Substituting for $p(i)$ and $E(i)$, we obtain

$$\sigma^2 = \sum_{i=1}^{N} \frac{i^2 p^i q^{N-i} N!}{i!(n-i)!} - (Np)^2 \qquad \text{(D-1)}$$

Bringing an Np to the left of the summation sign and canceling an i in the numerator and denominator, we get

$$\sigma^2 = Np \sum_{i=1}^{N} \frac{ip^{i-1} q^{N-i} (N-1)!}{(i-1)!(N-i)!} - (Np)^2 \qquad \text{(D-2)}$$

Letting $M = N - 1$ and $j = i - 1$ and substituting into Equation D-2, we have

$$\sigma^2 = Np \sum_{j=0}^{M} \frac{(j+1)p^j q^{M-j} M!}{j!(M-j)!} - (Np)^2 \qquad \text{(D-3)}$$

Expanding Equation D-3, we obtain

$$\sigma^2 = Np \sum_{j=0}^{M} j \frac{p^j q^{M-j} M!}{j!(M-j)!} + Np \sum_{j=0}^{M} \frac{p^j q^{M-j} M!}{j!(M-j)!} - (Np)^2 \qquad \text{(D-4)}$$

Now note that

$$\sum_{j=0}^{M} j \frac{p^j q^{M-j} M!}{j!(M-j)!}$$

is the expression for the expected value of a binomial distribution with parameters M and p; that is, it is equal to $Mp = (N-1)p$. Likewise,

$$\sum_{j=0}^{M} \frac{p^j q^{M-j} M!}{j!(M-j)!}$$

is (as noted in Digression 4-2) simply the sum of all probabilities for a binomial distribution with parameters M and p and therefore must equal 1.

Putting these pieces of information together, Equation D-4 becomes

$$\sigma^2 = Np(N-1)p + Np - (Np)^2$$

And the rest is just algebra:

$$\begin{aligned}\sigma^2 &= (Np)^2 - Np^2 + Np - (Np)^2 \\ &= Np - Np^2 \\ &= Np(1-P) \\ &= Npq\end{aligned}$$

PROBLEMS

1. Harvey has to go to the laundromat, supermarket, high school, bowling alley, and fortune teller all in one day.

a. In how many possible orders could Harvey do his five chores?
b. Suppose Harvey could only do three of his chores today, but the fortune teller *must* be done today. How many ways are there to choose which three chores will be done today?

2. Social security numbers on the island of Bileo consist of three letters followed by four digits followed by either an asterisk (*) or a dollar sign ($). One such number would be GRL4552*.
   a. What is the total number of different social security numbers possible?
   b. What is the number of social security numbers involving the letters ABC (in any order) followed by the digits 1234 (in any order) followed by an asterisk?
   c. What is the number of social security numbers involving the letters ABC (in that order) followed by the digits 1234 (in that order)?
3. This question concerns five-letter English-language words. For purposes of this question, assume that each letter of the word can be drawn from the 26-letter alphabet except that Q is always followed by U. (This means that Q can't appear in the last position.)
   a. How many words are there that contain the letters A, B, C, D, and E in any order?
   b. How many words are there that do not contain a Q?
   c. How many words are there that do contain at least a Q?
   d. How many words are there in all?
4. Consider the set of all eight-letter strings (QBAACNBL would be such a string).
   a. How many such strings are there with vowels in the first and fifth positions and consonants in the other positions?
   b. How many such strings are there such that each string contains exactly three vowels?
5. This question refers to telephone numbers. A telephone number may be thought of as a three-digit exchange plus a four-digit "ending." *There can be no zeros in the exchange.*
   a. How many possible telephone numbers are there?
   b. How many numbers are there that satisfy the following characteristics:
      (1) All digits in the exchange are the same.
      (2) There are exactly two 9s and one 7 in the ending.
   c. How many numbers are there such that all digits in the exchange are different from one another?
6. A student body has the following distribution of majors:
   Basket weaving: 20
   Martian studies: 20
   Psychology: 40
   History of billiards: 10
   Massage: 50

California languages: 50
Faucets: 10

A student committee is to be formed of 20 representatives, with representation being proportional to the number of majors (2 BW, 2 MS, 4 Psych, 1 HB, 5 M, 5 CL, and 1 F).

a. How many such committees are there?
b. How many committees are there that contain Lee, who majors in massage, and his girlfriend, Farrah, a faucet major?
c. What is the probability that neither Lee nor Farrah will be on the committee?
d. What is the probability that a given person will be chosen if that person majors in psychology? In California languages?

