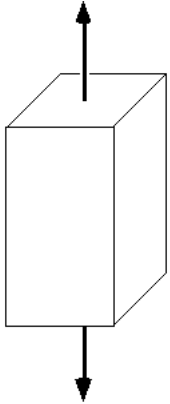


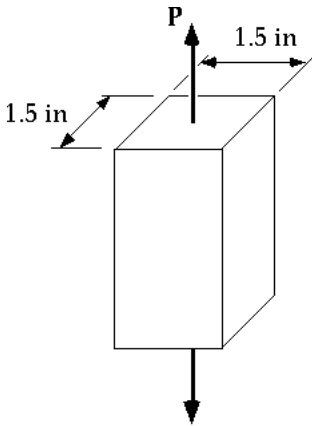
Simple Stress Example: 1



A 3D perspective drawing of a rectangular rod. A vertical arrow points upwards from the top face, and another vertical arrow points downwards from the bottom face, representing tensile forces.

A problem in shear and normal stress

2

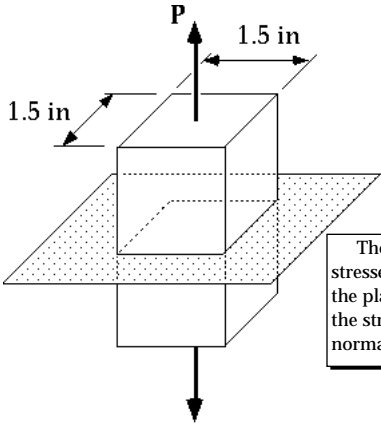


A 3D perspective drawing of a square rod. The top and bottom faces are labeled with a vertical arrow pointing up and a vertical arrow pointing down, both labeled P . The side length of the square cross-section is indicated as 1.5 in on both the top and left edges. Below the rod, a vertical arrow points down, labeled $P = 80,000 \text{ lbs}$.

Find the average shear and normal components of stress on a plane normal to the axis of the rod, and on a cut oriented 60° from the axis of the rod.

Consider the rod with a square cross-section shown at the left. We need to find the components of the average stress as indicated.

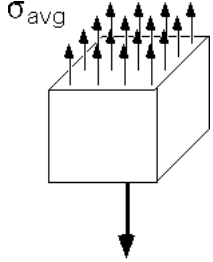
2 Hide Text



A 3D perspective drawing of the square rod from the previous slide, but now it is cut horizontally. A shaded horizontal plane passes through the middle of the rod. The top and bottom faces are labeled with a vertical arrow pointing up and a vertical arrow pointing down, both labeled P . The side length of the square cross-section is indicated as 1.5 in on both the top and left edges. Below the rod, a vertical arrow points down, labeled $P = 80,000 \text{ lbs}$.

The first step in determining stresses is to cut the material in the plane you want to calculate the stress. In this case it is a plane normal to the axis of the rod.

3 Hide Text



A free body diagram of the bottom portion of the rod. The bottom face is labeled with a vertical arrow pointing down, labeled $P = 80,000 \text{ lbs}$. The top face, which was the cut surface, is shown with a distribution of small vertical arrows pointing upwards, labeled σ_{avg} .

Next, we draw a free body diagram of the material below the cut. In order to satisfy equilibrium in the vertical direction there must be some distribution of normal stress on the cut such that it equilibrates the 80,000 lb load.

4 Hide Text

Simple Stress Example: 2

1.5 in

1.5 in

$A = 1.5 \text{ in} \times 1.5 \text{ in}$

We begin by calculating the area of the cut.

σ_{avg}

A

$P = 80,000 \text{ lbs}$

5 Hide Text

1.5 in

1.5 in

$A = 1.5 \text{ in} \times 1.5 \text{ in}$

$A = 2.25 \text{ in}^2$

Simple enough.

σ_{avg}

A

$P = 80,000 \text{ lbs}$

6 Hide Text

F_n

$A = 1.5 \text{ in} \times 1.5 \text{ in}$

$A = 2.25 \text{ in}^2$

$\sigma_{avg} = \frac{F_n}{A}$

Since you are only required to calculate the "average" stress on the section, the simplified formula:

$\sigma_{avg} = F_n / A$

may be applied.

