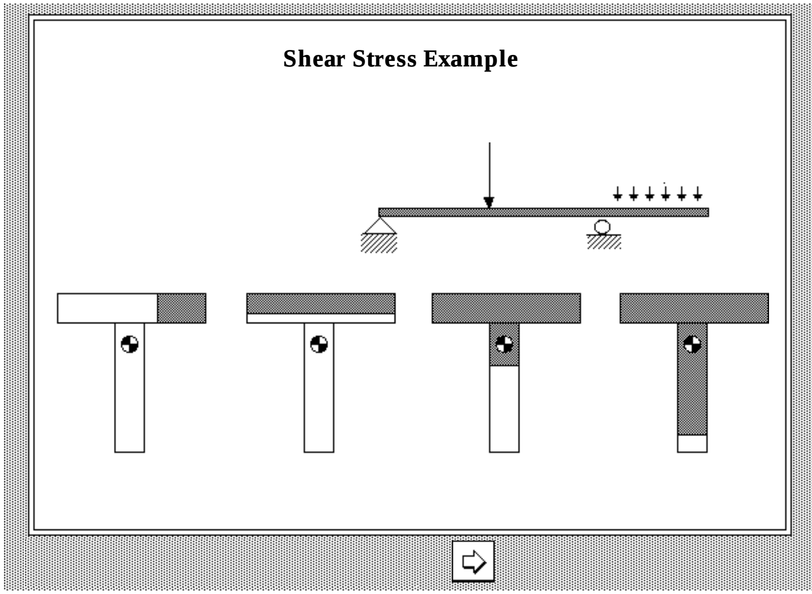
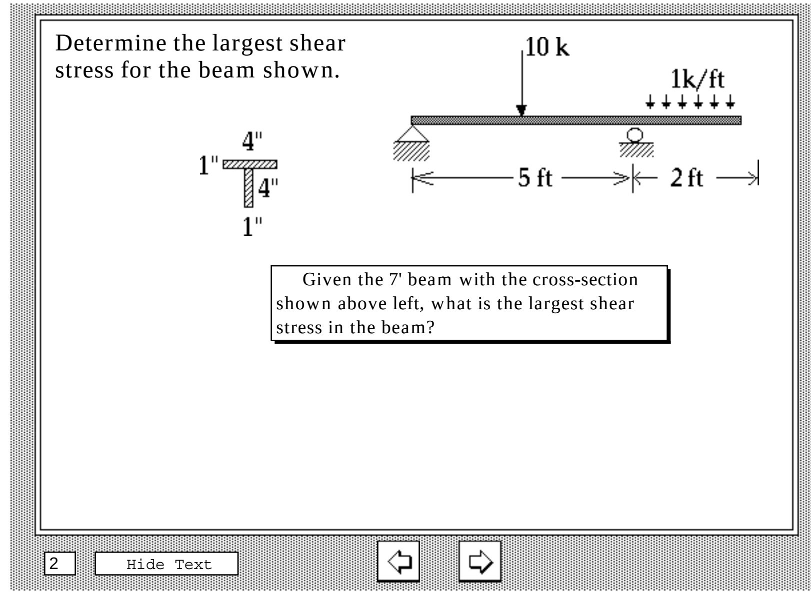
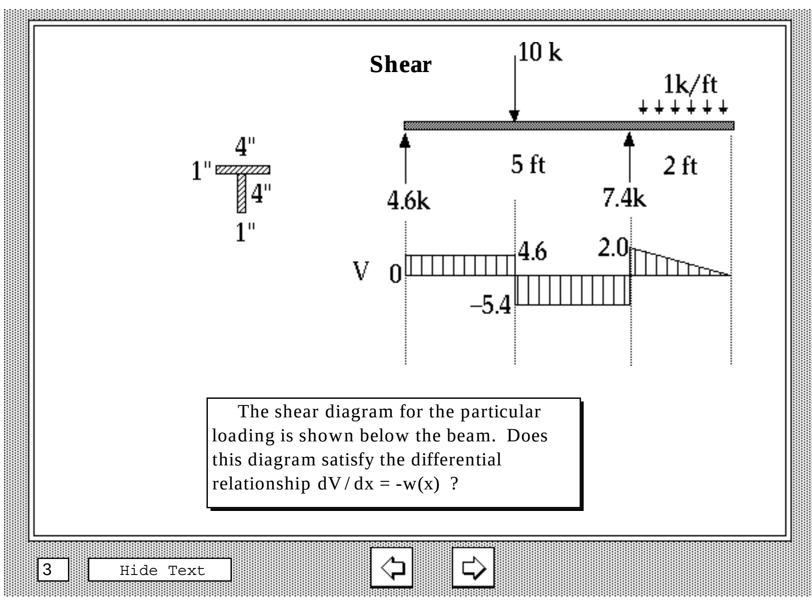
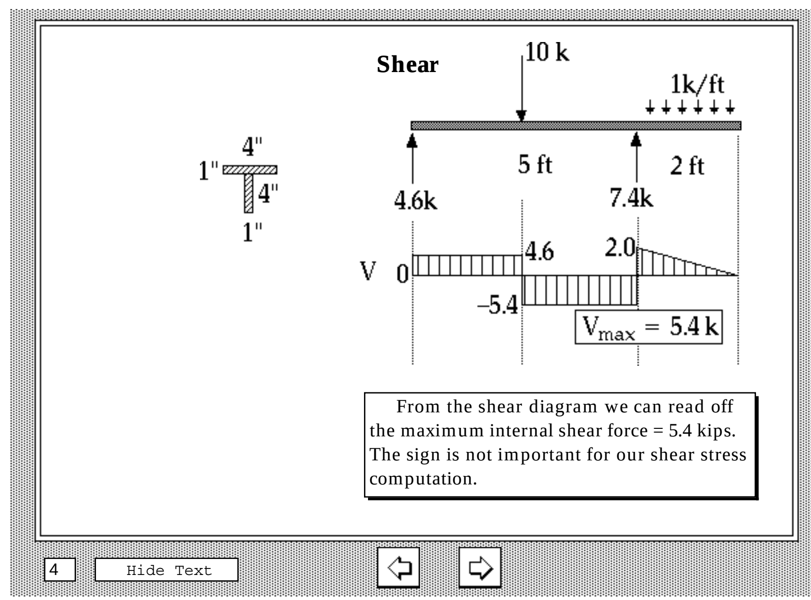

Shear Stress Example


Determine the largest shear stress for the beam shown.
10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
Given the 7' beam with the cross-section shown above left, what is the largest shear stress in the beam?
2
Hide Text


Shear
10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
The shear diagram for the particular loading is shown below the beam. Does this diagram satisfy the differential relationship dV / dx = -w(x) ?
3
Hide Text


Shear
10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
From the shear diagram we can read off the maximum internal shear force = 5.4 kips. The sign is not important for our shear stress computation.
4
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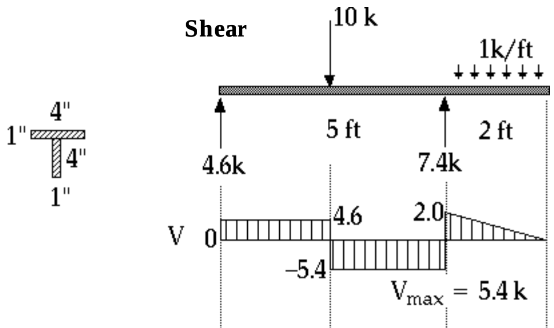
Shear

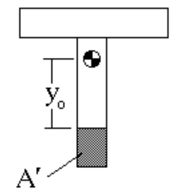


$$\tau = \frac{VQ}{It}$$

$$I = 18.2\ \text{in}^4$$

Recall the formula used to calculate shear stresses due to bending, $\tau = VQ/It$. We have just read the internal shear force, V, off of the shear diagram. We also already calculated the moment of inertia for this particular section. The remaining problem is that of calculating Q and t.

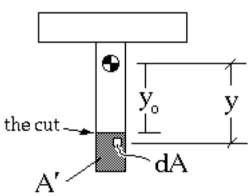
Calculating Q(y_o)



$$\int_{A'} y\, dA = Q(y_o)$$

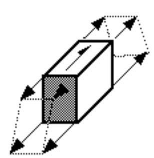
Generally, the most time consuming part of determining the shear stress in a beam is calculating the value of Q(y_o), the first moment of area about the centroid for the area above or below a cut located a distance y_o from the centroid.

Calculating Q(y_o)

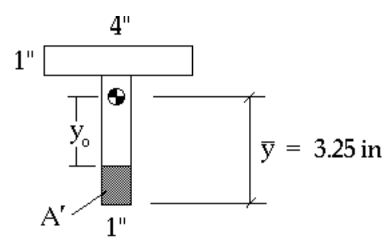


$$\int_{A'} y\, dA = Q(y_o)$$

Again, what we mean by the term Q(y_o) is the value of the integral of y·dA over the area A', where A' is the area above or below the cut, and y is the distance to the centroid of the entire

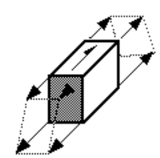


Calculating Q(y_o)

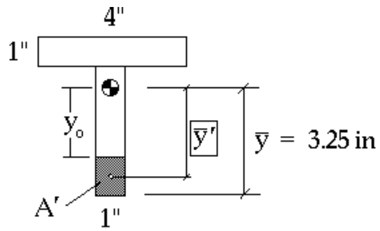


$$\int_{A'} y\, dA = Q(y_o)$$

Here are the dimensions of the cross section, and the location of the centroid we calculated earlier. These are the data we need to compute Q.

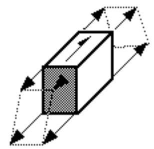


Calculating $Q(y_0)$



$$\int_{A'} y\, dA = Q(y_o)$$

For simple geometric regions, there is no need to actually do any integrals. The first moment of area can be calculated using the location of the centroid of A'.



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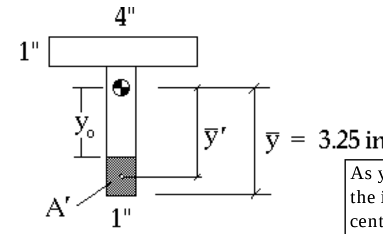
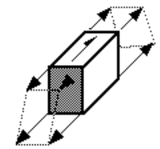
Calculating $Q(y_0)$



$$\int_{A'} y\, dA = Q(y_o)$$

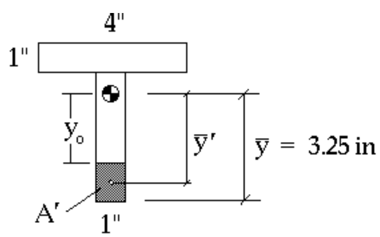
$$\bar{y}' A' = Q(y_o)$$

As you have learned in previous classes, the integral expression is equivalent to the centroidal distance times the area, A'.



10 Hide Text Skip the Details...

Calculating $Q(y_0)$

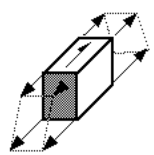


$$\int_{A'} y\, dA = Q(y_o)$$

$$\bar{y}' A' = Q(y_o)$$

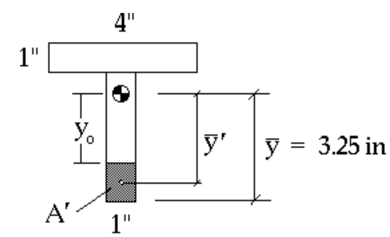
$$\boxed{\bar{y}' = \frac{3.25\text{ in} + y_o}{2}}$$

For this case, the centroid of A' is easily determined.



11 Hide Text

Calculating $Q(y_0)$



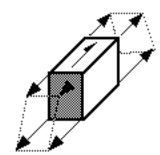
$$\int_{A'} y\, dA = Q(y_o)$$

$$\bar{y}' A' = Q(y_o)$$

$$\bar{y}' = \frac{3.25\text{ in} + y_o}{2}$$

$$\boxed{A' = (3.25\text{ in} - y_o)\,1"}$$

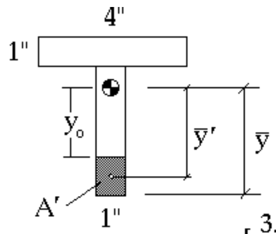
The area is calculated as shown. Note that A' is also a function of y_o.



