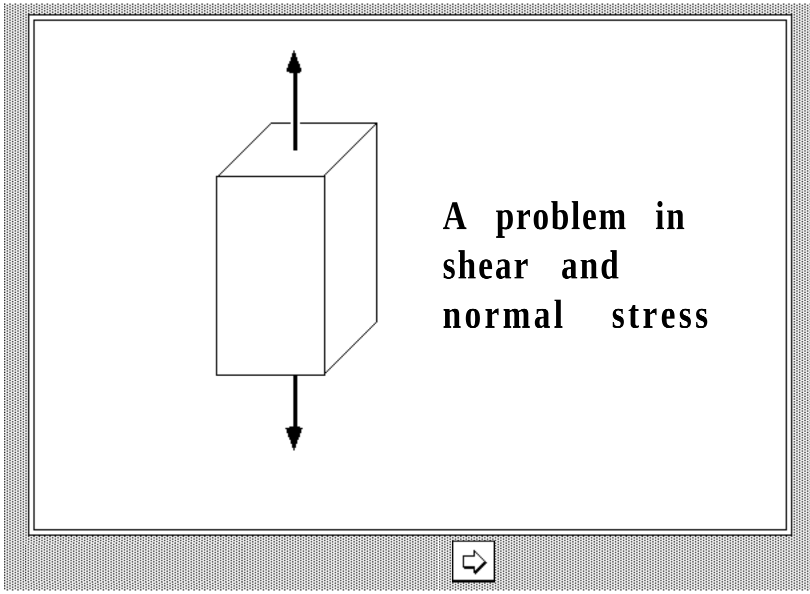
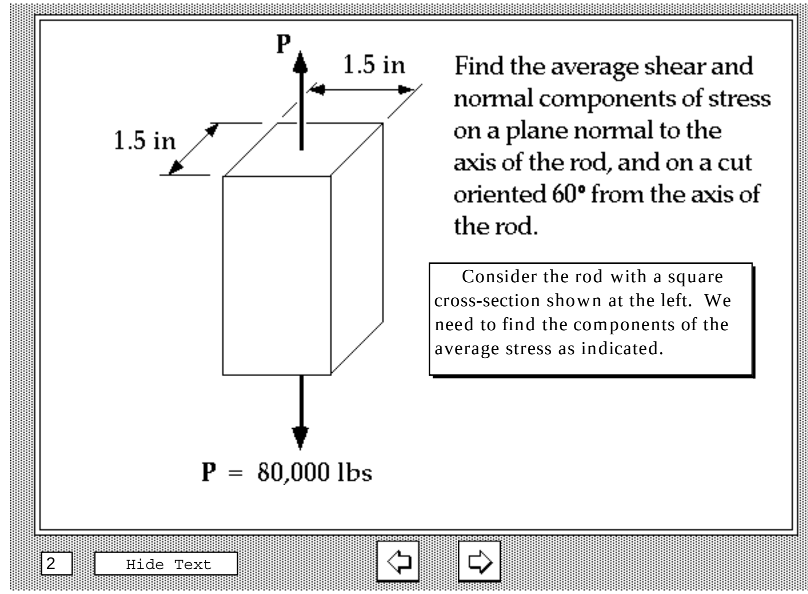
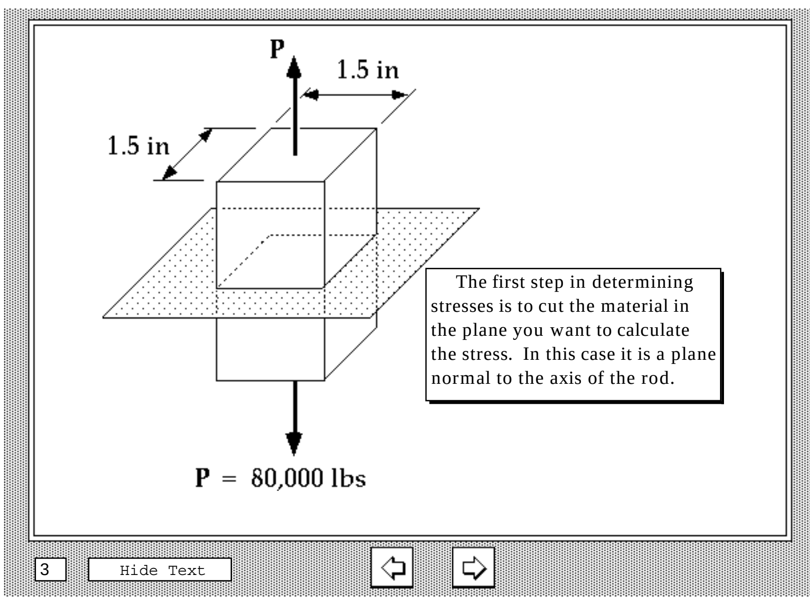
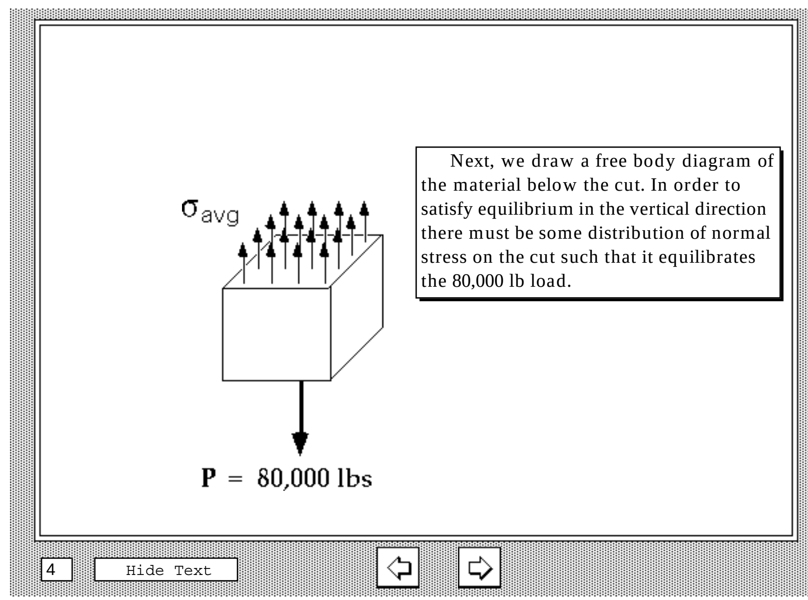
Simple Stress Example: 1

