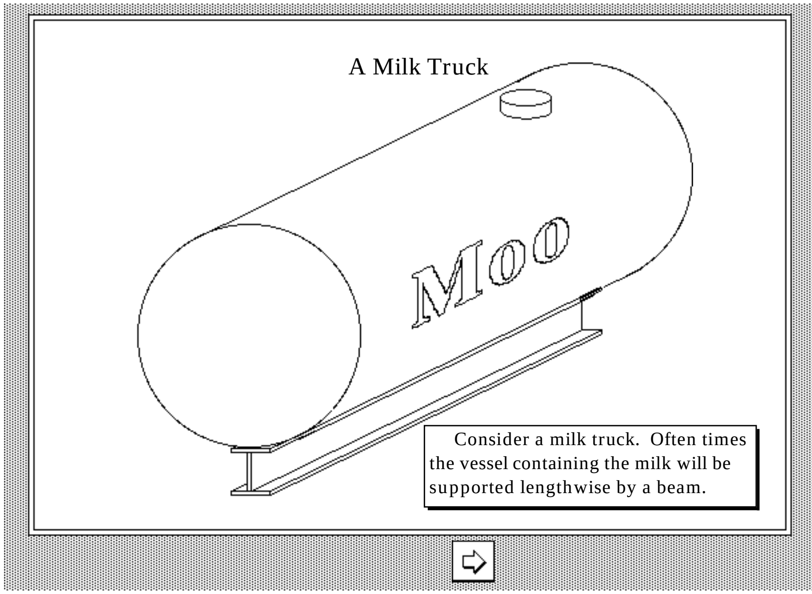
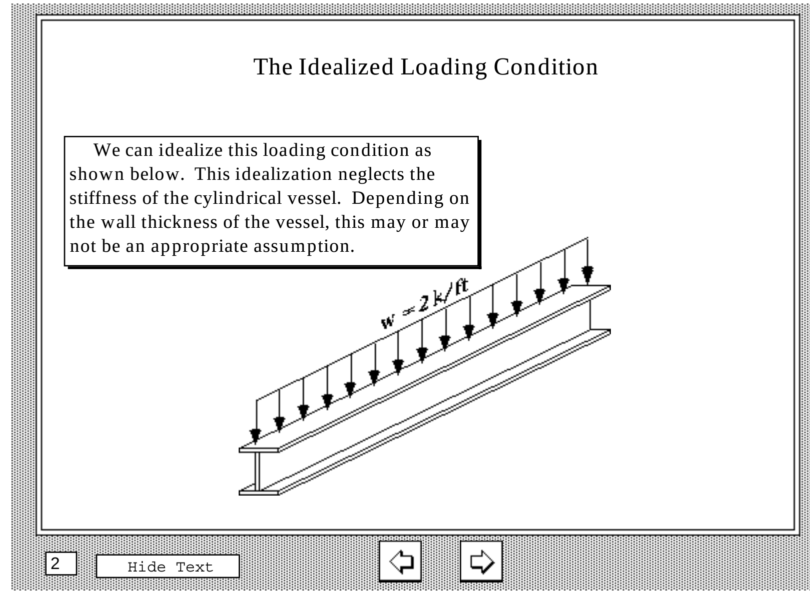

A Milk Truck
Moo
Consider a milk truck. Often times the vessel containing the milk will be supported lengthwise by a beam.