12 Hide Text

Shear Stress Example: 4 (3/30/00)

Calculating Q(y_0)



$\bar{y} = 3.25\text{ in}$

$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}' A' = Q(y_o)$$

$$\bar{y}' = \frac{3.25\text{ in} + y_o}{2}$$

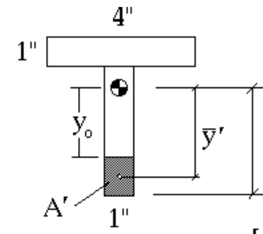
$$A' = (3.25\text{ in} - y_o)\,1"$$

$$\left[\frac{3.25\text{ in} + y_o}{2}\right](3.25\text{ in} - y_o)\,1" = Q(y_o)$$

Multiplying these two quantities together leads to the desired expression for Q as a function of y_0.

13 Hide Text

Calculating Q(y_0)



$\bar{y} = 3.25\text{ in}$

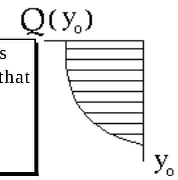
$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}' A' = Q(y_o)$$

$$\bar{y}' = \frac{3.25\text{ in} + y_o}{2}$$

$$A' = (3.25\text{ in} - y_o)\,1"$$

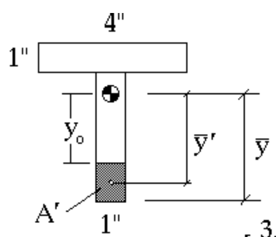
$$\left[\frac{3.25\text{ in} + y_o}{2}\right](3.25\text{ in} - y_o)\,1" = Q(y_o)$$



This function is a parabola, and its plot is shown to the right. Note that the function is zero at the outer edge of the section, and is a maximum at the centroid of the

14 Hide Text

Calculating Q(y_0)



$\bar{y} = 3.25\text{ in}$

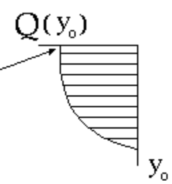
$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}' A' = Q(y_o)$$

$$\bar{y}' = \frac{3.25\text{ in} + y_o}{2}$$

$$A' = (3.25\text{ in} - y_o)\,1"$$

$$\left[\frac{3.25\text{ in} + y_o}{2}\right](3.25\text{ in} - y_o)\,1" = Q(y_o)$$



The maximum value is calculated setting $y_0 = 0$.

$$Q_{max} = \frac{(3.25\text{ in})^2}{2}\,1"$$

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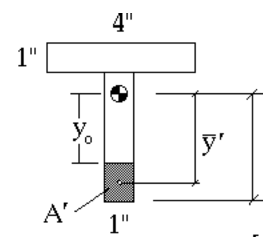
Calculating Q(y_0)



$\bar{y} = 3.25\text{ in}$

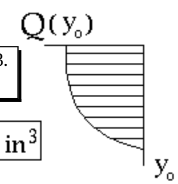
$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}' A' = Q(y_o)$$

$$\bar{y}' = \frac{3.25\text{ in} + y_o}{2}$$

$$A' = (3.25\text{ in} - y_o)\,1"$$

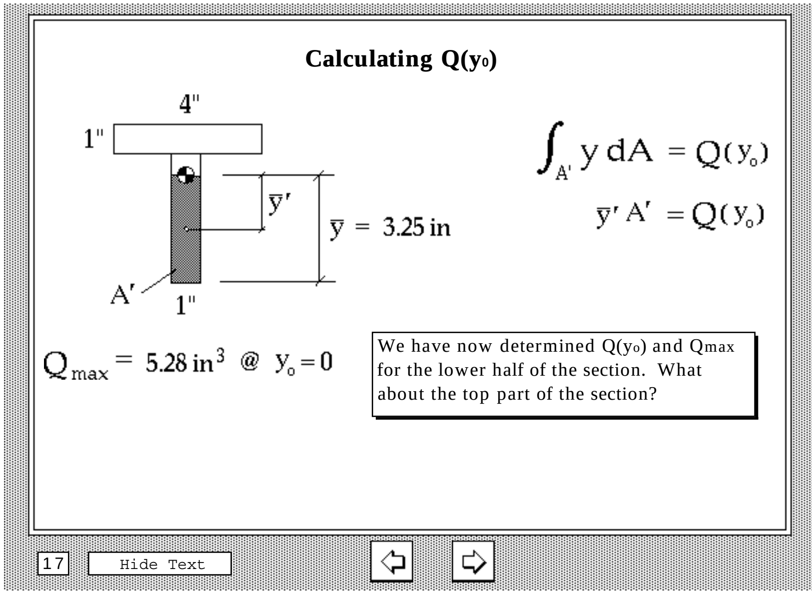
$$\left[\frac{3.25\text{ in} + y_o}{2}\right](3.25\text{ in} - y_o)\,1" = Q(y_o)$$



The numerical result is 5.28 in^3. Note the units.

$$Q_{max} = \frac{(3.25\text{ in})^2}{2}\,1" = \boxed{5.28\text{ in}^3}$$

16 Hide Text Review Details


Calculating Q(yo)
4"
1"
A′
1"
$\bar{y}'$
$\bar{y} = 3.25$ in
$\int_{A'} y\, dA = Q(y_o)$
$\bar{y}' A' = Q(y_o)$
$Q_{max} = 5.28\ in^3$ @ $y_o = 0$
We have now determined Q(yo) and Qmax for the lower half of the section. What about the top part of the section?
17
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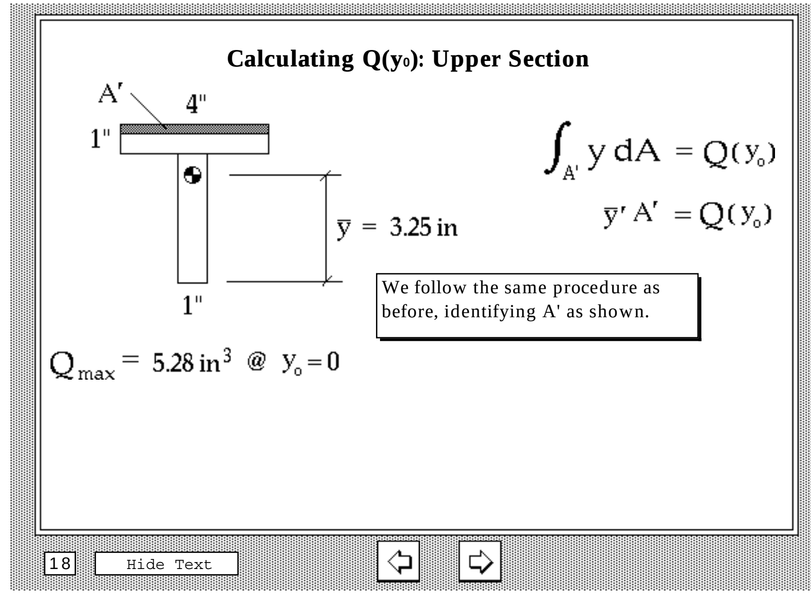
Calculating Q(yo): Upper Section
A′
4"
1"
1"
$\bar{y} = 3.25$ in
$\int_{A'} y\, dA = Q(y_o)$
$\bar{y}' A' = Q(y_o)$
$Q_{max} = 5.28\ in^3$ @ $y_o = 0$
We follow the same procedure as before, identifying A' as shown.
18
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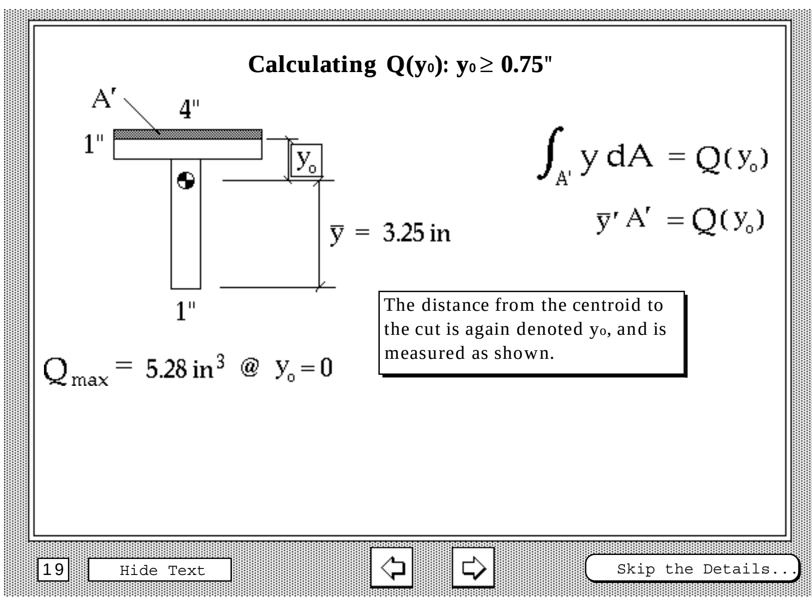
Calculating Q(yo): yo ≥ 0.75"
A′
4"
1"
y_o
$\bar{y} = 3.25$ in
1"
$\int_{A'} y\, dA = Q(y_o)$
$\bar{y}' A' = Q(y_o)$
$Q_{max} = 5.28\ in^3$ @ $y_o = 0$
The distance from the centroid to the cut is again denoted yo, and is measured as shown.
19
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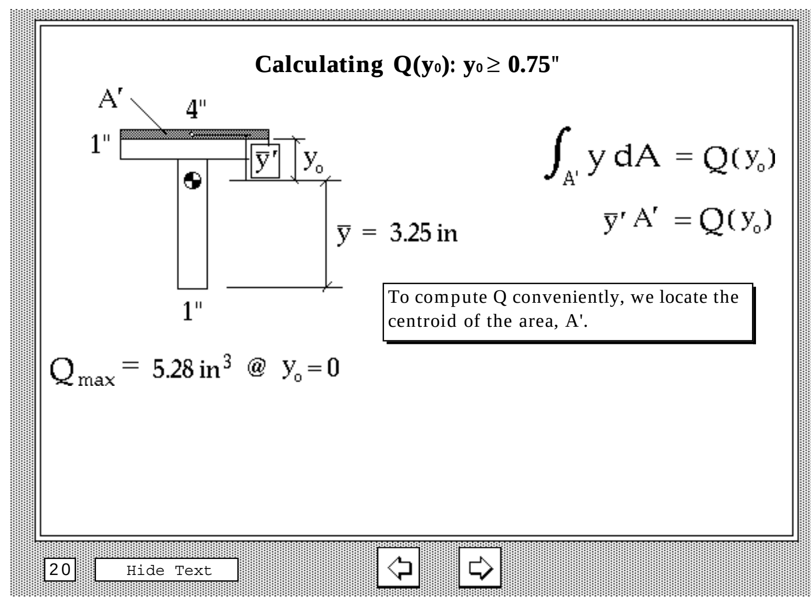
Skip the Details...


Calculating Q(yo): yo ≥ 0.75"
A′
4"
1"
$\bar{y}'$
y_o
$\bar{y} = 3.25$ in
1"
$\int_{A'} y\, dA = Q(y_o)$
$\bar{y}' A' = Q(y_o)$
$Q_{max} = 5.28\ in^3$ @ $y_o = 0$
To compute Q conveniently, we locate the centroid of the area, A'.
20
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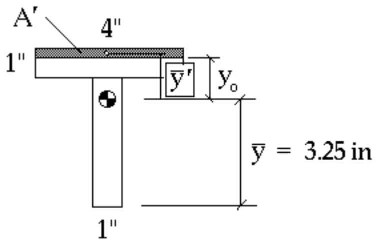
Calculating Q(y₀): y₀ ≥ 0.75"



$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}'A' = Q(y_o)$$

$$Q_{max} = 5.28\text{ in}^3 \ @\ y_o = 0$$

$$\bar{y}' = \frac{1.75\text{ in} + y_o}{2}$$

The calculation is similar to last time.