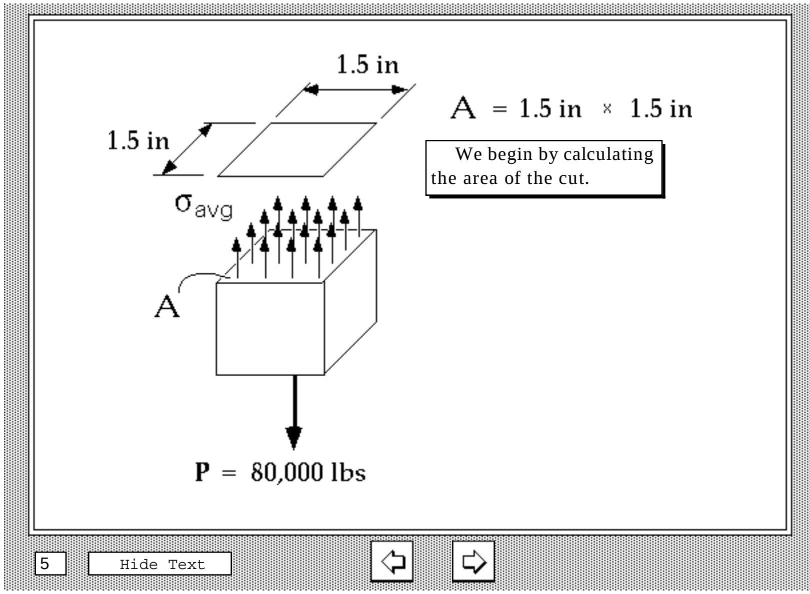


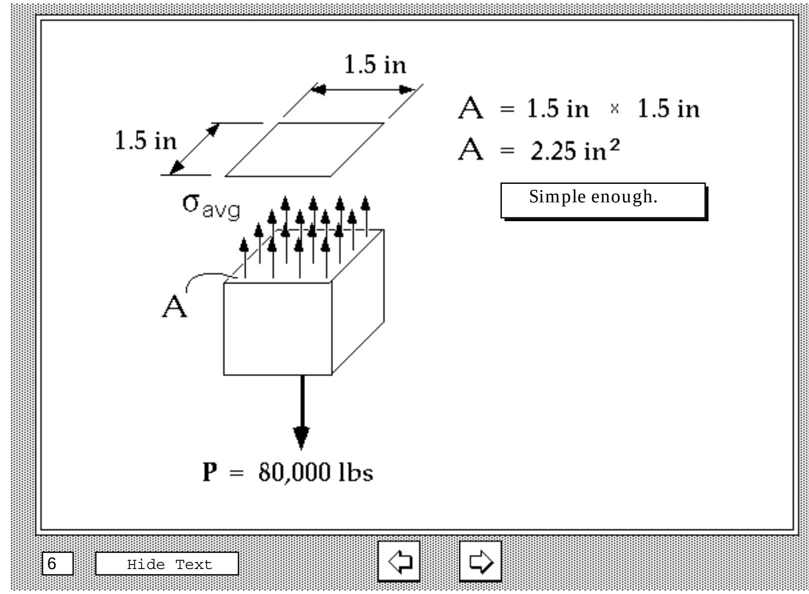


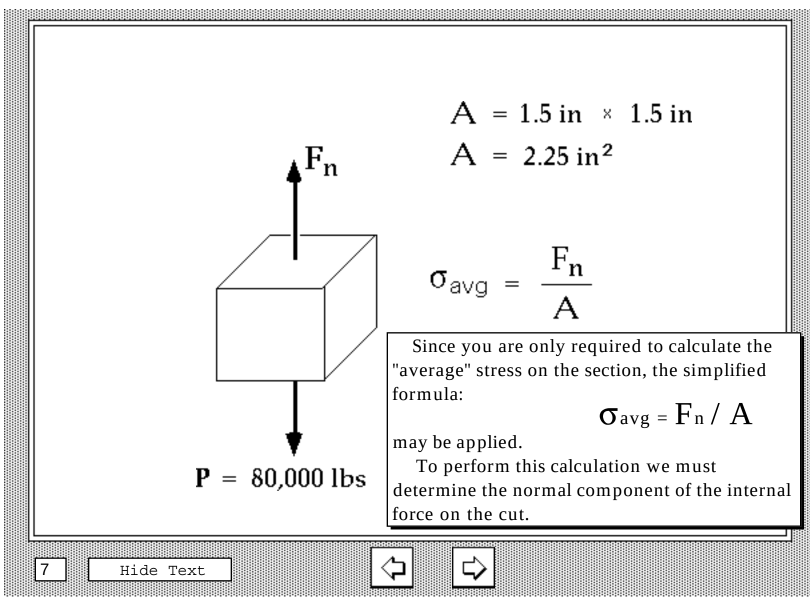


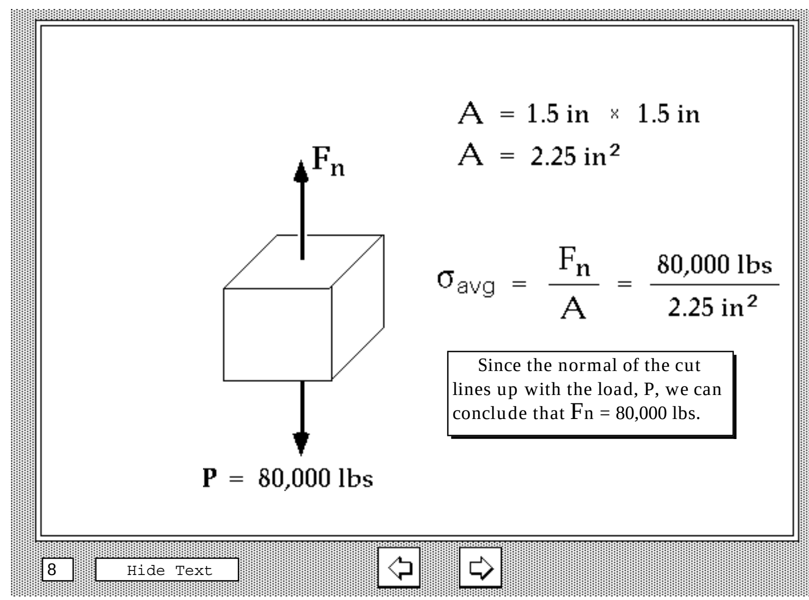


Simple Stress Example: 2

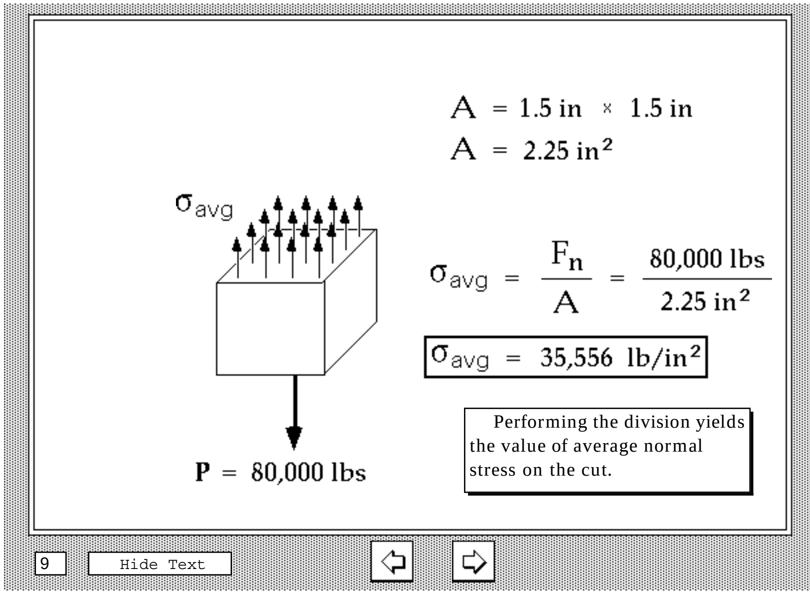


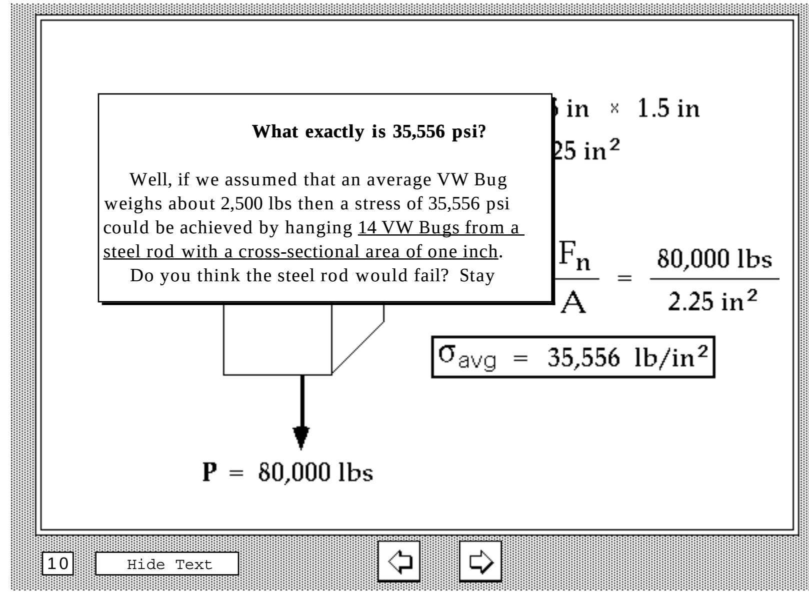


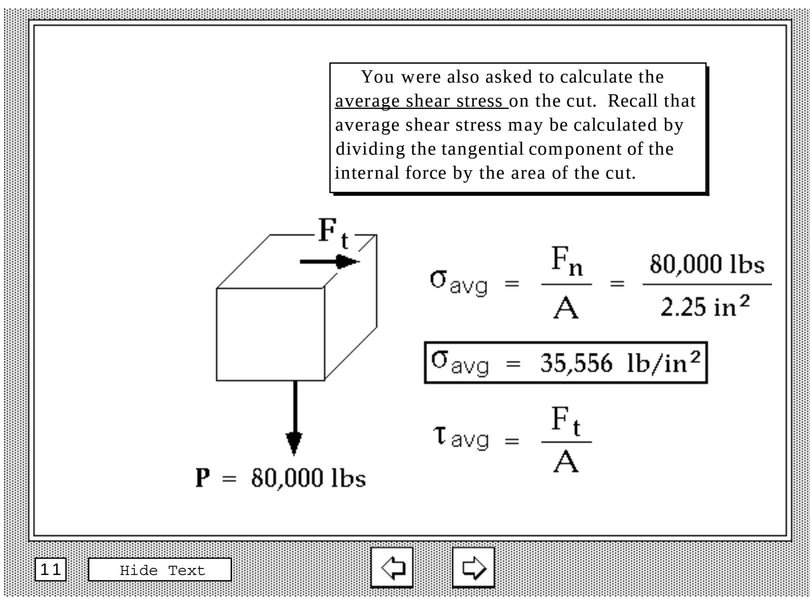


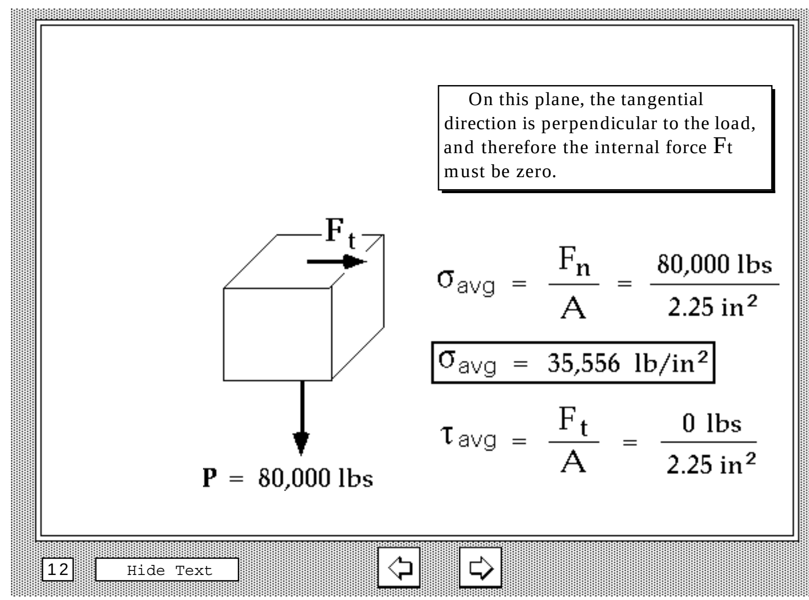


Simple Stress Example: 3





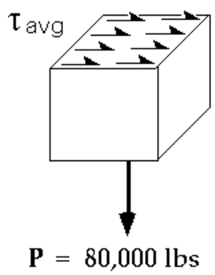




As you might have guessed, the shear stress on this cut is zero.

At this point we have answered the first half of the problem.

$$A = 1.5\text{ in} \times 1.5\text{ in}$$
$$A = 2.25\text{ in}^2$$