The Idealized Loading Condition
We can idealize this loading condition as shown below. This idealization neglects the stiffness of the cylindrical vessel. Depending on the wall thickness of the vessel, this may or may not be an appropriate assumption.
w = 2 k/ft
2
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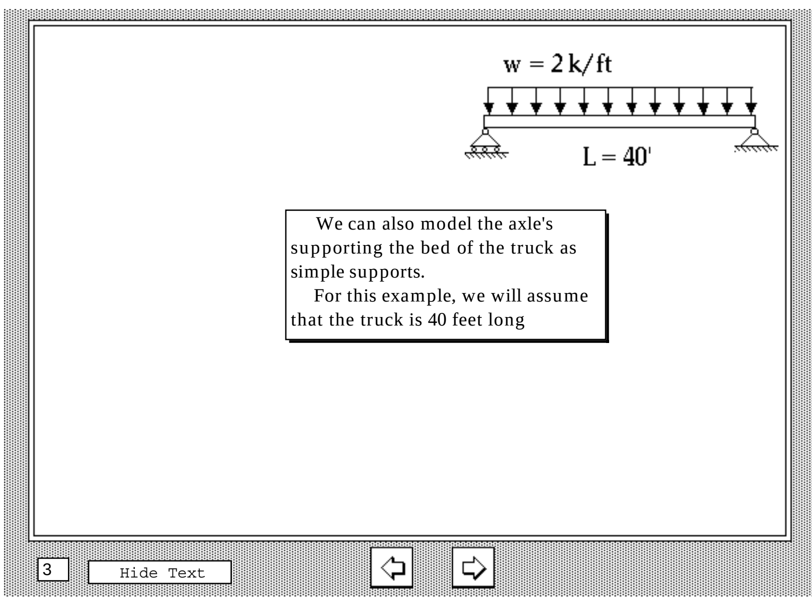
w = 2 k/ft
L = 40'
We can also model the axle's supporting the bed of the truck as simple supports.
For this example, we will assume that the truck is 40 feet long
3
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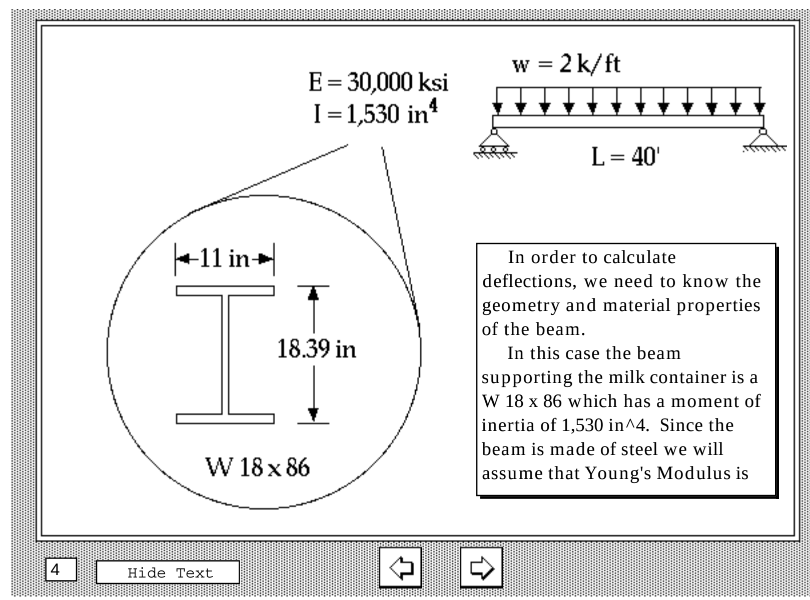
E = 30,000 ksi
I = 1,530 in^4
w = 2 k/ft
L = 40'
11 in
18.39 in
W 18 x 86
In order to calculate deflections, we need to know the geometry and material properties of the beam.
In this case the beam supporting the milk container is a W 18 x 86 which has a moment of inertia of 1,530 in^4. Since the beam is made of steel we will assume that Young's Modulus is
4
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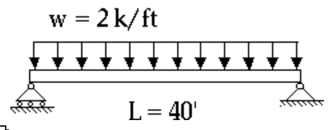
$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

E = 30,000 ksi
I = 1,530 in^4



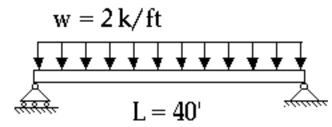
We begin the analysis by recalling the differential equation relating beam loading to beam deflection. In this equation, υ is the deflection, w is the load, and the coordinate x starts at the left of the beam and runs right.