To perform this calculation we must determine the normal component of the internal force on the cut.

$P = 80,000 \text{ lbs}$

7 Hide Text

F_n

$A = 1.5 \text{ in} \times 1.5 \text{ in}$

$A = 2.25 \text{ in}^2$

$\sigma_{avg} = \frac{F_n}{A} = \frac{80,000 \text{ lbs}}{2.25 \text{ in}^2}$

Since the normal of the cut lines up with the load, P, we can conclude that $F_n = 80,000 \text{ lbs}$.

$P = 80,000 \text{ lbs}$

8 Hide Text

Simple Stress Example: 3

$A = 1.5 \text{ in} \times 1.5 \text{ in}$
 $A = 2.25 \text{ in}^2$

$$\sigma_{avg} = \frac{F_n}{A} = \frac{80,000 \text{ lbs}}{2.25 \text{ in}^2}$$

$\sigma_{avg} = 35,556 \text{ lb/in}^2$

Performing the division yields the value of average normal stress on the cut.

$P = 80,000 \text{ lbs}$

9 Hide Text

What exactly is 35,556 psi?

Well, if we assumed that an average VW Bug weighs about 2,500 lbs then a stress of 35,556 psi could be achieved by hanging 14 VW Bugs from a steel rod with a cross-sectional area of one inch. Do you think the steel rod would fail? Stay

$A = 1.5 \text{ in} \times 1.5 \text{ in}$
 $A = 2.25 \text{ in}^2$

$$\frac{F_n}{A} = \frac{80,000 \text{ lbs}}{2.25 \text{ in}^2}$$

$\sigma_{avg} = 35,556 \text{ lb/in}^2$

$P = 80,000 \text{ lbs}$

10 Hide Text

You were also asked to calculate the average shear stress on the cut. Recall that average shear stress may be calculated by dividing the tangential component of the internal force by the area of the cut.

$$\sigma_{avg} = \frac{F_n}{A} = \frac{80,000 \text{ lbs}}{2.25 \text{ in}^2}$$

$\sigma_{avg} = 35,556 \text{ lb/in}^2$

$$\tau_{avg} = \frac{F_t}{A}$$

$P = 80,000 \text{ lbs}$

11 Hide Text

On this plane, the tangential direction is perpendicular to the load, and therefore the internal force F_t must be zero.

$$\sigma_{avg} = \frac{F_n}{A} = \frac{80,000 \text{ lbs}}{2.25 \text{ in}^2}$$

$\sigma_{avg} = 35,556 \text{ lb/in}^2$

$$\tau_{avg} = \frac{F_t}{A} = \frac{0 \text{ lbs}}{2.25 \text{ in}^2}$$

$P = 80,000 \text{ lbs}$

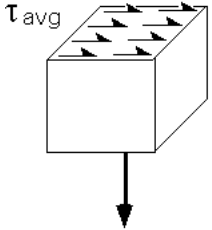
12 Hide Text

Simple Stress Example: 4

As you might have guessed, the shear stress on this cut is zero.
At this point we have answered the first half of the problem.

$A = 1.5 \text{ in} \times 1.5 \text{ in}$
 $A = 2.25 \text{ in}^2$

τ_{avg}

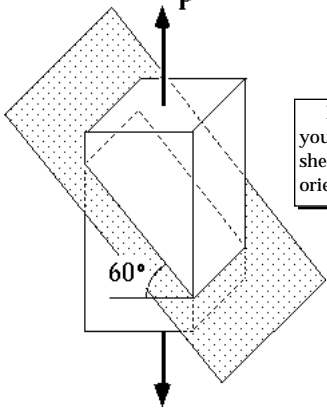


$P = 80,000 \text{ lbs}$

$\sigma_{\text{avg}} = \frac{F_n}{A} = \frac{80,000 \text{ lbs}}{2.25 \text{ in}^2}$
 $\sigma_{\text{avg}} = 35,556 \text{ lb/in}^2$

$\tau_{\text{avg}} = \frac{F_t}{A} = \frac{0 \text{ lbs}}{2.25 \text{ in}^2}$
 $\tau_{\text{avg}} = 0 \text{ lb/in}^2$

13 Hide Text

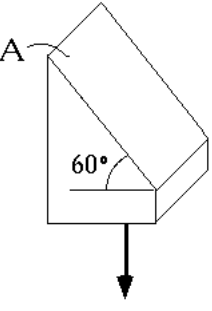


For the second part of this problem, you were asked to calculate the average shear and normal stresses on a plane oriented 60° from the original cut.

