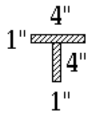
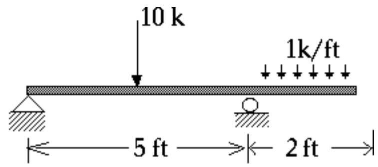
Bending Stress Example: 1

Determine the maximum bending stress.





How do we find the maximum normal stress in the loaded beam shown above due to bending?

The first thing to notice is that the beam is statically determinate. This means that we can solve directly for stresses without considering deflection of the beam. Therefore, we do not need to know the material or cross-sectional properties of the beam to calculate the maximum stress.

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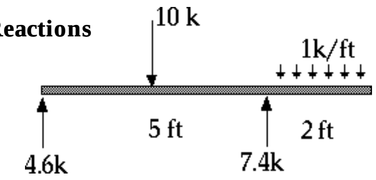
Reactions



We begin by determining the support reaction for the given loading. The reactions are determined by applying external equilibrium to the entire structure;

$\Sigma M = 0, \quad \Sigma F_y = 0.$

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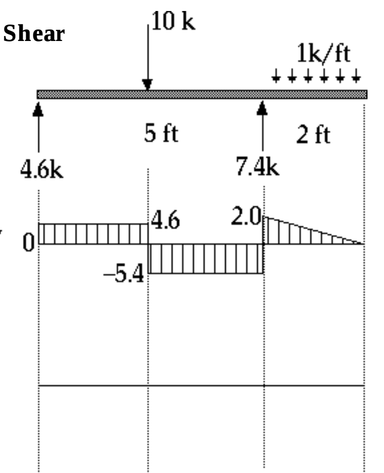
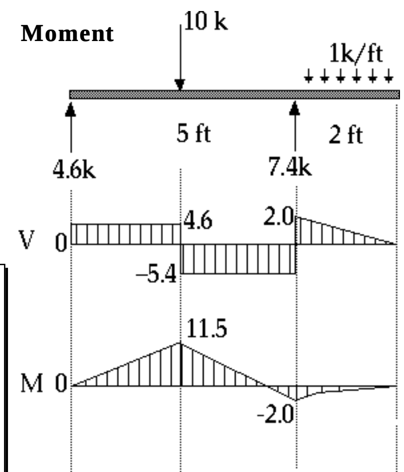
Shear



Knowing the reactions and the loading we can plot the shear diagram.

3 Hide Text

Moment



Using the shear diagram we can then plot the moment diagram.

If you do not remember how to plot shear and moment diagrams for beams, you should use the stack "Internal Forces" as a review. The ability to calculate internal shears and moments is

Shear & Moment Diagrams

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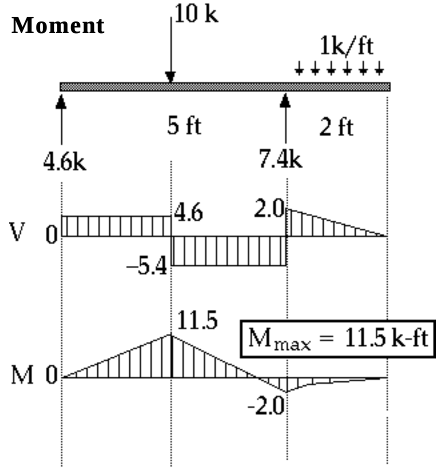
Moment



From the moment diagram we extract the maximum moment (negative or positive) experienced by the beam.

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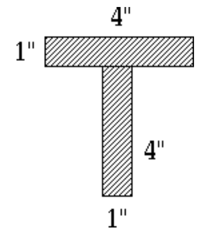
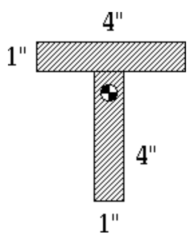
Cross Section Properties



Recall that the equation governing bending stresses in beams is $\sigma = My/I$. We have just determined the maximum M. In order to calculate y and I we must look at a section of the beam.

6 Hide Text Skip the details

Cross Section Properties: Centroid



Simple bending of a beam occurs about the beams centroid. We begin our evaluation of the cross section properties by calculating the location of the centroid.

