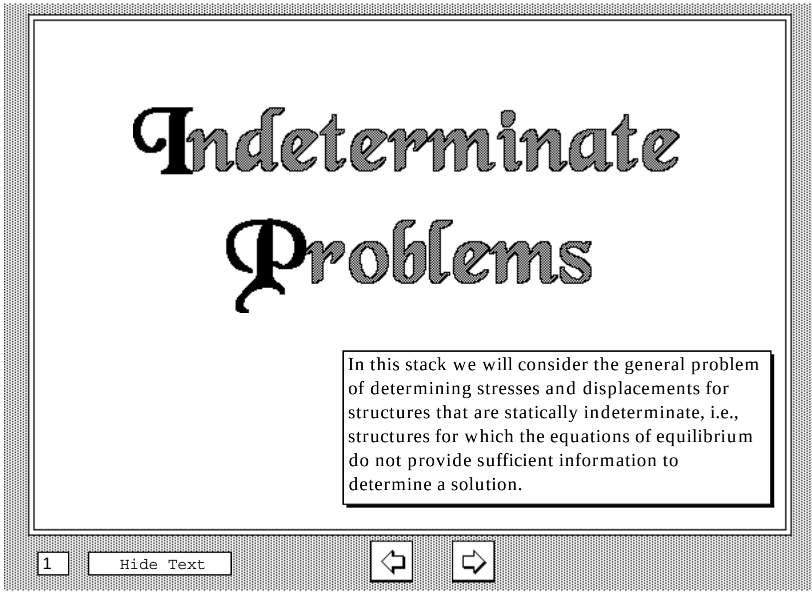
Indeterminate Problems
In this stack we will consider the general problem of determining stresses and displacements for structures that are statically indeterminate, i.e., structures for which the equations of equilibrium do not provide sufficient information to determine a solution.
1
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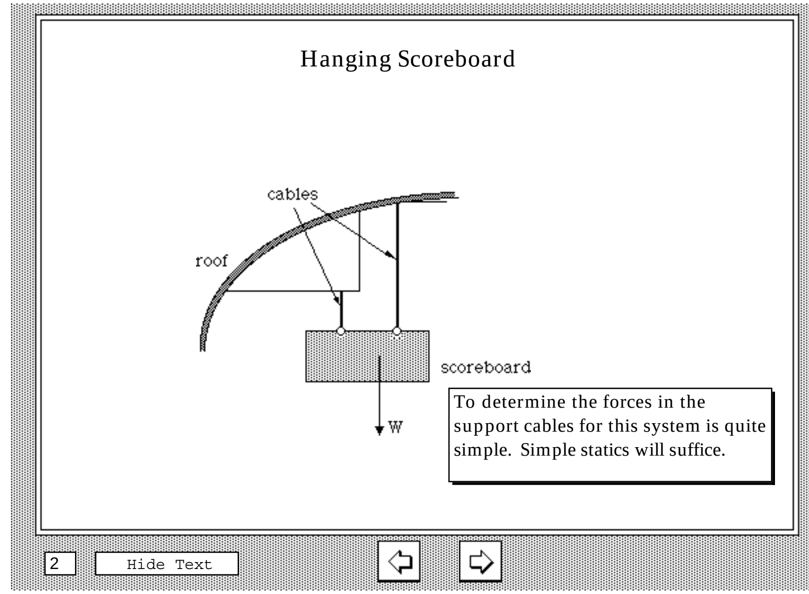
Hanging Scoreboard
cables
roof
scoreboard
W
To determine the forces in the support cables for this system is quite simple. Simple statics will suffice.
2
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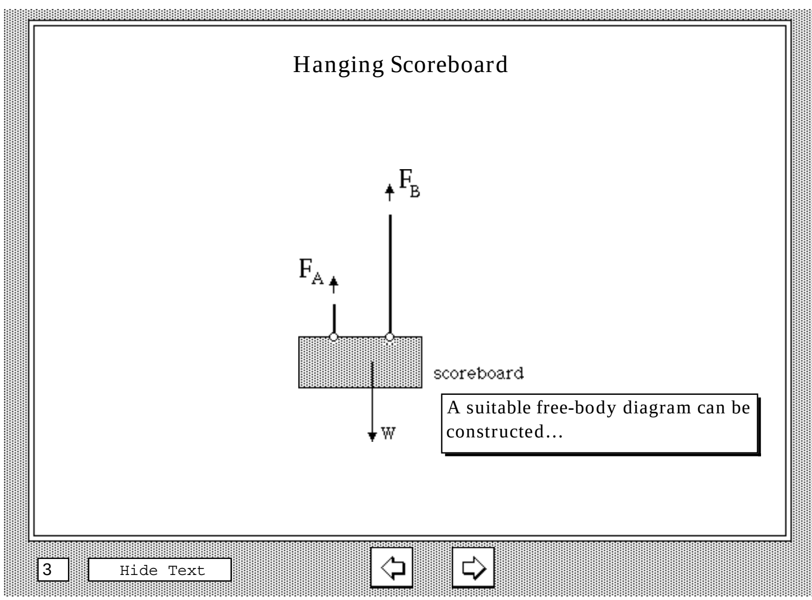
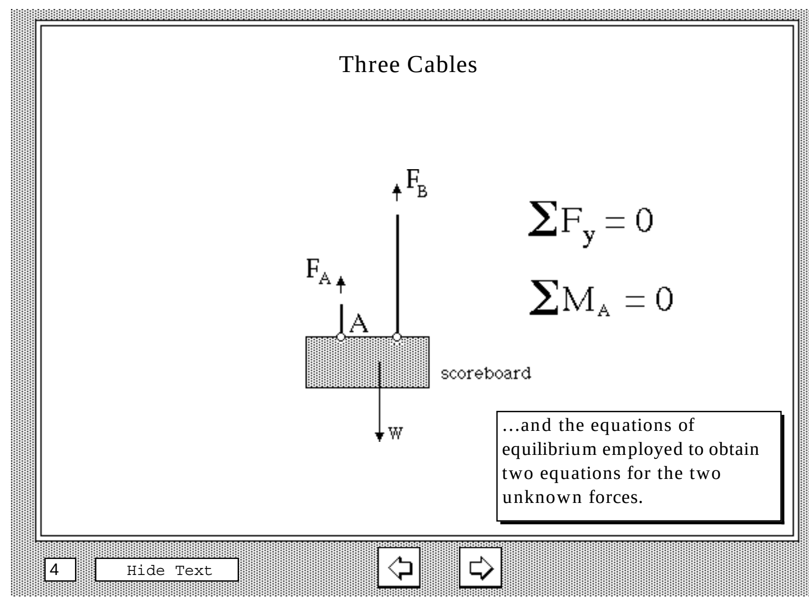