$$\sigma_{avg} = \frac{F_n}{A} = \frac{80{,}000\text{ lbs}}{2.25\text{ in}^2}$$

$$\boxed{\sigma_{avg} = 35{,}556\ \text{lb/in}^2}$$

$$\tau_{avg} = \frac{F_t}{A} = \frac{0\text{ lbs}}{2.25\text{ in}^2}$$

$$\boxed{\tau_{avg} = 0\ \text{lb/in}^2}$$

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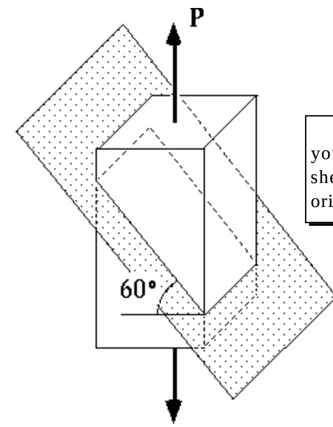

For the second part of this problem, you were asked to calculate the average shear and normal stresses on a plane oriented 60° from the original cut.

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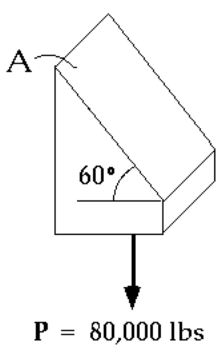
$$A = 1.5\text{ in} \times 1.5\text{ in} / \cos 60°$$

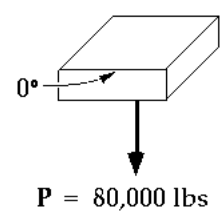
Again, we begin the problem by calculating the area of the cut.



15 Hide Text

$$A = 1.5\text{ in} \times 1.5\text{ in} / \cos 60°$$

$$A_{TEST} = 1.5\text{ in} \times 1.5\text{ in} / \cos 0° = 2.25\text{ in}^2 \ \checkmark$$



How did we know that it was the cosine of the angle that was needed to divide the area?

A good check is to examine extreme cases of the angle. If, for example, we set the angle to 0° then the area of the cut should equal the area calculated in the previous solution. Since the cosine of 0° is 1, we see that the equation above is correct.