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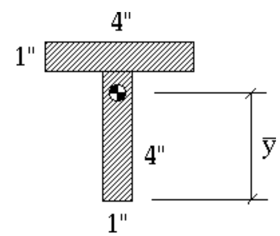
Cross Section Properties: Centroid



We will calculate the location of the centroid in reference to the bottom of the beam. It is important to always choose a reference point before attempting to calculate the centroid of a section.

(Notice that it is trivial to determine the x location of the centroid. Because the beam is symmetric about the y axis, we know that the centroid must lie on the y axis.)

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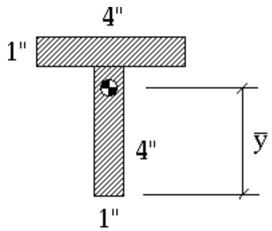
Bending Stress Example: 3

Cross Section Properties: Centroid

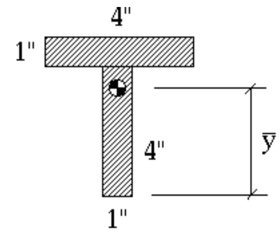


$$\bar{y} \sum A = \sum y A$$

The centroid is defined as the point about which the first moment of area is zero. The formula shown above may be used to calculate the y distance to the centroid.

If it is not clear where this formula came from, you might wish to visit the "Centroids" stack in the ENGR 210 folder.

Cross Section Properties: Centroid

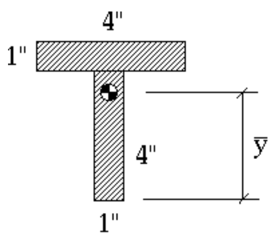


$$\bar{y} \sum A = \sum y A$$

$$\bar{y}\,[4"(1") + 4"(1")] = 4"(1")(2") + 4"(1")(4.5")$$

Plugging in the numbers...

Cross Section Properties: Centroid



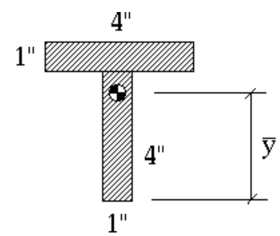
$$\bar{y} \sum A = \sum y A$$

$$\bar{y}\,[4"(1") + 4"(1")] = 4"(1")(2") + 4"(1")(4.5")$$

$$\boxed{\bar{y} = 3.25 \text{ in}}$$

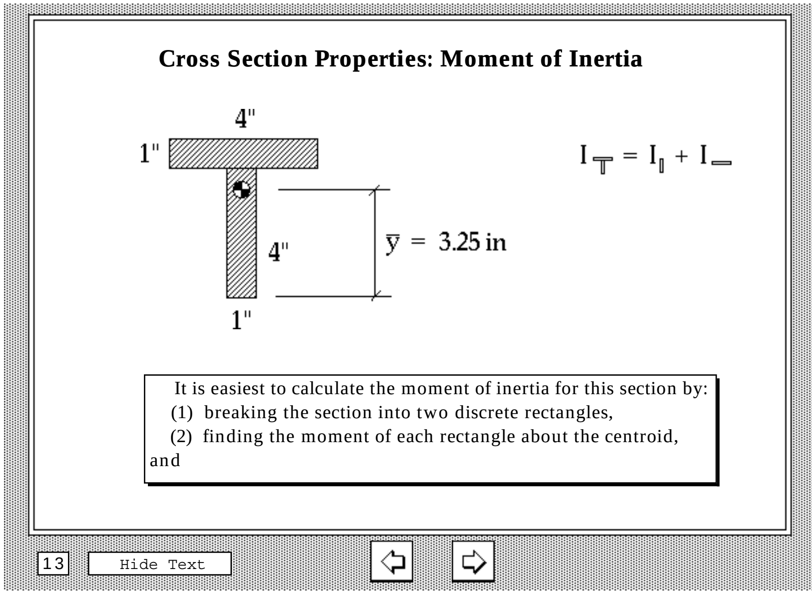
...we find that the distance from the bottom of the section to the centroid is 3.25 inches.