Hanging Scoreboard
F_B
F_A
scoreboard
W
A suitable free-body diagram can be constructed…
3
Hide Text


Three Cables
F_B
F_A
A
$\Sigma F_y = 0$
$\Sigma M_A = 0$
scoreboard
W
…and the equations of equilibrium employed to obtain two equations for the two unknown forces.
4
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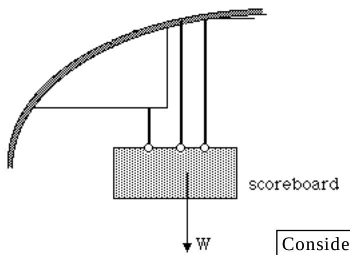
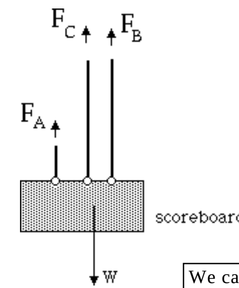
Three Cables



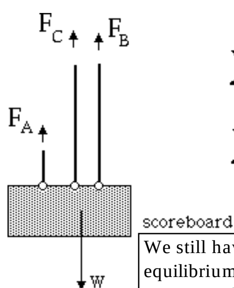
Consider the case in which three cables are used instead of two.

Three Cables



We can construct a free-body diagram as before. Note that we have three unknown forces.

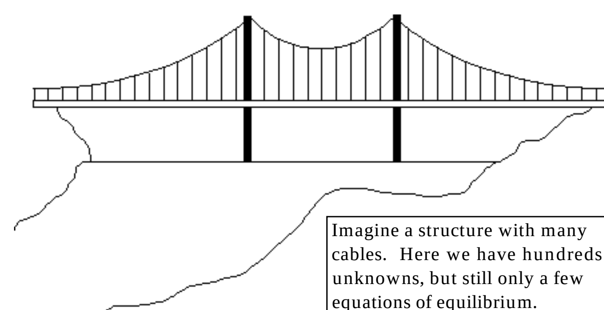
Three Cables



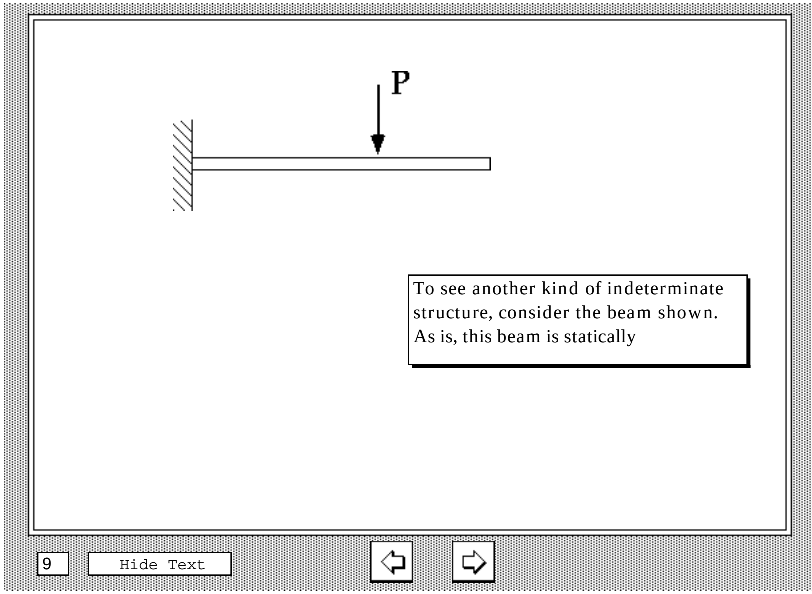
$$\sum F_y = 0$$

$$\sum M_A = 0$$

We still have only two equations of equilibrium (x-direction equilibrium is irrelevant for this problem). This means we have two equations for three unknowns – we can not determine the solution with this information alone.



Imagine a structure with many cables. Here we have hundreds of unknowns, but still only a few equations of equilibrium.


