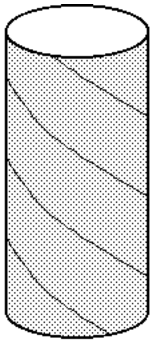
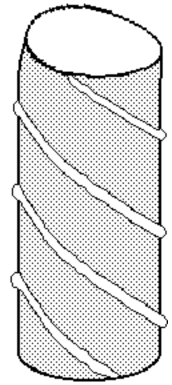
A Package of Ready-to-Cook Dinner Rolls



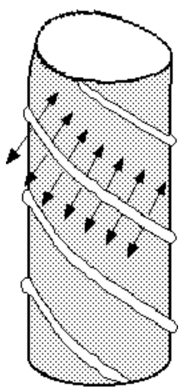
In this stack we will consider a problem illustrating the application of pressure vessel and stress transformation concepts. In particular, we will look at a common packaging method for dinner rolls. This consists of a cylinder as shown, which when struck on a counter top will burst open …

Easy Open Packaging



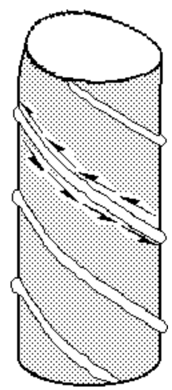
…the package is opened by literally bursting at its seams.

Tensile Strength of Glue



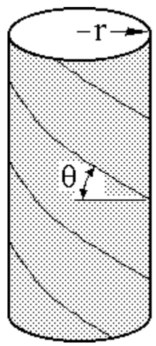
This bursting apart is caused by stresses acting across the glued joints. In general these stresses will consist of tensile and …

Shear Strength of Glue



… shear components as shown. Our task will be to analyze these stresses in terms of the internal pressure caused by the expanding dough.

Dimensions of Package



$\theta = 30°$

$r = 1.5"$

$t = 0.05"$

First, we identify the basic geometric parameters. These are the radius of the tube, the thickness of the cardboard, and the inclination angle of the seam.

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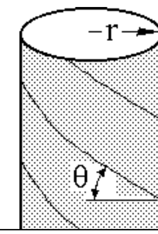
Is it "Thin Walled"?



$\theta = 30°$

$r = 1.5"$

$t = 0.05"$

Before doing an analysis, it is always important to make sure that the assumptions underlying the theory are valid. In this case we need to verify that the walls are indeed thin.

$$\frac{r}{t} = \frac{1.5"}{0.05"} = 30 > 10 \checkmark$$

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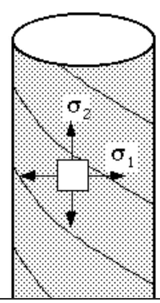
Calculate the Stresses



Longitudinal Stress

$$\sigma_2 = \frac{pr}{2t}$$

Hoop Stress

$$\sigma_1 = \frac{pr}{t}$$

We can now use our equations to calculate the hoop and longitudinal stress in the thin-walled cylinder.

A Quiz

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Internal Pressure?

To solve this problem, we need to determine a target internal pressure of the container at failure. After considering the action of the yeast in the dough, the change in pressure of the container during shipping, and the strength of our average customer, we have decided that the internal pressure should be about 20 psi when the container splits open.

$$p = 20 \text{ psi}$$

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Given the internal pressure and the geometric parameters, it is a simple matter to calculate the stresses.

Longitudinal Stress

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_2 = \frac{20\text{ psi} \times 1.5''}{2 \times 0.05''}$$

$$\boxed{\sigma_2 = 300\text{ psi}}$$

Hoop Stress

$$\sigma_1 = \frac{p\,r}{t}$$

$$\sigma_1 = \frac{20\text{ psi} \times 1.5''}{0.05''}$$

$$\boxed{\sigma_1 = 600\text{ psi}}$$

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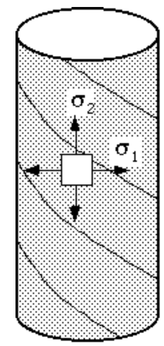
Calculated Stresses



Returning to the original figure, we now have evaluated the stresses as shown. But …

$$\sigma_1 = 600\text{ psi}$$
$$\sigma_2 = 300\text{ psi}$$

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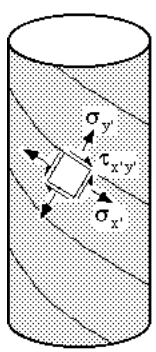
Desired Stresses