7. Consider the word *pancreas*. Suppose four of the seven letters in this word are randomly drawn.
   a. What is the probability that the four letters will all be different?
   b. What is the probability that two of the four letters will be the same?
   c. What is the probability that two of the four letters will be the same and the other two will be different?

8. Ten people meet and shake hands with one another.
   a. How many handshakings will be required such that each person has shaken hands with every other person?
   b. Suppose that the 10 persons consist of 5 women and 5 men. Women and men do not shake hands with one another, although women shake hands with women and men shake hands with men. Now how many handshakings will take place?

9. A psychology experiment contains four conditions: *A*, *B*, *C*, and *D*.
   a. Each subject is expected to participate in all four conditions. Naturally, a subject has to go through the conditions in a particular order (for example, *BCDA*). How many subjects will be necessary such that all possible orders are gone through exactly once?
   b. Suppose that due to lack of time, each subject can go through only two of the four conditions. How many subjects will be required in order that each possible pair of conditions is given to exactly one subject?
   c. Under the conditions of question b, how many subjects will be required such that each possible pair of conditions in each possible order is given to exactly one subject?

10. Residents of Mars have telephones of the following sort: Each number is made up of a two-digit exchange plus four letters, such as 27-FROG. (Martians, it turns out, have the same 10 digits and 26 letters that we do.)
   a. How many possible telephone numbers are there?
   b. How many numbers are there that contain the digits 3 and 4 and the letters A, B, C, and D in any order?
   c. How many numbers are there such that all digits and letters are different from one another?

d. Imagine that Martians are order-blind; that is, to them 13 and 31 would look just the same; likewise, FROG, GROF, ROGF, GRFO, and so on would all look the same. Now how many numbers are there that are different, from the Martians' point of view? (Assume all letters and numbers must be different from one another.)

11. I have 6 pairs of underpants, 8 pairs of socks, 2 pairs of pants, 10 shirts, and 3 hats. Define an "outfit" of a combination of 1 pair of underpants, 1 pair of socks, 1 pair of pants, 1 shirt, and 1 hat.
   a. How many different outfits can I create?
   b. Suppose 4 of my 10 shirts are identical to one another. Now how many different outfits can I create?
   c. Suppose I don't care whether or not my socks match. Now how many different outfits can I create?
   d. Suppose my early-rising brother has gotten up before me, made an outfit from my clothes, and left. Now how many different outfits can I create?
   e. Suppose I am in a hurry and don't have time to put on any underpants. Now how many different outfits can I create?

12. In a shooting contest shots are fired at a 64-square checkerboard. Each square has an equal probability of being hit, and each shot must hit the checkerboard somewhere.
   a. Suppose 10 shots are fired. What is the probability that they will all hit different squares?
   b. Suppose 100 shots are fired. What is the probability that three or more squares will remain unhit?

13. Suppose a standard bridge hand of 13 cards is randomly dealt from a standard deck. What is the probability that the hand will contain:
   a. Thirteen spades?
   b. No spades?
   c. Five spades?
   d. The 6, 7, and 2 of clubs; the king and jack of diamonds; the 8, 10, and queen of hearts; and the 7, 8, 10, jack, and queen of spades? How does your answer to this part relate to your answer to the first part?

14. Compute the expected value, variance, and standard deviation of the following binomial distributions. Then plot the distributions.

   $N = 4, p = 0.10$
   $N = 4, p = 0.50$
   $N = 4, p = 0.90$
   $N = 15, p = 0.10$

15. A fair die is thrown (one with equal probability of coming up 1, 2, 3, 4, 5, or 6).
   a. What is the probability of obtaining a 6?
   b. What is the probability of *not* obtaining a 6?
   c. Suppose this die is thrown 120 times. Characterize the distribution of the

number of 6s you should get. What is the *mean* and *variance* of this distribution?