Cross Section Properties: Moment of Inertia



$$\bar{y} = 3.25 \text{ in}$$

The second section property we must calculate is the moment of inertia. We will be using the location of the centroid to calculate the moment of inertia.


Cross Section Properties: Moment of Inertia
4"
1"
4"
1"
ȳ = 3.25 in
It is easiest to calculate the moment of inertia for this section by:
(1) breaking the section into two discrete rectangles,
(2) finding the moment of each rectangle about the centroid, and
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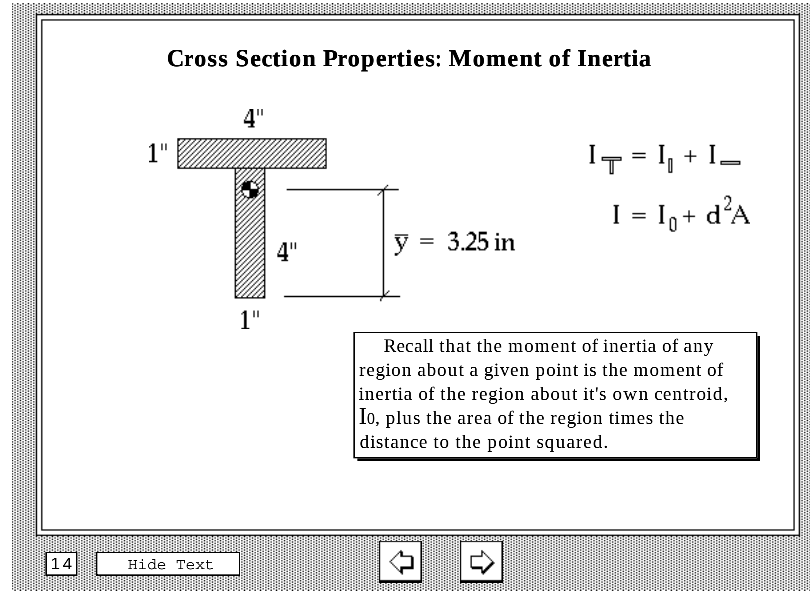
Cross Section Properties: Moment of Inertia
4"
1"
4"
1"
ȳ = 3.25 in
I = I0 + d2A
Recall that the moment of inertia of any region about a given point is the moment of inertia of the region about it's own centroid, I0, plus the area of the region times the distance to the point squared.
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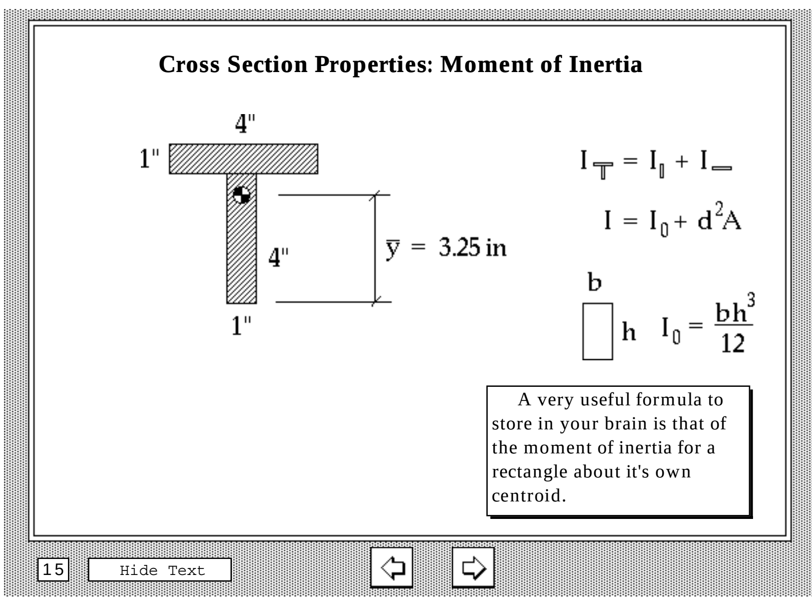