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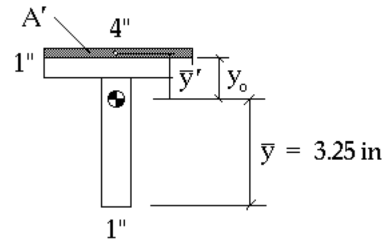
Calculating Q(y₀): y₀ ≥ 0.75"



$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}'A' = Q(y_o)$$

$$Q_{max} = 5.28\text{ in}^3 \ @\ y_o = 0$$

$$\bar{y}' = \frac{1.75\text{ in} + y_o}{2}$$

$$A' = (1.75\text{ in} - y_o)4"$$

A' is calculated next

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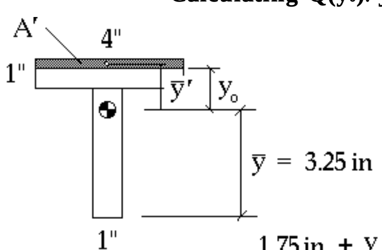
Calculating Q(y₀): y₀ ≥ 0.75"



$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}'A' = Q(y_o)$$

$$\left[\frac{1.75\text{ in} + y_o}{2}\right](1.75\text{ in} - y_o)4" = Q(y_o)$$

$$Q_{max} = 5.28\text{ in}^3 \ @\ y_o = 0$$

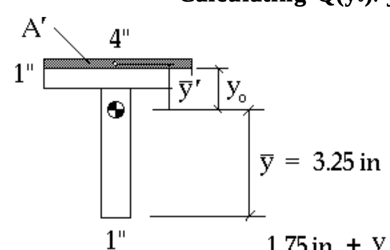
$$\bar{y}' = \frac{1.75\text{ in} + y_o}{2}$$

$$A' = (1.75\text{ in} - y_o)4"$$

Forming the product leads to the desired result. Of course, this expression is only valid in the upper part of the "T", where the width is 4".

23 Hide Text

Calculating Q(y₀): y₀ ≥ 0.75"



$$\int_{A'} y\,dA = Q(y_o)$$

$$\bar{y}'A' = Q(y_o)$$

$$\left[\frac{1.75\text{ in} + y_o}{2}\right](1.75\text{ in} - y_o)4" = Q(y_o)$$

$$Q_{max} = 5.28\text{ in}^3 \ @\ y_o = 0$$

$$\boxed{5.0\text{ in}^3 = Q(0.75)}$$

$$\bar{y}' = \frac{1.75\text{ in} + y_o}{2}$$

$$A' = (1.75\text{ in} - y_o)4"$$

At the junction of the vertical and horizontal portions of the section, Q takes on the value 5.0 in^3

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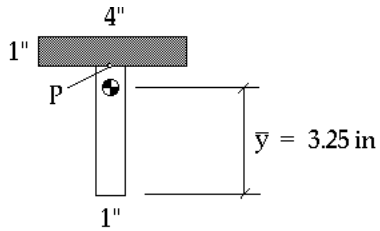
Calculating $Q(y_0)$: $y_0 \le 0.75"$



$$\int_{A'} y\, dA = Q(y_o)$$

$Q_{max} = 5.28\ in^3$ @ $y_o = 0$

$Q(P) = 5.0\ in^3$

Labeling this junction point P, we have the result shown.

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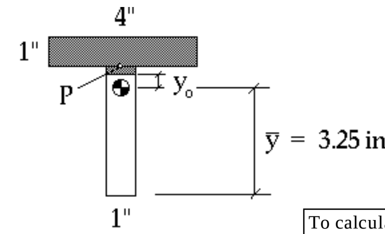
Calculating $Q(y_0)$: $y_0 \le 0.75"$



$$\int_{A'} y\, dA = Q(y_o)$$

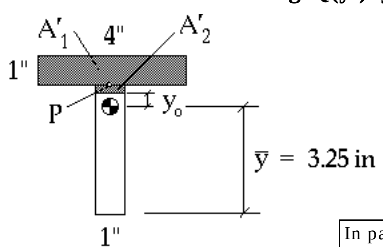
To calculate Q below the point P, we just keep doing the same sort of calculations, but they get slightly more complicated.

$Q_{max} = 5.28\ in^3$ @ $y_o = 0$

$Q(P) = 5.0\ in^3$

26 Hide Text

Calculating $Q(y_0)$: $y_0 \le 0.75"$



$$\int_{A'} y\, dA = Q(y_o)$$

$$\Sigma\, \bar{y}'_k A'_k = Q(y_o)$$

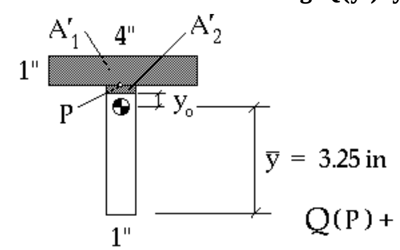
In particular, we need to calculate the total moment of area of the entire shaded region shown. This requires the slight generalization of the discrete equation as shown above.

$Q_{max} = 5.28\ in^3$ @ $y_o = 0$

$Q(P) = 5.0\ in^3$

27 Hide Text

Calculating $Q(y_0)$: $y_0 \le 0.75"$



$$\int_{A'} y\, dA = Q(y_o)$$

$$\Sigma\, \bar{y}'_k A'_k = Q(y_o)$$

$$Q(P) + (0.75"/2)(0.75")(1") = Q(0)$$

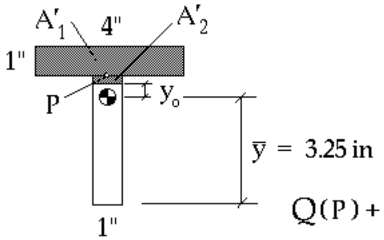
$Q_{max} = 5.28\ in^3$ @ $y_o = 0$

$Q(P) = 5.0\ in^3$

The result for A'_1 is just Q(P), and if we consider the case $y_o = 0$, we get the equation above.

28 Hide Text

Calculating Q(y₀): y₀ ≤ 0.75"



$$\int_{A'} y\, dA = Q(y_o)$$

$$\Sigma\, \bar{y}'_k A'_k = Q(y_o)$$

$$Q(P) + (0.75''/2)(0.75'')(1'') = Q(0)$$

$$5.0\ \text{in}^3 + 0.28\ \text{in}^3 = Q(0)$$

$Q_{max} = 5.28\ \text{in}^3$ @ $y_o = 0$

$Q(P) = 5.0\ \text{in}^3$

Putting in the numbers...

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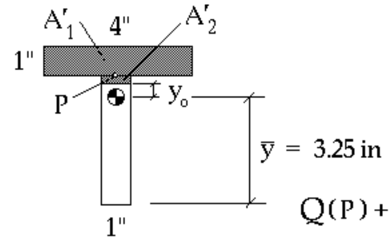
Calculating Q(y₀): y₀ ≤ 0.75"



$$\int_{A'} y\, dA = Q(y_o)$$

$$\Sigma\, \bar{y}'_k A'_k = Q(y_o)$$

$$Q(P) + (0.75''/2)(0.75'')(1'') = Q(0)$$

$$5.0\ \text{in}^3 + 0.28\ \text{in}^3 = Q(0)$$

$$\boxed{5.28\ \text{in}^3 = Q(0)}$$

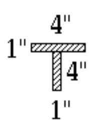
(checks)

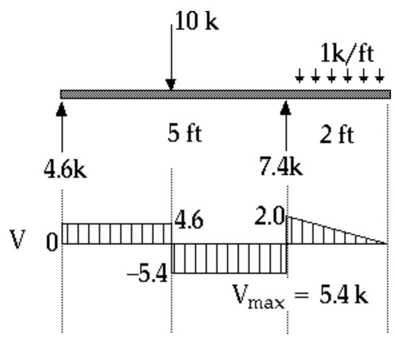
$Q_{max} = 5.28\ \text{in}^3$ @ $y_o = 0$

$Q(P) = 5.0\ \text{in}^3$

... leads us to the same result we had before for the bottom portion of the section. This is no accident: it is a direct result of the definition of the centroid.

30 Hide Text



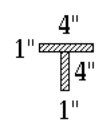


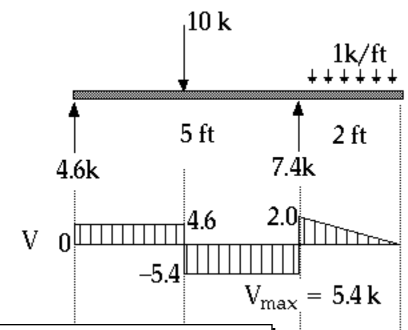
$$\tau = \frac{\overset{\checkmark}{V}\overset{\checkmark}{Q}}{\underset{\checkmark}{I}\, t}$$

$Q_{max} = 5.28\ \text{in}^3$ @ $y_o = 0$

We now have the maximum values of V and Q, and we already know I. What about t? We will consider t shortly, but it turns out that it is useful to consider VQ/I by itself.

31 Hide Text Review Details



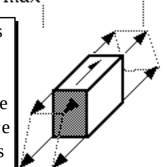


$$\tau = \frac{\overset{\checkmark}{V}\overset{\checkmark}{Q}}{\underset{\checkmark}{I}\, t}$$

$Q_{max} = 5.28\ \text{in}^3$ @

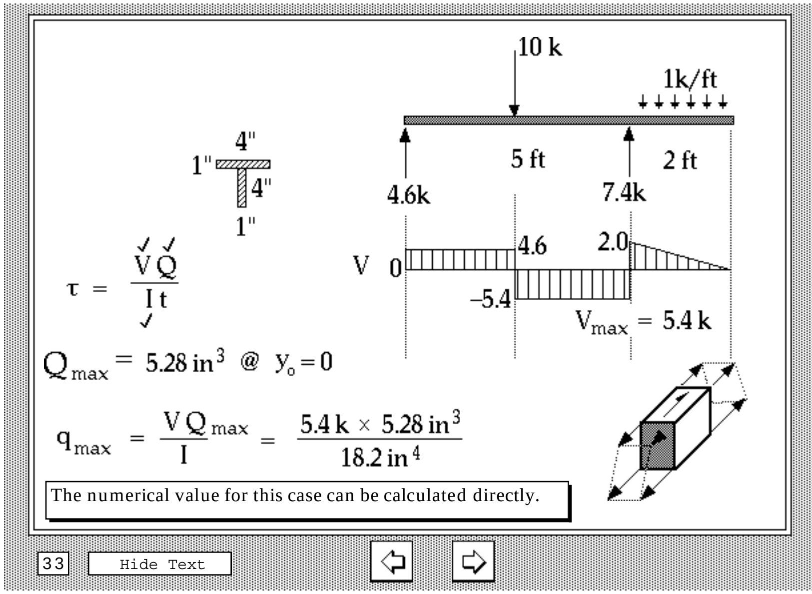
$$q_{max} = \frac{V\, Q_{max}}{I}$$

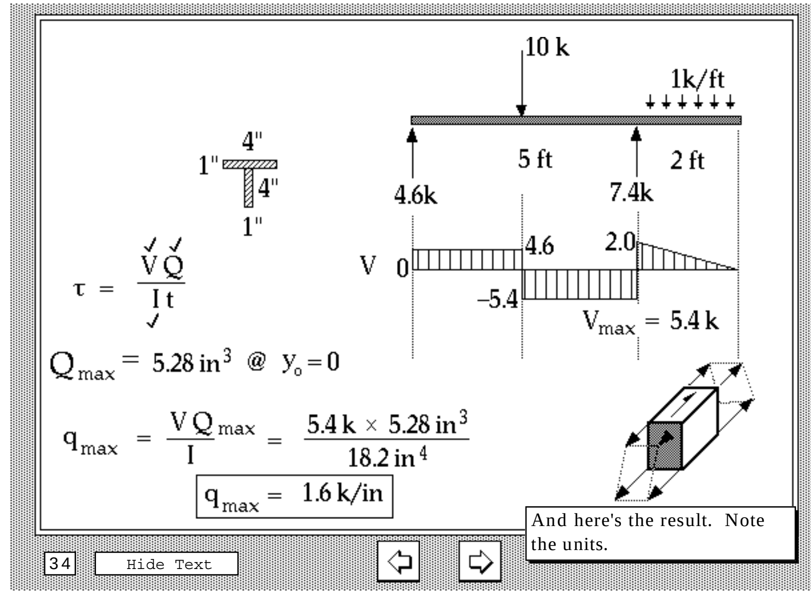
VQ/I is called the **shear flow**, and it is typically denoted q. This quantity is related to the amount of horizontal shear force we accumulate as we move along the beam. Later in this stack, we will see a important application for this quantity.