16. Suppose that for the next 100 days, the probability of rain in Seattle on a given day is $\frac{4}{5}$. Furthermore, the probability of rain on any given day is independent of the probability of rain on any other day.
    a. What is the expected number of rainy days during the next 100 days?
    b. What is the variance of the probability distribution of rainy days during the next 100 days?
    c. What is the probability that *all* of the next 100 days will be rainy (to three decimal places)?
    d. What is the probability that between 83 and 91 (inclusive) of the next 100 days will be rainy (to three decimal places)?

17. Joe Smith has enrolled at Central Puyallup State College. To get there from home each morning, he must go through seven traffic lights. Each light is green with a 0.3 probability and each light is independent of each other light. Compute the following probabilities.
    a. All lights are green.
    b. Between three and five (inclusive) lights are green.
    c. Fewer than four lights are green.
    d. At least one light is red or orange.
    e. At least two lights are red or orange.

18. For questions 16 and 17, why would the independence assumption probably be faulty?

19. Frank Jones claims that 60% of the people in his city will vote for him for mayor. Suppose that out of a random sample of 10 people, 3 claim they will vote for Jones and the other 7 claim they will vote for Tom Smith.

    If Jones is correct in saying that 60% of the people will vote for him:
    a. How probable is it that exactly 3 out of a sample of 10 people will vote for him?
    b. How probable is it that 3 or fewer out of a sample of 10 people will vote for him?

20. The Muy Munchy Mixed Nut Company puts out cans of mixed nuts that contain 40% walnuts, 30% peanuts, and 30% pecans. Suppose I dip into a Muy Munchy can and scoop out five random nuts. What are the following probabilities?
    a. At least one nut is a walnut.
    b. There is one pecan.
    c. Three of the nuts are peanuts.
    d. Four of the nuts are walnuts.

21. The Tacoma Tigers baseball team has a probability of 0.10 of winning each time they play. Furthermore, each game is independent of every other game.

    Suppose the Tigers play 10 games:

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a. How many
b. How many

a. What is the probability that they will win no games?
b. What is the probability that they will win at least 1 game? At least 2 games?
c. What is the probability that they will win fewer than 3 games?
d. What is the probability that they will win exactly 1 game?

22. The Charlie Tuna Company claims that any given tuna in the ocean has only a 10% chance of ingesting mercury. But a random sample of 10 tunas reveals that 4 of them have mercury poisoning. If the Charlie Tuna Company is correct in its claim, how probable is it that as many as 4 tuna would show mercury poisoning?

23. Little Wanda has a probability of 0.70 of spelling any given word correctly on her spelling test.
a. If the test contains 10 words, what is the probability that Wanda will get exactly 7 words correct?
b. If the test contains 100 words, what is the probability that Wanda will get exactly 70 words correct?
c. If the test contains 100 words, what is the probability that Wanda will get between 65 and 75 words correct?
d. What is the relationship among your answers to the previous questions? Can you think of any reason for this relationship?

24. Joe Smith has gone to Reno for vacation. Joe wanders into a casino to play blackjack. The following things are true:

Joe will bet \$1.00 on each hand (that is, if he wins the hand, he gains \$1.00; if he loses the hand, he loses \$1.00).

Joe's probability of winning any given hand is $\frac{1}{4}$.

Each hand Joe plays is independent of every other hand.

Now:
a. If Joe plays four hands, how many hands does he expect to win?
b. If Joe plays eight hands, how many hands does he expect to win?
c. Define a "profit" as Joe ending up with *more* money than he started with after playing N hands. What is the probability of a profit if $N = 4$?
d. Defining a profit as in the question above, what is the probability of a profit if $N = 8$?

25. Joe Smith is playing a chess match with his friend Bobby. Bobby gives Joe his choice: Either they can play three games or they can play nine games. In either case the person winning the majority of the games wins the match. (Assume that in either case they will play all the games—that is, if they decided to play nine games, they'd play all nine even if one of them won the first five in a row.) You may ignore the possibility of a draw.

Joe knows that his probability of winning any given chess game against Bobby is $\frac{1}{3}$.
a. How many games does Joe expect to win if they play three games?
b. How many games does Joe expect to win if they play nine games?

c. What is the variance of the number of games Joe expects to win if they play three games (to three decimal places)?
d. What is the variance of the number of games Joe expects to win if they play nine games (to three decimal places)?
e. If they play three games, what is the probability (to three decimal places) that Joe will win the match?
f. If they play nine games, what is the probability (to three decimal places) that Joe will win the match?