5 Hide Text

$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

$$EI\frac{d^3 \upsilon}{dx^3} = -2x + C_1$$

E = 30,000 ksi
I = 1,530 in^4



We proceed by integrating the differential equation four times to arrive at an expression for the deflection, υ.

One integration....

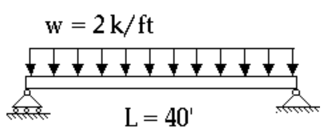
6 Hide Text

$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

$$EI\frac{d^3 \upsilon}{dx^3} = -2x + C_1$$

$$EI\frac{d^2 \upsilon}{dx^2} = -x^2 + C_1 x + C_2$$

E = 30,000 ksi
I = 1,530 in^4



...two integrations...

7 Hide Text

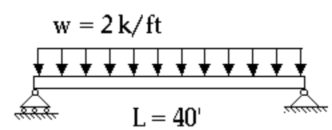
$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

$$EI\frac{d^3 \upsilon}{dx^3} = -2x + C_1$$

$$EI\frac{d^2 \upsilon}{dx^2} = -x^2 + C_1 x + C_2$$

$$EI\frac{d\upsilon}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_2 x + C_3$$

E = 30,000 ksi
I = 1,530 in^4



...three integrations...

8 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

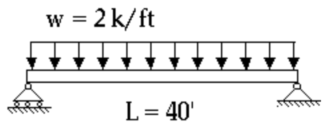
$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x + C_2$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_2x + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4



...four integrations!! We now have an expression for the deflection of the beam, v, in terms of the position, x, and four unknown constants. How do we solve for these constants ??

9 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

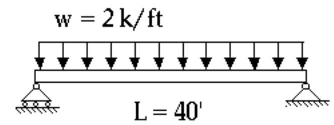
$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x + C_2 = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_2x + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4



Moment Boundary Conditions

That's right!! we apply BOUNDARY CONDITIONS.
We will begin by applying the moment boundary conditions. To do this we must remember that the moment is equal to the second derivative of the displacement.

10 Hide Text Moment curvature relations?

$$EI\frac{d^4v}{dx^4} = -w(x)$$

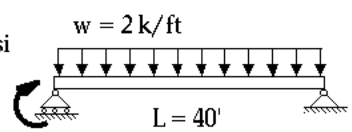
$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x + C_2 = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_2x + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4



Moment Boundary Conditions

(1) $M(0) = 0$

From your knowledge of supports, you should remember that the moment is zero at a roller

11 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

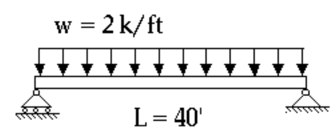
$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x + C_2 = M(x)$$

(x → 0)

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_2x + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4



Moment Boundary Conditions

(1) $M(0) = 0$

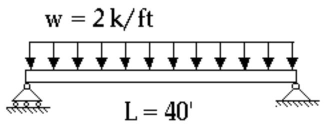
$$-(0)^2 + C_1(0) + C_2 = 0$$

If we substitute x = 0 into our equation for the moment in the beam...

12 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

E = 30,000 ksi
I = 1,530 in^4



$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x + C_2 = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_2x + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

Moment Boundary Conditions

(1) $M(0) = 0$

$$-(0)^2 + C_1(0) + C_2 = 0$$

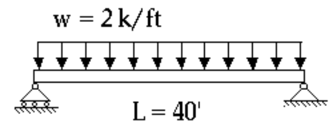
$$\Rightarrow C_2 = 0$$

...we find that the integration constant C2 is zero.

13 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

E = 30,000 ksi
I = 1,530 in^4



$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_3$$

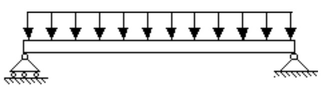
$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + C_3x + C_4$$

Moment Boundary Conditions

(1) $M(0) = 0$

We have now applied one boundary condition and eliminated one integration constant. It follows that to eliminate four integration constants we will need four boundary conditions.