$\theta = ?$

$\sigma_{x'} = ?$

$\sigma_{y'} = ?$

$\tau_{x'y'} = ?$

…to evaluate the stresses across the seam, we need to consider the stress components oriented as shown. We need to identify θ, and then use the transformation equations or Mohr's circle.

$$\sigma_1 = 600\text{ psi}$$
$$\sigma_2 = 300\text{ psi}$$

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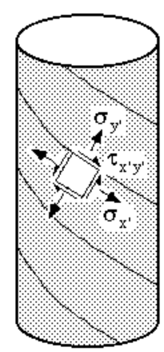
Desired Stresses



$\theta = -30°$

Assuming the block to have rotated clockwise, we take θ as – 30°.

$\sigma_{x'} = ?$

$\sigma_{y'} = ?$

$\tau_{x'y'} = ?$

$$\sigma_1 = 600\text{ psi}$$
$$\sigma_2 = 300\text{ psi}$$

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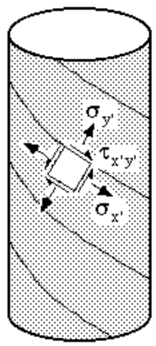
Desired Stresses



$$\theta = -30°$$

~~$\sigma_{x'} = ?$~~

$\sigma_{y'} = ?$

$\tau_{x'y'} = ?$

Examining the seam we can see that we do not need to consider $\sigma_{x'}$. To compute the remaining stress components, we recall the transformation

$\sigma_1 = 600$ psi

$\sigma_2 = 300$ psi

Recall The StressTransformation Equations

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x + \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Note that in this case our original stress components are $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, and $\tau_{xy} = 0$.

Transformation Equations for this Problem

$$\sigma_{y'} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 + \sigma_2}{2}\cos 2\theta - \cancel{\tau_{xy}\sin 2\theta}$$

$$\tau_{x'y'} = \frac{\sigma_2 - \sigma_1}{2}\sin 2\theta + \cancel{\tau_{xy}\cos 2\theta}$$

Therefore, we can eliminate the shear components to simplify the equations as shown.

Substituting...

$$\sigma_{y'} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 + \sigma_2}{2}\cos 2\theta - \cancel{\tau_{xy}\sin 2\theta}$$

$$\tau_{x'y'} = \frac{\sigma_2 - \sigma_1}{2}\sin 2\theta + \cancel{\tau_{xy}\cos 2\theta}$$

Recall the hoop and longitudinal stresses we calculated earlier.

$\sigma_1 = 600$ psi

$\sigma_2 = 300$ psi

Substituting...

$$\sigma_{y'} = \frac{600 + 300}{2} - \frac{600 + 300}{2}\cos 2\theta$$

$$\tau_{x'y'} = \frac{300 - 600}{2}\sin 2\theta$$

Recall θ: $\theta = -30°$

$\sigma_1 = 600$ psi

$\sigma_2 = 300$ psi

The Solution

Do the calculations:

$$\sigma_{y'} = \frac{600 + 300}{2} - \frac{600 + 300}{2}\cos 2\theta = 225 \text{ psi}$$

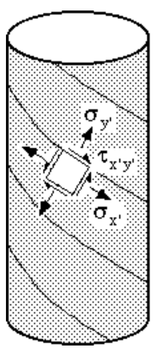
$$\tau_{x'y'} = \frac{300 - 600}{2}\sin 2\theta = 130 \text{ psi}$$

$\theta = -30°$

$\sigma_1 = 600$ psi

$\sigma_2 = 300$ psi

Final Results

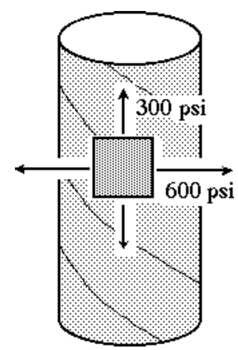


The resolved stress components acting across the glue line are as indicated. These values could be used to design the glue joint so that unintended opening of the package does not occur.