Cross Section Properties: Moment of Inertia
4"
1"
4"
1"
ȳ = 3.25 in
I = I0 + d2A
b
h
I0 = bh3/12
A very useful formula to store in your brain is that of the moment of inertia for a rectangle about it's own centroid.
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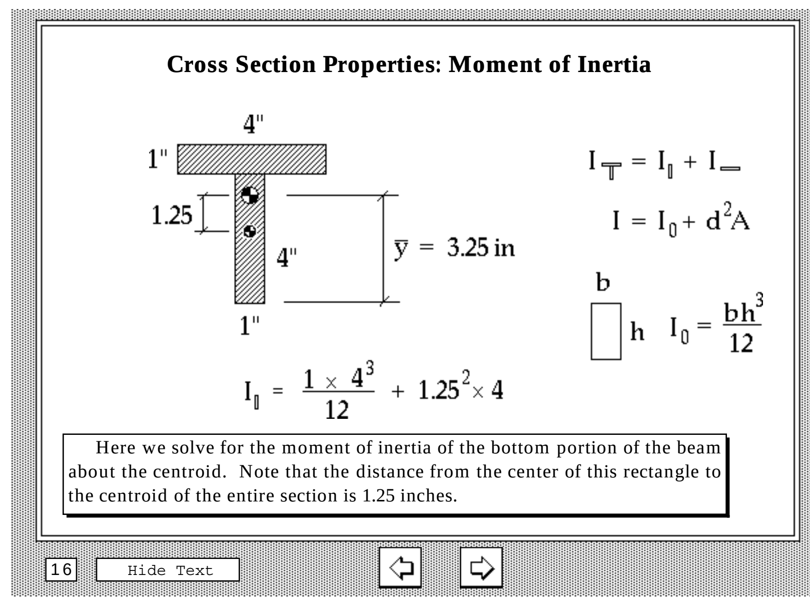
Cross Section Properties: Moment of Inertia
4"
1"
1.25
4"
1"
ȳ = 3.25 in
I = I0 + d2A
b
h
I0 = bh3/12
= (1 × 4^3)/12 + 1.25^2 × 4
Here we solve for the moment of inertia of the bottom portion of the beam about the centroid. Note that the distance from the center of this rectangle to the centroid of the entire section is 1.25 inches.
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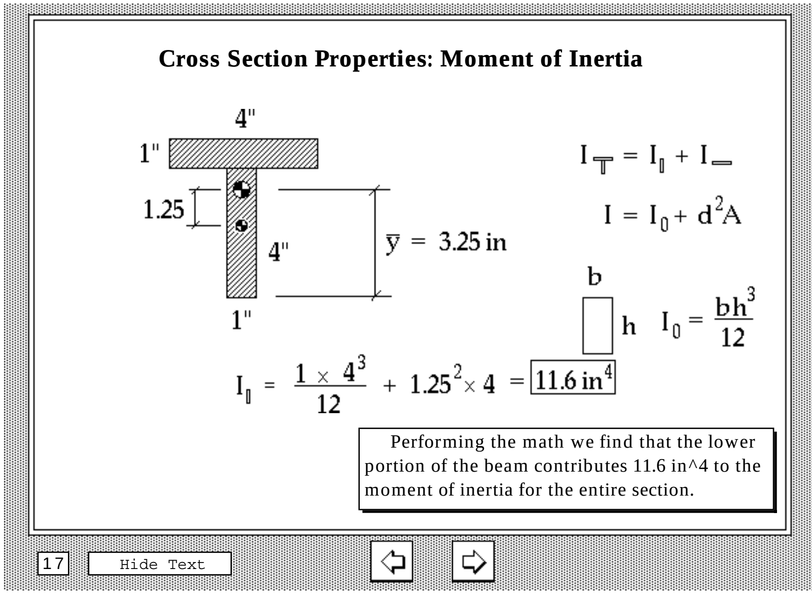
Cross Section Properties: Moment of Inertia
4"
1"
1.25
4"
1"
ȳ = 3.25 in
I_T = I_| + I_—
I = I_0 + d^2A
b
h
I_0 = bh^3/12
I_| = (1 × 4^3)/12 + 1.25^2 × 4 = 11.6 in^4