P
To see another kind of indeterminate structure, consider the beam shown. As is, this beam is statically
9
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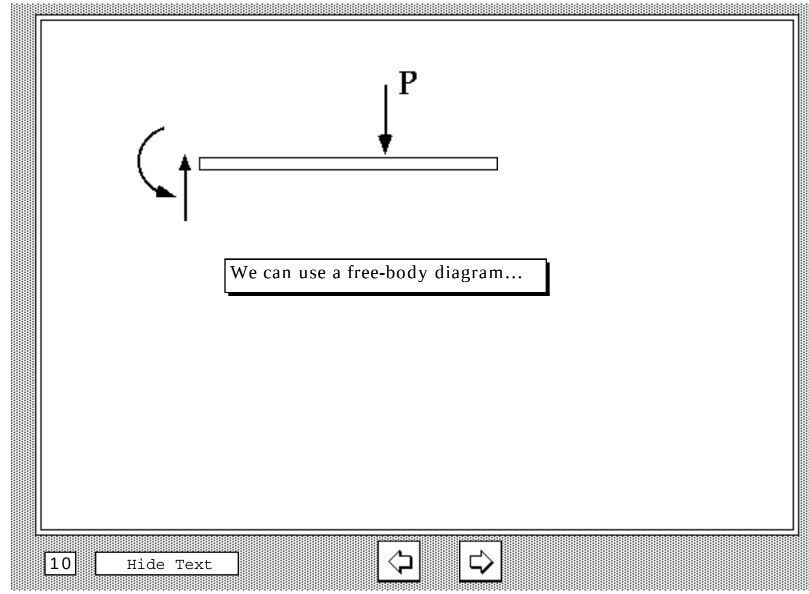
P
We can use a free-body diagram…
10
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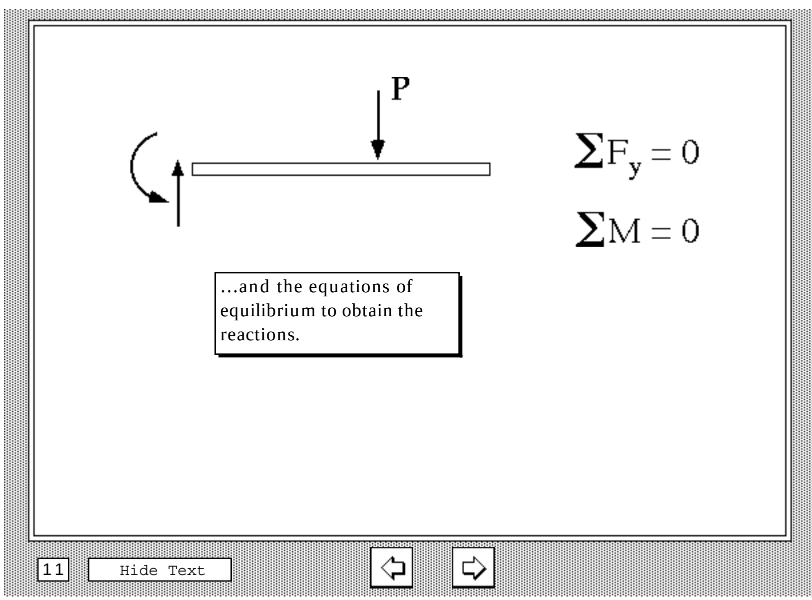
P
$\sum F_y = 0$
$\sum M = 0$
…and the equations of equilibrium to obtain the reactions.
11
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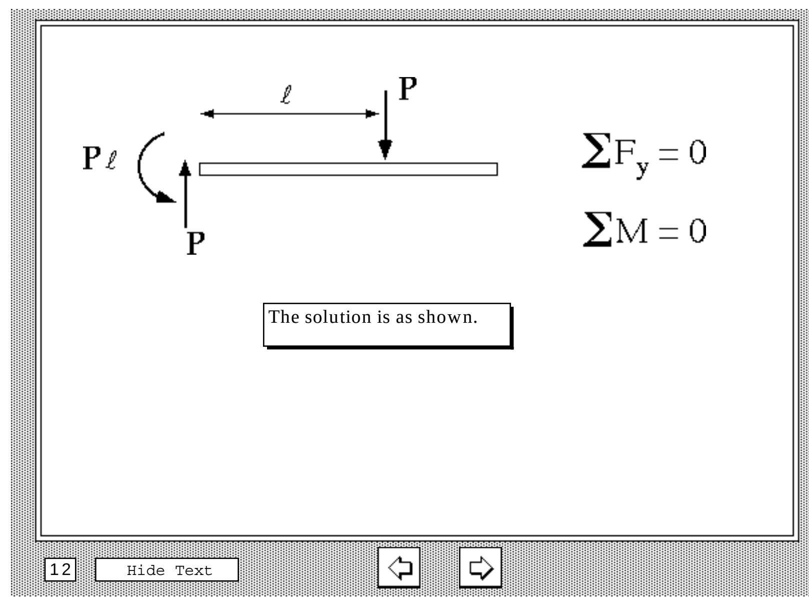
ℓ
P
Pℓ
P
$\sum F_y = 0$
$\sum M = 0$
The solution is as shown.
12
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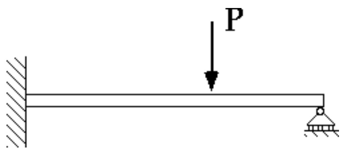
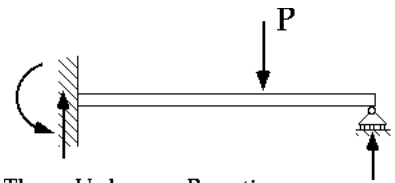


Now an additional support has been added. Note that this support is not required for the structure to function properly. It is extra or **redundant**. Redundant supports lead to static indeterminacy, as we will see.



Three Unknown Reactions

Two Equations of Equilibrium

$$\sum F_y = 0$$

$$\sum M = 0$$

As in the case of the cable problem, we now have more unknowns than equations.