$\sigma_{y'} = 225$ psi

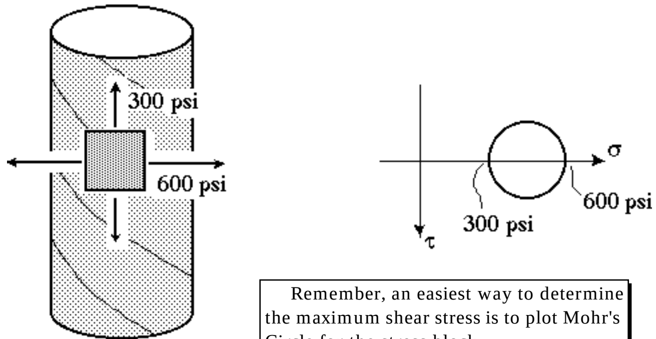
$\tau_{x'y'} = 130$ psi

Maximum Shear Stress?



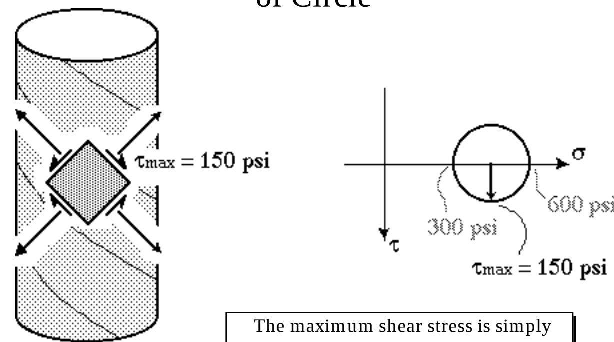
As long as we have the principal normal stresses for the package of dinner rolls, let's calculate the maximum shear stress experienced by the cardboard packaging.

Plot Mohr's Circle



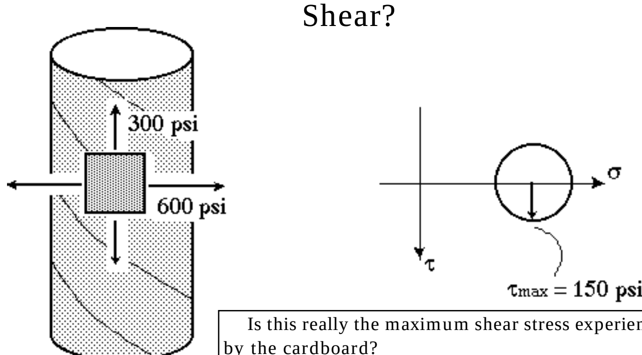
Remember, an easiest way to determine the maximum shear stress is to plot Mohr's Circle for the stress block.

Maximum Shear Stress is Radius of Circle



The maximum shear stress is simply the radius of Mohr's Circle. In this case the maximum shear stress is 150 psi.

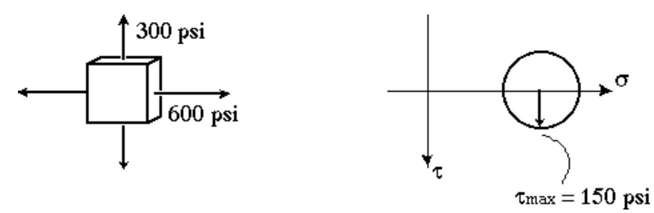
Did We Find the True Maximum Shear?



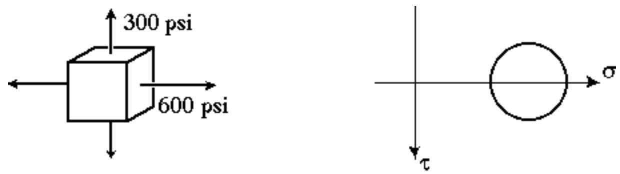
Is this really the maximum shear stress experienced by the cardboard?

Currently, we are only viewing the stresses in two-dimensions. Let's see what the stresses look like in three dimensions for this stress block.

Element is 3-D



Element is 3-D



What is the value of the normal stress acting on the face of the stress block oriented towards us?

Do you remember the assumption we made about this stress in solving the thin walled pressure vessel problem?

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Stress is Zero in Third Direction

We assumed that all stress acts tangential to the surface of the pressure vessel (plane stress). Therefore, a stress which is oriented normal to the surface of the pressure vessel is zero.