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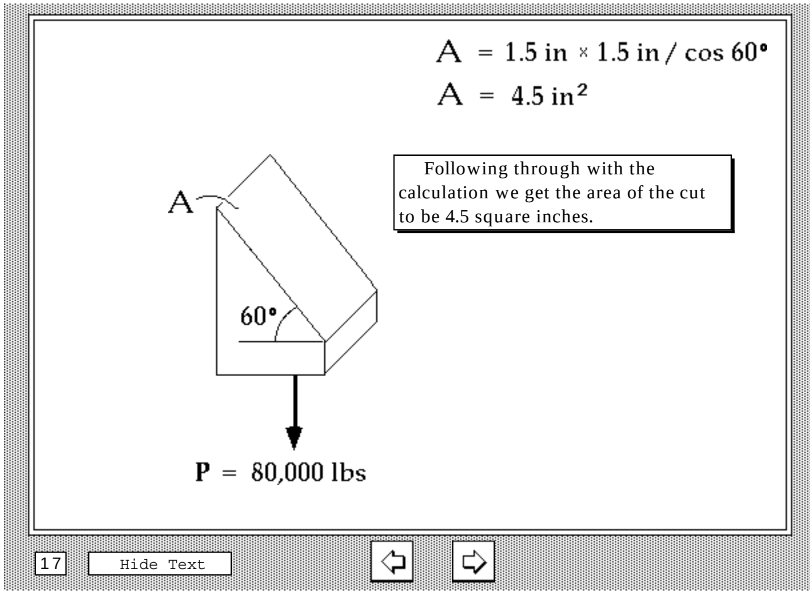
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
Following through with the calculation we get the area of the cut to be 4.5 square inches.
A
60°
P = 80,000 lbs
17
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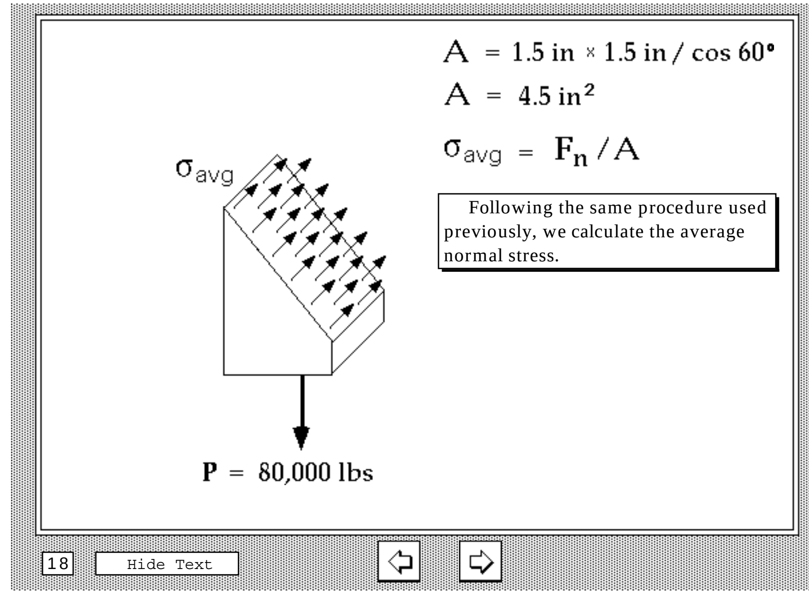
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
σavg = Fn / A
Following the same procedure used previously, we calculate the average normal stress.
σavg
P = 80,000 lbs
18
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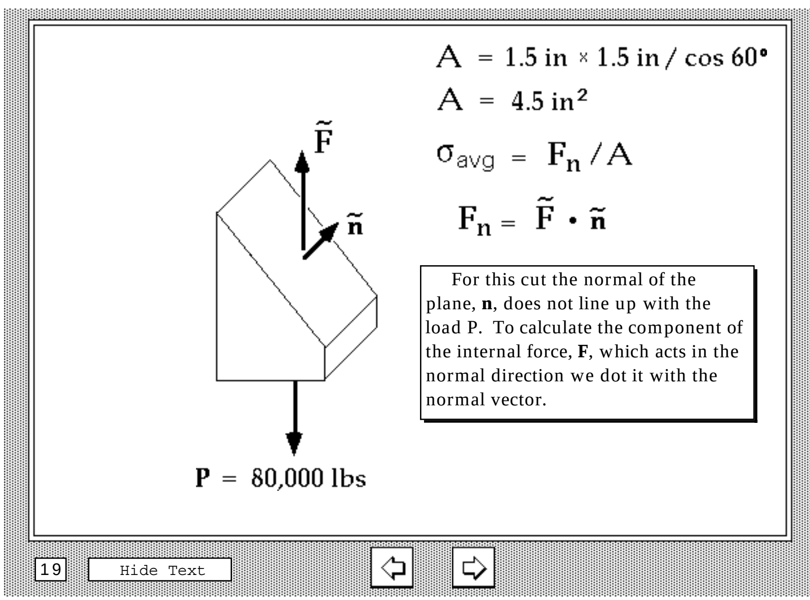
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
σavg = Fn / A
Fn = F̃ • ñ
For this cut the normal of the plane, n, does not line up with the load P. To calculate the component of the internal force, F, which acts in the normal direction we dot it with the normal vector.
F̃
ñ
P = 80,000 lbs
19
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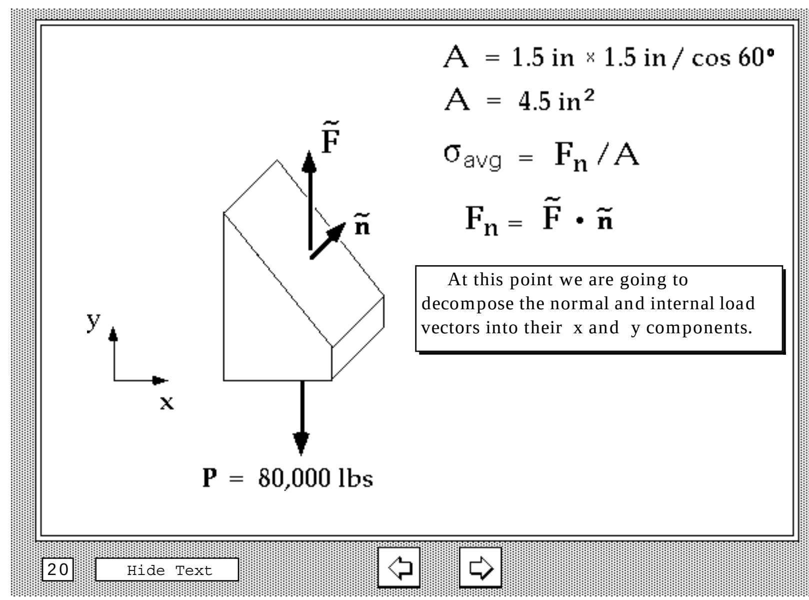
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
σavg = Fn / A
Fn = F̃ • ñ
At this point we are going to decompose the normal and internal load vectors into their x and y components.
F̃
ñ
y
x
P = 80,000 lbs
20
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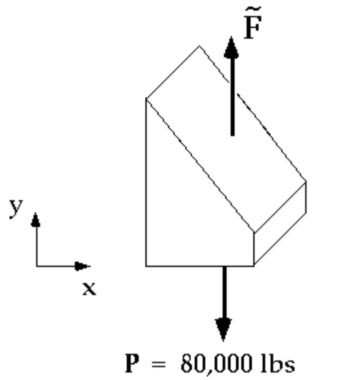

$$A = 1.5\text{ in} \times 1.5\text{ in} / \cos 60°$$
$$A = 4.5\text{ in}^2$$
$$\sigma_{avg} = F_n / A$$
$$F_n = \tilde{F} \cdot \tilde{n}$$
$$\tilde{F} = P\,\mathbf{j}$$

Since the internal load vector, **F**, acts entirely in the y direction it may be expressed as P**j**.