14 Hide Text

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$

Again, we begin the problem by calculating the area of the cut.



$P = 80,000 \text{ lbs}$

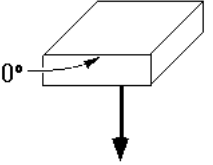
15 Hide Text

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$

$A_{\text{TEST}} = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 0^\circ = 2.25 \text{ in}^2 \checkmark$

How did we know that it was the cosine of the angle that was needed to divide the area?

A good check is to examine extreme cases of the angle. If, for example, we set the angle to 0° then the area of the cut should equal the area calculated in the previous solution. Since the cosine of 0° is 1, we see that the equation above is correct.



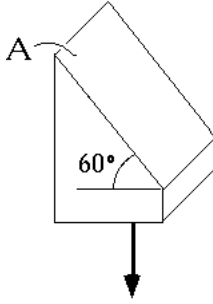
$P = 80,000 \text{ lbs}$

16 Hide Text

Simple Stress Example: 5

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$

Following through with the calculation we get the area of the cut to be 4.5 square inches.



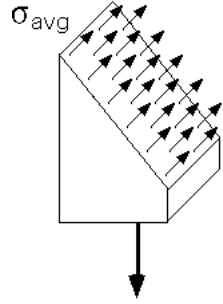
$P = 80,000 \text{ lbs}$

17 Hide Text

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$

$\sigma_{avg} = F_n / A$

Following the same procedure used previously, we calculate the average normal stress.



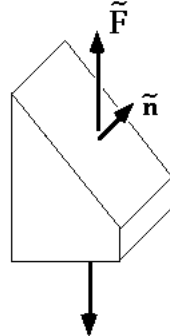
$P = 80,000 \text{ lbs}$

18 Hide Text

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$

$\sigma_{avg} = F_n / A$
 $F_n = \tilde{F} \cdot \tilde{n}$

For this cut the normal of the plane, \tilde{n} , does not line up with the load P . To calculate the component of the internal force, \tilde{F} , which acts in the normal direction we dot it with the normal vector.



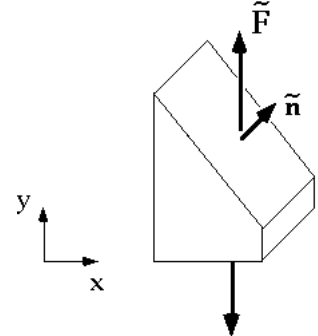
$P = 80,000 \text{ lbs}$

19 Hide Text

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$

$\sigma_{avg} = F_n / A$
 $F_n = \tilde{F} \cdot \tilde{n}$

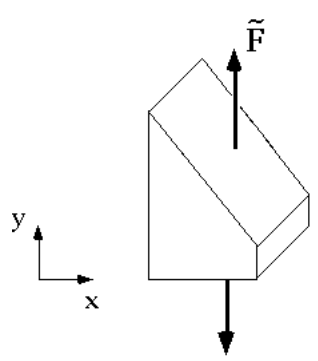
At this point we are going to decompose the normal and internal load vectors into their x and y components.



$P = 80,000 \text{ lbs}$

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Simple Stress Example: 6

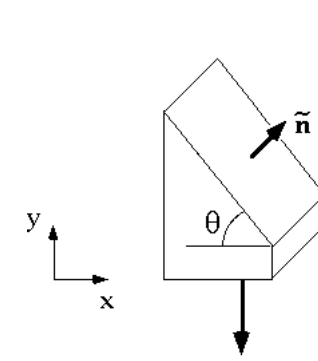


$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\sigma_{\text{avg}} = F_n / A$
 $F_n = \tilde{F} \cdot \tilde{n}$
 $\tilde{F} = P \mathbf{j}$

Since the internal load vector, F , acts entirely in the y direction it may be expressed as $P\mathbf{j}$.