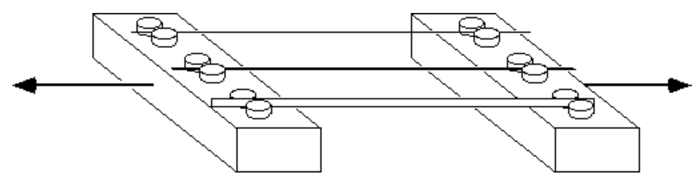
2 Equations, 3 unknowns

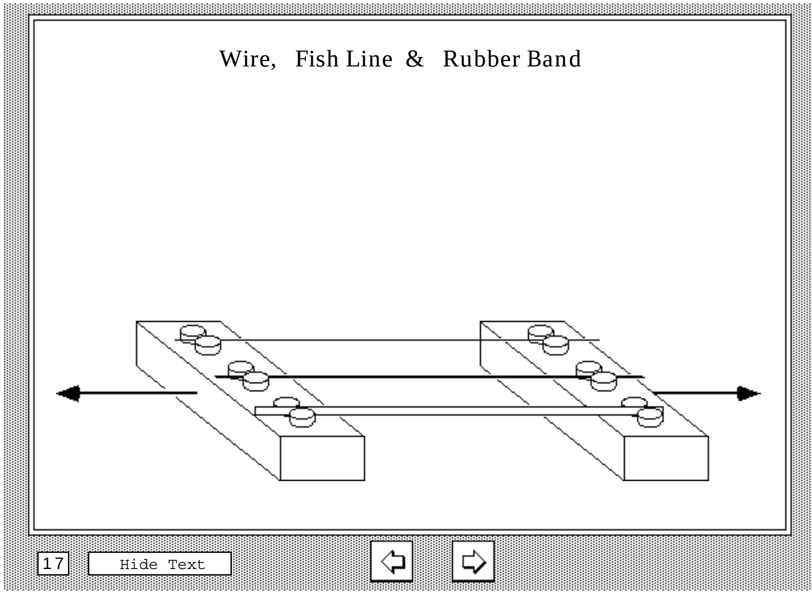
$$3x + 4y + 6z = 123$$
$$5x + 2y + 9z = 24$$

You may recall from your linear algebra studies that an underdetermined system has an infinite number of solutions. But clearly the structures we considered will have only one set of internal forces and reactions for a given loading. What additional information can we use to determine this unique solution?

Wire, Fish Line & Rubber Band

To answer this question, imagine a simple experiment in which steel wire, fish line and a rubber band are stretched as shown. The loading blocks do not rotate, and so each material experiences the same displacement or stretch.




Wire, Fish Line & Rubber Band
17
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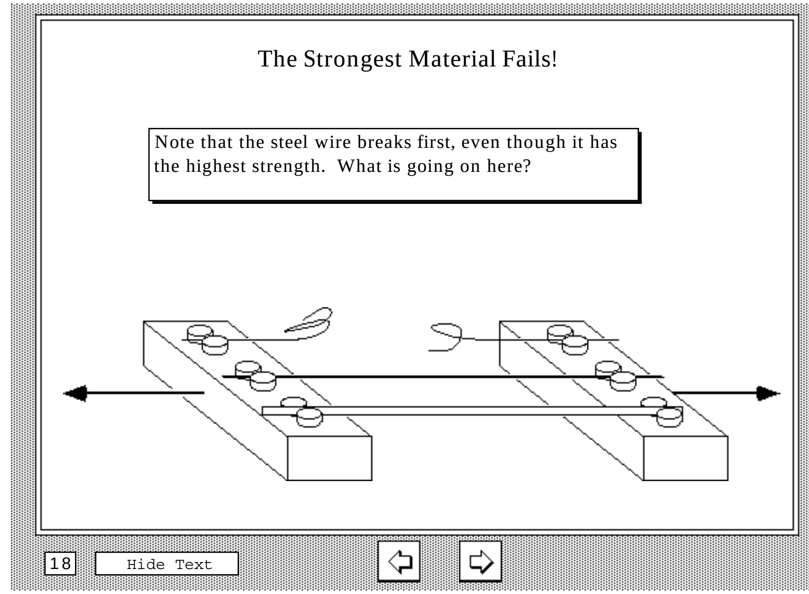
The Strongest Material Fails!
Note that the steel wire breaks first, even though it has the highest strength. What is going on here?
18
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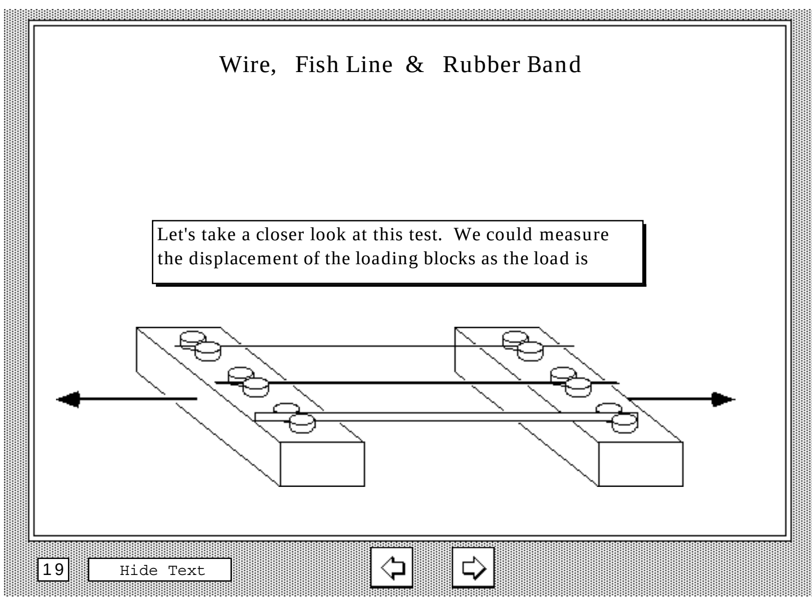
Wire, Fish Line & Rubber Band
Let's take a closer look at this test. We could measure the displacement of the loading blocks as the load is
19
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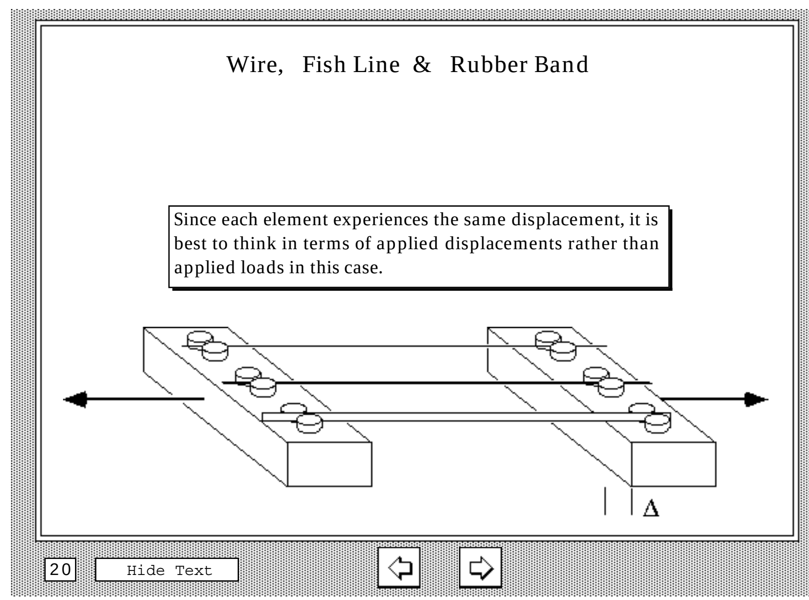
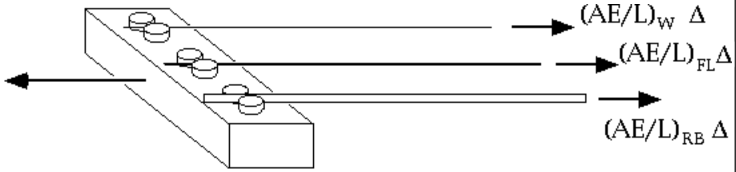