Let's now change our perspective on the stress block so we are looking directly down the 600 psi normal stress. We are NOT rotating the stress block, we are walking around the stress block to the right until our perspective is directly down the 600 psi normal stress

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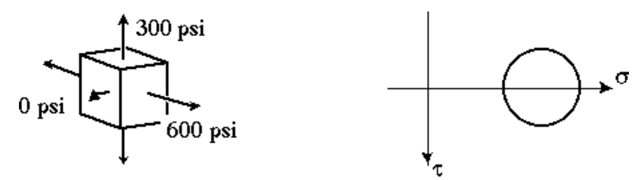
Change Perspective

27 Hide Text

Change Perspective



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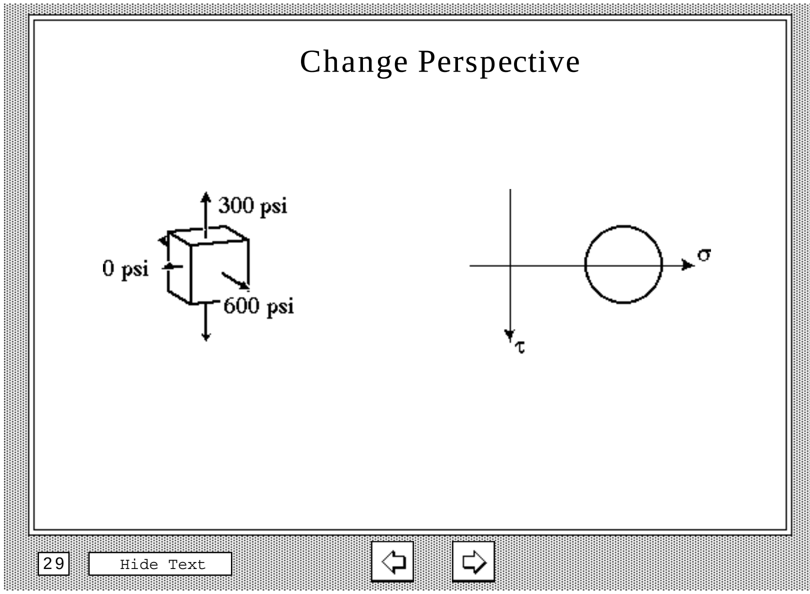
Change Perspective
300 psi
0 psi
600 psi
σ
τ
29
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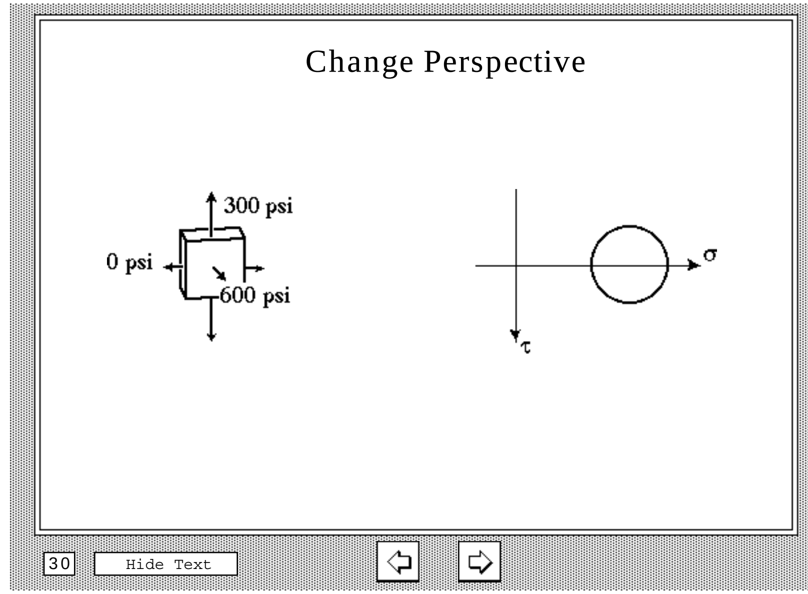
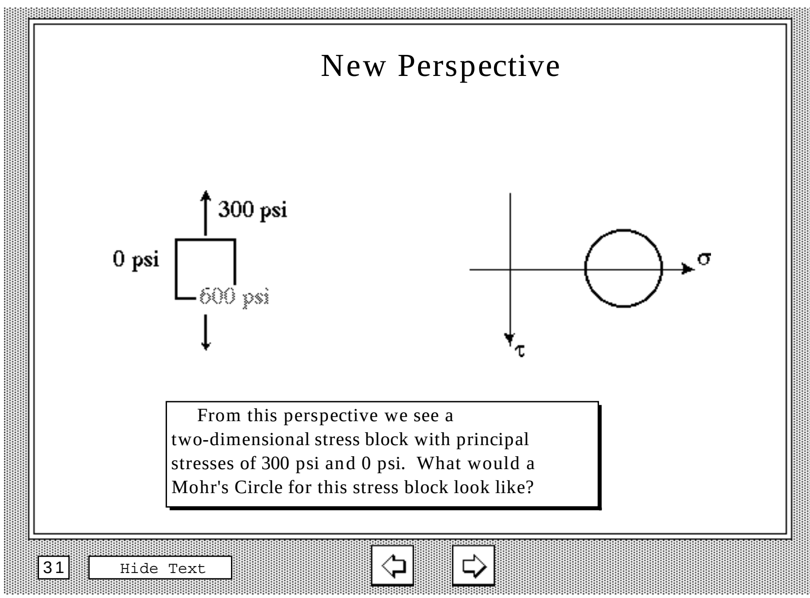