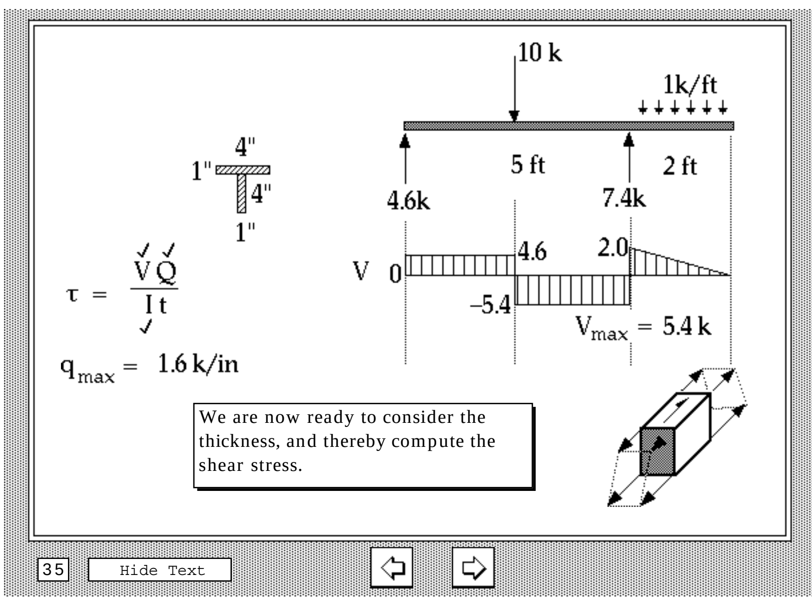


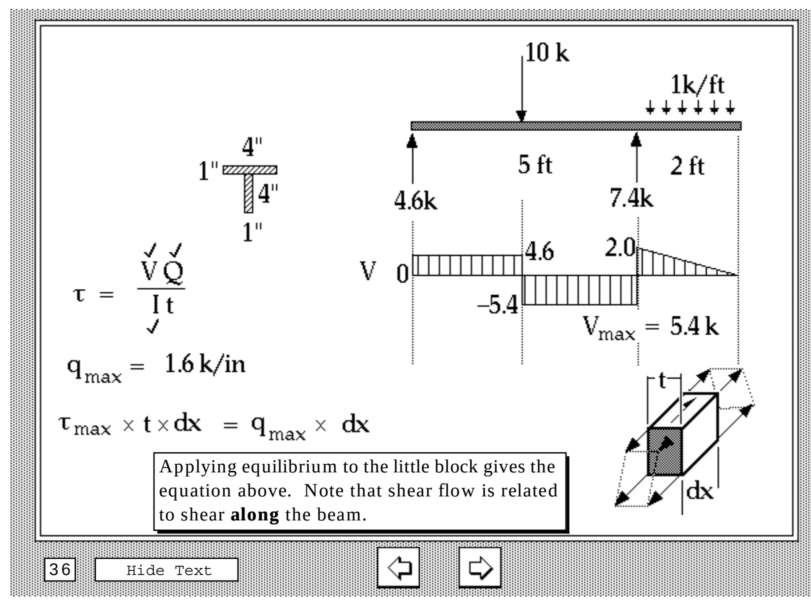
32 Hide Text

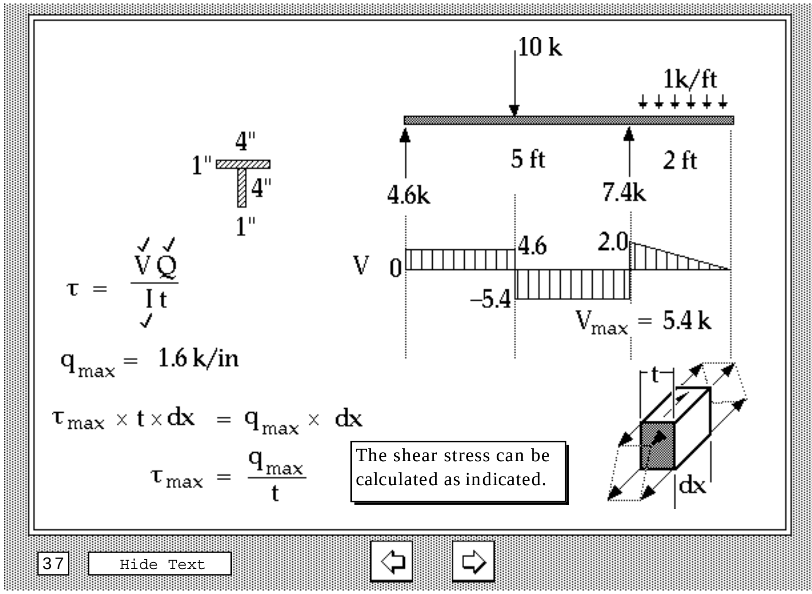
Shear Stress Example: 9 (3/30/00)