Wire, Fish Line & Rubber Band
Since each element experiences the same displacement, it is best to think in terms of applied displacements rather than applied loads in this case.
Δ
20
Hide Text

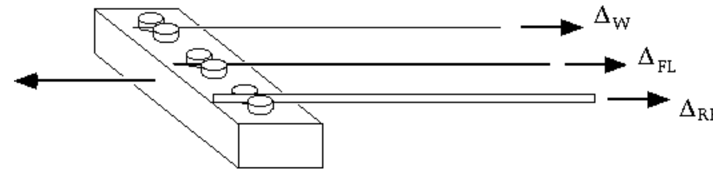
Wire, Fish Line & Rubber Band

The force in each member can be calculated from the known displacement, Δ, by simply rearranging the relation $\Delta = PL / AE$, and plugging in the appropriate values for each member. The key to finding the member forces, then, is to consider the displacements as well as the forces. In this case we can determine the member forces by using the fact that the displacement for each member is the same. This relation between displacements is analogous to the relations between forces required by equilibrium, and is called **compatibility**.



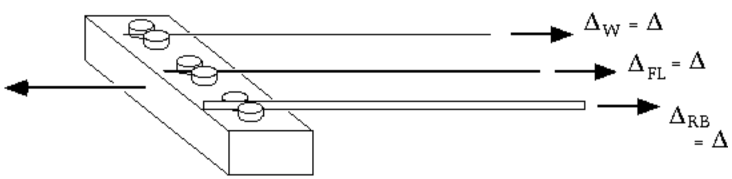
Wire, Fish Line & Rubber Band

We can put the compatibility conditions in a "relation" form as follows. We first assume that the displacement of each member is independent.

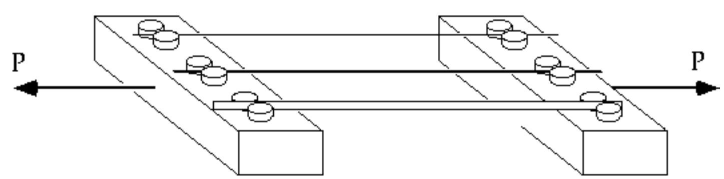


Wire, Fish Line & Rubber Band

We then impose the compatibility condition that each member displacement equals that of the loading block.

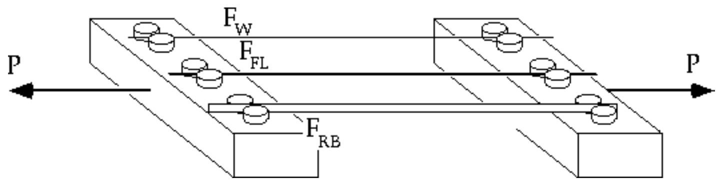


Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:



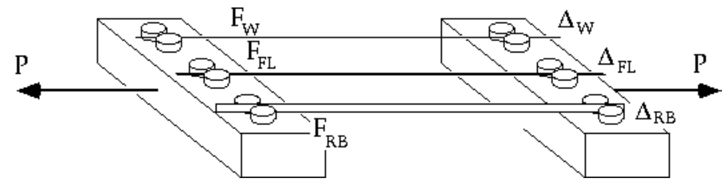
Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