Change Perspective
300 psi
0 psi
600 psi
σ
τ
30
Hide Text


New Perspective
300 psi
0 psi
600 psi
σ
τ
From this perspective we see a two-dimensional stress block with principal stresses of 300 psi and 0 psi. What would a Mohr's Circle for this stress block look like?
31
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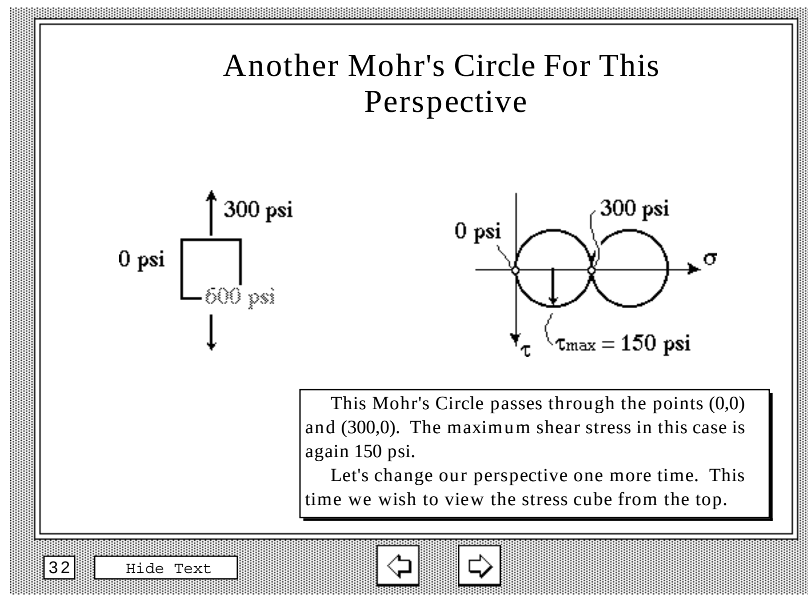
Another Mohr's Circle For This Perspective
300 psi
0 psi
600 psi
0 psi
300 psi
σ
τ