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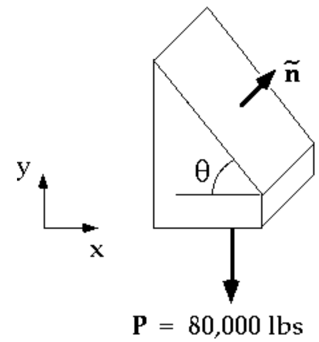

$$A = 1.5\text{ in} \times 1.5\text{ in} / \cos 60°$$
$$A = 4.5\text{ in}^2$$
$$\sigma_{avg} = F_n / A$$
$$F_n = \tilde{F} \cdot \tilde{n}$$
$$\tilde{F} = P\,\mathbf{j}$$
$$\tilde{n} = \sin 60°\,\mathbf{i} + \cos 60°\,\mathbf{j}$$

Determining the components of the normal vector is a little trickier. You should be able to confirm that the x component of **n** varies as the sine of the angle θ, and that the y component of **n** varies as cosθ.

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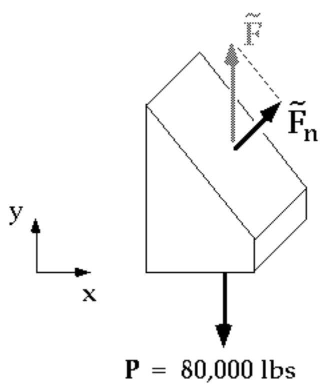

$$A = 1.5\text{ in} \times 1.5\text{ in} / \cos 60°$$
$$A = 4.5\text{ in}^2$$
$$\sigma_{avg} = F_n / A$$
$$F_n = \tilde{F} \cdot \tilde{n}$$
$$\tilde{F} = P\,\mathbf{j}$$
$$\tilde{n} = \sin 60°\,\mathbf{i} + \cos 60°\,\mathbf{j}$$
$$F_n = P \cos 60° = 40{,}000\text{ lbs}$$

Performing the dot product we find that the normal component of the internal force is 40,000 lbs.

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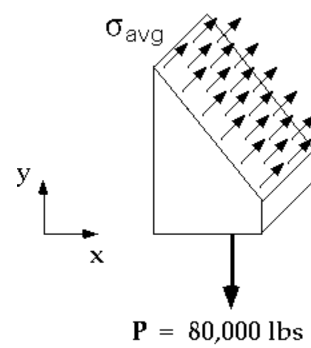
The average normal stress is then calculated by dividing the normal component of the internal force by the area of the cut.



$$A = 1.5\text{ in} \times 1.5\text{ in} / \cos 60°$$
$$A = 4.5\text{ in}^2$$
$$\sigma_{avg} = F_n / A$$
$$F_n = \tilde{F} \cdot \tilde{n}$$
$$\tilde{F} = P\,\mathbf{j}$$
$$\tilde{n} = \sin 60°\,\mathbf{i} + \cos 60°\,\mathbf{j}$$
$$F_n = P \cos 60° = 40{,}000\text{ lbs}$$
$$\sigma_{avg} = \frac{40{,}000\text{ lbs}}{4.5\text{ in}^2} = 8{,}889\text{ psi}$$

