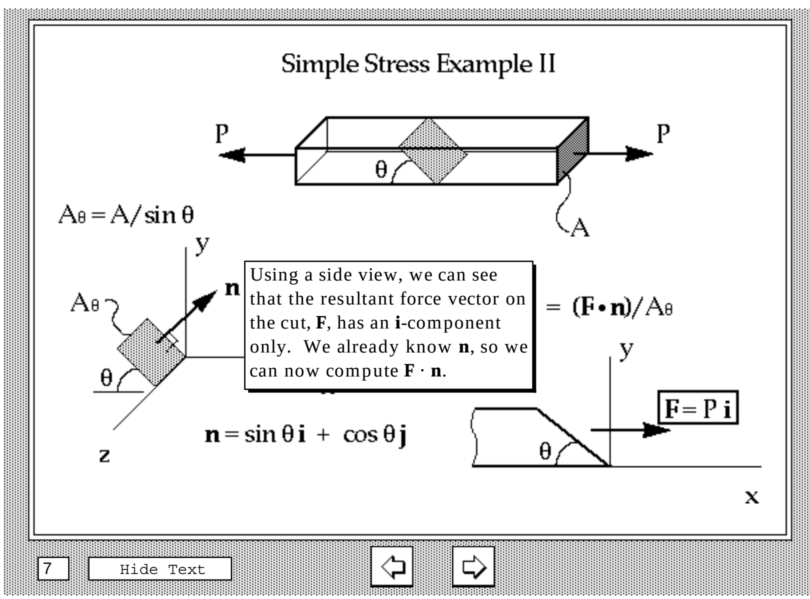
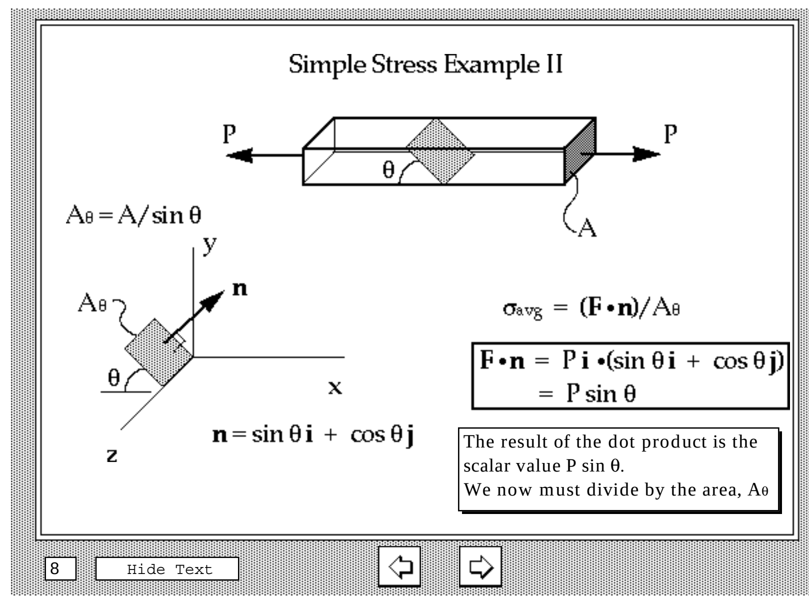
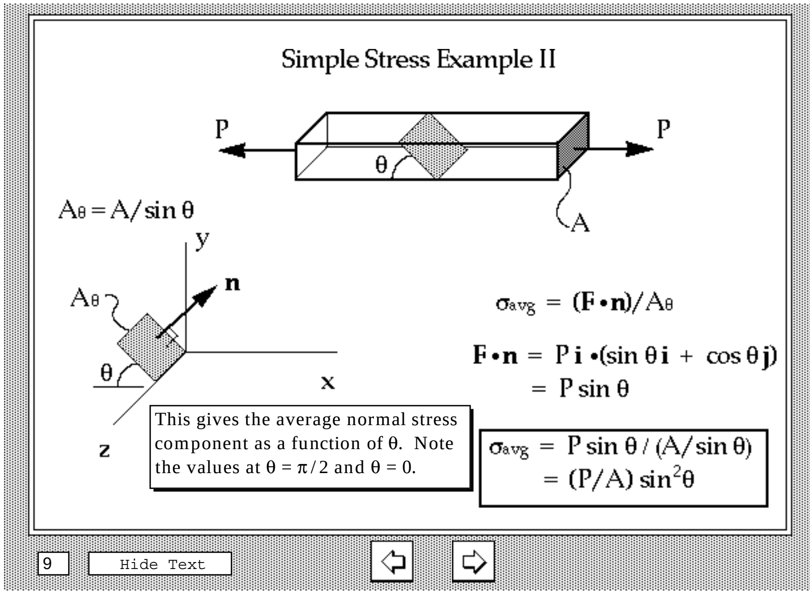
Simple Stress Example II: 2