1. The member forces (3 unknowns in this case)



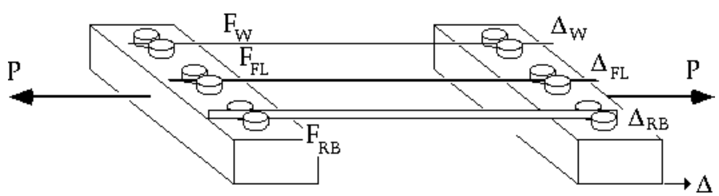
Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

1. The member forces (3 unknowns in this case)
2. The member displacements (3 unknowns in this case)



Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

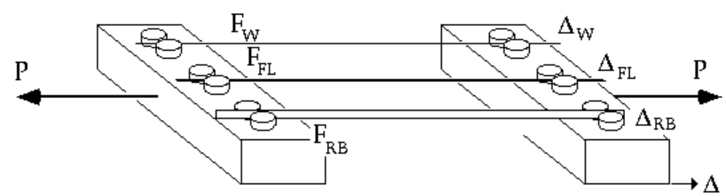
1. The member forces (3 unknowns in this case)
2. The member displacements (3 unknowns in this case)
3. The loading block displacement (1 unknown in this case).



Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

1. The member forces (3 unknowns in this case)
2. The member displacements (3 unknowns in this case)
3. The loading block displacement (1 unknown in this case).

To solve for these 7 unknowns we have the following relations:

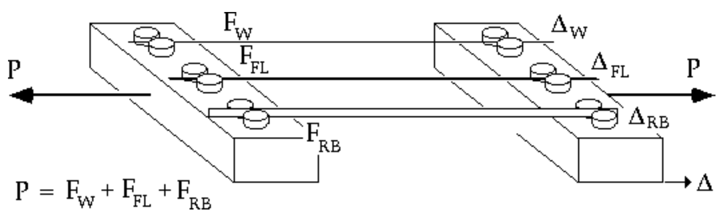


Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

1. The member forces (3 unknowns in this case)
2. The member displacements (3 unknowns in this case)
3. The loading block displacement (1 unknown in this case).

To solve for these 7 unknowns we have the following relations:

1. Equilibrium (1 equation in this case)



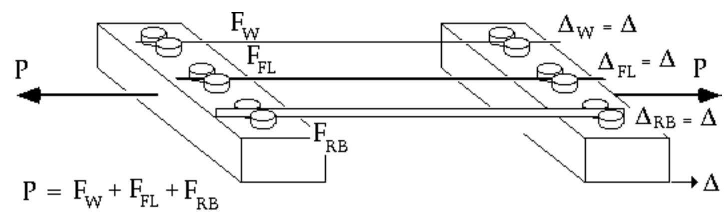
29 Hide Text

Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

1. The member forces (3 unknowns in this case)
2. The member displacements (3 unknowns in this case)
3. The loading block displacement (1 unknown in this case).

To solve for these 7 unknowns we have the following relations:

1. Equilibrium (1 equation in this case)
2. Compatibility (3 equations in this case)



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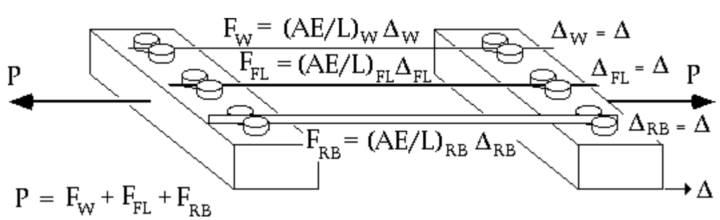
Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

1. The member forces (3 unknowns in this case)
2. The member displacements (3 unknowns in this case)
3. The loading block displacement (1 unknown in this case).

To solve for these 7 unknowns we have the following relations:

1. Equilibrium (1 equation in this case)
2. Compatibility (3 equations in this case)
3. Load-Displacement relations (3 equations in this case)



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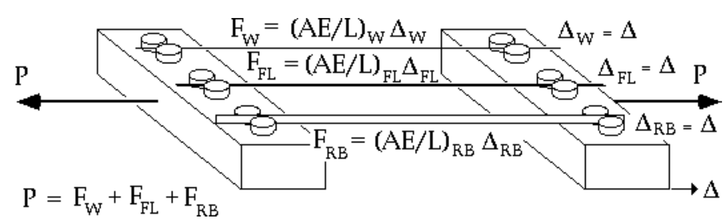
Let's take a more global perspective of this problem. In general we will have the following sets of unknowns:

1. The member forces (3 unknowns in this case)
2. The member displacements (3 unknowns in this case)
3. The loading block displacement (1 unknown in this case).

To solve for these 7 unknowns we have the following relations:

1. Equilibrium (1 equation in this case)
2. Compatibility (3 equations in this case)
3. Load-Displacement relations (3 equations in this case)

Thus we have an equal number of equations and unknowns, and so the problem can be solved. This is a general and important result.