Performing the math we find that the lower portion of the beam contributes 11.6 in^4 to the moment of inertia for the entire section.
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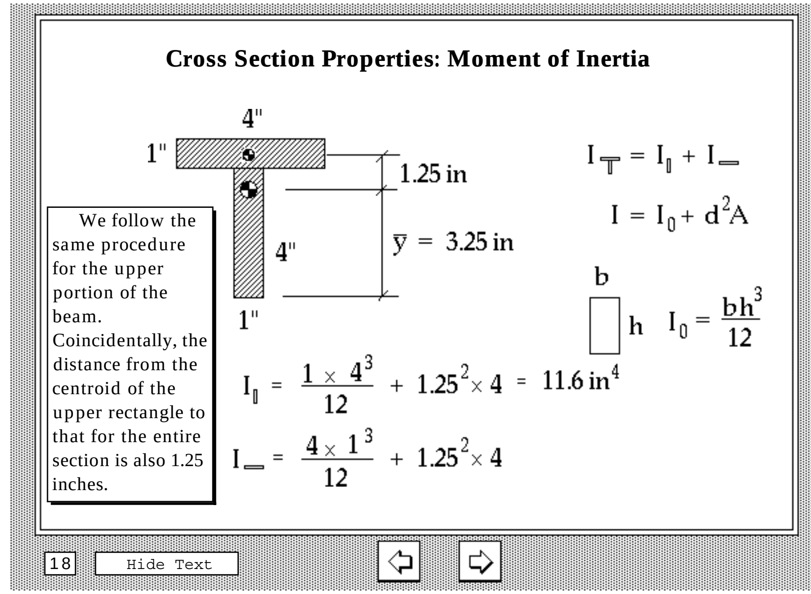
Cross Section Properties: Moment of Inertia
4"
1"
1.25 in
4"
1"
ȳ = 3.25 in
We follow the same procedure for the upper portion of the beam. Coincidentally, the distance from the centroid of the upper rectangle to that for the entire section is also 1.25 inches.
I_T = I_| + I_—
I = I_0 + d^2A
b
h
I_0 = bh^3/12
I_| = (1 × 4^3)/12 + 1.25^2 × 4 = 11.6 in^4
I_— = (4 × 1^3)/12 + 1.25^2 × 4
18
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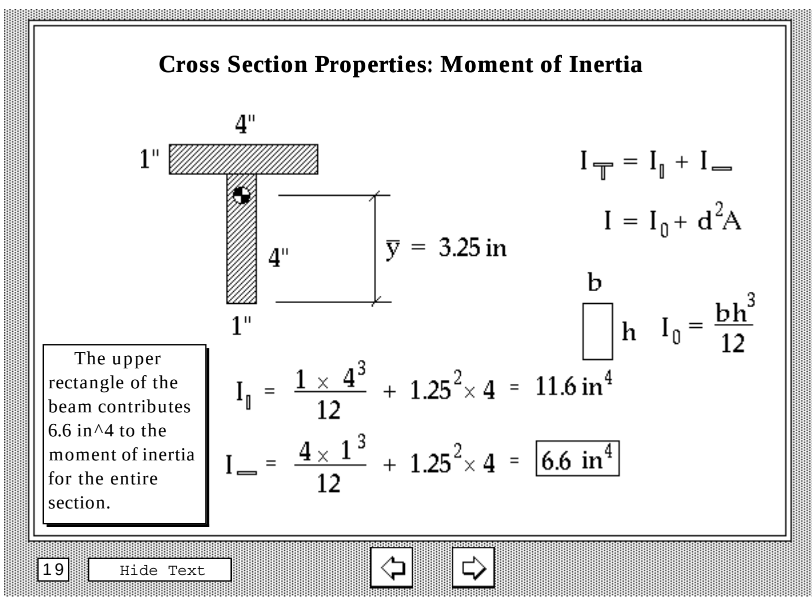
Cross Section Properties: Moment of Inertia
4"
1"
4"
1"
ȳ = 3.25 in
I_T = I_| + I_—
I = I_0 + d^2A
b
h
I_0 = bh^3/12
The upper rectangle of the beam contributes 6.6 in^4 to the moment of inertia for the entire section.
I_| = (1 × 4^3)/12 + 1.25^2 × 4 = 11.6 in^4
I_— = (4 × 1^3)/12 + 1.25^2 × 4 = 6.6 in^4