Simple Stress Example II
P
P
θ
A
$A_\theta = A/\sin\theta$
y
n
A_θ
θ
x
z
$\sigma_{avg} = (\mathbf{F} \cdot \mathbf{n})/A_\theta$
$\mathbf{F} \cdot \mathbf{n} = P\,\mathbf{i} \cdot (\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$
$= P\sin\theta$
This gives the average normal stress component as a function of θ. Note the values at θ = π/2 and θ = 0.
$\sigma_{avg} = P\sin\theta / (A/\sin\theta)$
$= (P/A)\sin^2\theta$
9
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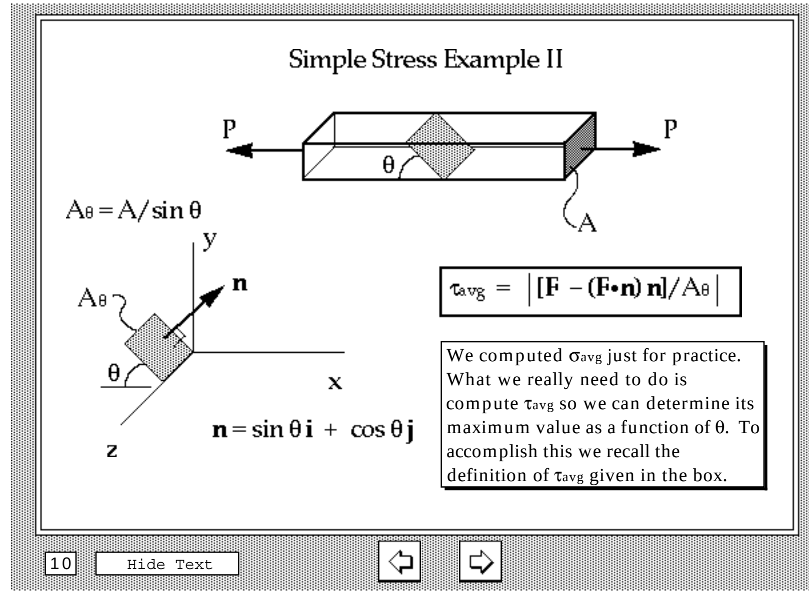
Simple Stress Example II
P
P
θ
A
$A_\theta = A/\sin\theta$
y
n
A_θ
θ
x
z
$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F} \cdot \mathbf{n})\,\mathbf{n}]/A_\theta\right|$
$\mathbf{n} = \sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$
We computed σ_{avg} just for practice. What we really need to do is compute τ_{avg} so we can determine its maximum value as a function of θ. To accomplish this we recall the definition of τ_{avg} given in the box.