14 Hide Text



v(0) = 0
dv / dx (0) = 0
M(0) = 0
V(0) = 0

v(L) = 0
dv / dx (L) = 0
M(L) = 0
V(L) = 0

Oh Boy, a QUIZ !!

We have already determined one of the four boundary conditions for this beam so it is already highlighted. Your job is to indicate the other three boundary conditions by clicking on the appropriate boxes.

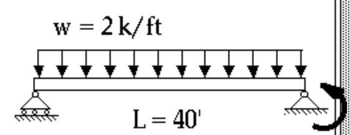
When you think you have highlighted the correct boundary conditions, click on the "I've Made My Choices"

I've Made My Choices

15 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

E = 30,000 ksi
I = 1,530 in^4



$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + C_3x + C_4$$

Moment Boundary Conditions

(1) $M(0) = 0$

(2) $M(40) = 0$

Continuing with the problem, we note that the moment at the right support must be zero.

16 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

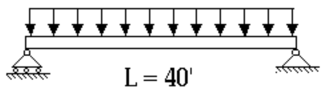
$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4

w = 2 k/ft



L = 40'

Moment Boundary Conditions

(1) M(0) = 0
(2) M(40) = 0

$$0 = -(40)^2 + C_1(40)$$

Substituting x = 40 into the equation for the moment in the beam...

17 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

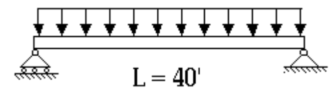
$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4

w = 2 k/ft



L = 40'

Moment Boundary Conditions

(1) M(0) = 0
(2) M(40) = 0

$$0 = -(40)^2 + C_1(40)$$

$$\Rightarrow C_1 = 40$$

...we find that the value of C_1 is

18 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = -2x + C_1$$

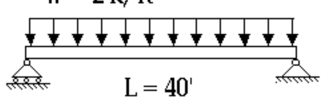
$$EI\frac{d^2v}{dx^2} = -x^2 + C_1x = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{C_1}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{C_1}{6}x^3 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4

w = 2 k/ft



L = 40'

Moment Boundary Conditions

(1) M(0) = 0
(2) M(40) = 0

$$0 = -(40)^2 + C_1(40)$$

$$\Rightarrow C_1 = 40$$

So we substitute the value for C_1 into our equations.

19 Hide Text

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = -2x + 40$$

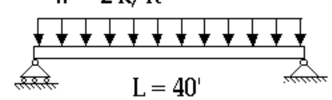
$$EI\frac{d^2v}{dx^2} = -x^2 + 40x = M(x)$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3x + C_4$$

E = 30,000 ksi
I = 1,530 in^4

w = 2 k/ft



L = 40'

Moment Boundary Conditions

(1) M(0) = 0
(2) M(40) = 0

$$0 = -(40)^2 + C_1(40)$$

$$\Rightarrow C_1 = 40$$

20 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

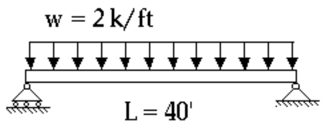
$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EI v = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3 x + C_4$$

$E = 30{,}000$ ksi
$I = 1{,}530\ in^4$



Moment Boundary Conditions

(1) $M(0) = 0$

(2) $M(40) = 0$

Displacement Boundary Conditions

The second two boundary conditions the beam must satisfy relate to the displaced shape of the

21 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

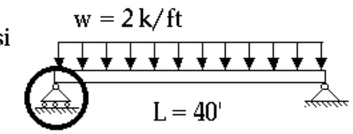
$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EI v = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3 x + C_4$$

$E = 30{,}000$ ksi
$I = 1{,}530\ in^4$



Moment Boundary Conditions

(1) $M(0) = 0$

(2) $M(40) = 0$

Displacement Boundary Conditions

(3) $v(0) = 0$

At the left end of the beam there is a roller support, therefore, the deflection at this end must be zero.