$P = 80,000 \text{ lbs}$

21 Hide Text

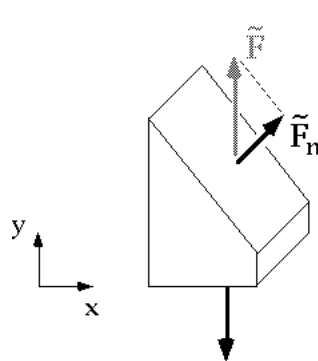


$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\sigma_{\text{avg}} = F_n / A$
 $F_n = \tilde{F} \cdot \tilde{n}$
 $\tilde{F} = P \mathbf{j}$
 $\tilde{n} = \sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}$

Determining the components of the normal vector is a little trickier. You should be able to confirm that the x component of \mathbf{n} varies as the sine of the angle θ , and that the y component of \mathbf{n} varies as $\cos\theta$.

$P = 80,000 \text{ lbs}$

22 Hide Text

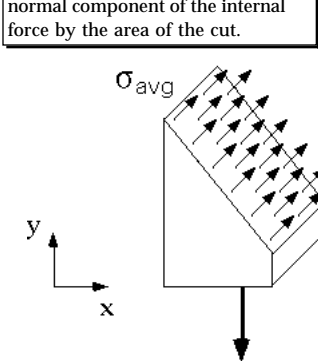


$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\sigma_{\text{avg}} = F_n / A$
 $F_n = \tilde{F} \cdot \tilde{n}$
 $\tilde{F} = P \mathbf{j}$
 $\tilde{n} = \sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}$
 $F_n = P \cos 60^\circ = 40,000 \text{ lbs}$

Performing the dot product we find that the normal component of the internal force is 40,000 lbs.

$P = 80,000 \text{ lbs}$

23 Hide Text



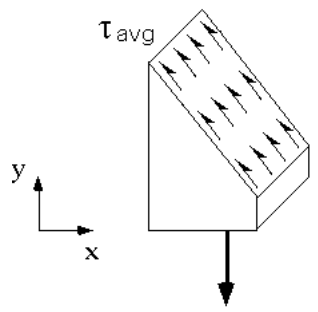
The average normal stress is then calculated by dividing the normal component of the internal force by the area of the cut.

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\sigma_{\text{avg}} = F_n / A$
 $F_n = \tilde{F} \cdot \tilde{n}$
 $\tilde{F} = P \mathbf{j}$
 $\tilde{n} = \sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}$
 $F_n = P \cos 60^\circ = 40,000 \text{ lbs}$
 $\sigma_{\text{avg}} = \frac{40,000 \text{ lbs}}{4.5 \text{ in}^2} = 8,889 \text{ psi}$

$P = 80,000 \text{ lbs}$

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Simple Stress Example: 7

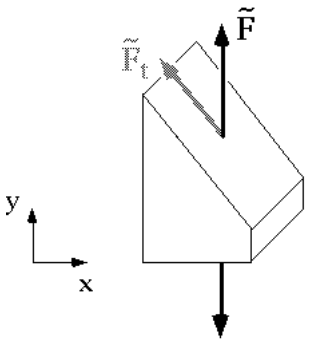


$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\tau_{\text{avg}} = F_t / A$

Finally, we will determine the average shear stress on the new

$P = 80,000 \text{ lbs}$

25 Hide Text



$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\tau_{\text{avg}} = F_t / A$
 $F_t = |\tilde{\mathbf{F}} - (\underbrace{\tilde{\mathbf{F}} \cdot \tilde{\mathbf{n}}}_{\mathbf{F}_n \text{ as a vector}}) \tilde{\mathbf{n}}|$

The tangential component of the internal force, F_t , is determined by subtracting the normal component of \mathbf{F} from \mathbf{F} itself.
 F_t may be thought of as the "shadow" cast by \mathbf{F} if the sun were coming from the \mathbf{n} direction.