10
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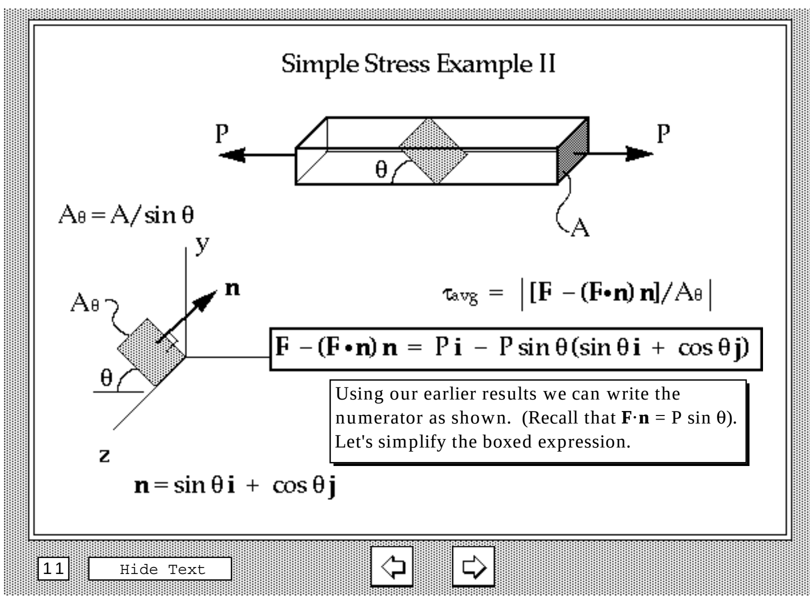
Simple Stress Example II
P
P
θ
A
$A_\theta = A/\sin\theta$
y
n
A_θ
θ
z
$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F} \cdot \mathbf{n})\,\mathbf{n}]/A_\theta\right|$
$\mathbf{F} - (\mathbf{F} \cdot \mathbf{n})\,\mathbf{n} = P\,\mathbf{i} - P\sin\theta(\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$
Using our earlier results we can write the numerator as shown. (Recall that $\mathbf{F} \cdot \mathbf{n} = P\sin\theta$).
Let's simplify the boxed expression.
$\mathbf{n} = \sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$
11
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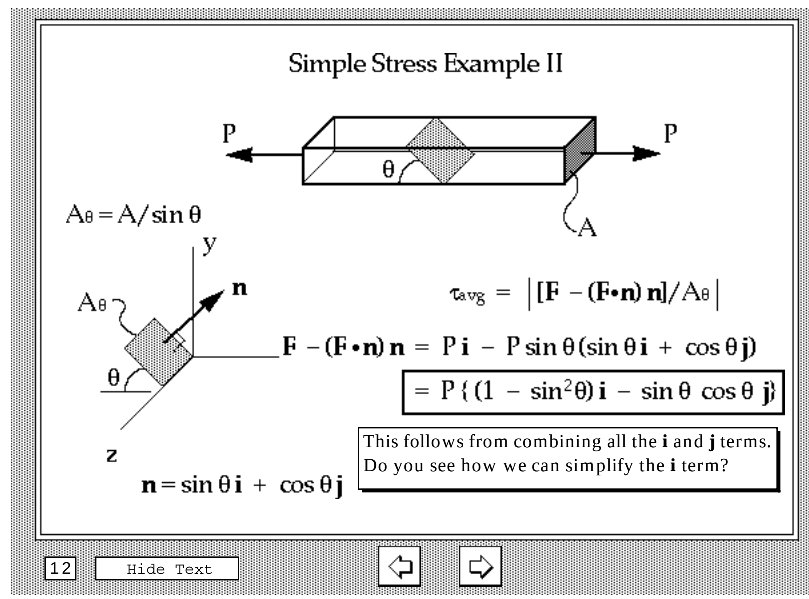
Simple Stress Example II
P
P
θ
A
$A_\theta = A/\sin\theta$
y
n
A_θ
θ
z
$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F} \cdot \mathbf{n})\,\mathbf{n}]/A_\theta\right|$