22 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

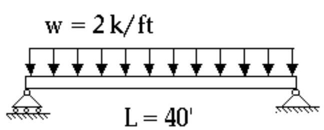
$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EI v = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3 x + C_4$$

$$0 = -\frac{(0)^4}{12} + \frac{40}{6}(0)^3 + C_3(0) + C_4$$

$E = 30{,}000$ ksi
$I = 1{,}530\ in^4$



Moment Boundary Conditions

(1) $M(0) = 0$

(2) $M(40) = 0$

Displacement Boundary Conditions

(3) $v(0) = 0$

Substituting x = 0 into the equation for the deflection of the beam...

23 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

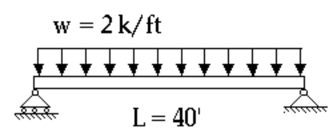
$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EI v = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3 x + C_4$$

$$0 = -\frac{(0)^4}{12} + \frac{40}{6}(0)^3 + C_3(0) + C_4$$

$$\Rightarrow C_4 = 0$$

$E = 30{,}000$ ksi
$I = 1{,}530\ in^4$



Moment Boundary Conditions

(1) $M(0) = 0$

(2) $M(40) = 0$

Displacement Boundary Conditions

(3) $v(0) = 0$

...we find that the integration constant C_4 is zero. Therefore, we can eliminate C_4.

24 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

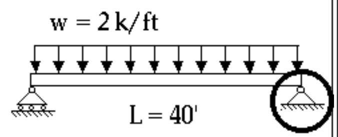
$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3 x$$

$E = 30{,}000$ ksi
$I = 1{,}530$ in^4



Moment Boundary Conditions

(1) $M(0) = 0$
(2) $M(40) = 0$

Displacement Boundary Conditions

(3) $v(0) = 0$
(4) $v(L) = 0$

The final boundary condition is that the deflection of the beam at the pin support is zero.

25 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

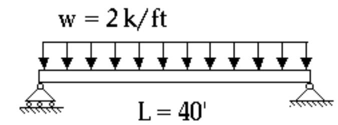
$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3 x$$

$$0 = -\frac{(40)^4}{12} + \frac{20}{3}(40)^3 + C_3(40)$$

$E = 30{,}000$ ksi
$I = 1{,}530$ in^4



Moment Boundary Conditions

(1) $M(0) = 0$
(2) $M(40) = 0$

Displacement Boundary Conditions

(3) $v(0) = 0$
(4) $v(L) = 0$

The same old algebra...

26 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

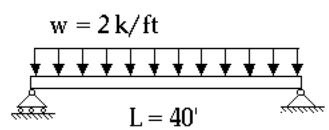
$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + C_3$$

$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 + C_3 x$$

$$0 = -\frac{(40)^4}{12} + \frac{20}{3}(40)^3 + C_3(40)$$

$$\Rightarrow C_3 = -5{,}333 \text{ k-ft}^2$$

$E = 30{,}000$ ksi
$I = 1{,}530$ in^4



Moment Boundary Conditions

(1) $M(0) = 0$
(2) $M(40) = 0$

Displacement Boundary Conditions

(3) $v(0) = 0$
(4) $v(L) = 0$

...and we find that the value of C_3 is -5,333. Although the units magically appeared at this point, they won't for you! Always try to carry the units with the numbers in your calculations.

27 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

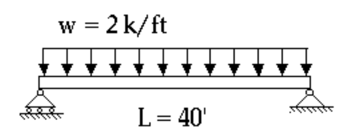
$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333$$

$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x$$

$E = 30{,}000$ ksi
$I = 1{,}530$ in^4



Moment Boundary Conditions

(1) $M(0) = 0$
(2) $M(40) = 0$

Displacement Boundary Conditions

(3) $v(0) = 0$
(4) $v(L) = 0$

28 Hide Text

$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

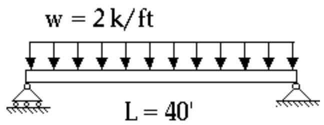
$$EI\frac{d^3 \upsilon}{dx^3} = -2x + 40$$

$$EI\frac{d^2 \upsilon}{dx^2} = -x^2 + 40x$$

$$EI\frac{d\upsilon}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333$$

$$\boxed{EI\upsilon = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x}$$

E = 30,000 ksi
I = 1,530 in^4



So this is it !! An expression for the deflection of the beam as a function of position. We now can calculate the deflection of the beam at any point along its length.