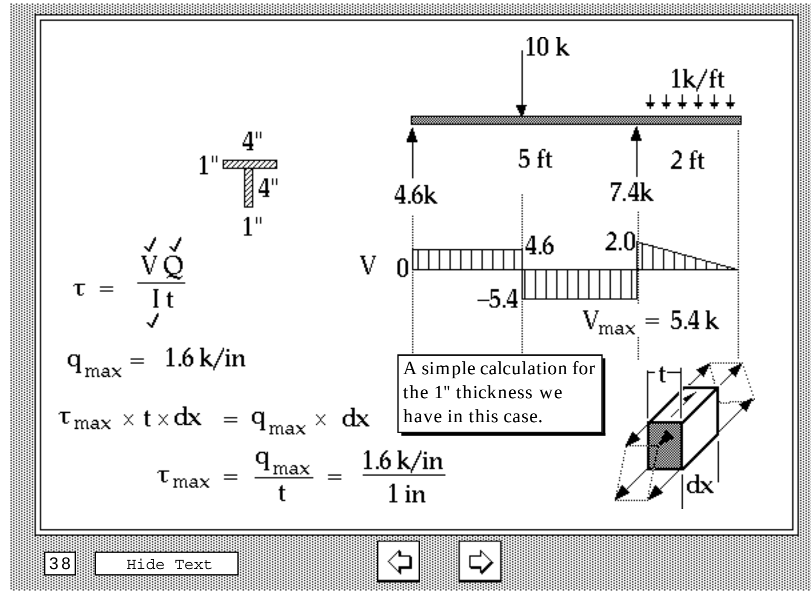


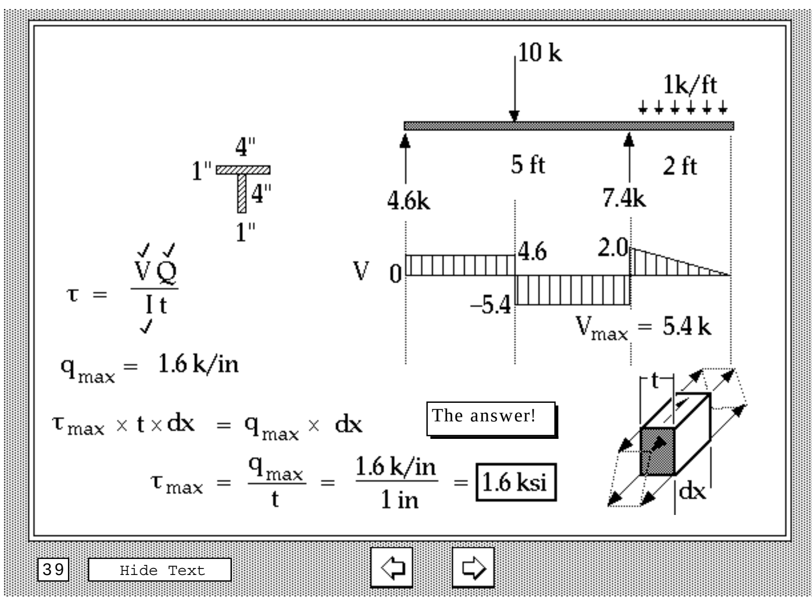


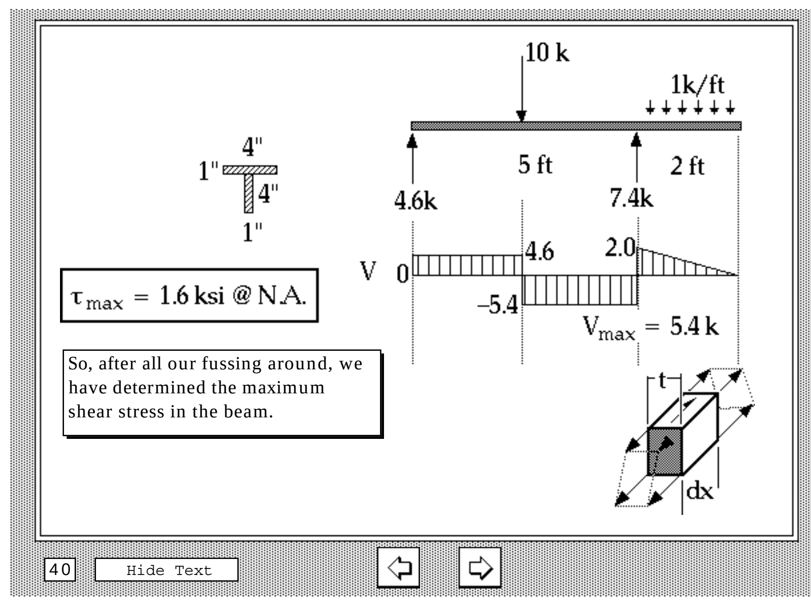


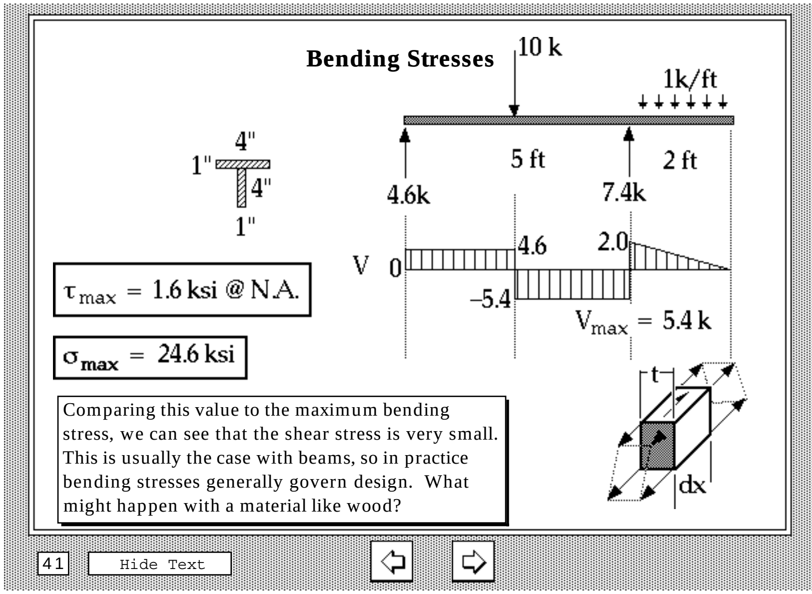


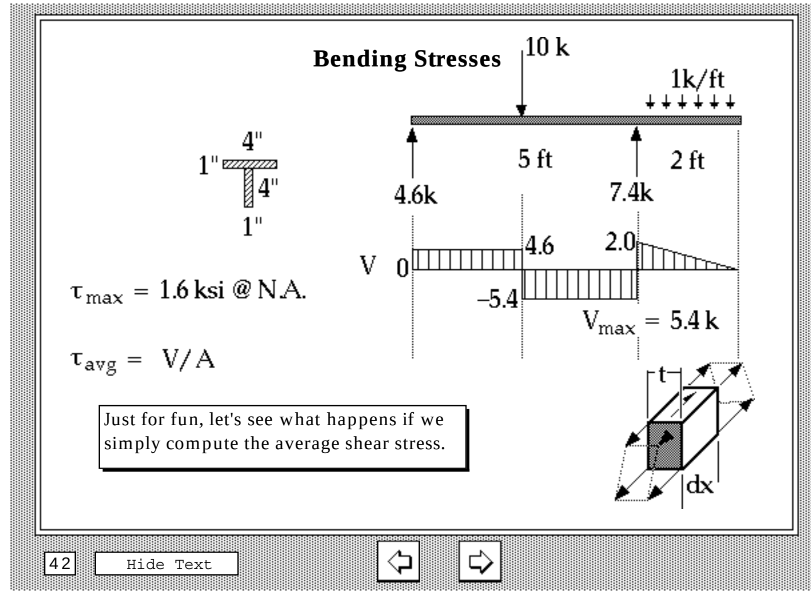

10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
$\tau = \frac{VQ}{It}$
q_{max} = 1.6 k/in
$\tau_{max} \times t \times dx = q_{max} \times dx$
$\tau_{max} = \frac{q_{max}}{t}$
The shear stress can be calculated as indicated.
t
dx
37
Hide Text

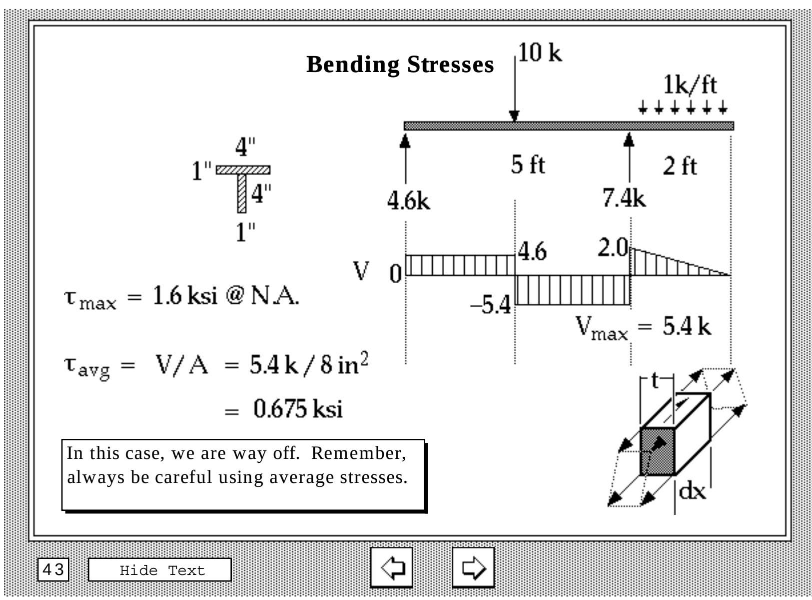

10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
$\tau = \frac{VQ}{It}$
q_{max} = 1.6 k/in
$\tau_{max} \times t \times dx = q_{max} \times dx$
$\tau_{max} = \frac{q_{max}}{t} = \frac{1.6\ k/in}{1\ in}$
A simple calculation for the 1" thickness we have in this case.
t
dx
38
Hide Text

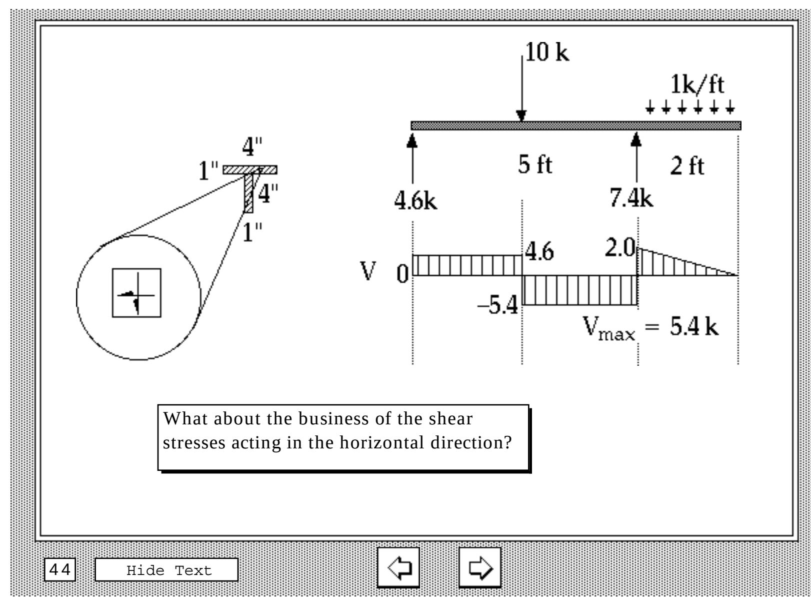

10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
$\tau = \frac{VQ}{It}$
q_{max} = 1.6 k/in
$\tau_{max} \times t \times dx = q_{max} \times dx$
$\tau_{max} = \frac{q_{max}}{t} = \frac{1.6\ k/in}{1\ in} = \boxed{1.6\ ksi}$
The answer!
t
dx
39
Hide Text

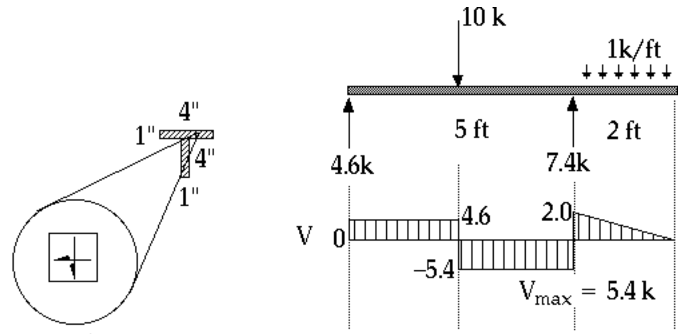

10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
τ_{max} = 1.6 ksi @ N.A.
So, after all our fussing around, we have determined the maximum shear stress in the beam.
t
dx
40
Hide Text


Bending Stresses
10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
τ_{max} = 1.6 ksi @ N.A.
σ_{max} = 24.6 ksi
t
dx
Comparing this value to the maximum bending stress, we can see that the shear stress is very small. This is usually the case with beams, so in practice bending stresses generally govern design. What might happen with a material like wood?
41
Hide Text


Bending Stresses
10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
τ_{max} = 1.6 ksi @ N.A.
τ_{avg} = V/A
t
dx
Just for fun, let's see what happens if we simply compute the average shear stress.
42
Hide Text


Bending Stresses
10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
τ_{max} = 1.6 ksi @ N.A.
τ_{avg} = V/A = 5.4 k / 8 in^2
= 0.675 ksi
t
dx
In this case, we are way off. Remember, always be careful using average stresses.
43
Hide Text

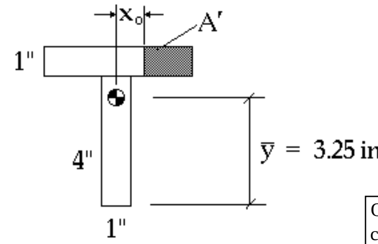

10 k
1k/ft
4"
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
V_{max} = 5.4 k
What about the business of the shear stresses acting in the horizontal direction?
44
Hide Text



$$q = \frac{\checkmark{V}Q}{\underset{\checkmark}{I}}$$

We use the same equations, except we need to calculate Q a bit differently.

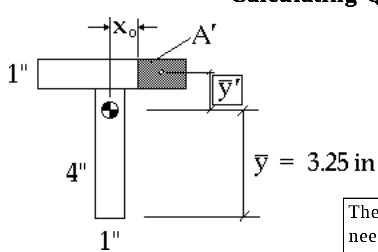
Calculating Q(x₀)



$$\int_{A'} y\,dA = Q(x_o)$$

Our area A' is as shown, and it is characterized in terms of x_o, rather than in terms of y_o.

Calculating Q(x₀)

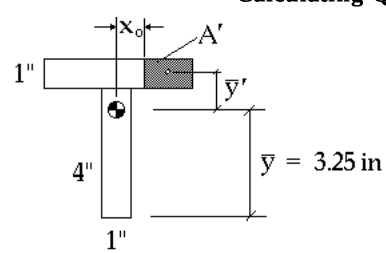


$$\int_{A'} y\,dA = Q(x_o)$$

$$\bar{y}'A' = Q(x_o)$$

The tricky part is to realize that we still need to compute the vertical moment of this area about the section centroid. Thus, we still use y-bar.

Calculating Q(x₀)



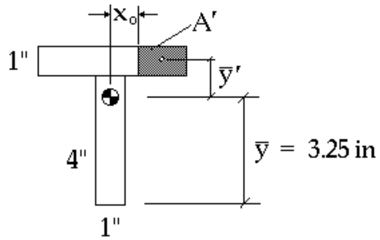
$$\int_{A'} y\,dA = Q(x_o)$$

$$\bar{y}'A' = Q(x_o)$$

$$\boxed{\bar{y}' = 0.5\text{ in} + 0.75\text{ in} = 1.25\text{ in}}$$

For this particular case, the centroid of A' is calculated as shown.

Calculating Q(x₀)



$$\int_{A'} y\,dA = Q(x_o)$$

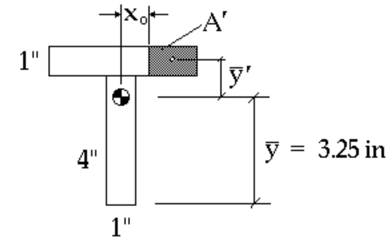
$$\bar{y}'A' = Q(x_o)$$

$$\bar{y}' = 0.5\text{ in} + 0.75\text{ in} = 1.25\text{ in}$$

$$\boxed{A' = (1\text{ in})(2 - x_o)}$$

A' itself can be expressed as shown.