$\mathbf{F} - (\mathbf{F} \cdot \mathbf{n})\,\mathbf{n} = P\,\mathbf{i} - P\sin\theta(\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$
$= P\{(1 - \sin^2\theta)\,\mathbf{i} - \sin\theta\cos\theta\,\mathbf{j}\}$
This follows from combining all the **i** and **j** terms.
Do you see how we can simplify the **i** term?
$\mathbf{n} = \sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$
12
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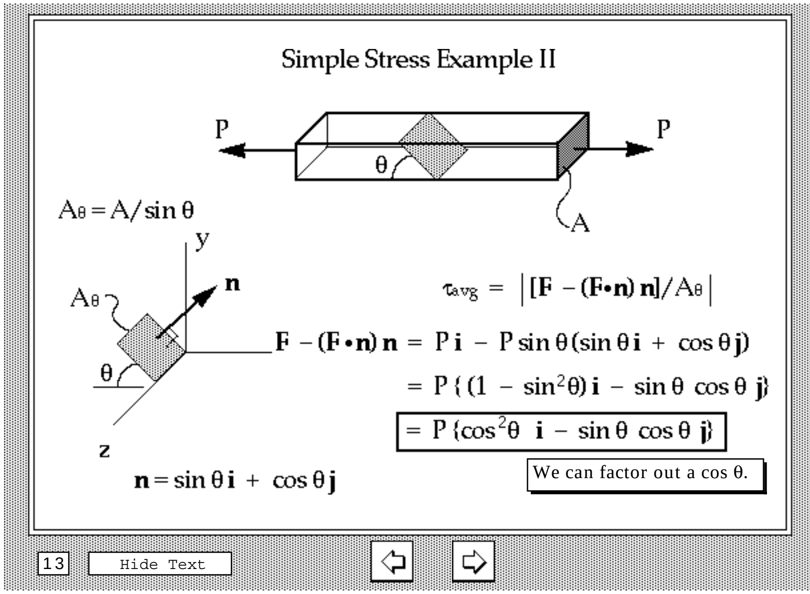
Simple Stress Example II
P
P
θ
A
Aθ = A/sin θ
y
n
Aθ
θ
z
τavg = |[F − (F•n) n]/Aθ|
F − (F•n) n = P i − P sin θ (sin θ i + cos θ j)
= P { (1 − sin²θ) i − sin θ cos θ j}
= P {cos²θ i − sin θ cos θ j}
n = sin θ i + cos θ j
We can factor out a cos θ.
13
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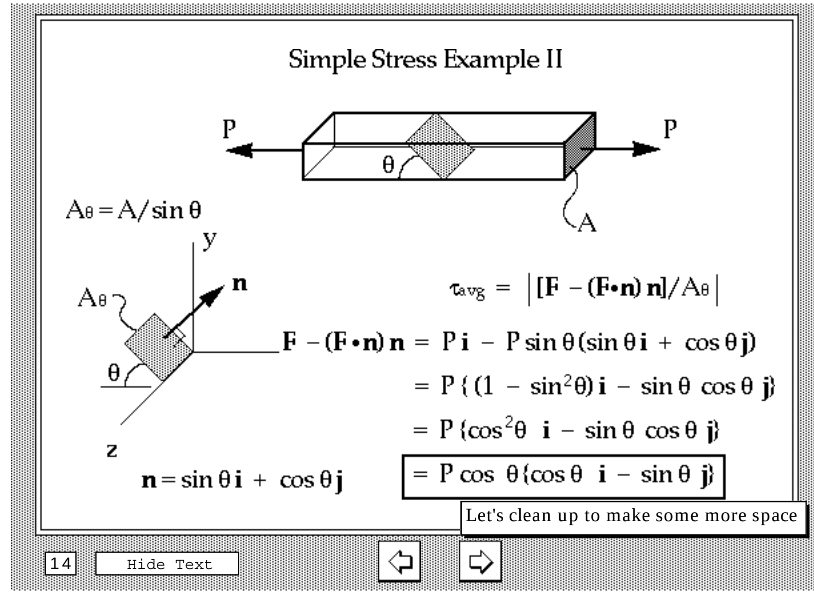
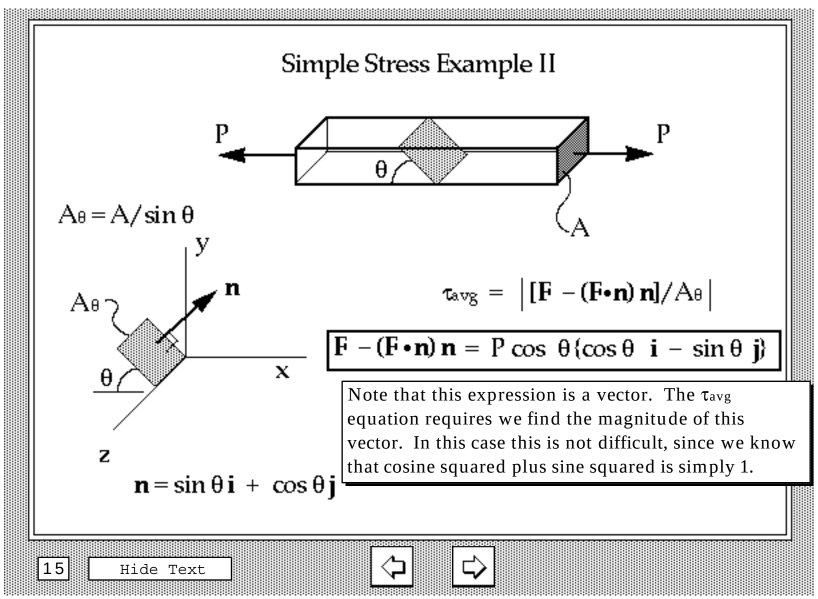
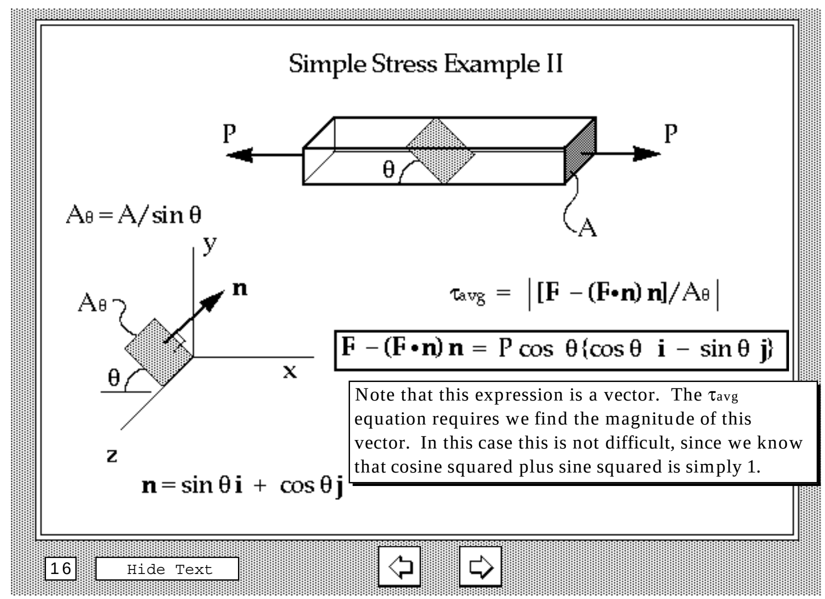
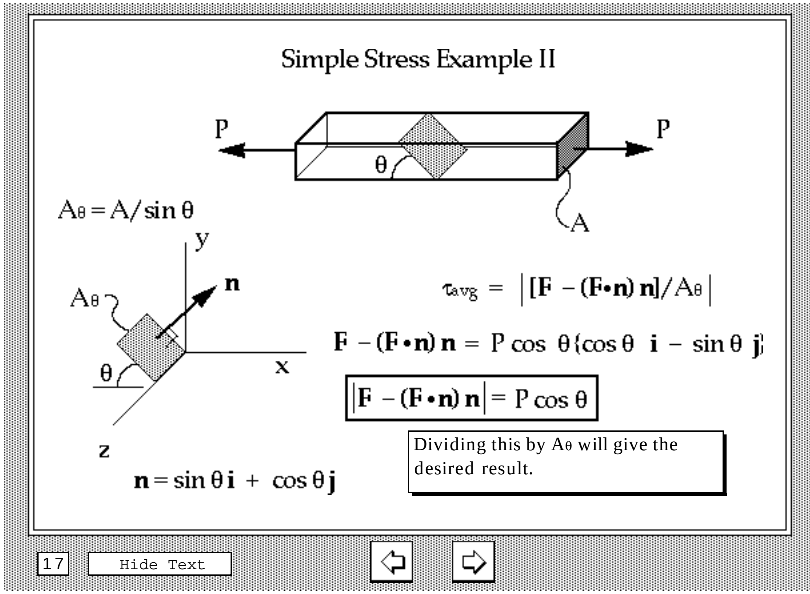