29 Hide Text

$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

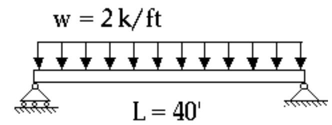
$$EI\frac{d^3 \upsilon}{dx^3} = -2x + 40$$

$$EI\frac{d^2 \upsilon}{dx^2} = -x^2 + 40x$$

$$EI\frac{d\upsilon}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333$$

$$EI\upsilon = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x$$

E = 30,000 ksi
I = 1,530 in^4



Find Maximum Deflection

It turns out that we are interested in checking the clearance of our milk truck, for it often travels down poorly maintained roads in order to deliver milk to the children in Tooleysville. This requires that we calculate the maximum deflection of the beam supporting the milk container.

30 Hide Text

$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

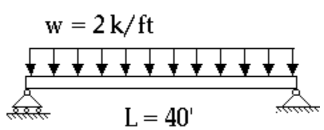
$$EI\frac{d^3 \upsilon}{dx^3} = -2x + 40$$

$$EI\frac{d^2 \upsilon}{dx^2} = -x^2 + 40x$$

$$EI\frac{d\upsilon}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333$$

$$EI\upsilon = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x$$

E = 30,000 ksi
I = 1,530 in^4



Find Maximum Deflection

$$\upsilon_{max} \ @ \ \frac{d\upsilon}{dx} = 0$$

Where does the maximum deflection occur? We equate the first derivative of the deflection to zero and solve for x.

31 Hide Text

$$EI\frac{d^4 \upsilon}{dx^4} = -w(x)$$

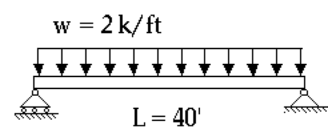
$$EI\frac{d^3 \upsilon}{dx^3} = -2x + 40$$

$$EI\frac{d^2 \upsilon}{dx^2} = -x^2 + 40x$$

$$EI\frac{d\upsilon}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333 \longrightarrow 0 = -\frac{x^3}{3} + 20x^2 + 5{,}333$$

$$EI\upsilon = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x$$

E = 30,000 ksi
I = 1,530 in^4



Find Maximum Deflection

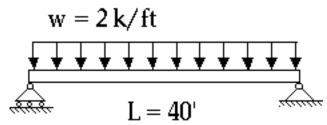
$$\upsilon_{max} \ @ \ \frac{d\upsilon}{dx} = 0$$

We already have the first derivative of the deflection, so we equate it to zero....

32 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

$E = 30{,}000$ ksi
$I = 1{,}530 \text{ in}^4$



$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

Find Maximum Deflection

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$v_{max} \; @ \; \frac{dv}{dx} = 0$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333 \longrightarrow 0 = -\frac{x^3}{3} + 20x^2 + 5{,}333$$

...and some non-trivial algebra leads us to the result that the maximum deflection occurs at x = 20'.

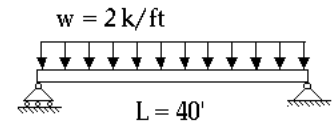
If we had been clever, we could also have noted that the loads and supports of this problem are symmetric, and therefore the maximum deflection must occur in the middle of the beam. The skill of identifying locations of maximum moment, shear and deflection is useful to develop.

$$\Rightarrow x = 20'$$

33 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

$E = 30{,}000$ ksi
$I = 1{,}530 \text{ in}^4$



$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

Find Maximum Deflection

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$v_{max} \; @ \; x = 20'$$

$$v(20) = \frac{1}{EI}\left[-\frac{(20)^4}{12} + \frac{20}{3}(20)^3 - 5{,}333(20)\right]$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333$$

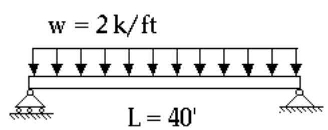
$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x$$

To find the maximum deflection of the beam, we now plug x = 20' into our equation for v.