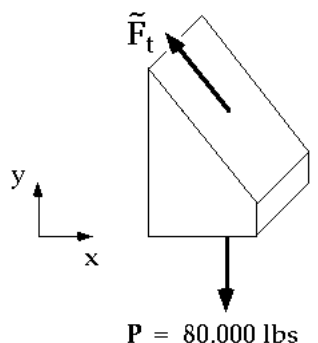
$P = 80,000 \text{ lbs}$

26 Hide Text

$\tilde{\mathbf{n}} = \sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}$

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\tau_{\text{avg}} = F_t / A$
 $F_t = |\tilde{\mathbf{F}} - (\tilde{\mathbf{F}} \cdot \tilde{\mathbf{n}}) \tilde{\mathbf{n}}|$
 $F_t = |80,000 \mathbf{j} - (40,000 \text{ lbs}) \tilde{\mathbf{n}}|$

Substituting for \mathbf{F} and $\mathbf{F} \cdot \mathbf{n} \dots$



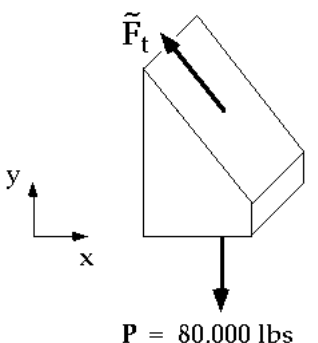
$P = 80,000 \text{ lbs}$

27 Hide Text

$\tilde{\mathbf{n}} = \sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}$

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$
 $\tau_{\text{avg}} = F_t / A$
 $F_t = |\tilde{\mathbf{F}} - (\tilde{\mathbf{F}} \cdot \tilde{\mathbf{n}}) \tilde{\mathbf{n}}|$
 $F_t = |80,000 \mathbf{j} - (40,000 \text{ lbs}) \tilde{\mathbf{n}}|$
 $F_t = |-34,641 \mathbf{i} + 60,000 \mathbf{j}|$

... we then multiply the second term by \mathbf{n} , and collect the \mathbf{i} and \mathbf{j} components.



$P = 80,000 \text{ lbs}$

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Simple Stress Example: 8

$\tilde{\mathbf{n}} = \sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}$
 $A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$

$\tau_{\text{avg}} = F_t / A$
 $F_t = |\tilde{\mathbf{F}} - (\tilde{\mathbf{F}} \cdot \tilde{\mathbf{n}}) \tilde{\mathbf{n}}|$
 $F_t = |80,000 \mathbf{j} - (40,000 \text{ lbs}) \tilde{\mathbf{n}}|$
 $F_t = |-34,641 \mathbf{i} + 60,000 \mathbf{j}|$
 $F_t = 69,282 \text{ lbs}$

The magnitude is calculated as the square root of the first term squared plus the second term squared.

$\mathbf{P} = 80,000 \text{ lbs}$

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We may now compute the average shear stress as the tangential component of the internal force divided by the area of the cut.

$A = 1.5 \text{ in} \times 1.5 \text{ in} / \cos 60^\circ$
 $A = 4.5 \text{ in}^2$

$\tau_{\text{avg}} = F_t / A$
 $F_t = |\tilde{\mathbf{F}} - (\tilde{\mathbf{F}} \cdot \tilde{\mathbf{n}}) \tilde{\mathbf{n}}|$
 $F_t = |80,000 \mathbf{j} - (40,000 \text{ lbs}) \tilde{\mathbf{n}}|$
 $F_t = |-34,641 \mathbf{i} + 60,000 \mathbf{j}|$
 $F_t = 69,282 \text{ lbs}$

$\tau_{\text{avg}} = \frac{69,282 \text{ lbs}}{4.5 \text{ in}^2} = 15,396 \text{ psi}$

$\mathbf{P} = 80,000 \text{ lbs}$

30 Hide Text

Here is something interesting. For the same rod under the same load the shear stress is zero for one cut and non-zero for another. In fact, both the normal stress and the shear stress are a function of

$\sigma_{\text{avg}} = 35,556 \text{ psi}$
 $\tau_{\text{avg}} = 0 \text{ psi}$

$\sigma_{\text{avg}} = 8,889 \text{ psi}$
 $\tau_{\text{avg}} = 15,396 \text{ psi}$

$\mathbf{P} = 80,000 \text{ lbs}$

$\mathbf{P} = 80,000 \text{ lbs}$

31 Hide Text

The End

↩