Simple Stress Example II
P
P
θ
A
Aθ = A/sin θ
y
n
Aθ
θ
z
τavg = |[F − (F•n) n]/Aθ|
F − (F•n) n = P i − P sin θ (sin θ i + cos θ j)
= P { (1 − sin²θ) i − sin θ cos θ j}
= P {cos²θ i − sin θ cos θ j}
= P cos θ {cos θ i − sin θ j}
n = sin θ i + cos θ j
Let's clean up to make some more space
14
Hide Text


Simple Stress Example II
P
P
θ
A
Aθ = A/sin θ
y
n
Aθ
θ
x
z
τavg = |[F − (F•n) n]/Aθ|
F − (F•n) n = P cos θ {cos θ i − sin θ j}
Note that this expression is a vector. The τavg equation requires we find the magnitude of this vector. In this case this is not difficult, since we know that cosine squared plus sine squared is simply 1.
n = sin θ i + cos θ j
15
Hide Text


Simple Stress Example II
P
P
θ
A
Aθ = A/sin θ
y
n
Aθ
θ
x
z
τavg = |[F − (F•n) n]/Aθ|
F − (F•n) n = P cos θ {cos θ i − sin θ j}
Note that this expression is a vector. The τavg equation requires we find the magnitude of this vector. In this case this is not difficult, since we know that cosine squared plus sine squared is simply 1.
n = sin θ i + cos θ j
16
Hide Text