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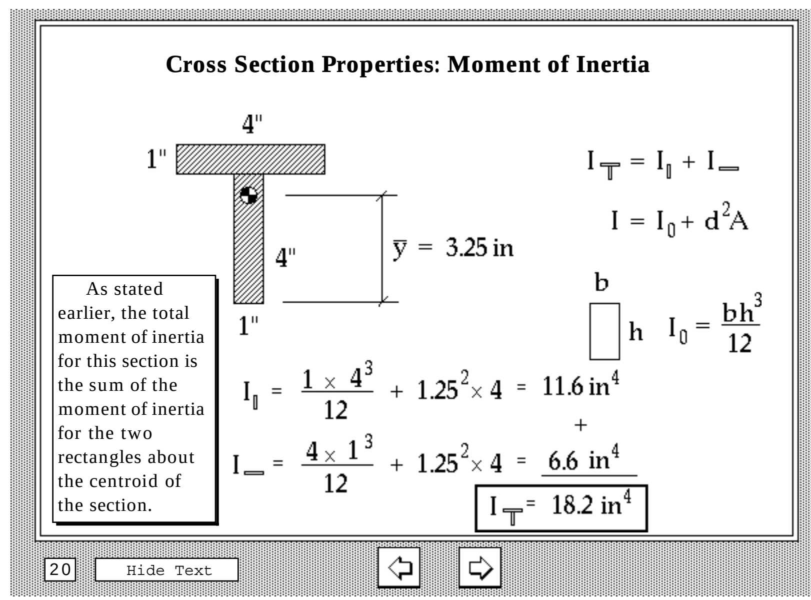
Cross Section Properties: Moment of Inertia
4"
1"
4"
1"
ȳ = 3.25 in
I_T = I_| + I_—
I = I_0 + d^2A
b
h
I_0 = bh^3/12
As stated earlier, the total moment of inertia for this section is the sum of the moment of inertia for the two rectangles about the centroid of the section.
I_| = (1 × 4^3)/12 + 1.25^2 × 4 = 11.6 in^4
+
I_— = (4 × 1^3)/12 + 1.25^2 × 4 = 6.6 in^4
I_T = 18.2 in^4
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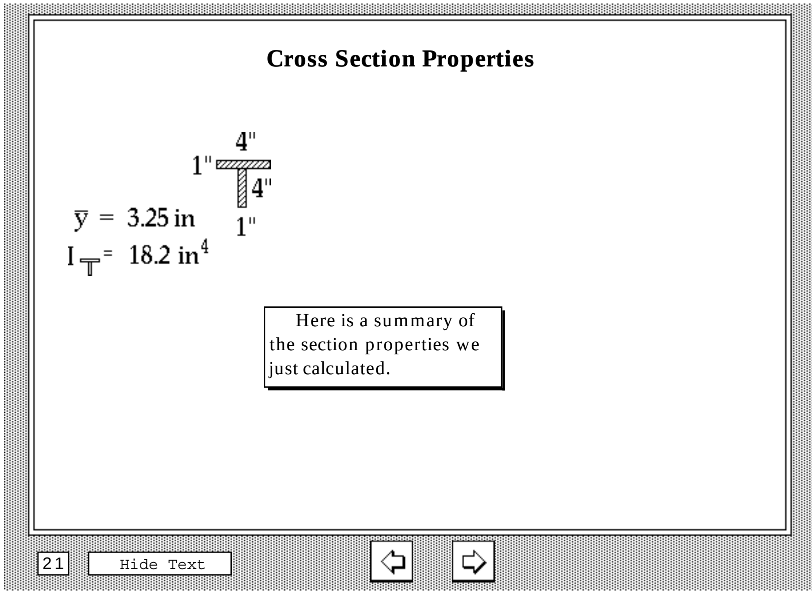
Cross Section Properties
4"
1"
4"
1"
ȳ = 3.25 in
I = 18.2 in4
Here is a summary of the section properties we just calculated.
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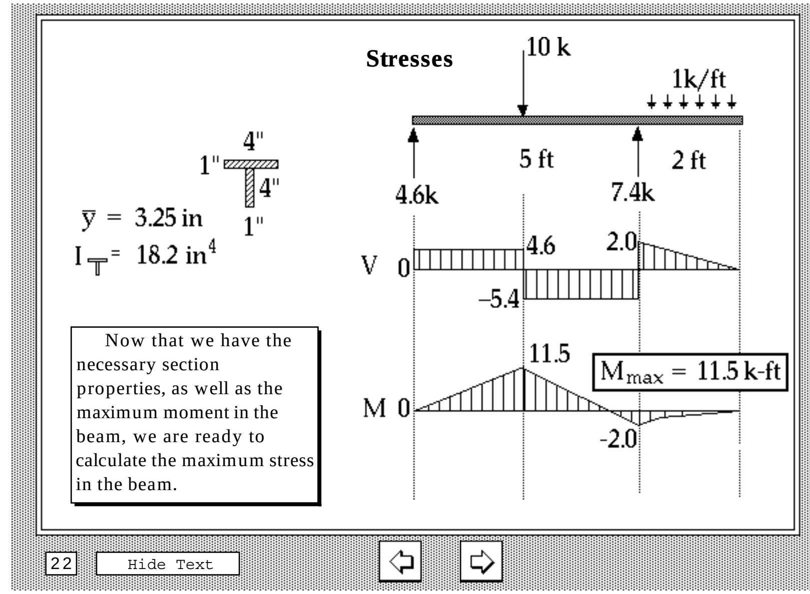
Stresses
10 k
1k/ft
4.6k
5 ft
7.4k
2 ft
4"
1"
4"
1"
ȳ = 3.25 in
I = 18.2 in4
V 0
4.6
2.0
–5.4
M 0
11.5
-2.0
Mmax = 11.5 k-ft