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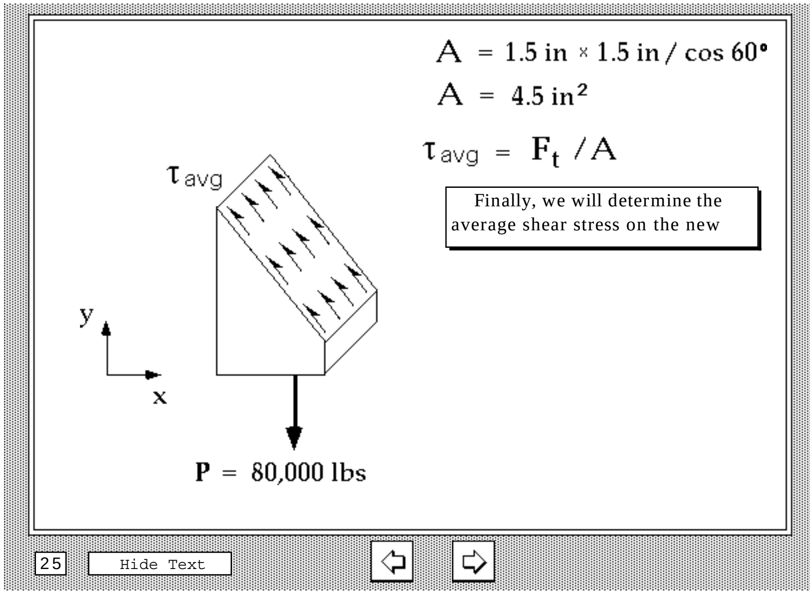
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
τavg = Ft / A
Finally, we will determine the average shear stress on the new
τavg
y
x
P = 80,000 lbs
25
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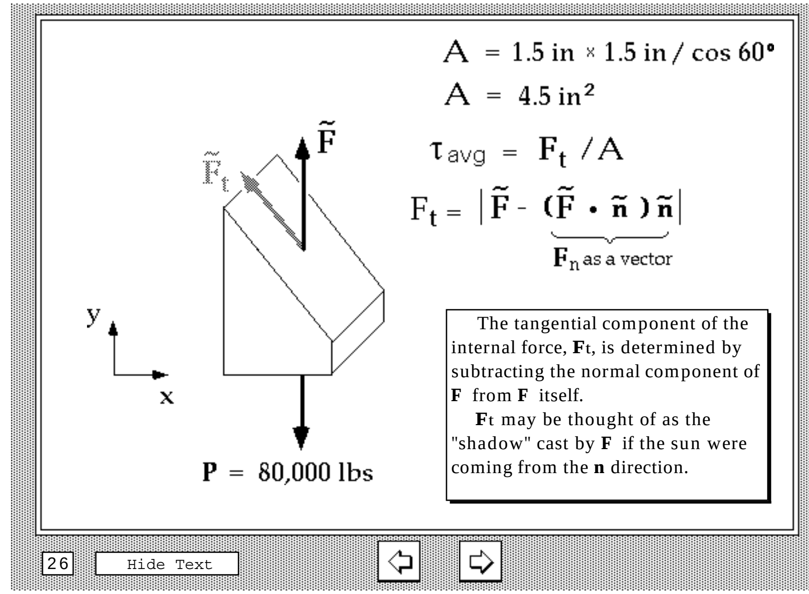
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
τavg = Ft / A
Ft = |F - (F • n) n|
Fn as a vector
F
Ft
y
x
P = 80,000 lbs
The tangential component of the internal force, Ft, is determined by subtracting the normal component of F from F itself.
Ft may be thought of as the "shadow" cast by F if the sun were coming from the n direction.
26
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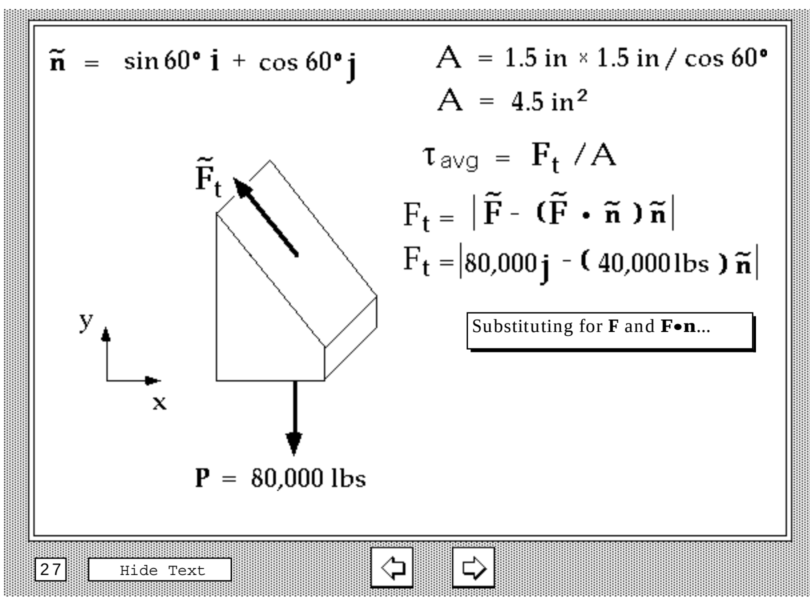
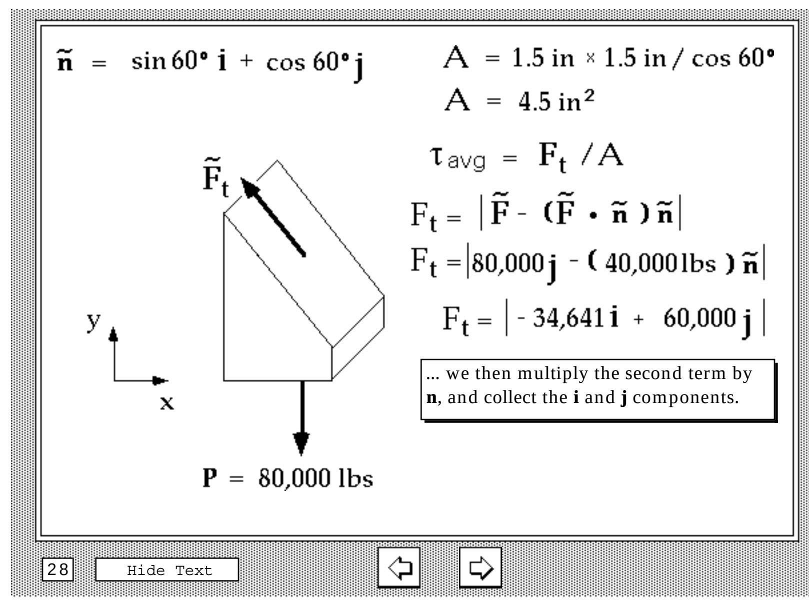
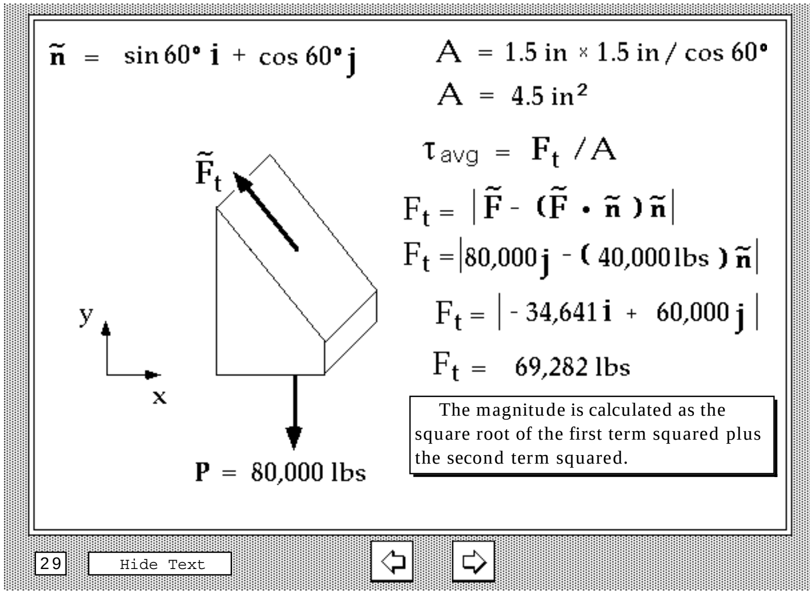

n = sin 60° i + cos 60° j
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
τavg = Ft / A
Ft = |F - (F • n) n|
Ft = |80,000 j - (40,000 lbs) n|
Substituting for F and F•n...
Ft
y
x
P = 80,000 lbs
27
Hide Text


n = sin 60° i + cos 60° j
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
τavg = Ft / A
Ft = |F - (F • n) n|
Ft = |80,000 j - (40,000 lbs) n|
Ft = | - 34,641 i + 60,000 j |
... we then multiply the second term by n, and collect the i and j components.
Ft
y
x
P = 80,000 lbs
28
Hide Text