Simple Stress Example II
P
P
θ
A
$A_\theta = A/\sin\theta$
y
n
A_θ
θ
x
z
$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n}]/A_\theta\right|$
$\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n} = P\cos\theta\{\cos\theta\ \mathbf{i} - \sin\theta\ \mathbf{j}\}$
$\left|\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n}\right| = P\cos\theta$
Dividing this by A_θ will give the desired result.
$\mathbf{n} = \sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$
17
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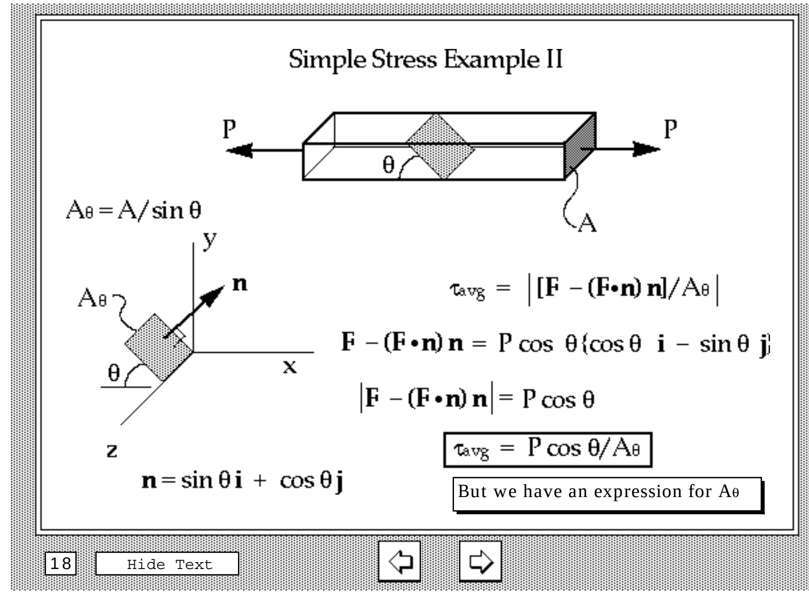
Simple Stress Example II
P
P
θ
A
$A_\theta = A/\sin\theta$
y
n
A_θ
θ
x
z
$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n}]/A_\theta\right|$
$\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n} = P\cos\theta\{\cos\theta\ \mathbf{i} - \sin\theta\ \mathbf{j}\}$
$\left|\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n}\right| = P\cos\theta$
$\tau_{avg} = P\cos\theta/A_\theta$
But we have an expression for A_θ
$\mathbf{n} = \sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$
18
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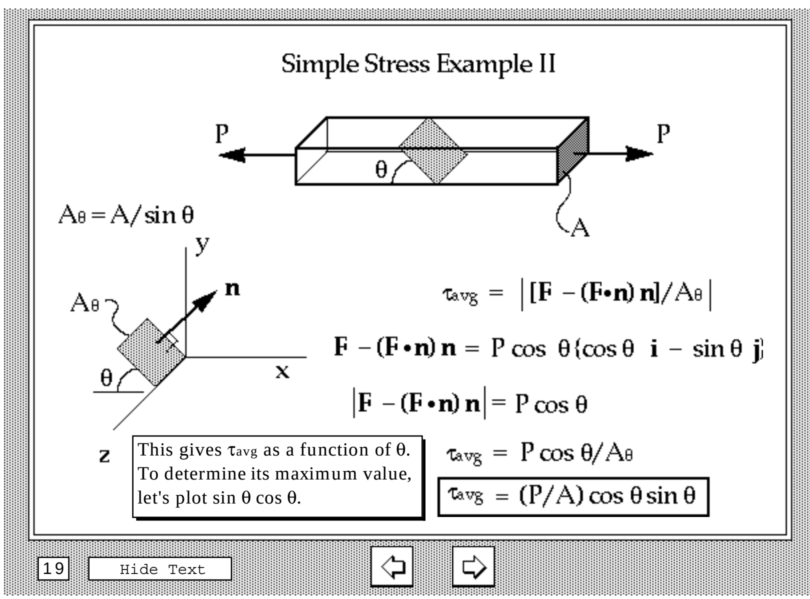
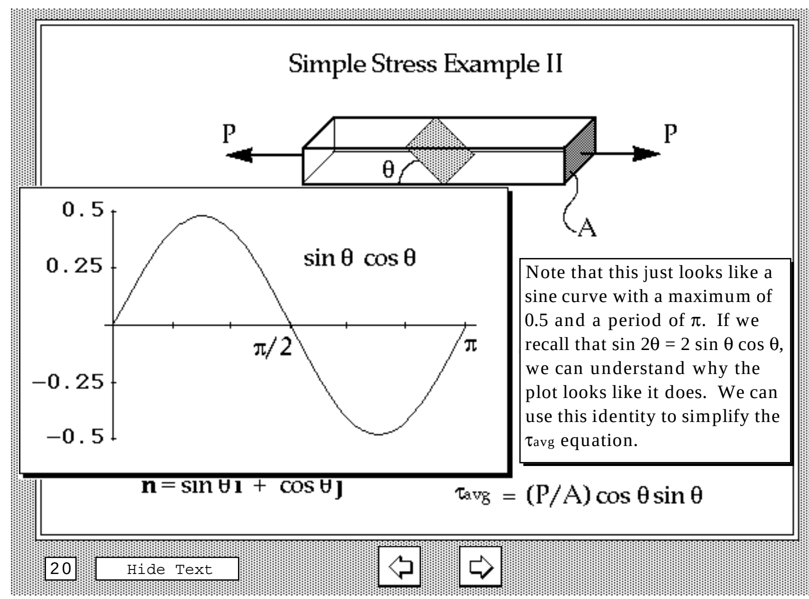