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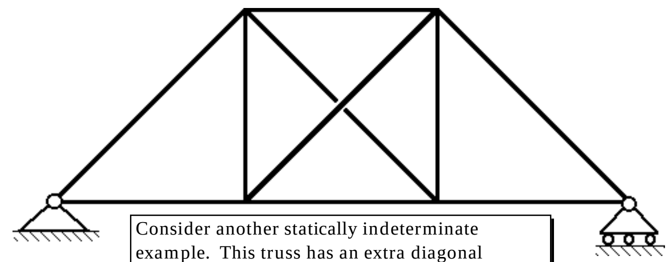
Equilibrium

Compatibility

Load-Displacement Relations

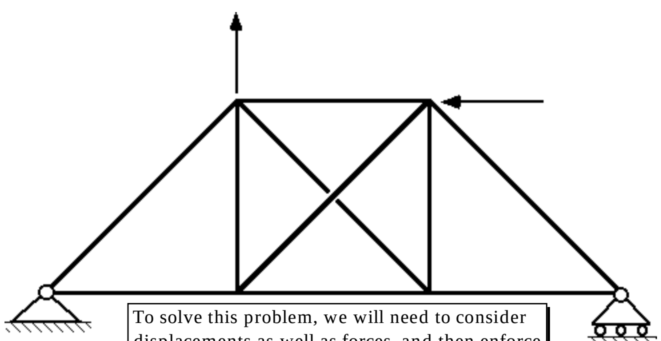
These are the three general ingredients for solving problems involving the mechanics of solids.

Indeterminate Truss



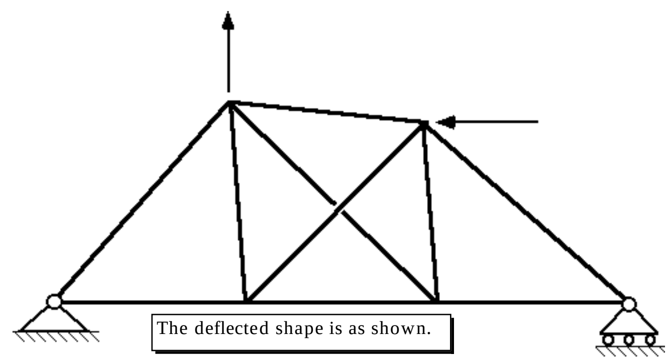
Consider another statically indeterminate example. This truss has an extra diagonal member, and so the member forces can not be determined by statics alone.

Load

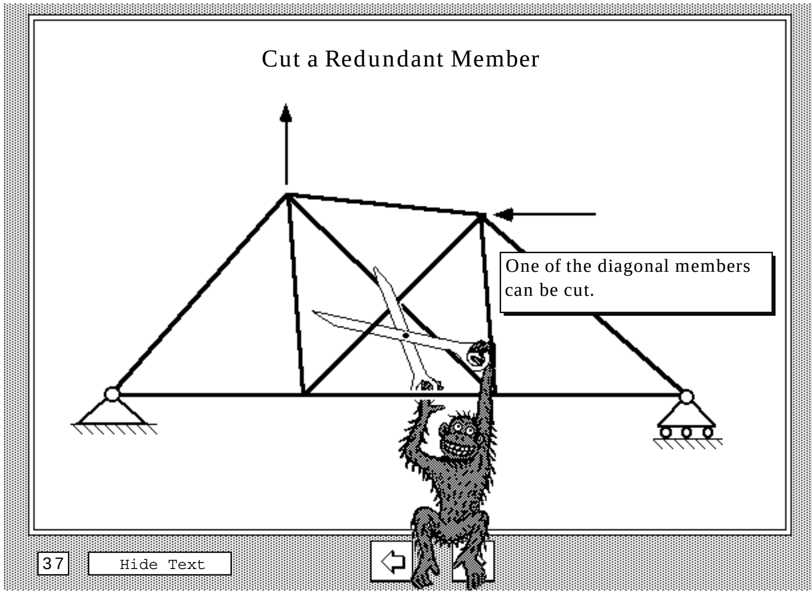


To solve this problem, we will need to consider displacements as well as forces, and then enforce equilibrium and compatibility, in combination with load-displacement relations.

Deflection



The deflected shape is as shown.