Calculating Q(x₀)



$$\int_{A'} y\,dA = Q(x_o)$$

$$\bar{y}'A' = Q(x_o)$$

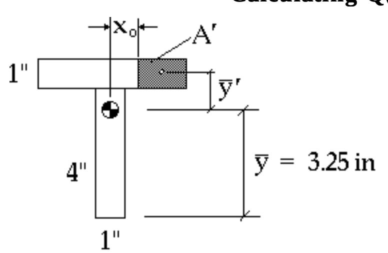
$$1.25\text{ in} \times (1\text{ in})^2 = Q\,(1\text{ in})$$

Forming the product gives Q. Here we have taken $x_o = 1$ in.

$$\bar{y}' = 0.5\text{ in} + 0.75\text{ in} = 1.25\text{ in}$$

$$A' = (1\text{ in})(2 - x_o)$$

Calculating Q(x₀)



$$\int_{A'} y\,dA = Q(x_o)$$

$$\bar{y}'A' = Q(x_o)$$

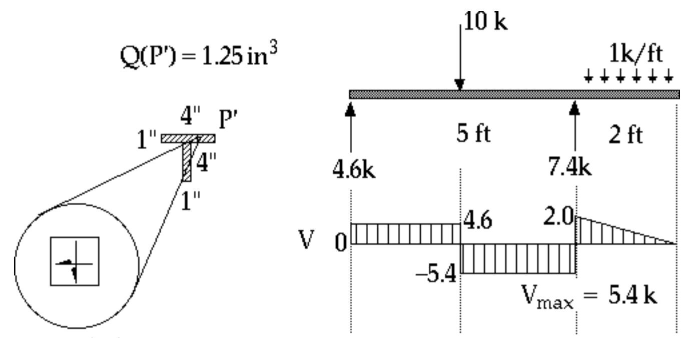
$$1.25\text{ in} \times (1\text{ in})^2 = Q\,(1\text{ in})$$

$$\boxed{1.25\text{ in}^3 = Q\,(1\text{ in})}$$

$$\bar{y}' = 0.5\text{ in} + 0.75\text{ in} = 1.25\text{ in}$$

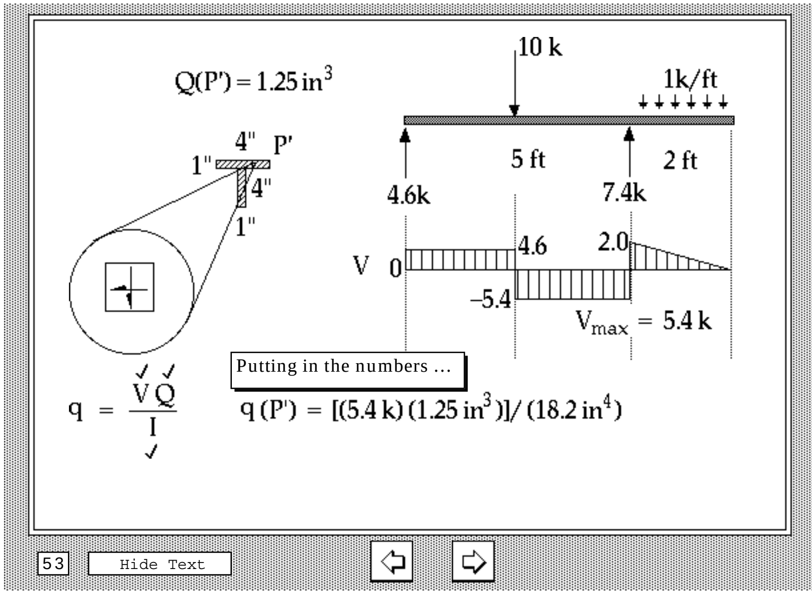
$$A' = (1\text{ in})(2 - x_o)$$

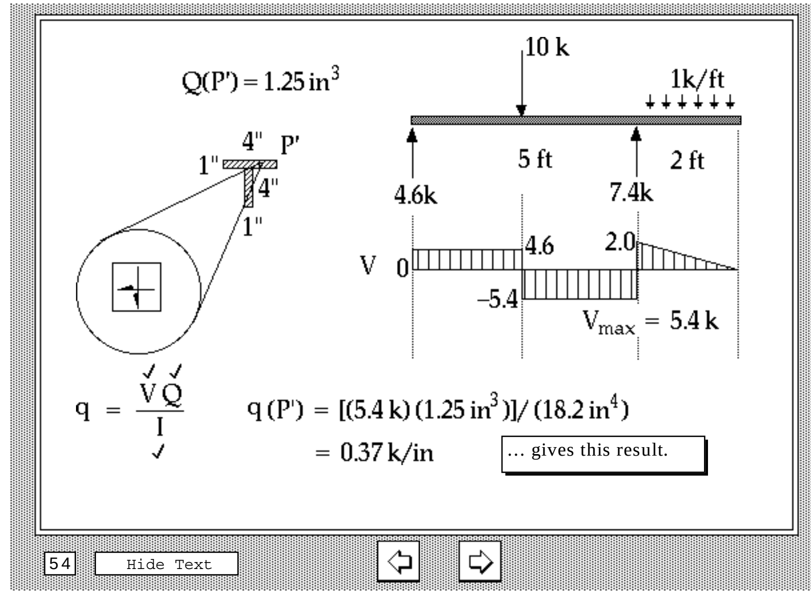
This is the result for this case. Values for other x_o's are computed similarly.

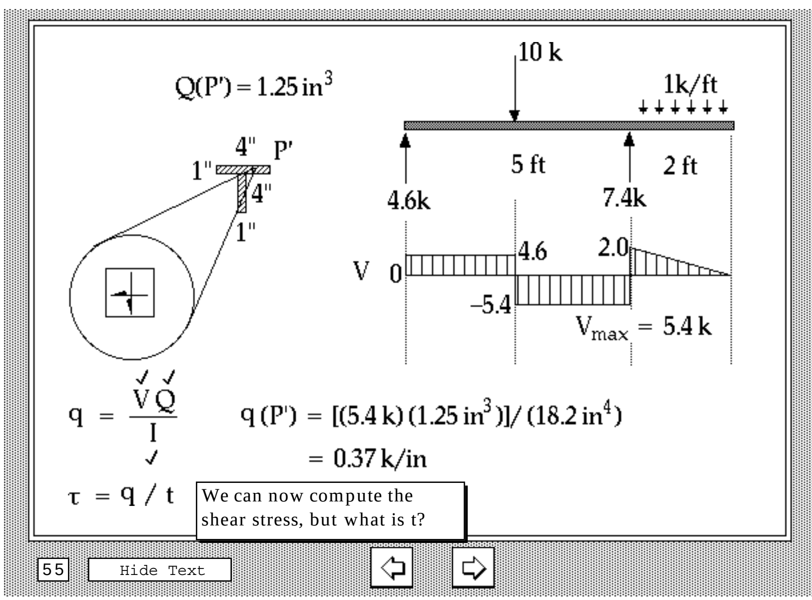


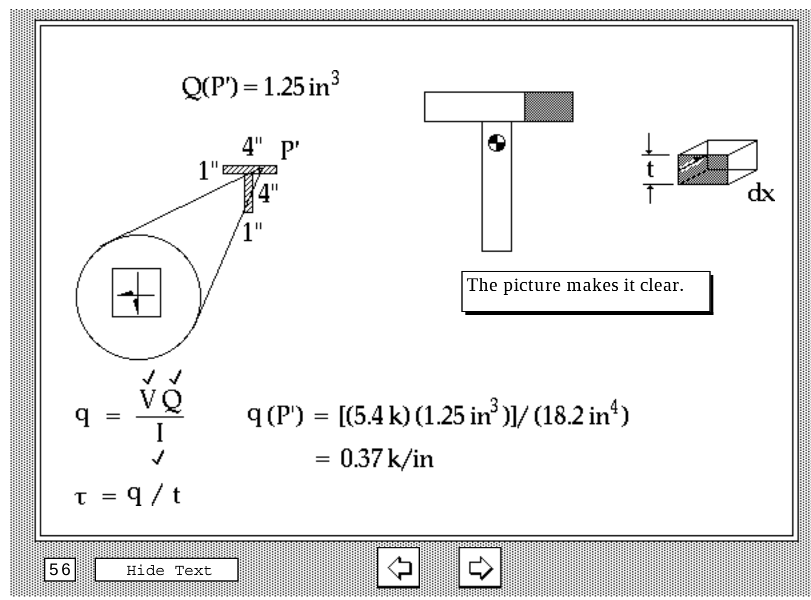
$$q = \frac{V\,Q}{I}$$

Now that we have Q, we can compute the shear flow, q.

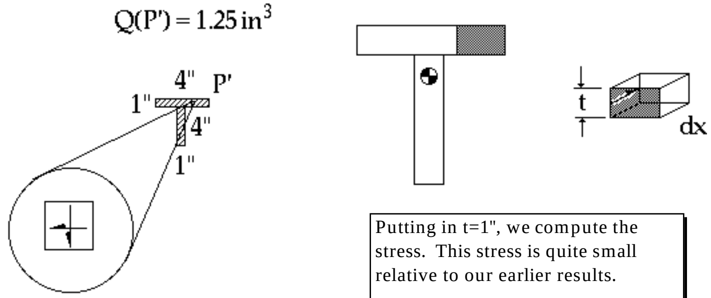

Q(P') = 1.25 in^3
10 k
1k/ft
4" P'
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V 0
4.6
2.0
–5.4
V_max = 5.4 k
Putting in the numbers ...
q = VQ / I
q (P') = [(5.4 k) (1.25 in^3)]/ (18.2 in^4)
53
Hide Text


Q(P') = 1.25 in^3
10 k
1k/ft
4" P'
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V 0
4.6
2.0
–5.4
V_max = 5.4 k
q = VQ / I
q (P') = [(5.4 k) (1.25 in^3)]/ (18.2 in^4)
= 0.37 k/in
... gives this result.
54
Hide Text


Q(P') = 1.25 in^3
10 k
1k/ft
4" P'
1"
4"
1"
5 ft
2 ft
4.6k
7.4k
V 0
4.6
2.0
–5.4
V_max = 5.4 k
q = VQ / I
q (P') = [(5.4 k) (1.25 in^3)]/ (18.2 in^4)
= 0.37 k/in
τ = q / t
We can now compute the shear stress, but what is t?
55
Hide Text


Q(P') = 1.25 in^3
4" P'
1"
4"
1"
t
dx
The picture makes it clear.
q = VQ / I
q (P') = [(5.4 k) (1.25 in^3)]/ (18.2 in^4)
= 0.37 k/in
τ = q / t
56
Hide Text

$Q(P') = 1.25\,in^3$



Putting in t=1", we compute the stress. This stress is quite small relative to our earlier results.

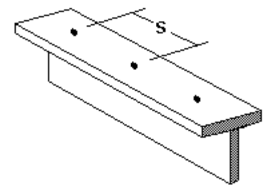
$$q = \frac{\checkmark V \checkmark Q}{I \checkmark} \qquad q(P') = [(5.4\,k)(1.25\,in^3)]/(18.2\,in^4)$$

$$= 0.37\,k/in$$

$$\tau = q/t = (0.37\,k/in)/(1\,in) = \boxed{0.37\,ksi}$$

57 Hide Text

Using the Shear Flow

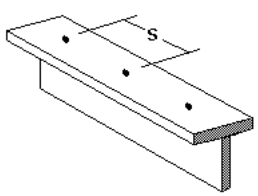


Determine the required connector spacing, s.

One of the important practical applications for shear stress/flow calculations is the determination of connector spacing in built-up sections. To see how this is done, we will assume that the T-section we have been considering is actually fabricated by bolting together the flange and web sections. Our task is to determine an adequate bolt spacing.

58 Hide Text

Using the Shear Flow



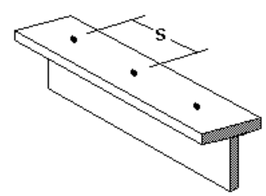
Determine the required connector spacing, s.

Connector capacity = 8.7 k/connector

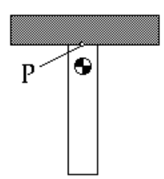
We will assume that we know the capacity of the bolts we will be using. In real life, you would most likely need to choose your own bolts size, and you would probably need to repeat the analysis to follow for a couple alternative choices.

59 Hide Text

Using the Shear Flow



Determine the required connector spacing, s.