Simple Stress Example II
P
P
θ
A
$A_\theta = A/\sin\theta$
y
n
A_θ
θ
x
z
$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n}]/A_\theta\right|$
$\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n} = P\cos\theta\{\cos\theta\ \mathbf{i} - \sin\theta\ \mathbf{j}\}$
$\left|\mathbf{F} - (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n}\right| = P\cos\theta$
This gives τ_{avg} as a function of θ. To determine its maximum value, let's plot sin θ cos θ.
$\tau_{avg} = P\cos\theta/A_\theta$
$\tau_{avg} = (P/A)\cos\theta\sin\theta$
19
Hide Text


Simple Stress Example II
P
P
θ
A
0.5
0.25
−0.25
−0.5
sin θ cos θ
π/2
π
Note that this just looks like a sine curve with a maximum of 0.5 and a period of π. If we recall that sin 2θ = 2 sin θ cos θ, we can understand why the plot looks like it does. We can use this identity to simplify the τ_{avg} equation.
$\tau_{avg} = (P/A)\cos\theta\sin\theta$
20
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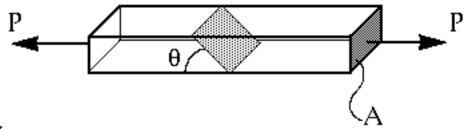
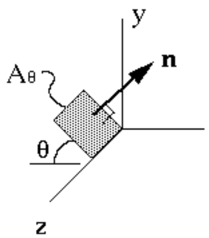
Simple Stress Example II