Now that we have the necessary section properties, as well as the maximum moment in the beam, we are ready to calculate the maximum stress in the beam.
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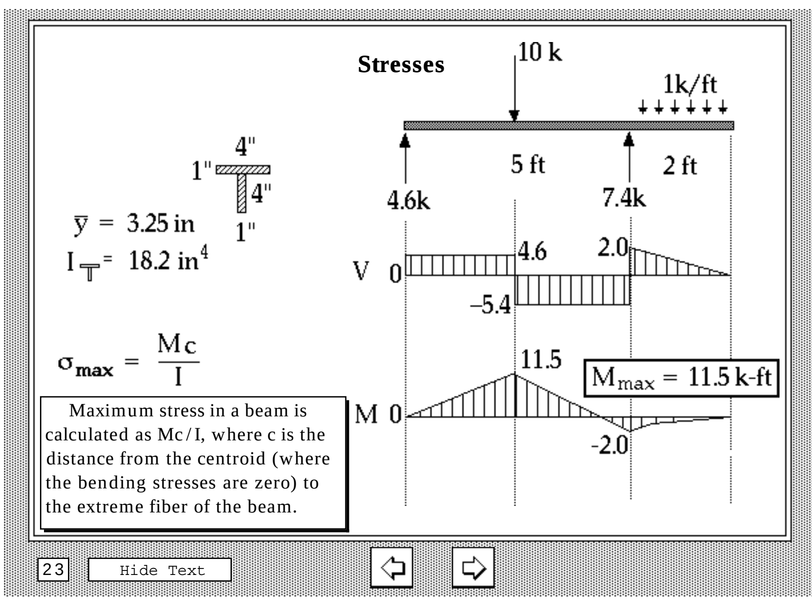
Stresses
10 k
1k/ft
4.6k
5 ft
7.4k
2 ft
4"
1"
4"
1"
ȳ = 3.25 in
I = 18.2 in4
V 0
4.6
2.0
–5.4
σmax = Mc / I
M 0
11.5
-2.0
Mmax = 11.5 k-ft
Maximum stress in a beam is calculated as Mc/I, where c is the distance from the centroid (where the bending stresses are zero) to the extreme fiber of the beam.
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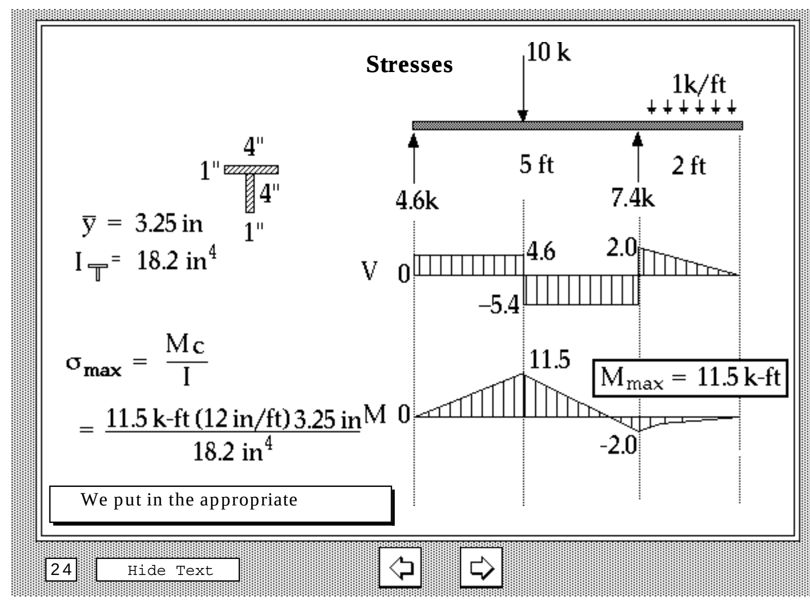
Stresses
10 k
1k/ft
4.6k
5 ft
7.4k
2 ft
4"
1"
4"
1"
ȳ = 3.25 in
I = 18.2 in4
V 0
4.6
2.0
–5.4
σmax = Mc / I
= 11.5 k-ft (12 in/ft) 3.25 in / 18.2 in4
M 0
11.5
-2.0
Mmax = 11.5 k-ft
We put in the appropriate
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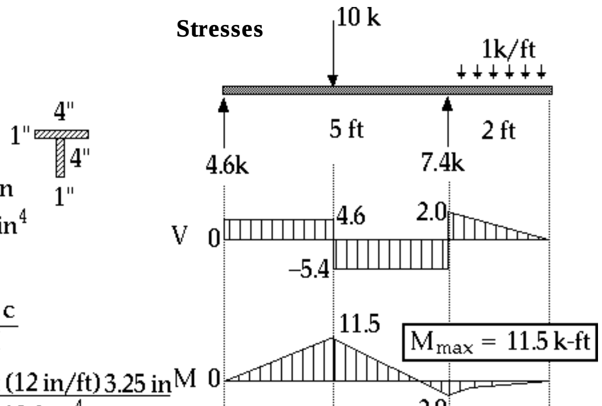
Stresses



$\bar{y}$ = 3.25 in

I_{T} = 18.2 in^4

$$\sigma_{max} = \frac{M c}{I}$$

$$= \frac{11.5 \text{ k-ft } (12 \text{ in/ft}) \, 3.25 \text{ in}}{18.2 \text{ in}^4}$$

$$= \boxed{24.6 \text{ ksi}}$$

... and find that the maximum stress in the beam is 24.6 ksi. This stress is near the limit for a safely designed beam made of mild steel.

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Summary

To solve for the maximum bending stress in a statically determinate, constant cross-section beam:

(1) Solve for the support reactions and plot the shear diagram to arrive at the Moment Diagram.
(2) From the moment diagram read off the maximum moment (negative or positive) in the beam, M_{max}.
(3) Calculate the location of the centroid of the section.
(4) Calculate the moment of inertia for the section about its centroid, I.
(5) Determine the distance from the centroid to the extreme fiber of the section, c.
(6) Calculate σ_{max} as:

$$\sigma_{max} = \frac{M c}{I}$$

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The End

Go back to the details