τmax = 150 psi
This Mohr's Circle passes through the points (0,0) and (300,0). The maximum shear stress in this case is again 150 psi.
Let's change our perspective one more time. This time we wish to view the stress cube from the top.
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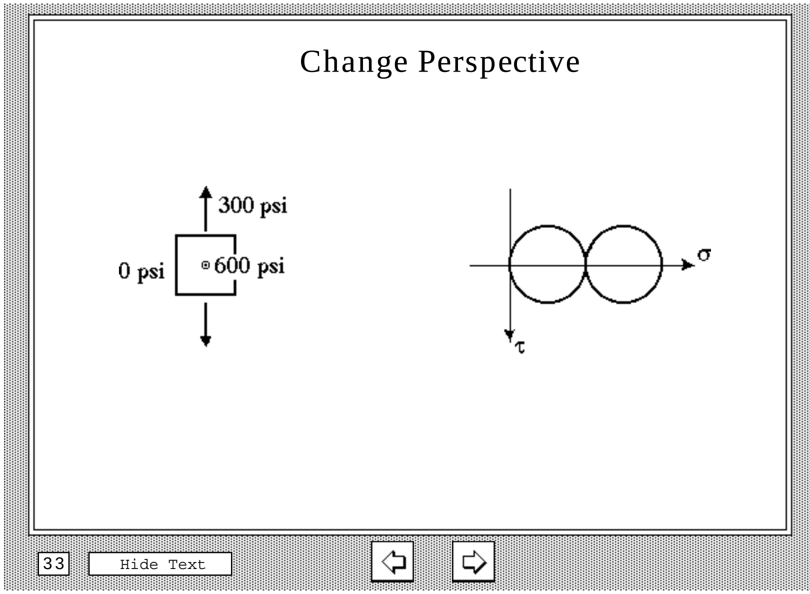
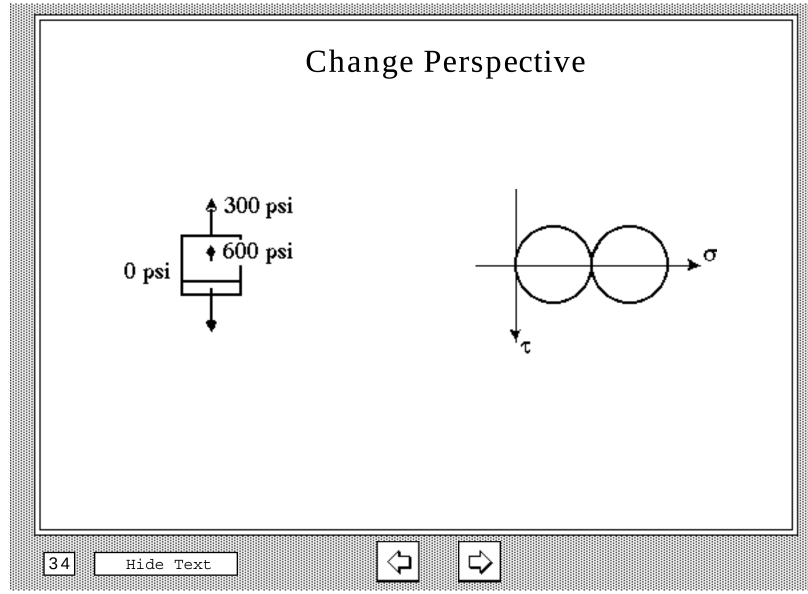
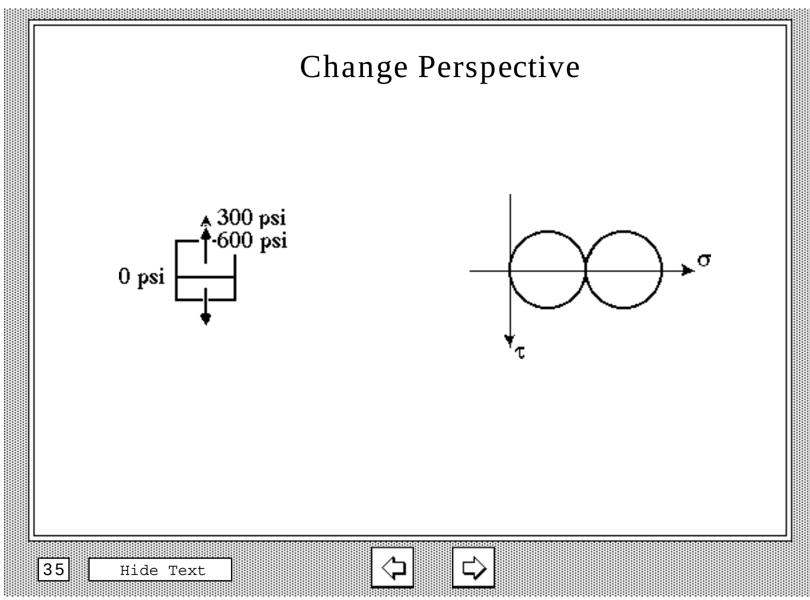
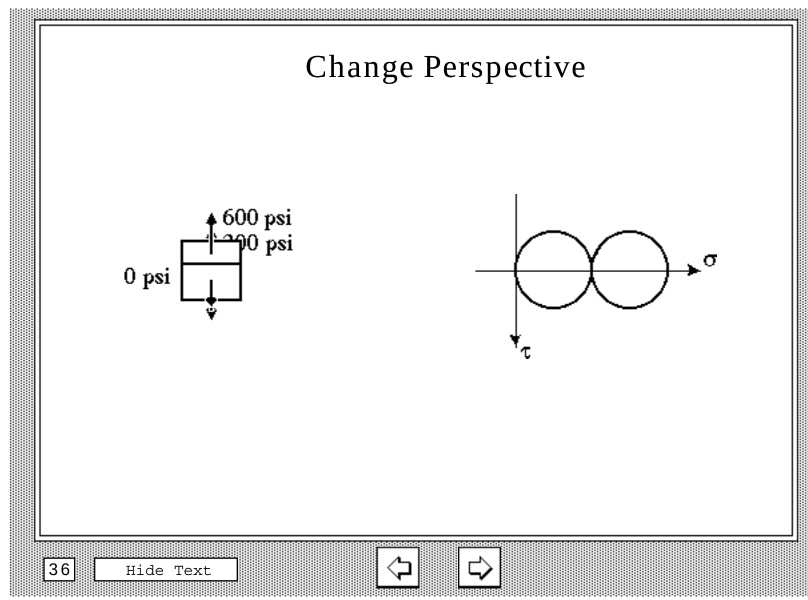