$A_\theta = A/\sin\theta$



$\mathbf{n} = \sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$

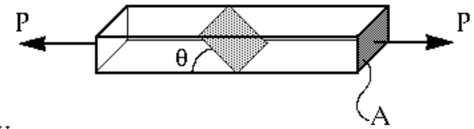
$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F}\bullet\mathbf{n})\,\mathbf{n}]/A_\theta\right|$

$\tau_{avg} = (P/A)\cos\theta\sin\theta$

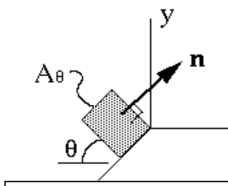
$\tau_{avg} = (1/2)(P/A)\sin 2\theta$

From this expression it is easy to determine the maximum.

Simple Stress Example II



$A_\theta = A/\sin\theta$



$\tau_{avg} = \left|[\mathbf{F} - (\mathbf{F}\bullet\mathbf{n})\,\mathbf{n}]/A_\theta\right|$

$\tau_{avg} = (P/A)\cos\theta\sin\theta$

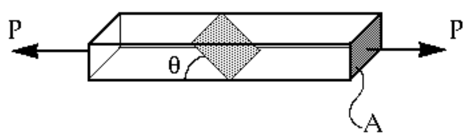
$\tau_{avg} = (1/2)(P/A)\sin 2\theta$

The maximum average shear stress is 1/2(P/A), and occurs on planes oriented at 45° from the horizontal loading plane.
(We will worry about the sign of τ later)

$@\,\theta = \pi/4,\ 3\pi/4$

$\tau_{avg} = (1/2)(P/A)$

Simple Stress Example II



Maximum Normal Stress

$\sigma_{avg} = P/A$

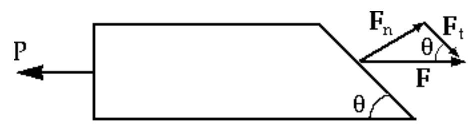
$@\,\theta = \pi/2$

Maximum Shear Stress

$\tau_{avg} = 1/2\,(P/A)$

$@\,\theta = \pi/4,\ 3\pi/4$

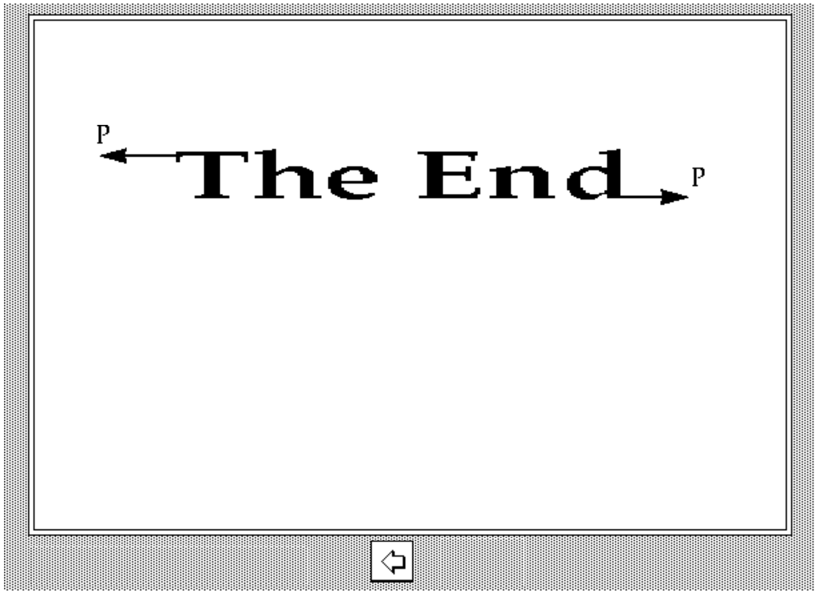
Notice that the maximum average shear stress occurs on planes oriented at 45° from the plane of maximum normal stress. Also, the maximum shear stress has a value of 1/2 the maximum average normal stress, P/A.



$F_n = P\sin\theta$

$F_t = P\cos\theta$

Note that for two-dimensional problems, like this one, the normal and shear components of the force may be calculated directly using the sine and cosine. While this technique is more direct than what we just did using vectors, it can be very difficult to apply in three dimensions. Ideally, you should be adept at using either technique.


P
The End
P