34 Hide Text

$$EI\frac{d^4 v}{dx^4} = -w(x)$$

$E = 30{,}000$ ksi
$I = 1{,}530 \text{ in}^4$



$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

Find Maximum Deflection

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$v_{max} \; @ \; x = 20'$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333$$

$$v(20) = \frac{1}{EI}\left[-\frac{(20)^4}{12} + \frac{20}{3}(20)^3 - 5{,}333(20)\right]$$

$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x \qquad = \frac{66{,}667}{EI} \qquad \text{UNITS ???}$$

The mathematics is easy enough, but how about the units of our result? We know that the units of E are ksi and I is expressed in inches^4. What are the units of 66,667 ?

35 Hide Text

Referring back to the point at which we calculated the integration constants you will find that we used units of kips and feet. We will multiply our displacement calculation by the factor 1,728 in^3/ft^3 to calculate the deflection in inches.

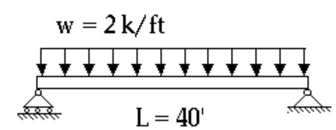
By not carrying the units along in our calculations, we have exposed ourselves to potential errors. Our excuse was that we didn't have the space on the screen – you don't have this excuse. In conclusion:

(1) Unitless numbers are potentially confusing,

(2) units are a good way of checking compatibility of terms in equations,

(3) factors of 12, 144, 1728, etc. can cause beams to collapse.

000 ksi
30 in^4



Find Maximum Deflection

$$v_{max} \; @ \; x = 20'$$

$$v(20) = \frac{1}{EI}\left[-\frac{(20)^4}{12} + \frac{20}{3}(20)^3 - 5{,}333(20)\right]$$

$$= \frac{66{,}667 \text{ k-ft}^3 \left(1{,}728 \frac{\text{in}^3}{\text{ft}^3}\right)}{30{,}000 \frac{\text{k}}{\text{in}^2} \bullet 1530 \text{ in}^4}$$

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$$EI\frac{d^4 v}{dx^4} = -w(x)$$

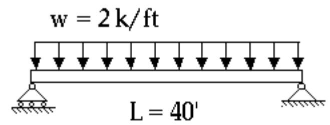
$$EI\frac{d^3 v}{dx^3} = -2x + 40$$

$$EI\frac{d^2 v}{dx^2} = -x^2 + 40x$$

$$EI\frac{dv}{dx} = -\frac{x^3}{3} + \frac{40}{2}x^2 + 5{,}333$$

$$EIv = -\frac{x^4}{12} + \frac{40}{6}x^3 - 5{,}333x$$

$E = 30{,}000$ ksi
$I = 1{,}530\ in^4$



Find Maximum Deflection

v_{max} @ $x = 20'$

$$v(20) = \frac{1}{EI}\left[-\frac{(20)^4}{12} + \frac{20}{3}(20)^3 - 5{,}333(20)\right]$$

$$= \frac{66{,}667\ k\text{-}ft^3 \left(1{,}728\frac{in^3}{ft^3}\right)}{30{,}000\frac{k}{in^2} \bullet 1530\ in^4} = \boxed{2.5\ in}$$

Ta Da. The maximum deflection of the milk truck is 2.5 inches. We can safely drive the truck on rugged roads. If you would like a check, click the "Dr. Beam" button below and use the Doc to confirm this result.

37 Hide Text Dr. Beam

Summary

Determing the deflections in a beam that is loaded uniformly is relatively straightforward: integrate EIv'''' = w_0, and then enforce the boundary conditions to determine the four constants of integration. The only trick is to be careful with your units when you are solving a numerical problem such as this one.

38 Hide Text

THE END