Change Perspective
300 psi
0 psi
600 psi
σ
τ
33
Hide Text


Change Perspective
300 psi
600 psi
0 psi
σ
τ
34
Hide Text


Change Perspective
300 psi
600 psi
0 psi
σ
τ
35
Hide Text


Change Perspective
600 psi
0 psi
σ
τ
36
Hide Text

Change Perspective

37 Hide Text

Final Perspective

From this perspective we see a two-dimensional stress block with principal stresses of 600 psi and 0 psi. Let's plot a Mohr's Circle for this stress block.

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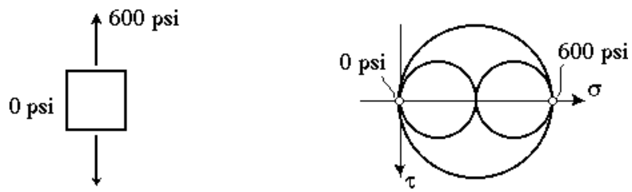
Mohr's Circle for Third Perspective



The Mohr's Circle for this stress block passes through the points (0,0) and (600,0). Do you think that the maximum shear stress for this perspective will be any different than the other two?

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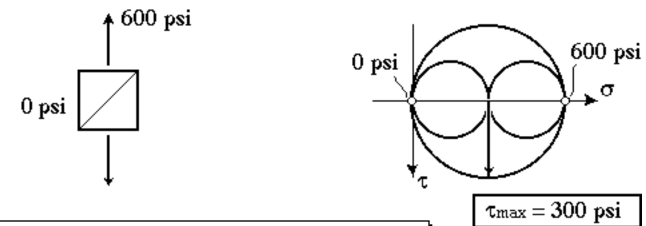
True Maximum Shear Stress



Wow. The maximum shear stress for our final Mohr's Circle is 300 psi!! The cardboard container is actually experiencing a shear stress twice as great as we had originally thought.

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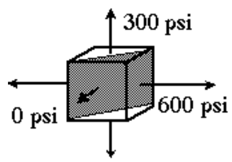
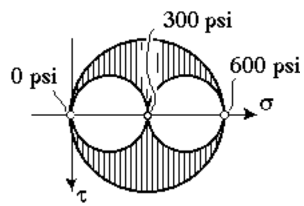
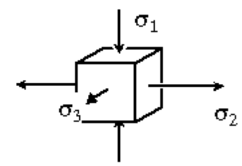
3-D Mohr's Circle

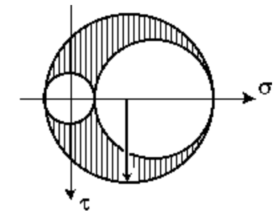




The plot we have just created is known as 3-D Mohr's Circle. The shaded region indicates the shear and normal stresses for every potential orientation of the stress cube. The plane of maximum shear stress is shown as shaded in the cube.

3-D Mohr's Circle





In general, we use 3-D Mohr's Circle to calculate the absolute maximum shear stress. If we know the principal stresses, we can calculate the absolute maximum shear stress as the maximum principal stress minus the minimum principal stress divided by two.

In the case of plane stress, 3-D Mohr's Circle is necessary in calculating the absolute maximum shear stress when the signs of the two principal stress are the same -- i.e. they are both tensile or both compressive.

The End