ñ = sin 60° i + cos 60° j
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
τavg = Ft / A
Ft = |F̃ - (F̃ • ñ) ñ|
Ft = |80,000 j - (40,000 lbs) ñ|
Ft = |- 34,641 i + 60,000 j|
Ft = 69,282 lbs
F̃t
y
x
P = 80,000 lbs
The magnitude is calculated as the square root of the first term squared plus the second term squared.
29
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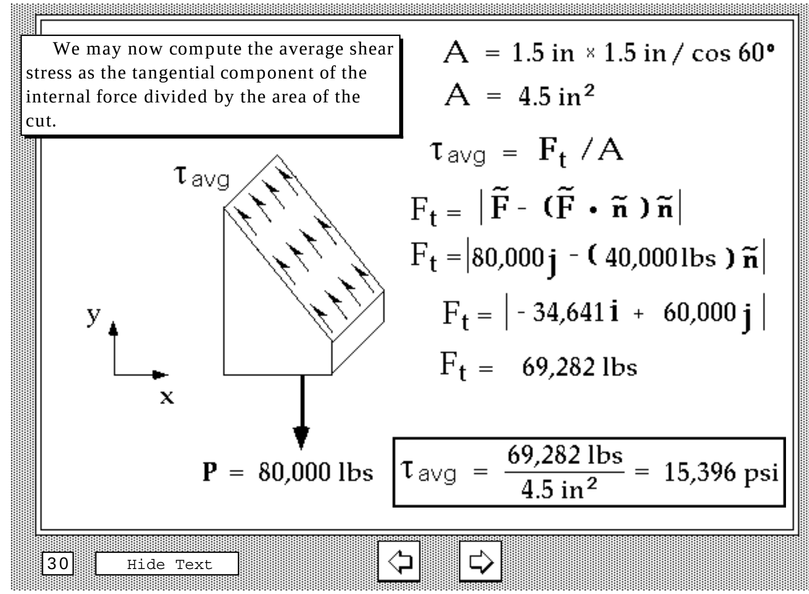
We may now compute the average shear stress as the tangential component of the internal force divided by the area of the cut.
A = 1.5 in × 1.5 in / cos 60°
A = 4.5 in²
τavg = Ft / A
Ft = |F̃ - (F̃ • ñ) ñ|
Ft = |80,000 j - (40,000 lbs) ñ|
Ft = |- 34,641 i + 60,000 j|
Ft = 69,282 lbs
τavg
y
x
P = 80,000 lbs
τavg = 69,282 lbs / 4.5 in² = 15,396 psi
30
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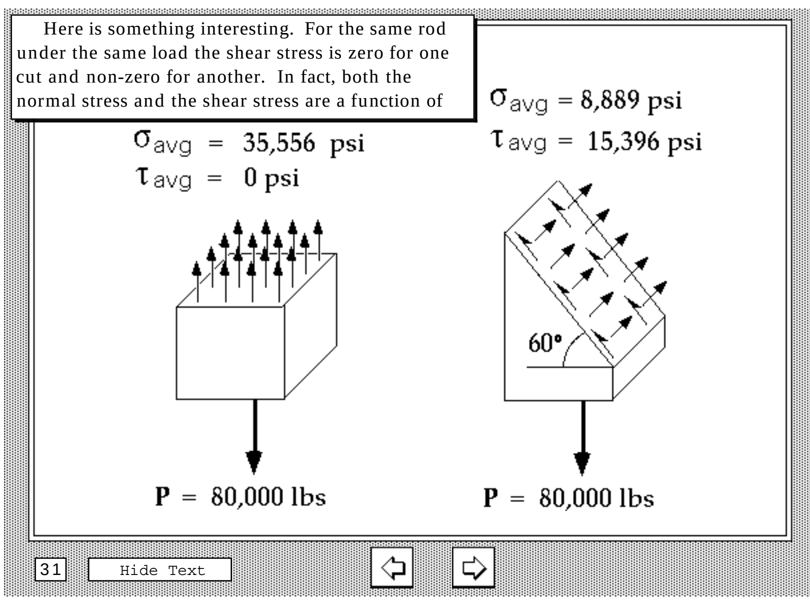
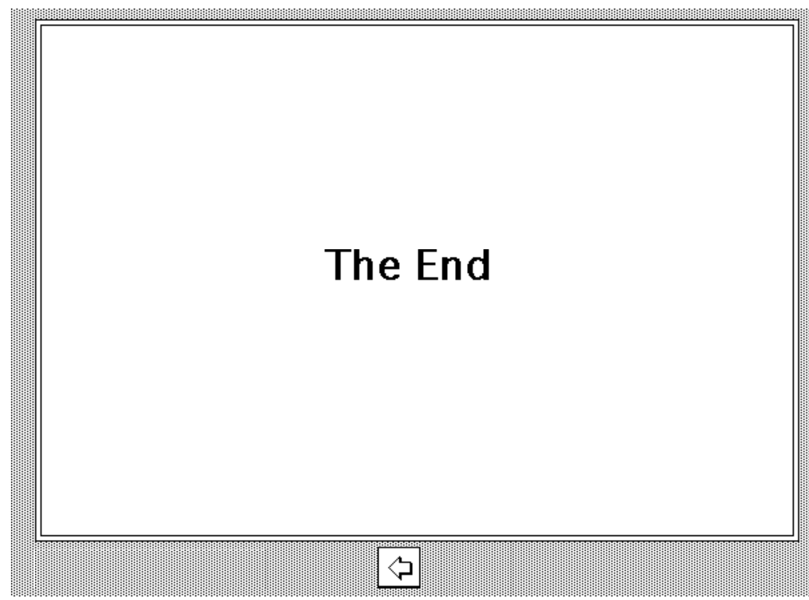

Here is something interesting. For the same rod under the same load the shear stress is zero for one cut and non-zero for another. In fact, both the normal stress and the shear stress are a function of
σavg = 35,556 psi
τavg = 0 psi
σavg = 8,889 psi
τavg = 15,396 psi
60°
P = 80,000 lbs
P = 80,000 lbs
31
Hide Text


The End