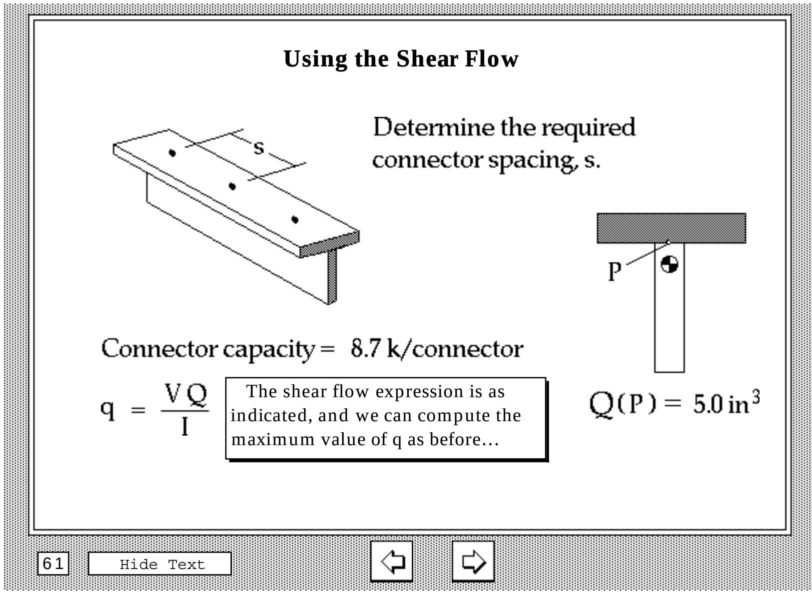


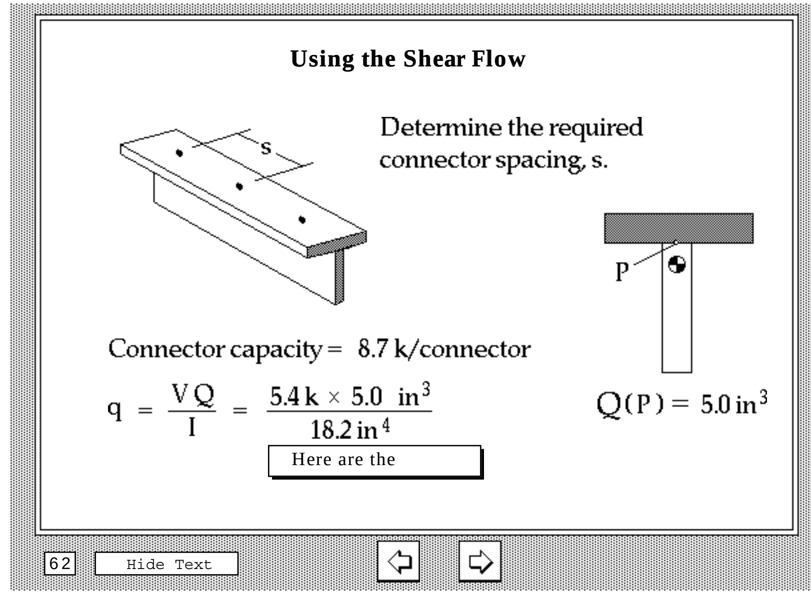
Connector capacity = 8.7 k/connector

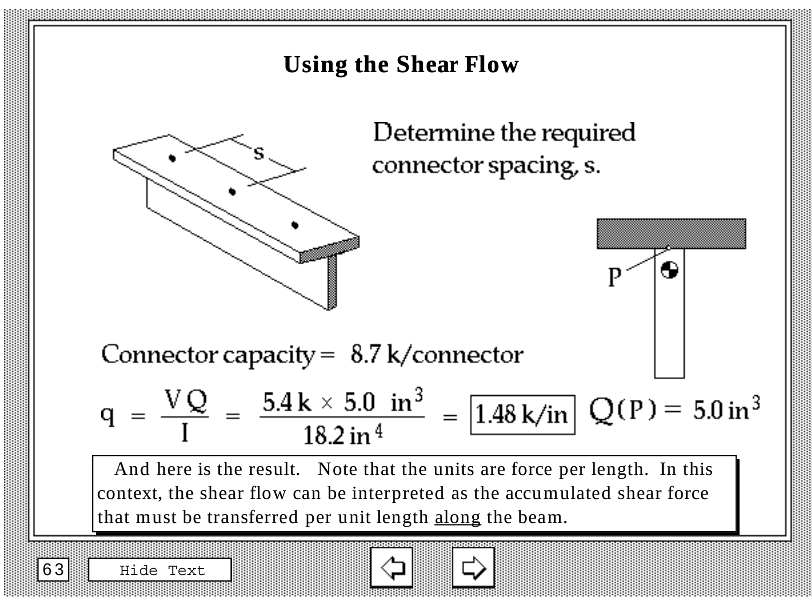
$Q(P) = 5.0\,in^3$

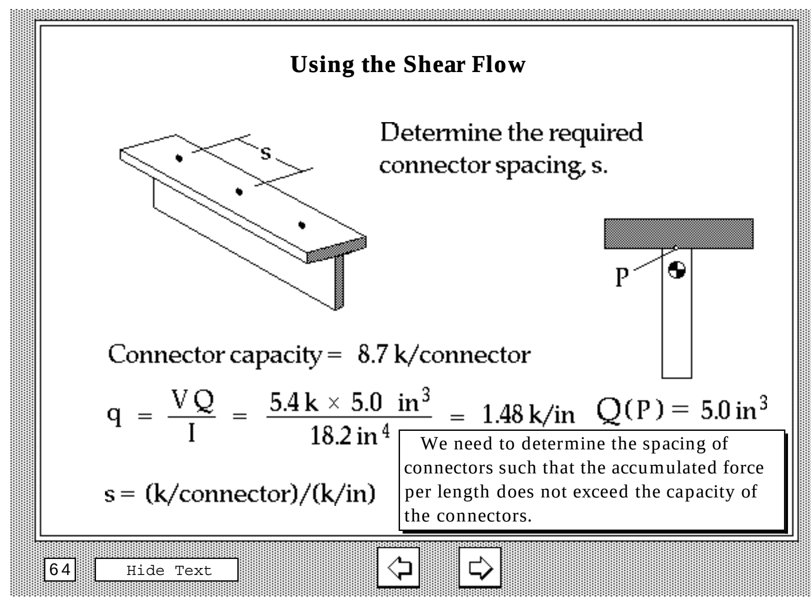
We need to determine the shear flow that must be transferred across the web/flange connection, and so we recall the Q corresponding to this location.

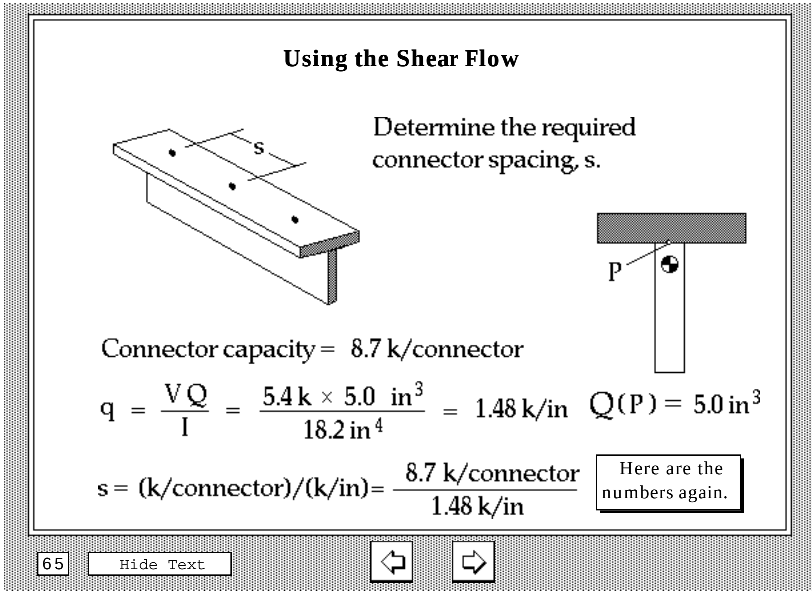
60 Hide Text

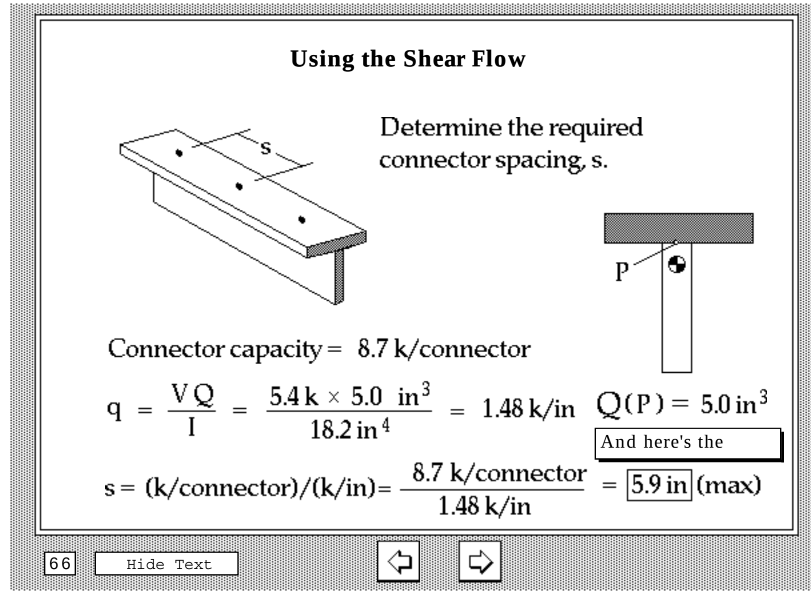

Using the Shear Flow
Determine the required connector spacing, s.
s
P
Connector capacity = 8.7 k/connector
$q = \frac{VQ}{I}$
The shear flow expression is as indicated, and we can compute the maximum value of q as before...
$Q(P) = 5.0\ in^3$
61
Hide Text

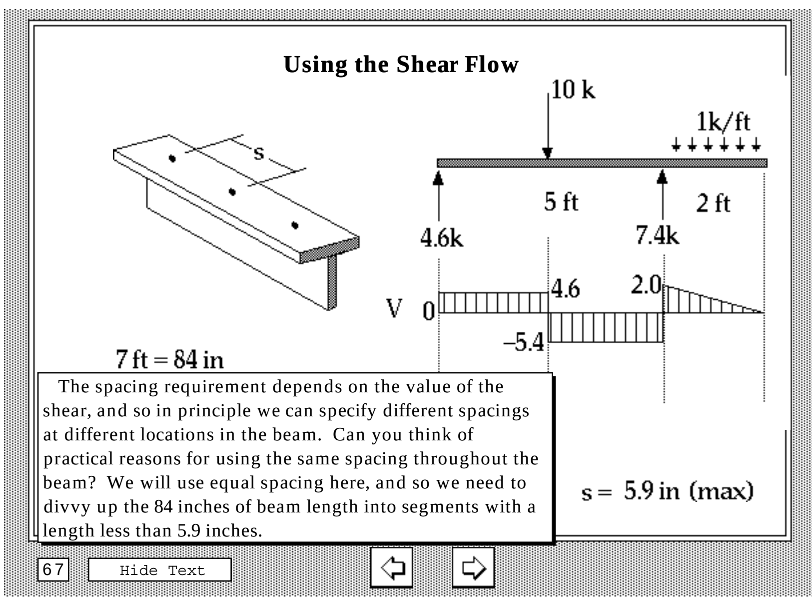

Using the Shear Flow
Determine the required connector spacing, s.
s
P
Connector capacity = 8.7 k/connector
$q = \frac{VQ}{I} = \frac{5.4\ k \times 5.0\ in^3}{18.2\ in^4}$
Here are the
$Q(P) = 5.0\ in^3$
62
Hide Text


Using the Shear Flow
Determine the required connector spacing, s.
s
P
Connector capacity = 8.7 k/connector
$q = \frac{VQ}{I} = \frac{5.4\ k \times 5.0\ in^3}{18.2\ in^4} = \boxed{1.48\ k/in}$
$Q(P) = 5.0\ in^3$
And here is the result. Note that the units are force per length. In this context, the shear flow can be interpreted as the accumulated shear force that must be transferred per unit length along the beam.
63
Hide Text


Using the Shear Flow
Determine the required connector spacing, s.
s
P
Connector capacity = 8.7 k/connector
$q = \frac{VQ}{I} = \frac{5.4\ k \times 5.0\ in^3}{18.2\ in^4} = 1.48\ k/in$
$Q(P) = 5.0\ in^3$
s = (k/connector)/(k/in)
We need to determine the spacing of connectors such that the accumulated force per length does not exceed the capacity of the connectors.
64
Hide Text