Cut a Redundant Member
One of the diagonal members can be cut.
37
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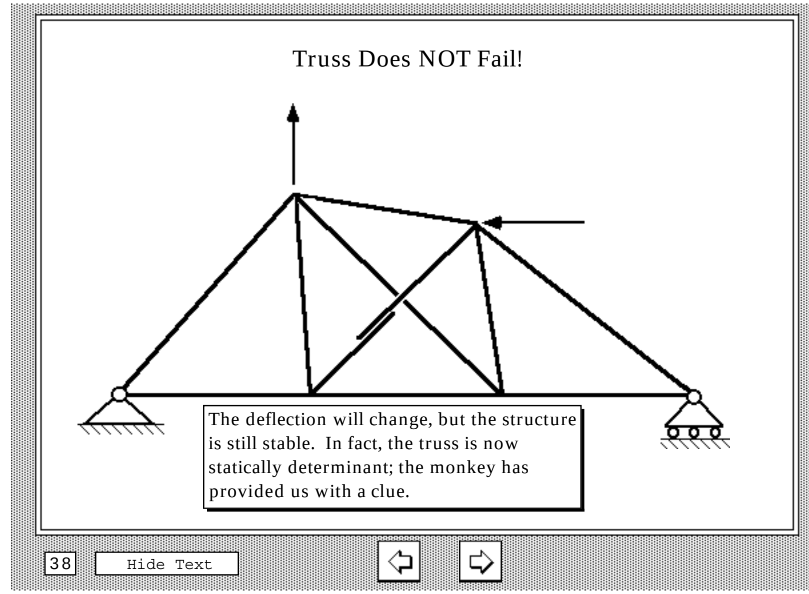
Truss Does NOT Fail!
The deflection will change, but the structure is still stable. In fact, the truss is now statically determinant; the monkey has provided us with a clue.
38
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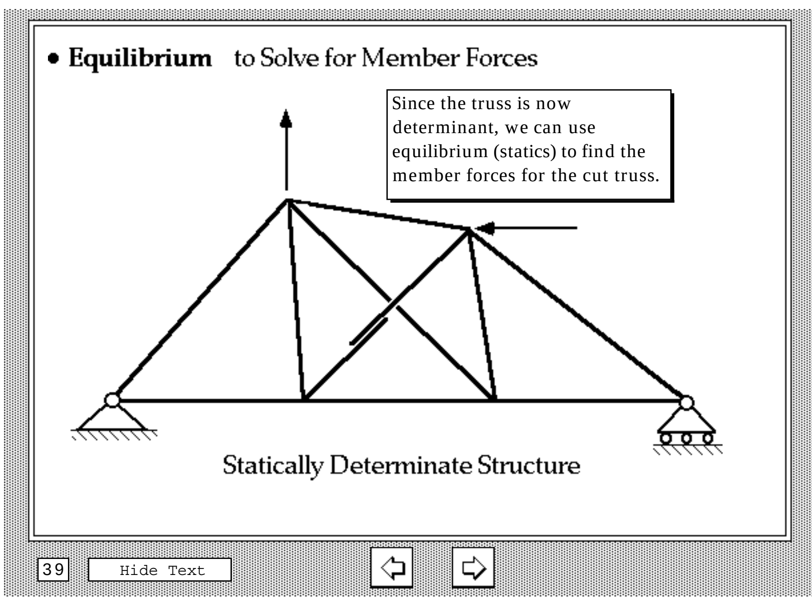
• Equilibrium to Solve for Member Forces
Since the truss is now determinant, we can use equilibrium (statics) to find the member forces for the cut truss.
Statically Determinate Structure
39
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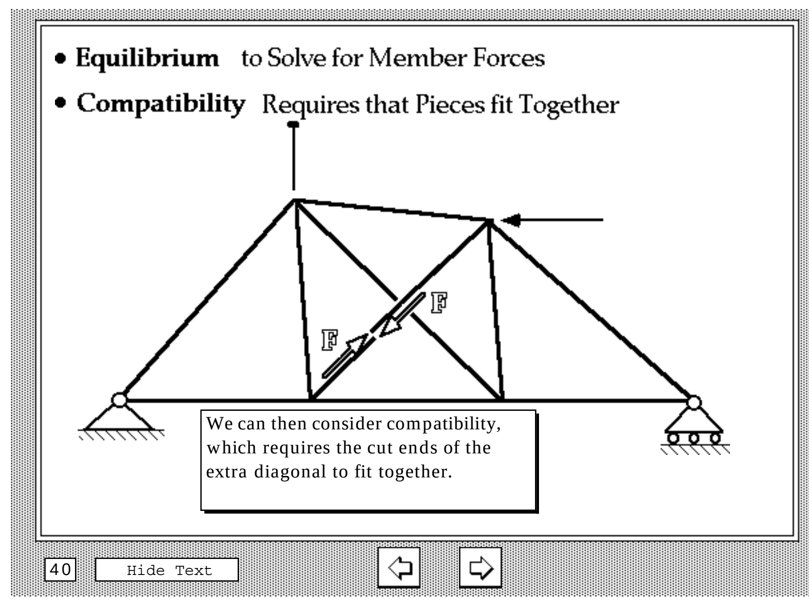
• Equilibrium to Solve for Member Forces
• Compatibility Requires that Pieces fit Together
F
F
We can then consider compatibility, which requires the cut ends of the extra diagonal to fit together.
40
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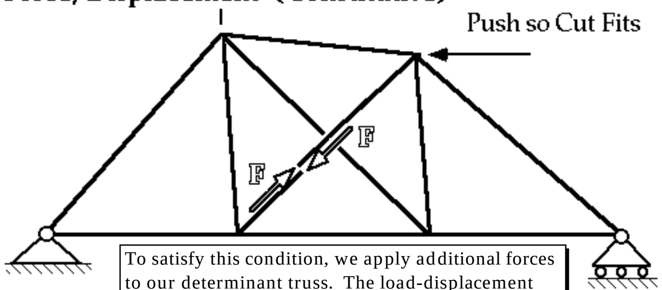
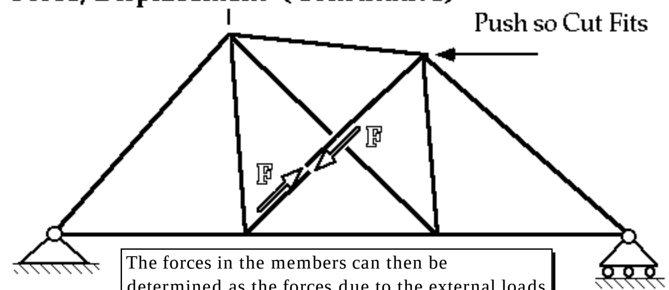
- **Equilibrium** to Solve for Member Forces
- **Compatibility** Requires that Pieces fit Together
- **Force/Displacement (Constitutive)** Tells How Hard to Push so Cut Fits



to

To satisfy this condition, we apply additional forces to our determinant truss. The load-displacement relations tell us how hard we need to push to make the structure fit together again.

- **Equilibrium** to Solve for Member Forces
- **Compatibility** Requires that Pieces fit Together
- **Force/Displacement (Constitutive)** Tells How Hard to Push so Cut Fits



The forces in the members can then be determined as the forces due to the external loads plus the forces due to the extra applied load, F.

Summary

Statically indeterminate systems are those for which the equations of equilibrium alone are insufficient to determine all the internal forces. To solve such problems in general, it is necessary to first introduce additional unknowns into the problem (the displacements) and then employ the full set of fundamental principles to obtain a sufficient number of equations:

1. Equilibrium relates forces to one another.
2. Compatibility relates displacements to one another.
3. Stress-Strain relations link forces and displacements.

The End