Using the Shear Flow
Determine the required connector spacing, s.
s
P
Connector capacity = 8.7 k/connector
q = VQ / I = 5.4 k × 5.0 in³ / 18.2 in⁴ = 1.48 k/in
Q(P) = 5.0 in³
s = (k/connector)/(k/in) = 8.7 k/connector / 1.48 k/in
Here are the numbers again.
65
Hide Text


Using the Shear Flow
Determine the required connector spacing, s.
s
P
Connector capacity = 8.7 k/connector
q = VQ / I = 5.4 k × 5.0 in³ / 18.2 in⁴ = 1.48 k/in
Q(P) = 5.0 in³
And here's the
s = (k/connector)/(k/in) = 8.7 k/connector / 1.48 k/in = 5.9 in (max)
66
Hide Text


Using the Shear Flow
10 k
1k/ft
s
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
7 ft = 84 in
The spacing requirement depends on the value of the shear, and so in principle we can specify different spacings at different locations in the beam. Can you think of practical reasons for using the same spacing throughout the beam? We will use equal spacing here, and so we need to divvy up the 84 inches of beam length into segments with a length less than 5.9 inches.
s = 5.9 in (max)
67
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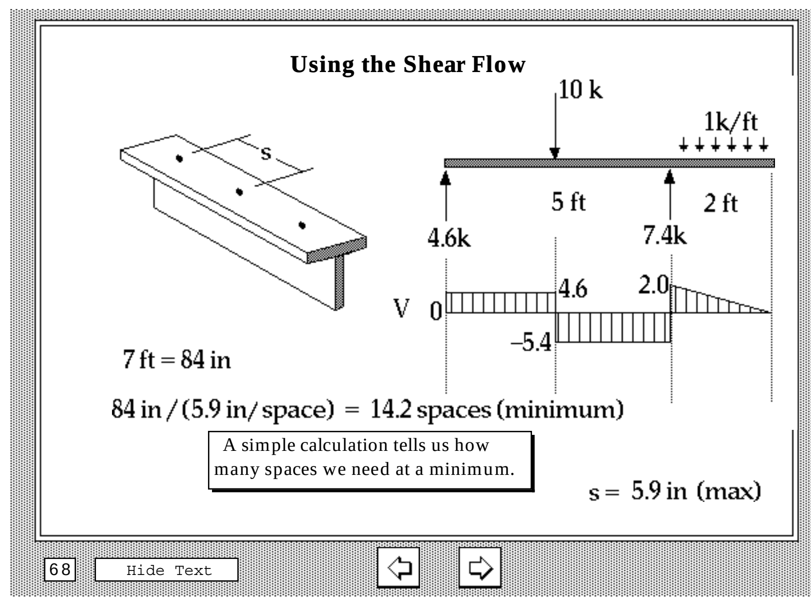
Using the Shear Flow
10 k
1k/ft
s
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
7 ft = 84 in
84 in / (5.9 in/ space) = 14.2 spaces (minimum)
A simple calculation tells us how many spaces we need at a minimum.
s = 5.9 in (max)
68
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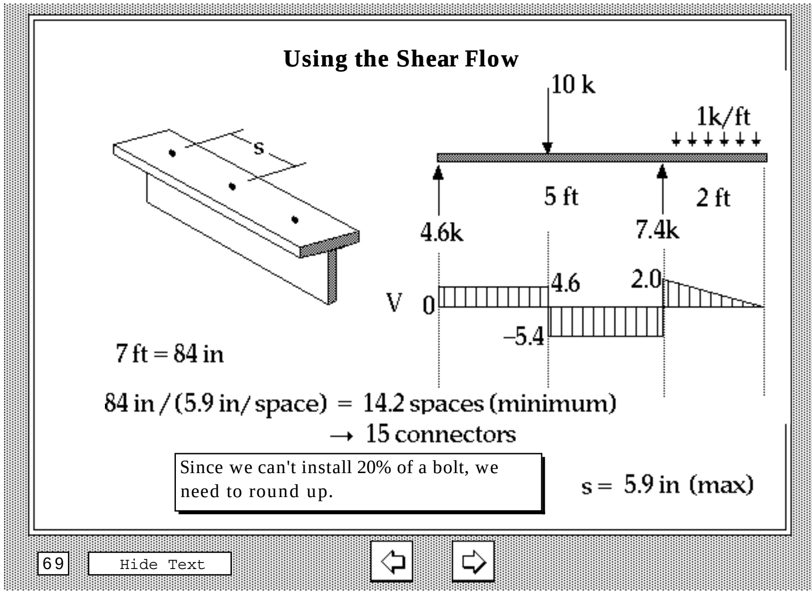
Using the Shear Flow
10 k
1k/ft
s
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
7 ft = 84 in
84 in / (5.9 in/ space) = 14.2 spaces (minimum)
→ 15 connectors
Since we can't install 20% of a bolt, we need to round up.
s = 5.9 in (max)
69
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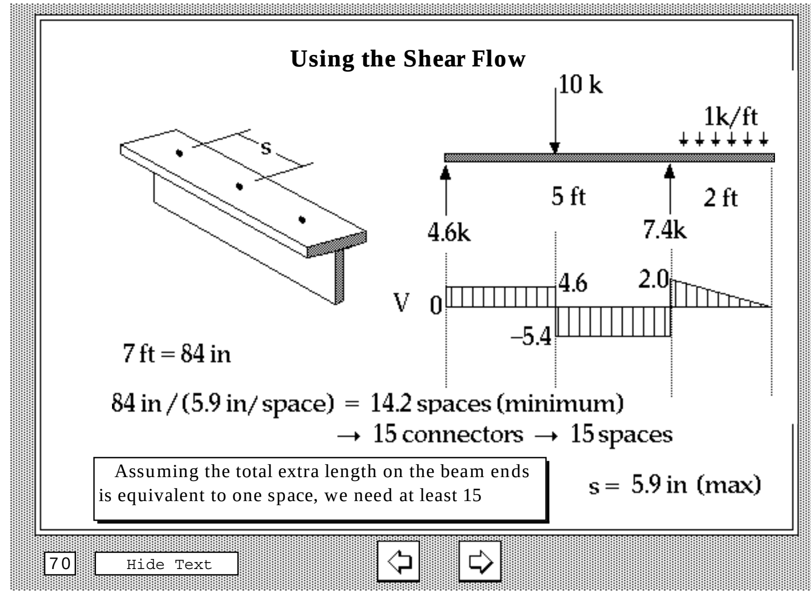
Using the Shear Flow
10 k
1k/ft
s
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
7 ft = 84 in
84 in / (5.9 in/ space) = 14.2 spaces (minimum)
→ 15 connectors → 15 spaces
Assuming the total extra length on the beam ends is equivalent to one space, we need at least 15
s = 5.9 in (max)
70
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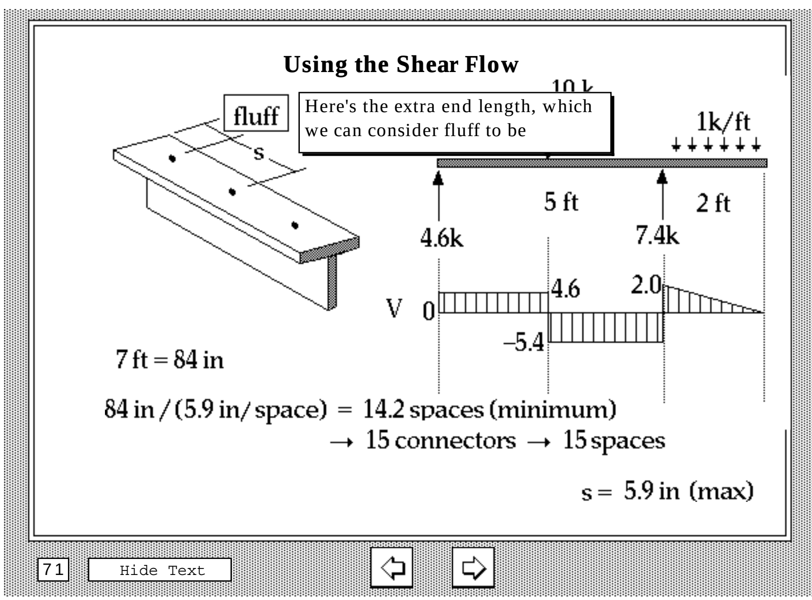
Using the Shear Flow
fluff
Here's the extra end length, which we can consider fluff to be
1k/ft
s
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
7 ft = 84 in
84 in / (5.9 in/ space) = 14.2 spaces (minimum)
→ 15 connectors → 15 spaces
s = 5.9 in (max)
71
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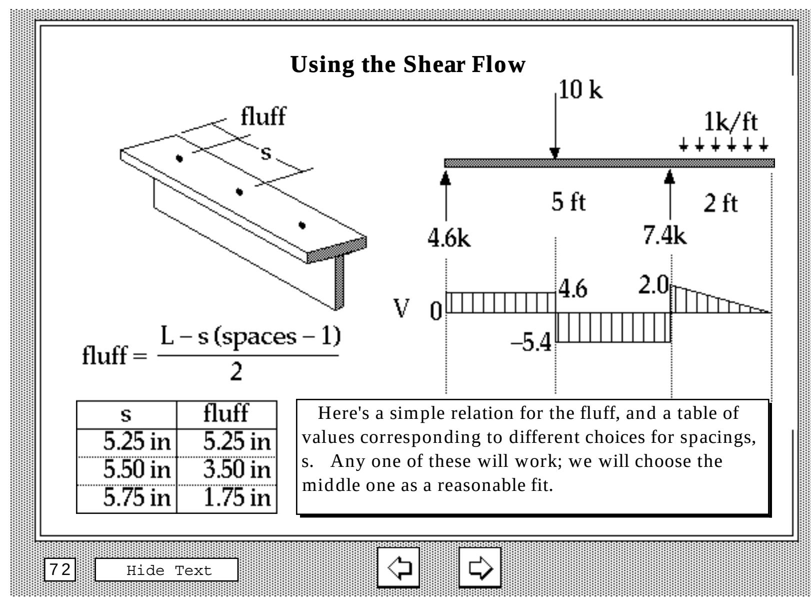
Using the Shear Flow
10 k
fluff
1k/ft
s
5 ft
2 ft
4.6k
7.4k
V
0
4.6
2.0
–5.4
fluff = (L – s (spaces – 1)) / 2
s | fluff
5.25 in | 5.25 in
5.50 in | 3.50 in
5.75 in | 1.75 in
Here's a simple relation for the fluff, and a table of values corresponding to different choices for spacings, s. Any one of these will work; we will choose the middle one as a reasonable fit.
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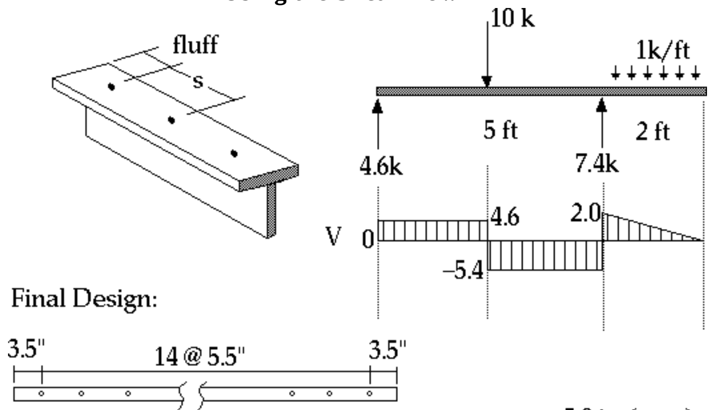
Using the Shear Flow



Here is the final configuration, ready for fabrication.

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Summary

In summary, we can see from these examples that the main trick to computing shear stress in beams is properly characterizing Q, and being careful in identifying the thickness, t.

In most cases, shear stresses in beams are of secondary importance in design. The principal exception is in the case of built-up members, for which the shear flow governs the connector spacing.

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End

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